ECONOMICS OF RECOMMENDER SYSTEMS IN ONLINE MARKETPLACES

by

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To my family and advisors.

ECONOMICS OF RECOMMENDER SYSTEMS
IN ONLINE MARKETPLACES

by

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IN ONLINE MARKETPLACES

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Electronic marketplaces such as Amazon Marketplace and Ebay deploy recommender systems as sales support tools to help consumers find their ideal product among the vast variety of products sold in these platforms. Recommender systems affect consumer decision making by informing consumers about products they may not be aware of and enlarging the consumers' consideration set ("informative role"). In this dissertation, we study the impacts of recommender systems on different players in an online channel structure where a dominant e-commerce platform sells competing products from different manufacturers, and simultaneously recommends a subset of these products to consumers. The dissertation consists of three essays.

The first essay highlights how recommender system design affects the upstream competition between manufacturers and the consequent implications for the recommendation strategy to be adopted by the retail platform. In our setting, consumers are differentiated with respect to their preference for the two products and awareness about the two products. A recommender system is designed to select the recommendation based on a weighted sum of expected retailer profit and expected consumer value. We find that the recommender system may benefit or hurt the retailer and manufacturers, depending on its design and market characteristics. We

show that the retailer's optimal recommendation strategy is mildly profit oriented in the sense that it assigns a larger but not too-large a weight to retailer profit than consumer value, and that under the optimal strategy, the price competition is less intense and the retailer profit is higher compared to when there is no recommender system.

The informative role of the recommender system deployed by an electronic marketplace functions as a medium for targeted advertising for sellers, analogous to traditional advertising media such as TV, newspaper, and the Internet. In the second essay, we examine how a recommender system affects competing sellers in electronic marketplaces regarding their advertising and pricing decisions. We find that sellers advertise less (advertising effect) on their own and decreases product prices (competition effect) in the presence of a recommender system. As a result of these two effects, sellers are more likely to benefit from the recommender system only when it has a high precision.

While the first two essays consider a recommender system that recommends competing products to help consumers find a better alternative, the third essay considers a mixed recommender system where both competing products and complementary products are included. In particular, we consider four sellers that sell four products in two categories via a common retail platform. Products in the same category are substitutes, and products in different categories are complements. We show that the recommender system does not necessarily benefit the marketplace; on one side, the recommender system increases the total sales, but on the other hand, the recommender system alters the competition in each category. In the presence of the recommender system, the price and profit of each seller critically depends on the degree of complementary among products.

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CHAPTER 1

INTRODUCTION

Recommender systems, which have become ubiquitous in electronic marketplaces, are touted as sales support tools that help consumers find their "ideal" product among the vast variety of products sold in those platforms(Hennig-Thurau et al., 2012). It has been reported that over 35% of sales on Amazon.com and more than 60% of the rentals on Netflix result from recommendations (Hosanagar et al., 2013). A typical recommender system uses information sources such as item's content, consumer's behavior history, and consumer's demographic information to predict a consumer's preference and recommends the product the consumer is most likely to buy or prefer(Resnick and Varian, 1997). However, some recommender systems are designed not just to benefit consumers but also to steer them toward products that serve the seller's own interests (Häubl and Murray, 2006).

Despite the ubiquity of recommender systems, the effect of these systems on the competition between products remains unclear, especially in the context of a dominant e-commerce platform that sells competing products from different manufacturers while simultaneously recommending a subset of these products. Such a two-level channel structure with one dominant e-commerce platform is commonly observed in practice (e.g., Amazon's online marketplace). Using a game theoretical model of an online channel structure in which two competing manufacturers sell their products through a common retail platform, we study the effect of recommender systems on the retail platform (hereafter referred to as the retailer), manufacturers, consumer surplus, and social welfare. In our setting, consumers are differentiated with respect to their preference for the two products (locational differentiation) and awareness about the two products (informational differentiation). A recommender system selects the recommendation based on a recommendation score which is a weighted sum of expected retailer profit and expected consumer value. We find that the recommender system may benefit or hurt the retailer and the manufacturers, depending on the signs and magnitudes of substitution effect and demand effect of the recommender system. The substitution effect of the recommender system either intensifies or softens the price competition between two manufacturers through two forces—the direct influence of the recommender system alters the informational differentiation of consumers, which affects the markup that manufacturers can charge. The strategic influence of the recommender system motivates the manufacturers to use price as a lever to attract more recommendations in their favor—when an increase in price increases (decreases) the recommendation score, manufacturers have an incentive to increase (decrease) prices in order to attract more recommendation. The demand effect of the recommender system increases overall consumer awareness, but, depending on the substitution effect, may increase or decrease the demand. The recommendation strategy, viz., the relative weight assigned to retailer profit vis-a-vis consumer value in computing the recommendation score, along with recommender system precision and relative sizes of consumers with different awareness level, determines whether the retailer benefits from the recommender system and by how much. We find that the retailer's optimal recommendation strategy is mildly profit oriented in the sense that it assigns a larger but not too-large a weight to retailer profit than consumer value, and that under the optimal strategy, the price competition is less intense and the retailer profit is higher compared to when there is no recommender system. Furthermore, an increase in either the recommender system precision or the fraction of consumers that are aware of at least one product induces the retailer to adopt a more profit oriented recommendation strategy.

Similar to traditional advertising, recommender systems can introduce consumers to new products and increase the market size which benefits sellers. The informative role of recommender systems in electronic marketplaces seems attractive to sellers because sellers do not pay the marketplaces for receiving recommendations. However, in the second essay, we show that the sellers do not necessarily benefit from the free exposure provided by the recommender system when sellers can strategically change advertising and pricing decisions to respond to retailer's deployment of the recommender system. In particular, in a marketplace that deploys a recommender system helping consumers discover the product that provides them the highest expected net utility, the impacts of the recommender system are the result of a subtle interaction between advertising effect and competition effect. The advertising effect causes sellers to advertise less on their own and the competition effect causes them to decrease prices in the presence of a recommender system. Essentially, sellers pay in the form of more intense price competition because of the recommender system. Furthermore, the competition effect is exacerbated by the advertising effect because the recommender system alters a seller's own strategies related to advertising intensity and price from being strategic substitutes in the absence of the recommender system to being strategic complements in the presence of the recommender system. As a result of these two effects, we find that sellers are more likely to benefit from the recommender system only when it has a high precision. The results do not change qualitatively whether sellers use targeted advertising or uniform advertising. However, we find that a recommender system that benefits sellers when sellers do not employ targeted advertising may actually hurt sellers when sellers adopt targeted advertising with a high precision. On the other hand, in the electronic marketplace with the recommender system, an increase in sellers' targeting precision beyond a threshold softens price competition, increases seller profits, and reduces consumer surplus.

Lastly, online marketplaces typically deploy a variety of recommender systems. One type of recommender systems display hyperlinks to related products—both competing products and complementary products—when consumers are browsing a focal product. For example, Amazon recommends competing products using the "customers who viewed this item also viewed" feature and recommends complementary products using the "customers who bought this item also bought" feature. Such recommendations broaden consumers' search space because they become aware of products they may not be aware of otherwise. As a result, these recommender systems affect the magnitude and the nature of influence that products have on each other's demand levels. In the third essay, we examine the economic impact of recommender system that recommends both competing products and complementary products. The system is applied in a competitive environment where independent sellers set their prices respectively. In particular, we choose a channel structure with a common e-commerce platform and four sellers selling products in two categories. Our results indicate that such recommender system does not necessarily benefit the marketplace, on one side, recommender system increases the total sales, on the other hand, recommender system alters the competition in each category. In the presence of recommender system, the price and profit of each seller critically depends on the degree of complementary among products. Surprisingly, sellers are better off when products have the same level of match (symmetric case) than the case when products have different level of match (asymmetric case).

CHAPTER 2

RECOMMENDER SYSTEM RETHINK: IMPLICATIONS FOR AN ELECTRONIC MARKETPLACE WITH COMPETING MANUFACTURERS

2.1 Synopsis

Recommender systems, which have become ubiquitous in eCommerce platforms, are touted as sales support tools that help consumers find their "ideal" product among the vast variety of products sold in those platforms (Hennig-Thurau et al., 2012). A recommender system provides consumers with product recommendations based on criteria such as user-specific preference, a user's shopping history, or choices made by other consumers with similar profiles (Xiao and Benbasat, 2007). It has been reported that over 35% of sales on Amazon.com and more than 60% of the rentals on Netflix result from recommendations (Hosanagar et al., 2013).

Commercial recommender systems vary along different dimensions such as the algorithm used to generate recommendations, the timing related to when recommendations are presented to a consumer, the manner in which recommendations are presented, the type of products—complementary or competing—recommended, and the objective to be achieved through the recommendation. For instance, while content-based recommender systems use product characteristics (e.g., genre, mood, author, and price in the case of books) to recommend items that are similar to products that a target consumer previously bought or liked, collaborative filter-based recommender systems recommend products based on purchase history or taste of similar consumers. On the timing and product type dimensions, while some systems recommend complementary products after a consumer has made a purchase, others recommend competing products before the purchase (e.g., when the consumer visits the platform or searches for a product). Although it is widely assumed that recommender systems are designed to benefit consumers by recommending the products they desire (Xiao and Benbasat, 2007), some are designed not just to benefit consumers but also to steer them toward products that serve the seller's own interests (Häubl and Murray, 2006). For example, instead of recommending solely based on preference match, firms can bias the recommendation by recommending products with certain characteristics (e.g., high profit margin products, soon-to-be discontinued products) in order to attain higher profits (Das et al., 2010; Xiao and Benbasat, 2015).

Research indicates that recommender systems affect consumer decision making (Tam and Ho, 2006). For instance, recommender systems could inform consumers about products they are unaware of and enlarge the consumers' consideration set ("informative role"), or increase the purchase probability of the recommended product if the consumer is already aware of it ("persuasive role") (Gretzel and Fesenmaier, 2006; Tam and Ho, 2005; Fleder and Hosanagar, 2009). Research has also examined the impact of recommender systems on product sales. For instance, some studies argue that recommender systems contribute to the long tail effect by exposing consumers to niche products, but others argue that some recommender system designs increase the popularity of already popular products (Mooney and Roy, 2000; Fleder and Hosanagar, 2009). Departing from these studies, in this paper we aim to study how a recommender system deployed by a retail platform affects the competition between manufacturers that sell competing products via the platform and the payoffs of players such as manufacturers, platform, consumers, and society, and the consequent implications for the recommendation strategy to be adopted by the retail platform.

The question of how recommender systems affect the price competition between substitutable products is important to both practitioners and academics. The question becomes especially important in the context of a dominant e-commerce platform that sells competing products from different manufacturers while simultaneously recommending a subset of these products. Such a two-level channel structure with one dominant e-commerce platform is commonly observed in practice (e.g., Amazon's online marketplace). In such contexts, although each manufacturer views the substitutable products from other manufacturers as competitors, the platform may view the products as satisfying the different needs of different consumers. Therefore, from a platform's perspective, an analysis of recommender systems' effects on both consumers (demand side) and manufacturers (supply side) is essential for a more complete understanding of the implications of recommender systems. In particular, while extant empirical research finds evidence for increased sales for recommended products, it is unknown whether and under what conditions recommender systems increase the platform's profitability. The effect of recommender systems on the manufacturers also remains unexplored and unclear despite the ubiquity of these systems in e-commerce platforms. Furthermore, how recommender system design affects the platform and manufacturers is yet another important question for the platform.

To address these questions, we develop an analytical model in which two manufacturers sell substitutable products through a common retail platform (hereafter referred to as the retailer).[1] Manufacturers set the sales prices and the retailer charges a percentage of the sales price as a fixed commission. Consumers visiting the retailer have heterogeneous preferences. While some are loyal consumers that buy only from the manufacturer they are loyal to, others are shoppers that are seeking to buy the product that offers a higher surplus. Consumers are heterogeneous in their awareness of the two products and it is a consequence of different advertising venues such as TV, newspapers, and the Internet accessible to manufacturers and consumers. We distinguish three types of consumers based on their awareness about products when they visit the retailer: partially informed consumers who are aware of only one product, fully informed consumers who are aware of both products, and uninformed consumers who are aware of neither product. The retailer employs a recommender system that ranks the products based on a recommendation score which is a weighted sum of expected consumer value and expected retailer profit if the product is

[1] The results do not require these sellers to be manufacturers. If they are resellers, the production cost in our model should be interpreted as the procurement cost for these resellers.

recommended. While making the recommendation, the retailer knows the profit that it will receive from the sale of a product, but is uncertain about the consumer preference and the product the consumer will buy after the recommendation. We model the uncertainty using recommender system precision which refers to the accuracy with which the recommender system infers the consumer preference. For the above context, we first derive the impacts of a recommender system (with exogenously specified characteristics) on the retailer, manufacturers, consumers, and the society, and examine how the recommender system characteristics and market characteristics such as the product awareness levels among consumers in the absence of recommender system affect the impacts. This analysis allows us to articulate the key drivers of recommender system impacts. Then, we investigate the retailer's optimal recommendation strategy in terms of the relative weight the retailer should assign to its own profit while choosing the recommendation. This analysis allows us to derive significant implications pertaining to retailer's choice of recommendation strategy.

One key finding from our analysis is that when manufacturers strategically respond to retailer's deployment of the recommender system, the retailer and manufacturers may benefit or hurt, depending on recommender system and market characteristics. The impacts of the recommender system are dictated by the signs and magnitudes of *substitution effect* which either intensifies or softens the price competition between manufacturers, and the *demand effect* which either increases or decreases the demand.

The substitution effect arises because of the strategic interactions between manufacturers, and can be attributed to two influences. First, the direct influence of the recommender system in changing the price competition is that recommendations alter the relative size of consumer group that manufacturers compete for (*common turf*) and that of consumer group the manufacturers cannot compete for (*monopoly turf*), which affect the markup that manufacturers can charge. On the one hand, recommendations convert uninformed shoppers to partially informed shoppers, which increases each manufacturer's monopoly turf.

On the other hand, recommendations convert some partially informed shoppers to fully informed shoppers, which makes more shoppers aware of both products and increases the common turf. Reacting to the changes in the sizes of the monopoly turf and common turf, manufacturers price differently in the presence of the recommender system. Second, the strategic influence of the recommender system in changing the price competition is that the recommender system presents manufacturers an opportunity to use price as a lever to attract more recommendations in their favor—when an increase in price increases (decreases) the chance of being recommended, manufacturers have an incentive to increase (decrease) prices in order to attract more recommendations. Depending on the magnitudes of the two influences, the *substitution effect* may be positive for the sellers (softening the price competition) or negative (intensifying the price competition).

The demand effect of the recommender system is attributed to the following. The recommender system increases the product awareness among shoppers which increases the demand from shoppers; however, the demand from loyal consumers may increase or decrease depending on whether the substitution effect increases or decreases the prices. Consequently, the overall demand can be higher or lower in the presence of the recommender system than in its absence.

Whether the recommender system benefits or hurts the sellers depends on the recommender system precision, the recommendation strategy viz., the relative weight assigned to retailer profit vis-a-vis consumer value in computing the recommendation score, and market characteristics such as the relative sizes of the three consumer segments pertaining to product awareness. We identify two types of recommendation strategies that generally have qualitatively different impacts on sellers and consumers. When the relative weight assigned to the retailer profit is high (low) such that an increase in price increases (decreases) the recommendation score for the product, we refer to it as a profit-oriented (consumer-oriented) recommender system. While a highly profit oriented recommender system induces manufacturers to set high prices which provides high profit margins to the retailer and manufacturers, it may significantly decrease the demand from loyal consumers. On the other hand, a highly consumer oriented recommender system induces manufacturers to undercut each other's price which can spur the demand from loyal consumers, but it can hurt the manufacturers and the retailer by creating severe price competition. Furthermore, when the recommender system is consumer oriented, an increase in precision softens the price competition and increases the sellers' benefit from the recommender system; however, when the recommender system is profit oriented, an increase in precision intensifies the price competition and decreases the sellers' benefit from the recommender system if the profit orientation is excessive. Similarly, the impacts of consumer awareness levels also depend on whether the recommender system is consumer oriented or profit oriented. Therefore, from the retailer's perspective, the choice of an optimal recommendation strategy that accounts for recommender system precision and market characteristics is critical.

We find that a mildly profit oriented recommendation strategy offers the highest payoff to the retailer. Under the optimal recommendation strategy, the price competition is less intense and the retailer profit is higher compared to when there is no recommender system. Furthermore, an increase in either the recommender system precision or the fraction of consumers that are aware of at least one product induces the retailer to adopt a more profit oriented recommendation strategy.

Consumers benefit from the recommender system only when the recommendation strategy does not assign too large a relative weight to the retailer profit. Ironically, an increase in precision improves the consumer surplus when the recommender system is profit oriented, but may not improve consumer surplus if the recommender system is consumer oriented. Consequently, if the retailer deploys the optimal recommendation strategy, then the consumers are also better off if the recommender system's precision is high. The impacts of the recommender system on the social welfare are qualitatively similar to those on consumers.

Altogether, our results reveal that when the recommender system is highly profit oriented, all players in the marketplace—retailer, manufacturers, consumers, and the society—are hurt by the recommender system. On the other hand, if the retailer deploys the optimal recommendation strategy that maximizes its own profit, every player in the market place—retailer, manufacturers, consumers, and the society—is better off in the presence of the recommender system than in its absence if the recommender system has a high precision.

The rest of this paper is organized as follows. In the next section, we review the related literature. In Section 2.3, we develop an analytical framework to model the impact of a recommender system in a channel structure where two competing sellers sell on a common platform. Section 2.4 examines the implication of the recommender system by comparing the scenario in which consumers purchase without recommendations with the scenario with recommendation. In Section 2.5, we examine the impact of recommender system and market characteristics on the results. In Section 2.6, we examine the characteristics and impacts of the optimal recommendation strategy that maximizes the retailer's payoff. Section 2.7 considers an extension in which the recommender system recommends both products to a consumer in the order of recommendation score. Section 2.8 concludes the paper with a discussion on managerial implications.

2.2 Related Literature

The existing literature on recommender systems generally falls into three streams. The first stream focuses on recommendation algorithms. Much of the work in this stream has focused on predicting consumer preference (e.g., Häubl and Trifts, 2000; Hostler et al., 2005).[2] Adomavicius and Tuzhilin (2005) provide a review of this work. Burke (2002) proposes a

[2]See http://www.digitalbookworld.com/2012/consumers-like-and-trust-amazon-book-recommendations-despite-industry-jitters/ for an insider look at recommendation algorithms used by Amazon.com. According to former Amazon engineers who were involved in the design and development of Amazon's recommendation engines, it is unlikely that Amazon's recommendation engines are set up to do anything other than recommend products that users are most likely to buy—contrary to the rumors about whether it is used for marketing purposes. They stated, "Anything that doesn't maximize the usefulness to customers of the recommendations will hurt sales, and it is hard to make up for that huge cost any other way."

recommender system that considers attributes such as price, quality, and delivery date while choosing the recommendation for a user. Wang and Zhang (2011) develop an algorithm that uses consumer net utility and show that their algorithm outperforms the widely used collaborative filtering-based recommender systems that do not use net utility. Algorithms to increase recommendation diversity have also been proposed (McNee et al., 2006; Vargas and Castells, 2011; Pu et al., 2011). Algorithms that focus on consumer preference or utility do not explicitly incorporate firm's revenue or profit in their recommendation algorithm, and they implicitly assume that recommending the product that offers the highest value to the consumer benefits the firm both in the short run based on the notion that recommendations which are not likely to be followed by the users are worthless and in the long run based on the idea that good recommendations increase consumer satisfaction and trust. Some recent studies have focused on revenue-driven recommendations. Chen et al. (2008) model purchase probabilities, and recommend k-highest ranked items in terms of expected revenue (purchase probability times price). Das et al. (2010) consider the trust between users and the recommender system to predict the purchase probability. Hosanagar et al. (2008) examine two trade-offs faced by a vendor when designing a recommender—the trade-off between the relevance of products to a consumer and the firm's margins from selling the product and the tradeoff between increasing near-term profit versus the long-run future profit. Shani et al. (2002) treat the problem of generating recommendations as a sequential optimization problem that takes into account the long-term effects and short-term effects of a recommendation. Unlike the studies in this stream of research, we focus on the economic impact of recommender systems on upstream manufacturers and the consequence of strategic manufacturer responses to the retailers. Consistent with the two broad foci of recommender systems—consumer and seller—, we adopt a recommendation strategy that uses a weighted sum of seller revenue and consumer value.

The second stream of literature focuses on the impact of recommender systems on consumers' decision making and choices. Senecal and Nantel (2004) show that recommended products are selected twice as often as non-recommended products. This influence is moderated by the type of recommendation source and the type of product. Cooke et al. (2002) find that context and familiarity can affect a consumer's reaction to recommendations. Consistent with empirical findings in this stream of research, we develop a model in which recommended products have a higher average chance of being bought than non-recommended products, but whether the recommended product is purchased depends on consumer preference, price, and awareness about the competing product.

The third stream of research examines recommender systems' effect on product sales and sales diversity. Using a simulation model, Fleder and Hosanagar (2009) show that recommendations made on the basis of sales and ratings reinforce the popularity of already popular products. Hosanagar et al. (2013) empirically show that recommender systems can lead to consumers purchasing more similar items. Using data from a video-on-demand retailer, Hinz and Eckert (2010) show how different classes of search and recommendation tools affect the distribution of sales across products, total sales, and consumer surplus. Brynjolfsson et al. (2011) find that a firm's online sales channel has a slightly higher diversity than its offline channel and they attribute the difference to recommender systems. Oestreicher-Singer and Sundararajan (2012a) empirically show that recommender systems induce significantly flatter demand and revenue distributions. Oestreicher-Singer and Sundararajan (2012b) show that, on average, the explicit visibility of a co-purchase relationship can amplify the influence of complementary products on each other's demands. Pathak et al. (2010) find that the strength of recommendations has a positive effect on sales and prices and that this effect is moderated by the recency effect. Jabr and Zheng (2014) analyze the effect of recommendations and word-of-moth reviews on product sales in a competitive environment and show that higher referral centrality of competing products is associated with lower product sales. In contrast to this stream of research, we examine the impact of recommender systems on competing products using an analytical model and study upstream effects of downstream deployment of a recommender system.

Much of the previously discussed research uses a technical or empirical research methodology. Theoretical research that has examined the impact of recommender systems is limited. Hervas-Drane (2009) shows that when recommender systems based on consumer taste are introduced alongside traditional word of mouth, there is a positive impact on consumers' interest in niche products and a decrease in market concentration. Bergemann and Ozmen (2006) use a two-stage game to show how a firm can strategically choose its price in the first stage to generate recommendations in the second stage. Our study departs from these in that we examine the effect of recommender systems in the setting of competing sellers in a channel structure, while previous studies consider a single seller and ignore the strategic interactions between sellers.

Our study is also related to the marketing literature on informative advertising and competition. Bester and Petrakis (1995) and Grossman and Shapiro (1984) predict an inverse relationship between advertising level and prices in a differentiated product market when advertising provides uninformed consumers with price information. Soberman (2004) extends Grossman and Shapiro (1984) to show that informative advertising alone can lead to either higher or lower prices depending on the level of differentiation between competing firms. While these studies consider uniform advertising, another stream of literature considers targeted advertising. Iyer et al. (2005) investigate how competing firms in a horizontally differentiated market choose the advertising strategy when they can target consumer segments according to their preferences. Gal-Or et al. (2006) examine how an advertiser should allocate resources to increase the quality of targeting. Gal-Or and Gal-Or (2005) study firms' advertising strategy when they use a single media distributor such as television cable company as the channel for advertising. Different from these studies, our focus is on the the impact of various recommender systems on competing sellers' profits.

2.3 Model

We consider a two-level channel structure with two manufacturers (A and B), one common retailer (R), and a continuum of consumers with heterogeneous preferences. Manufacturer A (B) produces product A (B) and sells the product via R. Each manufacturer sets the price of its product, and the retailer charges the manufacturers a commission equal to α fraction of the price on each sale.[3] We assume that the fixed and marginal production costs are zero.

The two products are horizontally differentiated and have different levels of misfit to different consumers. In particular, we assume that the products are located at the two end points of a Hotelling line of a unit length, with product A being at 0 and product B being at 1. Consumers are uniformly distributed along the Hotelling line. The distance between a product and a consumer measures the degree of misfit of the product to the consumer. We refer to consumers' differentiation along the Hotelling line as *locational differentiation.*

We distinguish two types of consumers—loyal consumers and shoppers—that differ with respect to whether and how recommendations affect their purchase decisions. Loyal consumers consider purchasing only the product from the manufacturer that they are loyal to. They are aware of the product from the manufacturer they are loyal to, but they may or may not purchase the product depending on its price and their misfit cost. On the other hand, shoppers are looking to purchase a product but are not loyal to any specific manufacturer. They will buy the product that offers the highest net utility from the set of products they are aware of. Thus, while the purchase decisions of shoppers are influenced by recommendations, those of loyal consumers are not.

[3] An alternative setting is one in which R buys from manufacturers and resells to consumers. Amazon.com uses such a wholesale scheme for some products and the platform scheme we consider in this paper for other products. It is noteworthy that Amazon.com sells 93% of products in the electronics category, 83.3% of Shoes, 96.9% of products in Sports & Outdoors category, and 96.8% of products in the Jewelry category using the platform scheme (Jiang et al., 2011).

Loyal Consumers: For a loyal consumer, the utility of the product she is loyal to is v_L and the unit misfit cost is t_L.[4] Thus, for a consumer loyal to product A and located at z on the Hotelling line, the net utility from buying product A is $U_{AL} = v_L - zt_L - p_A$. Analogously, for a consumer loyal to product B and located at z, the net utility from buying product B is $U_{BL} = v_L - (1-z)t_L - p_B$.

Shoppers: For a shopper, the utility of either product is v_S and the unit misfit cost is t_S. Thus, for a shopper located at z, the net utility from buying product A is $U_{AS} = v_S - zt_S - p_A$ and from buying product B is $U_B = v_S - (1-z)t_S - p_B$. Shoppers may have different awareness of the two products: fully informed shoppers are aware of both products, uninformed shoppers are aware of neither product, and partially informed shoppers include two groups—consumers who are only aware of product A and consumers who are only aware of product B. In the absence of recommendations, we assume that the fraction of the fully informed shopper segment is θ_b, the fraction of each partially informed shopper group is θ, and, the proportion of uninformed shoppers is $1 - 2\theta - \theta_b$. This basic awareness structure is implied by the existence of advertising venues accessible to manufacturers and consumers, such as TV, newspapers, and the Internet. A shopper's awareness is independent of her location.

We refer to consumers' differentiation along the awareness dimension as *informational differentiation*. We normalize the number of shoppers in the market to one and let γ be the number of consumers loyal to each manufacturer.

Recommendation Precision: The recommender system is uncertain about a consumer's true preference or location on the Hotelling line, and uses information such as purchase data, rating data, and profile data to estimate her location. The estimate may be imperfect, and we use a commonly used approach to model this estimation (e.g., Lewis and Sappington 1994; Johnson and Myatt 2006). In particular, we assume that the retailer observes a signal

[4]The utility for the product she is not loyal to can be assumed to be zero.

s regarding consumer's location. The signal could be viewed as the output of the consumer preference prediction model that is ubiquitous in recommender systems. These predictive models typically estimate the likely consumer rating of a product based on other available information and the rating is used as a measure of the likely consumer preference. The signal equals the consumer's true location with probability β, and with probability $(1-\beta)$ the signal is uninformative and follows the prior (uniform) distribution of consumer location. That is, for a consumer whose true location is y, $P(s = y|z = y) = \beta$ and $P(s \neq y|z = y) = 1 - \beta$, where $y \in [0, 1]$. As shown in the appendix, using Bayesian updating, we can derive the consumer's expected location given the signal as

$$\mathbb{E}(z|y) = \left(\frac{1-\beta}{2} + \beta y\right) \tag{2.1}$$

Thus, the model indicates that the signal is informative (i.e., provides useful information for the retailer to estimate the consumer's preference) but noisy (i.e., does not perfectly reveal the true preference). We refer to β as the precision of the recommender system.

Recommendation Strategy: In the base model, we consider that when a consumer comes to the retailer's site, the retailer recommends one product to this consumer. In the extension, we also examine the case where the retailer recommends both products. The retailer considers two factors when deciding which product to recommend: retailer's profit and consumer's net utility. The retailer's profit from a product is determined by the price of the product and the commission rate for each transaction. If the consumer purchases product i, the retailer gets a profit of αp_i, $i \in \{A, B\}$. We assume that the retailer assigns a relative weight of w to its own profit vis-a-vis consumer's net utility. A high (low) value for w suggests that the recommender system is more profit (consumer) oriented. Because the retailer does not know the consumer's type (loyal consumer or shopper) or true location, we define the score used by the recommender system for product i using expected retailer profit and expected consumer net utility as follows:

$$R_i = \mathbb{E}\left(\text{consumer net utility}|i \text{ is recommended}\right) + w\mathbb{E}\left(\text{retailer profit}|i \text{ is recommended}\right) \tag{2.2}$$

Table 2.1. Summary of Notations

Notation	*Definition and Comments*
v_S	utility for a product of a shopper
t_S	unit misfit cost of a shopper
v_L	utility for a product of a loyal consumer
t_L	unit misfit cost of a loyal consumer
γ	the size of the loyal consumers for a product
h	$h \equiv \frac{\gamma v_L}{t_L}$, indicate the potential demand from the loyal consumers for a product
τ	$\tau \equiv \frac{\gamma}{t_L}$, indicate the price sensitivity of the loyal customers for a product
θ	proportion of each partially informed consumer segment
θ_b	proportion of fully informed consumers
β	probability of getting a correct signal about consumer location
w	weight on profit in the recommendation score
R_i	recommendation score for product i, $i \in \{A, B\}$
p_i	price of product i, $i \in \{A, B\}$
α	commission rate charged by the retailer

The retailer recommends the product that has a higher score.

The sequence of events is as follows. In Stage 1, manufacturers set prices p_A and p_B simultaneously. In Stage 2, consumers visit the platform and make their purchase decisions. Two scenarios are considered: one without the recommender system and the other with the recommender system. We use the scenario without the recommender system as the benchmark to analyze the effect of the recommender system. In the scenario without the recommender system, consumers make their purchase decisions based on their awareness and preferences of the products. In the scenario with the recommender system, the retailer recommends one product to each consumer in Stage 2, and consumers make their purchase decisions with this additional information.

We assume that the cost of developing the recommender system and the cost of providing a recommendation are zero. A consumer's type, her awareness about a product, and her preference are private information. All other model parameters are common knowledge. All players are risk neutral. Table 2.1 summarizes the parameters used in the paper. Finally we make the following technical assumptions for our base model to rule out trivial or unrealistic cases.

Assumption 1: $\max\{\frac{(2\theta+\theta_b)t_S}{(\theta_b+2\gamma t_S/t_L)}, \frac{\beta t_S}{[(1-\alpha w)(1-\theta-\theta_b)+\beta(\theta+\theta_b)+2\beta\gamma t_S/t_L]}\} < v_L < t_L$

Assumption 2: $v_S > \max\{t_S\frac{2(\theta+\theta_b)+2\gamma(2t_S+v_L)/t_L}{\theta_b+4\gamma t_S/t_L}, t_S\frac{(1-\alpha w)(1-\theta-\theta_b)+\beta(1+\theta+\theta_b)+2\beta\gamma(2t_S+v_L)/t_L}{(1-\alpha w)(1-\theta-\theta_b)+\beta(\theta+\theta_b)+4\beta\gamma t_S/t_L}\}$

Assumption 3: $\beta > \frac{(1-\theta-\theta_b)(\alpha w-1)}{\theta+\theta_b+2\gamma t_S/t_L}$

Assumption 1 ensures that the demand from loyal consumers is positive in the equilibrium; that is, sellers do not target only at shoppers and loyal consumers are not fully covered. Assumption 2 ensures that all shoppers except the uninformed ones buy. Assumption 3 ensures that sellers' profit functions in the presence of recommendation are concave in their respective prices, which guarantees a unique pure strategy equilibrium.

2.4 Impacts of Recommender System

In this section, using backward induction, we first derive the subgame perfect equilibrium outcomes for the case without the recommender system and for the case with the recommender system. We then analyze the impacts of the recommender system by comparing the equilibrium in the two cases.

2.4.1 Benchmark Case (No Recommendation)

A loyal consumer buys the product that she is loyal to if and only if her net utility is non-negative. Thus, we can formulate the demand for product i from its loyal customers as $D_{iL} = \frac{\gamma}{t_L}(v_L - p_i)$. For expositional clarity, we define $h \equiv \gamma v_L/t_L$ and $\tau \equiv \gamma/t_L$. Then, we have

$$D_{iL} = h - \tau p_i \tag{2.3}$$

in which τ indicates the price sensitivity of its loyal consumers and h indicates the potential market size of loyal consumers for a product.

A shopper buys the product that offers a higher net utility from the set of products she is aware of. We denote as z_0 the location of the marginal shopper who would be indifferent between the two products if she were fully informed. Based on the utility function, we have

$$z_0 = \frac{p_B - p_A + t_S}{2t_S} \tag{2.4}$$

Shoppers located at $z < z_0$ would, if fully informed, purchase product A, and shoppers located at $z > z_0$ purchase product B. Therefore, the demands for the two products from shoppers can be formulated as $D_{AS} = \theta + z_0\theta_b$ and $D_{BS} = \theta + (1 - z_0)\theta_b$. The total demand for product i is given by:

$$D_i = D_{iS} + D_{iL} \tag{2.5}$$

The manufacturers maximize their profits by choosing their optimal prices:

$$\max_{p_i} \pi_i = (1 - \alpha)p_i D_i \tag{2.6}$$

Based on their best response to each other, we obtain the equilibrium price and demand for each manufacturer. The following lemma summarizes the equilibrium outcome.

Lemma 1. *In the absence of the recommender system, the equilibrium prices, demands, manufacturer profits, retailer profit, consumer surplus, and social welfare are as follows:*

(a) Price:

$$\bar{p}_A^* = \bar{p}_B^* = t_S + 2t_S \left(\tfrac{\theta + h - 2\tau t_S}{\theta_b + 4\tau t_S}\right) \tag{2.7}$$

(b) Demand:

$$\bar{D}_A^* = \bar{D}_B^* = \left(\tfrac{2\theta + \theta_b}{2} + h\right)\left(1 - \tfrac{2\tau t_S}{\theta_b + 4\tau t_S}\right) \tag{2.8}$$

(c) Manufacturer profit:

$$\bar{\pi}_A^* = \bar{\pi}_B^* = (1 - \alpha)t_S \left(\tfrac{2\theta + \theta_b + 2h}{\theta_b + 4\tau t_S}\right)^2 \left(\tfrac{\theta_b}{2} + \tau t_S\right) \tag{2.9}$$

(d) Retailer profit:

$$\bar{\pi}_R^* = \alpha t_S \left(\frac{2\theta+\theta_b+2h}{\theta_b+4\tau t_S}\right)^2 (\theta_b + 2\tau t_S) \tag{2.10}$$

(e) Consumer surplus:

$$\bar{CS}^* = (2\theta + \theta_b) \left[v_S - t_S \frac{2\theta+\theta_b+2h}{\theta_b+4\tau t_S} \right] - t_S \frac{4\theta+\theta_b}{4} + \frac{(\theta_b h-(2\theta+\theta_b-2h)\tau t_S)^2}{\tau(\theta_b+4\tau t_S)^2} \tag{2.11}$$

(f) Social welfare:

$$\bar{W}^* = (2\theta + \theta_b) v_S - t_S \frac{4\theta+\theta_b}{4} + \frac{[\theta_b h-(2\theta+\theta_b-2h)\tau t_S][\theta_b h+(6\theta+3\theta_b+10h)\tau t_S]}{\tau(\theta_b+4\tau t_S)^2} \tag{2.12}$$

Proof. All proofs are in the appendix unless indicated otherwise. □

The price expression in Lemma 1(a) has a simple interpretation. The first term is the price when there are only shoppers and all consumers are aware of both products. That is, the first term is the equilibrium price when the market includes only consumers that both manufacturers compete for; in other words, the market consists only of *common turf* (of consumers) that manufacturers compete in. The second term is the markup manufacturers can charge because of the presence of *monopoly turf* in which there is no competition between manufacturers. Two sources contribute to the presence of monopoly turf for a manufacturer: consumers loyal to the manufacturer, and partially informed shoppers who are only aware of this manufacturer. Effectively, the informational differentiation that exists among consumers—because of heterogeneity in awareness among shoppers and loyalty to one product among loyal consumers—shapes the equilibrium price, and the fraction $\frac{\theta+h-2t_S\tau}{\theta_b+4t_S\tau}$ measures, in some sense, the extent of informational differentiation. A higher informational differentiation results in a lower elasticity of demand; that is, holding everything else constant, a higher informational differentiation among consumers softens the competition and increases the price. Therefore, the equilibrium price and profit are increasing in θ and h but decreasing in θ_b. Intuitively, any increase in the size of the monopoly turf softens the price competition and an increase in the size of the common turf intensifies the price competition between manufacturers. It is also intuitive that an increase in either θ, θ_b or h increases the demand for both manufacturers because the total market size increases in this case.

2.4.2 With Recommender System

When the recommender system is in place, each consumer is recommended the product that has a higher recommendation score given the signal received by the retailer regarding the consumer's location. In this section, we first derive the recommendation score of each product based on the signal that the retailer receives about a consumer's location. We then formulate the demand function for each manufacturer and solve for the equilibrium outcome.

We consider the retailer observes signal y regarding a consumer's location. With probability $\frac{\gamma}{1+2\gamma}$, this consumer is a loyal consumer of product i, and with probability $\frac{1}{1+2\gamma}$, this consumer is a shopper. Suppose the consumer is a shopper. If product A is recommended, the consumer becomes fully informed with probability $\theta+\theta_b$ because this case occurs if this consumer was only aware of B or fully informed before recommendation. Otherwise, this consumer becomes informed about A only, which occurs with probability $1-\theta-\theta_b$. Analogously, if product B is recommended to this consumer, the consumer becomes fully informed with probability $\theta+\theta_b$ and becomes informed about B only with probability $1-\theta-\theta_b$. On the other hand, if the consumer is a loyal one, then regardless of the recommendation, she makes the same purchase decisions. Thus, the recommendation score for each product can be derived as follows.

$$\begin{aligned} R_A = \quad & \tfrac{1-\theta-\theta_b}{1+2\gamma}\left[(v_S - \mathbb{E}(z|y)t_S - p_A) + w\alpha p_A\right] + \tfrac{\theta+\theta_b}{1+2\gamma}\int_0^{z_0}\left[(v_S - zt_S - p_A) + w\alpha p_A\right] f(z|y)dy \\ & + \tfrac{\theta+\theta_b}{1+2\gamma}\int_{z_0}^{1}\left[(v_S - (1-z)t_S - p_B) + w\alpha p_B\right] f(z|y)dy + \tfrac{\gamma}{1+2\gamma}\int_0^{\frac{v_L-p_A}{t_L}}\left[(v_L - zt_L - p_A) + w\alpha p_A\right] f(z|y)dy \\ & + \tfrac{\gamma}{1+2\gamma}\int_0^{\frac{v_L-p_B}{t_L}}\left[(v_L - (1-z)t_L - p_B) + w\alpha p_B\right] f(z|y)dy \end{aligned} \tag{2.13}$$

$$
\begin{aligned}
R_B = \ & \tfrac{1-\theta-\theta_b}{1+2\gamma}\left[\left(v_S-\left(1-\mathbb{E}(z|y)\right)-p_B\right)t_S+w\alpha p_B\right]+\tfrac{\theta+\theta_b}{1+2\gamma}\int_0^{z_0}\left[\left(v_S-zt_S-p_A\right)+w\alpha p_A\right]f(z|y)dy \\
& +\tfrac{\theta+\theta_b}{1+2\gamma}\int_{z_0}^{1}\left[\left(v_S-(1-z)t_S-p_B\right)+w\alpha p_B\right]f(z|y)dy+\tfrac{\gamma}{1+2\gamma}\int_0^{\frac{v_L-p_A}{t_L}}\left[\left(v_L-zt_L-p_A\right)+w\alpha p_A\right]f(z|y)dy \\
& +\tfrac{\gamma}{1+2\gamma}\int_0^{\frac{v_L-p_B}{t_L}}\left[\left(v_L-(1-z)t_L-p_B\right)+w\alpha p_B\right]f(z|y)dy
\end{aligned}
\tag{2.14}
$$

where $f(z|y)$ is the probability density function of true location z conditional on signal y. By substituting $\mathbb{E}(z|y)$ from Equation (2.1), we get the difference in recommendation score as:

$$
R_A - R_B = (1-\theta-\theta_b)\Big[\beta(1-2y)t_S + (1-w\alpha)(p_A-p_B)\Big] \tag{2.15}
$$

We denote as y_0 the marginal signal under which the recommendation scores for the two products are equal (i.e., $R_A = R_B$), which, based on Equation (2.15), can be derived as

$$
y_0 = \frac{1}{2} + \frac{(p_B-p_A)(1-\alpha w)}{2\beta t_S} \tag{2.16}
$$

If the recommender system receives a signal less than y_0, the retailer recommends A to the consumer; otherwise, the retailer recommends product B.

Next, we formulate the demand function for each manufacturer. Since recommendations do not affect the demand from loyal consumers, the loyal consumer demand remains the same as that given in Equation (2.3). For the demand from shoppers, we provide the derivation details for the case $y_0 \geq z_0$. The derivation for the case $y_0 < z_0$ is analogous. We first consider the shopper segment that is only aware of product A prior to receiving the recommendation. Within this segment, a shopper whose location is less than z_0 buys product A regardless of the recommendation she receives. A shopper whose location is between z_0 and 1 buys A if she is recommended A because she will be aware of only A. If she is recommended B and so is aware of both products, she buys product B. A shopper located between z_0 and y_0 receives a recommendation for product A (and thus buys product A) with probability $(1-\beta)y_0+\beta$. A shopper located between y_0 and 1 receives recommendation for product A

(and thus buys product A) with probability $(1-\beta)y_0$. Combining all together, the demand for A from shoppers who are only aware of product A is:

$$d_{AS} = \theta\Big[z_0 + \int_{z_0}^{y_0} P(s \le y_0|z)dz + \int_{y_0}^{1} P(s \le y_0|z)dz\Big] = \theta\left[y_0 + z_0(1-y_0)(1-\beta)\right]$$

and the demand for product B is $d_{BS} = \theta - d_{AS}$.

Using a similar logic, we can derive the demand functions for other shopper segments. Aggregating the demand functions for all shopper segments, we derive the demand for product i from shoppers as:

$$D_{iS} = \frac{1}{2} + \frac{\beta(\theta+\theta_b) + (1-\theta-\theta_b)\left(1-\alpha w\right)}{2\beta t_S}\left(p_{\bar{i}} - p_i\right) \tag{2.17}$$

The total demand for product i is $D_i = D_{iS} + D_{iL}$, in which D_{iL} is as in Equation (2.10). Similar to the benchmark case, we can formulate the manufacturers' optimization problems. Based on their best response to each other, we obtain the equilibrium price and demand for each manufacturer. The following lemma summarizes the equilibrium outcomes.

Lemma 2. *When the retailer uses the recommender system, the equilibrium prices, demands, manufacturer profits, retailer profit, consumer surplus, and social welfare are as follows:*

(a) Price:

$$p_A^* = p_B^* = \frac{\beta(1+2h)t_S}{\left(1-\alpha w\right)\left(1-\theta-\theta_b\right) + \beta(\theta+\theta_b+4\tau t_S)} \tag{2.18}$$

(b) Demand:

$$D_A^* = D_B^* = \left(\tfrac{1}{2}+h\right)\left(1 - \tfrac{2\beta\tau t_S}{(1-\alpha w)(1-\theta-\theta_b)+\beta(\theta+\theta_b+4\tau t_S)}\right) \tag{2.19}$$

(c) Manufacturer profit:

$$\pi_A^* = \pi_B^* = (1-\alpha)\beta t_S\left(\tfrac{1+2h}{(1-\alpha w)(1-\theta-\theta_b)+\beta(\theta+\theta_b+4\tau t_S)}\right)^2 \tfrac{(1-\theta-\theta_b)(1-\alpha w)+\beta(\theta+\theta_b+2\tau t_S)}{2} \tag{2.20}$$

(d) Retailer profit:

$$\pi_R^* = \alpha\beta t_S \tfrac{(1+2h)^2\left[(1-\theta-\theta_b)(1-\alpha w)+\beta(\theta+\theta_b+2\tau t_S)\right]}{\left((1-\alpha w)(1-\theta-\theta_b)+\beta(\theta+\theta_b+4\tau t_S)\right)^2} \tag{2.21}$$

(e) *Consumer surplus:*

$$CS^* = v_S - \frac{t_S[1+(1-\beta)(1-\theta-\theta_b)]}{4} - \frac{(1+2h)\beta t_S}{(1-\alpha w)(1-\theta-\theta_b)+\beta(\theta+\theta_b+4\tau t_S)} + \frac{(h[(1-\alpha w)(1-\theta-\theta_b)+\beta(\theta+\theta_b)]-\beta(1-2h)\tau t_S)^2}{\tau[(1-\alpha w)(1-\theta-\theta_b)+\beta(\theta+\theta_b+4\tau t_S)]^2} \tag{2.22}$$

(f) *Social welfare:*

$$W^* = v_S - \frac{t_S[1+(1-\beta)(1-\theta-\theta_b)]}{4} + \frac{(h[(1-\alpha w)(1-\theta-\theta_b)+\beta(\theta+\theta_b)]-\beta(1-2h)\tau t_S)(h[(1-\alpha w)(1-\theta-\theta_b)+\beta(\theta+\theta_b)]+\beta(3+10h)\tau t_S)}{\tau[(1-\alpha w)(1-\theta-\theta_b)+\beta(\theta+\theta_b+4\tau t_S)]^2} \tag{2.23}$$

The price expression given in Lemma 2(a) provides insights into how the price competition between manufacturers is shaped by the recommender system. The price expression enables easier interpretation when written as

$$p_A^* = p_B^* = \frac{\beta(\theta+\theta_b+4\tau t_S)}{(1-\alpha w)(1-\theta-\theta_b)+\beta(\theta+\theta_b+4\tau t_S)}\left[t_S + 2t_S\left(\frac{\frac{1-\theta+\theta_b}{2}+h-2\tau t_S}{\theta+\theta_b+4\tau t_S}\right)\right] \equiv K\left[t_S + 2t_S\left(\frac{\frac{1-\theta-\theta_b}{2}+h-2\tau t_S}{\theta+\theta_b+4\tau t_S}\right)\right] \tag{2.24}$$

As in the price expression for the benchmark case, t_S is the price under full information when there are only shoppers. In the equilibrium, $1-\theta-\theta_b$ proportion of shoppers are partially informed and $\theta+\theta_b$ proportion of shoppers are fully informed. The second term inside the bracket, $2t_S\left(\frac{\frac{1-\theta-\theta_b}{2}+h-2t_S\tau}{\theta+\theta_b+4t_S\tau}\right)$, is the markup that manufacturers can charge because of the informational differentiation that exists after the recommendation. Therefore, the expression in the bracket, $t_S + 2t_S\left(\frac{\frac{1-\theta+\theta_b}{2}+h-2\tau t_S}{\theta+\theta_b+4\tau t_S}\right)$, can be viewed as the myopic price when manufacturers ignore the effect of pricing on which product would be recommended. The myopic prices take the same form as the ones in the benchmark case, although the sizes of partially informed shoppers and fully informed shoppers changes in the presence of the recommender system, which is the *direct influence* of the recommender system on pricing. Another main difference from the benchmark case lies in that the myopic price term in the bracket is moderated by a coefficient, K, which captures the *strategic influence* of the recommender system on pricing. The strategic influence arises because, in addition to informing shoppers, the recommender system also alters manufacturers' pricing behavior as the prices might affect which product to be recommended for a user. When $w < 1/\alpha$, the coefficient is less then 1, and the strategic influence tends to decrease the price because a lower price for a product leads to a higher recommendation score and thus more recommendations in its favor. In contrast, when $w > 1/\alpha$, the coefficient is greater than 1, and the strategic influence tends to increase the price.

We find that several qualitative results regarding the impacts of the recommender system depend on whether $w < 1/\alpha$. Therefore, we refer to the recommender system with $w < 1/\alpha$ as *consumer-oriented recommender system* and that with $w > 1/\alpha$ as *profit-oriented recommender system.*

The implication of θ or θ_b for price competition is different from the case with no recommender system. On one side, an increase in θ or θ_b decreases the informational differentiation and the markup that manufacturers can charge (i.e., $\left(\frac{\frac{1-\theta+\theta_b}{2}+h-2\tau t_S}{\theta+\theta_b+4\tau t_S}\right)$ decreases in both θ and θ_b). On the other side, an increase in θ or θ_b increases (decreases) the strategic force (coefficient K) when the recommender system is consumer (profit) oriented. Consequently, an increase in θ or θ_b increases the price only when w is small, or, the consumer orientation is high (i.e., $w < \frac{1-\beta}{\alpha}$).

2.4.3 Recommender System Impacts

We can now assert the impacts of the recommender system by comparing equilibrium outcomes in the scenario without the recommender system with those in the scenario with the recommender system.

Proposition 1. *Compared to the scenario without the recommender system, in the presence of the recommender system,*

(a) each product's price is lower (i.e., $p_i^* < \bar{p}_i^*$, $i \in \{A, B\}$) if and only if

$$w < \frac{1}{\alpha} - \beta \left[\frac{\theta_b-(2\theta+\theta_b)(\theta_b+\theta)-2\theta h+4\tau(1-2\theta-\theta_b)t_S}{\alpha(1-\theta-\theta_b)(2\theta+\theta_b+2h)}\right] \tag{2.25}$$

(b) each product's demand is higher (i.e., $D_i^* > \bar{D}_i^*$) if and only if

$$w < \tfrac{1}{\alpha} + \tfrac{\beta(\theta+\theta_b)+2\beta\tau t_S}{\alpha(1-\theta-\theta_b)} - \tfrac{2\beta\tau(\theta_b+2t\tau)t_S}{\alpha(1-\theta-\theta_b)}\left[\tfrac{2\theta+\theta_b+2h}{(1-2\theta-\theta_b)(\theta_b+2t_S\tau)+2t_S\tau(1+2h)}\right] \tag{2.26}$$

(c) the retailer and manufacturers are better off (i.e., $\pi_i^* > \bar{\pi}_i^*$) if and only if

$$w_1 < w < w_2 \tag{2.27}$$

where

$$w_1 = \tfrac{1}{\alpha} + \tfrac{\beta[(\theta+\theta_b)+4t_S\tau]}{\alpha(1-\theta-\theta_b)} - \tfrac{\beta}{\alpha(1-\theta-\theta_b)}\left(\frac{1+\sqrt{1-8t_S\tau\left(\frac{2\theta+\theta_b+2h}{(\theta_b+4t_S\tau)(1+2h)}\right)^2(\theta_b+2t_S\tau)}}{2\left(\frac{2\theta+\theta_b+2h}{(\theta_b+4t_S\tau)(1+2h)}\right)^2(\theta_b+2t_S\tau)}\right)$$

$$w_2 = \tfrac{1}{\alpha} + \tfrac{\beta[(\theta+\theta_b)+4t_S\tau]}{\alpha(1-\theta-\theta_b)} - \tfrac{\beta}{\alpha(1-\theta-\theta_b)}\left(\frac{1-\sqrt{1-8t_S\tau\left(\frac{2\theta+\theta_b+2h}{(\theta_b+4t_S\tau)(1+2h)}\right)^2(\theta_b+2t_S\tau)}}{2\left(\frac{2\theta+\theta_b+2h}{(\theta_b+4t_S\tau)(1+2h)}\right)^2(\theta_b+2t_S\tau)}\right)$$

(d) consumer surplus is higher (i.e., $CS^* \geq \bar{CS}^*$) if and only if $w < w_{cs}$, where w_{cs} is the root of

$$(1-2\theta-\theta_b)\left[v_S - \tfrac{(2-\beta)t_S}{4} - p_i^*\right] + \tfrac{\theta(1+\beta)t_S}{4} + (\bar{p}_i^* - p_i^*)\left[2\theta+\theta_b+2h-\tau(\bar{p}_i^*+p_i^*)\right] = 0 \tag{2.28}$$

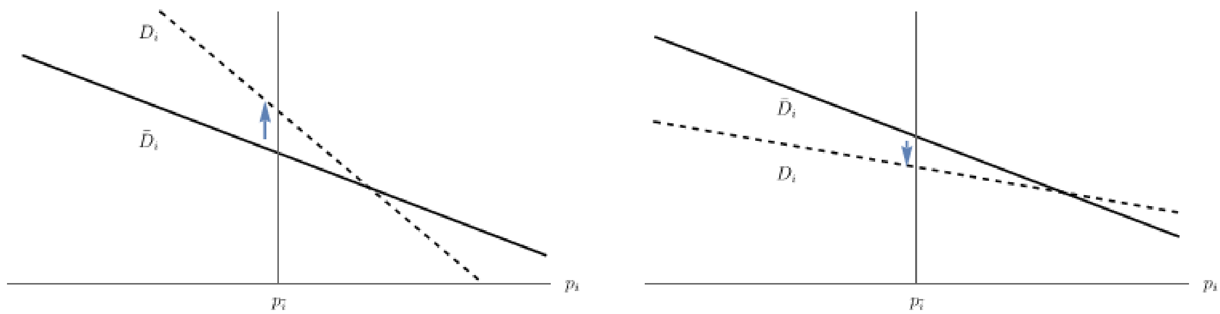
(e) social welfare is higher (i.e., $W^* > \bar{W}^*$) if $\frac{1-\theta-\theta_b}{\beta} + (\theta+\theta_b) < (2+\frac{3}{h})\tau t_S$ and $w < w_w$, where w_w is the root of

$$(1-2\theta-\theta_b)\left[v_S - \tfrac{(2-\beta)t_S}{4}\right] + \tfrac{\theta(1+\beta)t_S}{4} + (\bar{p}_i^* - p_i^*)\left[3\tau(\bar{p}_i^*+p_i^*)-2h\right] = 0 \tag{2.29}$$

Proposition 1(a) reveals that the price competition between manufacturers can be intensified or softened by the recommender system, depending on model parameters. Such an effect of the recommender system is captured by the change in the elasticity of the demand function, or the change in the slopes of the demand functions, as illustrated in Figure 2.1. Figure 2.1 shows the demand of product i as a function of p_i given the competitor's price $p_{\bar{i}}$. We refer to the change in slope of the demand function as the *substitution effect* of the recommender system. Clearly, the recommender system can make the demand function more elastic inducing a negative substitution effect which intensifies price competition or less elastic inducing a positive substitution effect which softens the price competition. We find that when the recommendation strategy does not assign too large a weight on the retailer's profit in deciding which product to recommend, recommender system intensifies the price competition, and when the weight on the retailer's profit is large, the recommender system softens the price competition.

The comparison of the equilibrium prices with and without the recommender system alludes to the direct influence and the strategic influence of the recommender system on pricing, which together contribute to the substitution effect. First, the recommender system changes the informational differentiation among consumers. The recommender system converts some partially informed shoppers to fully informed shoppers, and, simultaneously, converts uninformed shoppers to partially informed shoppers. This change in the informational differentiation because of recommendation alters the elasticity of firm's demand functions, which is the direct influence of the recommender system. Second, the recommender system's strategic influence on manufacturers induces them to use price as a lever to increase the likelihood of its product being recommended and this also alters the elasticity of demand functions. Whether the two influences together cause a positive or a negative substitution effect depends critically on the weight assigned to retailer's profit. Corollary 1 reveals the sharp contrast between a consumer-oriented recommender system and a profit-oriented recommender system regarding how the recommender system affects the price competition.

Corollary 1. *(a) When the recommender system is consumer oriented (i.e., if $w < \frac{1}{\alpha}$), each product's price is lower in the presence of the recommender system than in its absence if and only if* $\theta > \frac{\sqrt{(3\theta_b+8t_S\tau+2h)^2+8(4t_S\tau+\theta_b)(1-\theta_b)}-3\theta_b-8t_S\tau-2h}{4}$ *or*
$\beta < \frac{(1-\theta-\theta_b)(1-\alpha w)(2\theta+\theta_b+2h)}{\theta_b-(2\theta+\theta_b)(\theta+\theta_b)+4t_S\tau(1-2\theta-\theta_b)-2\theta h}$.



(a) Negative Substitution Effect and Positive Demand Effect

(b) Positive Substitution Effect and Negative Demand Effect

Figure 2.1. Demand and Substitution Effects

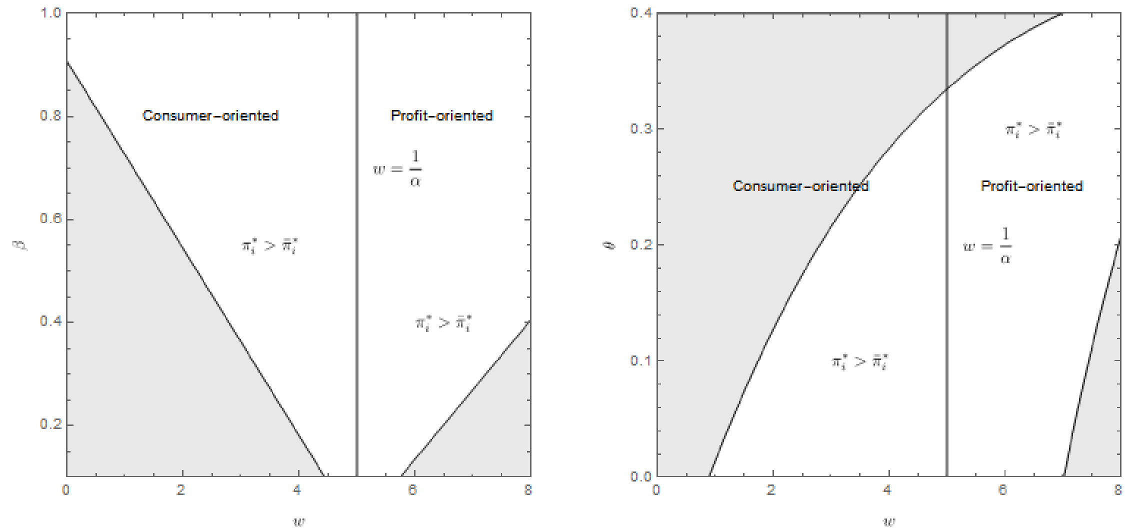
(b) When the recommender system is profit oriented (i.e., if $w > \frac{1}{\alpha}$), each product's price is lower in the presence of the recommender system than in its absence if and only if $\theta > \frac{\sqrt{(3\theta_b+8t_S\tau+2h)^2+8(4t_S\tau+\theta_b)(1-\theta_b)}-3\theta_b-8t_S\tau-2h}{4}$ *and* $\beta > \frac{(1-\theta-\theta_b)(1-\alpha w)(2\theta+\theta_b+2h)}{\theta_b-(2\theta+\theta_b)(\theta+\theta_b)+4t_S\tau(1-2\theta-\theta_b)-2\theta h}$

When the recommender system is consumer oriented, if the relative size of partially informed shoppers in the absence of recommender system is large (or, alternatively, the size of uninformed shoppers is small), the recommender system decreases the informational differentiation because more shoppers are likely to be fully informed after the recommendation, which tends to increase the competition and decrease the price. Moreover, when the recommender system is consumer oriented, manufacturers have strategic incentives to lower the prices to increase the chance of recommendations for their products. Therefore, if θ is large (as prescribed in Corollary 1(a)), each product's price is lower in the presence of the recommender system. If θ is small, the direct influence of recommender system tends to increase the informational differentiation and soften the competition. On the other other hand, the strategic influence still tends to increase the competition. If the strategic influence dominates the direct influence (i.e., if the condition on β, as prescribed in Corollary 1(a), is satisfied), each product's price is lower in the presence of the recommender system.

When the recommender system is profit oriented, the direct influence of the recommender system works in the same way as when the recommender system is consumer oriented: if the relative size of partially informed consumers in the absence of recommender system is large (small), the recommender system decreases (increases) the informational differentiation, which tends to drive down (up) the price. However, in contrast to the case with consumer-oriented recommender system, manufacturers have strategic incentives to increase the prices to increase the chance of recommendations for their product. Therefore, each product's price is lower in the presence of the recommender system only if direct influence tends to drive down the price (i.e., if θ is large) and the strategic influence is mild (i.e., if β is large).

Proposition 1(b) shows that the recommender system increases (decreases) the total demand when w is low (high). Two factors contribute to the recommender system's impact on the demand. First, the recommender system increases shoppers' awareness of at least one product which increases the demand among shoppers. Second, the recommender system either increases or decreases the demand among loyal consumers depending on whether it decreases the price or increases the price as indicated in Proposition 1(a). Clearly, if the recommender system induces a decrease in price, then the demand for each product increases because demand from loyal consumers as well as shoppers increases; on the other hand, if the recommender system induces a price increase, then the demand increases only if the demand increase from shoppers offsets the demand decrease from loyal consumers. We refer to the change in demands as the *demand effect* of the recommender system. In Figure 2.1, the change in the intercept of the demand functions with the vertical axis (i.e., $p_A - p_B = 0$) reflects the demand effect. Contrary to substitution effect, the demand effect tends to be positive (negative) when w is small (large).

Proposition 1(c) shows that the retailer and manufacturers do not necessarily benefit from the recommender system; whether the retailer and manufacturers benefit from the recommender system depends on the signs and magnitudes of the substitution effect and the



(a) $\theta = 0.2$ (b) $\beta = 0.4$

Figure 2.2. Impact of Recommender System on Profits ($\theta_b = 0.2$, $t = 0.3$, $\alpha = 0.2$, $\tau = 1.5$,$h = 2.5$)

demand effect. A positive (negative) demand effect and a positive (negative) substitution effect benefit (hurts) the retailer and manufacturers. Figure 2.2 illustrates the impacts of the recommender system on seller profits. The conditions stated in Proposition 1(c) show that the sellers benefit from the recommender system only when the weight assigned to the profit is neither too high nor too small. More importantly, as shown in Figure 2.2, neither a consumer-oriented recommender system nor a profit-oriented recommender system ensures that the sellers are better off with the recommender system than without. The following result illustrates the contrast between consumer-oriented recommender system and profit-oriented recommender system regarding the conditions that favor a higher seller profit with the recommender system.

Corollary 2. *(a) When the recommender system is consumer oriented (i.e., when $w < \frac{1}{\alpha}$), the retailer and manufacturers are better off in the presence of the recommender system than in its absence if and only if $w > \min\{w_1, \frac{1}{\alpha}\}$.*

(b) When the recommender system is profit oriented (i.e., when $w > \frac{1}{\alpha}$), the retailer and manufacturers are better off in the presence of the recommender system than in its absence if and only if $\max\{w_1, \frac{1}{\alpha}\} < w < w_2$.

Corollary 2(a) shows that when the recommender system is consumer oriented, the sellers are worse off if the recommender system is highly consumer oriented (i.e., $w < w_1$). As seen in Proposition 5(a), a highly consumer oriented recommender system intensifies the price competition. Even though the demand increases in this case, the increase in demand does not offset the loss in profit margin. A similar observation applies when the recommender system is highly profit oriented. In this case, while the profit margin increases, the decline in demand from loyal consumers offsets the gain from higher profit margin.

Propositions 1(d) and 1(e) show that the recommendation strategy that assigns a very high weight on retailer profit hurts consumer surplus and social welfare. Intuitively, a low weight on retailer profit benefits consumers because in this case consumers benefit from lower prices, higher demand from both loyal consumers and shoppers, as well as lower misfit costs for shoppers in the presence of the recommender system. Analogously, the lower misfit costs for shoppers and higher demand from loyal consumers and shoppers benefit the social welfare when the recommender system uses a low weight on the retailer profit. Interestingly, we find when the recommender system is very highly profit oriented(i.e., when $w > max\,\{w_2, w_{cs}, w_w\}$), all players in the marketplace—retailer, manufacturers, consumers, and the society—are hurt by the recommender system.

In summary, the analysis in this section reveals that when manufacturers strategically respond to retailer's deployment of the recommender system,the impacts on sellers, consumers, and the society depend critically on the recommendation strategy, the recommender system precision, and consumers' awareness about products.

We examine the roles of these factors on the recommender system impacts more closely in the next section.

2.5 Roles of Recommender System and Market Characteristics

In our model, the recommender system is characterized by its precision (β) and recommendation strategy (w). The market characteristics is primarily captured by the proportion of partially informed consumers (θ) in the absence of the recommender system. In this section, we examine the roles of these parameters on the impacts of the recommender system. We denote the benefit from the recommender system to seller i as $\Delta\pi_i = {\pi^*}_i - \bar{\pi}_i^*$, $i \in \{A, B, R\}$, the benefit to consumers as $\Delta CS = CS^* - \bar{CS}^*$, the benefit to society as $\Delta W = W^* - \bar{W}^*$, and denote the impact on price competition as $\Delta p_i = p_i^* - \bar{p}_i^*$.

We further note that since the benchmark is unaffected by recommender system parameters, the results regarding the roles of recommender system parameters can also be interpreted how a change in these parameters affect the these various players (i.e., manufacturers, retailer, consumers, and the society) in the presence of the recommender system.

2.5.1 Role of Recommender System Precision

Proposition 2. *When recommender system precision β increases,*

(a) the effect of the recommender system on softening price competition increases (i.e., $\frac{\partial \triangle p_i}{\partial \beta} > 0$) if and only if the recommender system is consumer oriented;

(b) the effect of the recommender system on demand enhancement increases (i.e., $\frac{\partial \triangle D_i}{\partial \beta} > 0$) if and only if the recommender system is profit oriented;

(c) the benefit of the recommender system to a seller increases (i.e., $\frac{\partial \triangle \pi_i}{\partial \beta} > 0$) if and only if the recommender system is consumer oriented, or the recommender system is profit oriented and $w > \frac{1}{\alpha} + \frac{\beta(\theta+\theta_b)}{\alpha(1-\theta-\theta_b)}$;

(d) the effect of the recommender system on consumer surplus decreases, (i.e., $\frac{\partial \triangle CS}{\partial \beta} < 0$) if and only if the recommender system is consumer oriented and

$(2h+1)^2 > \frac{[(1-\alpha w)(1-\theta-\theta_b)+\beta(\theta+\theta_b+4t_s\tau)]^3}{4(1-\alpha w)[(1-\alpha w)(1-\theta-\theta_b)+\beta(\theta+\theta_b+2\tau t_S)]}$

(e) the effect of the recommender system on social welfare decreases, (i.e., $\frac{\partial \triangle W}{\partial \beta} < 0$) if and

only if

$$8(1+2h)(1-\alpha w)\left(3\beta t_S\tau - h\left[(1-\alpha w)\left(1-\theta-\theta_b\right)+\beta(\theta+\theta_b-2t_s\tau)\right]\right) > \left[(1-\alpha w)\left(1-\theta-\theta_b\right)+\beta(\theta+\theta_b+4t_s\tau)\right]^3$$

Propositions 2(a), 2(b), and 2(c) reveal that the impact of β on sellers is different under different recommendation strategies. If the recommender system is consumer oriented or highly profit oriented ($w > \frac{\beta(\theta+\theta_b)+(1-\theta-\theta_b)}{\alpha(1-\theta-\theta_b)}$), the sellers benefit from improving the recommender system's precision. Under these conditions, in particular, if the recommender system benefits the sellers, an improvement in β enhances the benefit, and if the recommender system hurts the sellers, an improvement in β mitigates this negative impact. If the recommender system is mildly profit oriented, the sellers are hurt from improving the recommender system's precision. The recommender system precision affects the seller profits via its impact on demand effect (Proposition 2(b)) and substitution effect (Proposition 2(a)).

We note from Equation (2.18), that an improvement in precision does not alter the direct influence but alters the strategic influence of the recommender system. When the recommender system is consumer (profit) oriented, the strategic influence represented by K in Equation (2.18) increases (decreases) in β. The intuition is as follows. Under a consumer-oriented (profit-oriented) recommender system, a decrease (an increase) in the price of a product, ceteris paribus, increases the likelihood of that product being recommended. However, the marginal increase in this likelihood is greater when the recommender system precision is low than when that precision is high. When the precision is low, the recommender system cannot estimate consumer preference well and highly relies on prices to recommend product because the recommender system perceives most consumers being concentrated in the middle of the Hotelling line. As a result, a small decrease (increase) in the price of a product induces recommendation of that product to a large number of consumers if the recommender system is consumer (profit) oriented. On the other hand, when the precision is high, from the recommender system's perspective, the consumers are spread more evenly throughout the line; therefore, a decrease (increase) in price leads to recommendation of that product to a smaller number of consumers if the recommender system is consumer (profit) oriented. Consequently, when the recommender system is consumer (profit) oriented, an increase in precision reduces manufacturers' incentive to decrease (increase) price, which softens (intensifies) price competition. On the other hand, the softening (intensification) of the price competition by an improvement in precision reduces (increases) the demand when the recommender system is consumer (profit) oriented. The net impact of recommender system precision is that sellers benefit only when the recommender system is consumer oriented or highly profit oriented.

An improvement in recommender system precision enhances the benefit or mitigates the hurt to consumers if the recommender system is profit oriented on the one hand. On the other hand, if the recommender system is consumer oriented, an improvement in precision exacerbates the hurt or diminishes the benefit to consumers when the market potential of loyal consumers is large compared to the number of shoppers. We note that the recommender system can benefit consumers in three ways: demand enhancement, intensification of price competition, and reduction in misfit costs. An improvement in precision always reduces the misfit costs whether the recommender system is consumer oriented or profit oriented. If the recommender system is profit oriented, since an improvement in precision intensifies the price competition (Proposition 2(a)) and enhances the demand, consumers end up benefiting from it. On the other hand, if the recommender system is consumer oriented, the adverse impacts of precision on demand and price competition offset the beneficial impact on the misfit costs if the market potential of loyal consumers is large relative to the number of shoppers who are the ones that benefit from a reduction in misfit costs because of recommender system. The impact of precision on social welfare is analogous to that on consumer surplus except that only the impacts on the demand and misfit costs play a role.

2.5.2 Role of Recommendation Strategy

Proposition 3. *When the weight assigned to the retailer profit component of recommendation score w increases,*

(a) the effect of the recommender system on softening price competition increases (i.e., $\frac{\partial \triangle p_i}{\partial w} > 0$);

(b) the effect of the recommender system on increasing demand decreases (i.e., $\frac{\partial \triangle D_i}{\partial w} < 0$);

(c) the benefit of the recommender system to seller increases (i.e., $\frac{\partial \triangle \pi_i}{\partial w} > 0$) if and only if $w < \frac{1}{\alpha} + \frac{\beta(\theta+\theta_b)}{\alpha(1-\theta-\theta_b)}$;

(d) the benefit of the recommender system on consumer surplus decreases (i.e., $\frac{\partial \triangle CS}{\partial w} < 0$);

(e) the benefit of the recommender system on social welfare decreases (i.e., $\frac{\partial \triangle W}{\partial w} < 0$) if and only if $w > \frac{1}{\alpha} + \frac{\beta(\theta+\theta_b)}{\alpha(1-\theta-\theta_b)} - \frac{\beta(2+\frac{3}{h})t_S\tau}{\alpha(1-\theta-\theta_b)}$.

Similar to recommender system precision, the recommendation strategy does not affect the direct influence of the recommender system on price competition. On the other hand, as w increases, the strategic influence of recommender system increases—manufacturers have greater incentives to increase their prices in order to increase the number of recommendations in their favor. As a result, the equilibrium price increases, which explains Proposition 3(a). However, as the price goes up, fewer loyal consumers would purchase in the presence of recommender system. Consequently, when w exceeds a threshold, the demand decreasing effect dominates the price increasing effect, an increase in w hurts retailer's profit. Proposition 3(c) implies that there exist an optimal w that maximizes retailer profit, and we explore the characteristics of this optimal recommendation strategy in Section 2.6. Furthermore, the adverse impact of w from consumer perspective, both in terms of softening of price competition and demand reduction, hurts consumer surplus, and a demand reduction caused by an increase in w hurts social welfare.

2.5.3 Role of the Size of Partially Informed Shoppers

Proposition 4. *When the proportion of partially informed consumers θ increases,*

(a) the effect of the recommender system on softening price competition increases (i.e., $\frac{\partial \triangle p_i}{\partial \theta} > 0$) if and only if $w < \frac{1-\beta}{\alpha}$ and $\theta > \frac{1-\alpha w+4\beta t_s \tau}{1-\alpha w-\beta} - \sqrt{\frac{\beta(1+2h)(\theta_b+4t_s\tau)}{2(1-\alpha w-\beta)}} - \theta_b$;

(b) the effect of the recommender system on increasing demand increases (i.e., $\frac{\partial \triangle D_i}{\partial \theta} > 0$) if and only if $w > \frac{1-\beta}{\alpha}$ and $\theta < \frac{\alpha w-1-4\beta t_S \tau}{\alpha w+\beta-1} + \sqrt{\frac{\beta t\tau(1+2h)(\theta_b+4t_S\tau)}{(\theta_b+2t_S\tau)(\alpha w+\beta-1)}} - \theta_b$;

(c) the benefit of recommender system to sellers (i.e., $\frac{\partial \triangle \pi_i}{\partial \theta} > 0$) increases if and only if $(1-\alpha w-\beta)\left[(1-\alpha w)(1-\theta-\theta_b)+\beta(\theta+\theta_b)\right] > 0$ and $\frac{(1+2h)^2}{(2\theta+\theta_b+2h)} > \frac{4(2\tau t_S+\theta_b)[(1-\alpha w)(1-\theta-\theta_b)+\beta(\theta+\theta_b)+4\beta\tau t_S]^3}{\beta(4\tau t_S+\theta_b)^2(1-\alpha w-\beta)[(1-\alpha w)(1-\theta-\theta_b)+\beta(\theta+\theta_b)]}$;

(d) the benefit of recommender system to consumers increases (i.e., $\frac{\partial \triangle CS}{\partial \theta} > 0$) if and only if $2v_S - \frac{5-\beta}{4}t_S \leq 4t_S\frac{(2\theta+\theta_b+2h)(\theta_b+3t_S\tau)}{(4t_S\tau+\theta_b)^2} - \beta t_S(1+2h)^2(1-\alpha w-\beta)\frac{(1-\alpha w)(1-\theta-\theta_b)+\beta(\theta+\theta_b)+2\beta t_S\tau}{[(1-\alpha w)(1-\theta-\theta_b)+\beta(\theta+\theta_b)+4\beta t_S\tau]^3}$;

(e) the benefit of recommender system to social welfare decreases (i.e., $\frac{\partial \triangle W}{\partial \theta} > 0$) if and only if $2v_S - \frac{5-\beta}{4}t_S \leq 4t_S\frac{\tau t_S(6\theta+3\theta_b+2h)-\theta_b h}{(4\tau t_S+\theta_b)^2} + 2t_S\beta(1+2h)(1-\alpha w-\beta)\frac{(3+2h)\beta t_S\tau-h[(1-\alpha w)(1-\theta-\theta_b)+\beta(\theta+\theta_b)]}{[(1-\alpha w)(1-\theta-\theta_b)+\beta(\theta+\theta_b)+4\beta\tau t_S]^3}$.

An increase in θ moderates the substitution effect by affecting both the direct influence and the strategic influence of the recommender system. When θ increases, on the one hand, informational differentiation among shoppers without recommender system (i.e., $\frac{\theta}{\theta_b}$) increases and the informational differentiation among shoppers with recommender system (i.e., $\frac{1-\theta-\theta_b}{2(\theta+\theta_b)}$) decreases, and, hence, the direct influence of the recommender system on softening competition decreases. On the other side, with a larger θ, fewer uninformed consumers are influenced through recommendations (which in turn are influenced by price) and more partially informed consumers might become fully informed because of recommendations. As a result, manufacturers have less incentive to use price as a lever to attract recommendations. In particular, an increase in θ increases (decreases) the strategic influence of the recommender system on softening competition under consumer (profit) oriented recommender system. Together, as stated in Proposition 4, an increase in θ reduces the substitution effect on softening price competition when $w > (1-\beta)/\alpha$. When $w < (1-\beta)/\alpha$, an increase in

θ can either enhance or diminish the effect. In particular, if the consumer orientation is not too high, the result obtained for a profit-oriented recommender system still applies. The strategic influence dominates the direct influence if the value of θ is big enough, and thus an increase in θ increases the effect of the recommender system on softening price competition, as prescribed by the conditions in Proposition 4(a).

An increase in θ reduces the demand gain from shoppers, but may increase or decrease the demand gain from loyal consumers depending on the substitution effect. The effect on the benefit to the sellers can be positive only if a decrease in the demand effect is dominated by an increase in the substitution effect or an increase in the demand effect dominates an decrease in the substitution effect, which requires the conditions stated in Proposition 4(c). The reduction in demand from shoppers also has negative effect on consumers, but the change in the substitution effect might be in favor of consumers. Proposition 4(d) characterizes the condition—when the reduction in consumer utility (i.e., $2v_S - \frac{5-\beta}{4}t_S$) is not too high—, under which the benefit of recommender system increases when θ increases.

2.6 Retailer's Optimal Recommendation Strategy

The results of the previous sections demonstrate that the recommendation strategy employed by the retailer, among other factors, determines whether the recommender system benefits the retailer. In practice, a retailer might not employ a recommendation strategy that hurts it. In this section, we endogenize the recommendation strategy by allowing the retailer to choose w that maximizes its profit in the presence of the recommender system.

Lemma 3. *When the retailer uses the optimal recommender system, the equilibrium prices, demands, manufacturer profits, retailer profit, consumer surplus, and social welfare are as follows:*

(a) Weight assigned to retailer profit

$$w^* = \frac{\beta(\theta+\theta_b)+(1-\theta-\theta_b)}{\alpha(1-\theta-\theta_b)} \tag{2.30}$$

(b) Prices

$$p_A^* = p_B^* \ = \frac{1+2h}{4\tau} \tag{2.31}$$

(c) Demand:

$$D_A^* = D_B^* = \tfrac{1}{4} + \tfrac{h}{2} \tag{2.32}$$

(d) Manufacturer profit:

$$\pi_A^* = \pi_B^* = (1-\alpha)\tfrac{(1+2h)^2}{16\tau} \tag{2.33}$$

(e) Retailer profit:

$$\pi_R^* = \alpha\tfrac{(1+2h)^2}{8\tau} \tag{2.34}$$

(f) Consumer surplus:

$$CS = v_S - \tfrac{t_S}{4}\left[1 + (1-\beta)(1-\theta-\theta_b)\right] + \tfrac{4h^2-12h-3}{16\tau} \tag{2.35}$$

(g) Social welfare:

$$W = v_S - \tfrac{t_S}{4}\left[1 + (1-\beta)(1-\theta-\theta_b)\right] + \tfrac{(2h-1)(10h+3)}{16\tau} \tag{2.36}$$

We make the following key observations based on Lemma 3. First, we notice that $w^* > 1/\alpha$, which means the retailer will choose a profit oriented recommender system. Meanwhile, the retailer does not prefer a recommender system with a too large profit orientation. A high profit orientation provides excessive incentives to manufacturers to increase their prices, which hurts the retailer by decreasing the demand from loyal consumers. Therefore, it is in the best interest of the retailer to limit w. On the other hand, a low w may provide manufacturers incentives to engage in excessive price competition which also hurts the retailer. Second, the extent of profit orientation (ie.., $w^* - \frac{1}{\alpha}$) in the optimal recommendation strategy depends on the recommender system precision as well as the market characteristics captured by $\theta + \theta_b$. Surprisingly, the retailer will choose a more profit oriented recommender system if the precision is higher. An improvement in precision induces the manufacturers to decrease prices if the recommender system is profit oriented, as seen from Proposition 2(a). On the other hand, an increase in w induces manufacturers to increase prices, as observed from Proposition 3 (a). Consequently, the optimal w increases in recommender system precision. A similar reasoning applies for the result that w^* increases in $\theta + \theta_b$.

We can now assert the impact of the optimal recommender system by comparing equilibrium quantities in the scenario without the recommender system and the scenario with the optimal recommender system.

Proposition 5. *Compared to the scenario without the recommender system, in the presence of the recommender system with the optimal recommendation strategy,*

(a) Each product's price is higher (i.e., $p_i^* > \bar{p}_i^*$);

(b) Each product's demand is higher (i.e., $D_i^* > \bar{D}_i^*$) if and only if $h < \frac{(1-2\theta-\theta_b)(2\theta_b+4t_S\tau)}{2\theta_b} - \frac{1}{2}$;

(c) The retailer and manufacturers are better off (i.e., $\pi_i^* > \bar{\pi}_i^*$);

(d) Consumer's surplus increases if and only if

$$\beta > \frac{[(1+2h)\theta_b+4\tau t_S(1-2\theta-\theta_b)][(6h-1+8\theta+4\theta_b)\theta_b+4t_s\tau(4h-1+6\theta+3\theta_b)]}{4\tau t_S(\theta_b+4\tau t_S)^2(1-\theta-\theta_b)} - \frac{\left(4v_S-\frac{1+2h+2t_S}{\tau}\right)(1-2\theta-\theta_b)}{t_S(1-\theta-\theta_b)} - \frac{\theta}{1-\theta-\theta_b}$$

(e) Social welfare increases if and only if

$$\beta > \frac{[(1+2h)\theta_b+4\tau t_S(1-2\theta-\theta_b)][(3-2h)\theta_b+4t_S\tau(3+6\theta+3\theta_b+4h)]}{4\tau t_S(\theta_b+4\tau t_S)^2(1-\theta-\theta_b)} - \frac{4\left(4v_S-\frac{2t_S}{\tau}\right)(1-2\theta-\theta_b)}{t_S(1-\theta-\theta_b)} - \frac{\theta}{1-\theta-\theta_b}$$

We find that the optimal recommendation strategy relaxes the price competition between manufacturers and benefits the retailer and manufacturers, as compared to the case without recommendation. However, the optimal recommender strategy does not necessarily increase the demand. More importantly, we find that both consumer surplus and social welfare

are higher under the optimal recommendation strategy than under the benchmark if the recommender system precision is high enough. This result is especially noteworthy because it shows that if the retailer deploys the optimal recommendation strategy that maximizes its own profit, every player in the market place—retailer, manufacturers, consumers, and the society—can be better off in the presence of the recommender system than in its absence. This is in contrast to the result that, when the recommendation strategy is not optimal, all players can actually be worse off, as seen in the discussion following Corollary 2.

2.7 Model Extension: Multiple Recommendations in a Ranked Order

In the baseline model, we assume that the retailer recommends one product—the one with the higher recommendation score. In this section, we consider a model in which the retailer recommends both products to a consumer, in light of the practice of retailers recommending multiple products to a consumer in some contexts.

When a retailer recommends multiple products, the recommendations are generally presented in a ranked order. In general, when presented in a ranked order, products at prominent positions are more likely to be examined by a consumer. For instance, in the sponsored search context, the products at the top positions typically have higher click rates than those at the bottom (Ghose and Yang, 2009). Studies on recommender systems document a similar phenomenon for a horizontal list of recommendations (Rodrigo et al., 2015). In view of these observations, we assume that in recommending the two products, recommendations are ranked based on recommendation scores so that the one that has a higher score is displayed at a more prominent position.

Consumers generally incur costs to evaluate the recommended products. Following the literature (e.g., Lizhen et al., 2011), we assume two types of customers in the market: one type of customers has a high search cost and they only check the recommended product on the more prominent position before they make the purchase decisions, and the other type of customers have a low search cost and they check both recommended products. The proportions of these two groups of customers are λ and $1-\lambda$. The consumers' search costs are independent of their awareness and preferences. We note that this model reduces to the base model when $\lambda = 1$. In the other extreme case of $\lambda = 0$, all consumers are fully informed after receiving the recommendations, regardless of the order in which they are presented.

Using an approach similar to that in the base model, we derive the demand functions of manufacturer A and manufacturer B in the presence of the recommender system as follows:

$$D_{iS} = \tfrac{1}{2} + \tfrac{\lambda(1-\theta-\theta_b)(1-\alpha w)+[1-\lambda(1-\theta-\theta_b)]\beta}{2\beta t_S}(p_B - p_A) \tag{2.37}$$

The total demand of product i is $D_i = D_{iS} + D_{iL}$. Similar to the base model, we impose restrictions on β and v to derive a pure-strategy equilibrium, as stated in the appendix. The equilibrium results are summarized by the following lemma.

Lemma 4. *When the retailer recommends two products in a ranked order, the equilibrium prices, demands, manufacturer profits, retailer profit, consumer surplus, and social welfare are as follows:*

(a) Price:

$$p_A^* = p_B^* = \tfrac{\beta(1+2h)t_S}{(1-\alpha w)\lambda(1-\theta-\theta_b)+\beta(1-\lambda(1-\theta-\theta_b)+4\tau t_S)} \tag{2.38}$$

(b) Demand:

$$D_A^* = D_B^* = \left(\tfrac{1}{2}+h\right)\left(1 - \tfrac{2\beta\tau t_S}{(1-\alpha w)\lambda(1-\theta-\theta_b)+\beta(1-\lambda(1-\theta-\theta_b)+4\tau t_S)}\right) \tag{2.39}$$

(c) Manufacturer profit:

$$\pi_A^* = \pi_B^* = (1-\alpha)\beta t_S \left(\tfrac{1+2h}{(1-\alpha w)\lambda(1-\theta-\theta_b)+\beta(1-\lambda(1-\theta-\theta_b)+4\tau t_S)}\right)^2 \tfrac{(1-\alpha w)\lambda(1-\theta-\theta_b)+\beta(1-\lambda(1-\theta-\theta_b)+2\tau t_S)}{2} \tag{2.40}$$

(d) Retailer profit:

$$\pi_R^* = \alpha\beta t_S \frac{(1+2h))^2[(1-\alpha w)\lambda(1-\theta-\theta_b)+\beta(1-\lambda(1-\theta-\theta_b)+2\tau t_S)]}{((1-\alpha w)\lambda(1-\theta-\theta_b)+\beta(1-\lambda(1-\theta-\theta_b)+4\tau t_S)^2} \tag{2.41}$$

(e) Consumer surplus:

$$CS = v_S - \frac{t_S[1+(1-\beta)(1-\theta-\theta_b)\lambda]}{4} - p_i^* + \frac{(h-p_i^*\tau)^2}{\tau} \tag{2.42}$$

(f) Social welfare:

$$W = v_S - \frac{t_S[1+(1-\beta)(1-\theta-\theta_b)\lambda]}{4} + \frac{(h-p_i^*\tau)(h+3p_i^*\tau)}{\tau} \tag{2.43}$$

As in the baseline case, the price expression in Lemma 4 can be written as

$$p_A^* = p_B^* \equiv \tilde{K}\left[t_S + 2t_S\left(\frac{\frac{\lambda(1-\theta-\theta_b)}{2}+h-2\tau t_S}{1-\lambda(1-\theta-\theta_b)+4\tau t_S}\right)\right] \tag{2.44}$$

where $\tilde{K} \equiv \frac{\beta(1-\lambda(1-\theta-\theta_b)+4\tau t_S)}{(1-\alpha w)\lambda(1-\theta-\theta_b)+\beta(1-\lambda(1-\theta-\theta_b)+4\tau t_S)}$. Notice that in this setting the fraction of consumers with high search costs λ plays a role in the price competition by affecting the direct influence and the strategic influence of the recommender system. On one side, an increase in λ increases the informational differentiation of sellers, hence inducing sellers to charge higher markups. On the other side, an increase in λ implies that more consumers' purchase decisions are influenced by the recommendation strategy, hence encouraging sellers to use price to attract recommendations. In particular, if the recommender system is consumer oriented, the strategic influence increases the price competition and has a negative impact on sellers' profits (i.e., $\tilde{K} < 1$), and increasing λ further exacerbates the negative effect (i.e., $\frac{\partial \tilde{K}}{\partial \lambda} < 0$). If the recommender system is profit oriented, the strategic driving force reduces price competition and has a positive impact on sellers' profits ($\tilde{K} > 1$), and increasing λ further enhances the positive impact (i.e., $\frac{\partial \tilde{K}}{\partial \lambda} > 0$).

The overall impact of λ on the substitution effect depends on the direct influence and the strategic influence. If the recommender system has a low degree of consumer orientation (i.e., $w > \frac{1-\beta}{1+\alpha-\beta}$), an increase in λ softens the price competition. If a recommender system has a high degree of consumer orientation (i.e., $w < \frac{1-\beta}{1+\alpha-\beta}$), an increase in λ intensifies the price competition.

Next we compare the case with multiple recommendations to the no recommender system case.

Proposition 6. *Compared to the no recommender system case, in the case of recommender system with multiple recommendations,*

(a) Each product's price is lower (i.e., $p_i^* < \bar{p}_i^*$) if and only if

$$w < \frac{1}{\alpha} - \beta\frac{\theta_b - (2\theta+\theta_b)[1-\lambda(1-\theta-\theta_b)] - 2h[1-\theta_b-\lambda(1-\theta-\theta_b)] + 4\tau t_S(1-2\theta-\theta_b)}{\lambda\alpha(1-\theta-\theta_b)(2\theta+\theta_b+2h)}$$

(b) Each product's demand is higher (i.e., $D_i^* > \bar{D}_i^*$, $i \in \{A, B\}$) if and only if

$$w < \frac{1}{\alpha} + \frac{\beta\big[1-\lambda(1-\theta-\theta_b)\big]+2\beta\tau t_S}{\alpha\lambda(1-\theta-\theta_b)} - \frac{2\beta\tau(\theta_b+2t\tau)t_S}{\alpha\lambda(1-\theta-\theta_b)}\left[\frac{2\theta+\theta_b+2h}{(1-2\theta-\theta_b)(\theta_b+2t_S\tau)+2t_S\tau(1+2h)}\right]$$

(c) The retailer and manufacturers are better off (i.e., $\pi_i^* > \bar{\pi}_i^*$) if and only if

$$w_1 < w < w_2$$

where

$$w_1 = \frac{1}{\alpha} + \frac{\beta[1-\lambda(1-\theta-\theta_b)+4t_S\tau]}{\alpha\lambda(1-\theta-\theta_b)} - \frac{\beta}{\alpha\lambda(1-\theta-\theta_b)}\left(\frac{1+\sqrt{1-8t_S\tau\left(\frac{2\theta+\theta_b+2h}{(\theta_b+4t_S\tau)(1+2h)}\right)^2(\theta_b+2t_S\tau)}}{2\left(\frac{2\theta+\theta_b+2h}{(\theta_b+4t_S\tau)(1+2h)}\right)^2(\theta_b+2t_S\tau)}\right)$$

$$w_2 = \frac{1}{\alpha} + \frac{\beta[1-\lambda(1-\theta-\theta_b)+4t_S\tau]}{\alpha\lambda(1-\theta-\theta_b)} - \frac{\beta}{\alpha\lambda(1-\theta-\theta_b)}\left(\frac{1-\sqrt{1-8t_S\tau\left(\frac{2\theta+\theta_b+2h}{(\theta_b+4t_S\tau)(1+2h)}\right)^2(\theta_b+2t_S\tau)}}{2\left(\frac{2\theta+\theta_b+2h}{(\theta_b+4t_S\tau)(1+2h)}\right)^2(\theta_b+2t_S\tau)}\right)$$

(d) Consumer surplus is higher (i.e., $CS \geq \bar{CS}$) if and only if $w < w_{cs}$, where w_{cs} is the root of

$$(1-2\theta-\theta_b)\left[v_S - \frac{(1+\lambda-\lambda\beta)t_S}{4} - p_i^*\right] + \frac{\theta(2-\lambda+\lambda\beta)t_S}{4} + (\bar{p}_i^* - p_i^*)\left[2\theta+\theta_b+2h-\tau\left(\bar{p}_i^*+p_i^*\right)\right] = 0 \quad (2.45)$$

(e) Social welfare is higher (i.e., $W > \bar{W}$) if $\frac{(1-\theta-\theta_b)(1-\beta)\lambda}{\beta} + 1 < (2+\frac{3}{h})\tau t_S$ and $w < w_w$, where w_w is the root of

$$(1-2\theta-\theta_b)\left[v_S - \frac{(1+\lambda-\lambda\beta)t_S}{4}\right] + \frac{\theta(2-\lambda+\lambda\beta)t_S}{4} + (\bar{p}_i^* - p_i^*)\left[3\tau\left(\bar{p}_i^*+p_i^*\right)-2h\right] = 0 \quad (2.46)$$

Comparing the above proposition with Proposition 1, we find that the above results are qualitatively similar to those in the base model, and the insights we discussed in Sections 2.3 and 2.4 generally carry over to the case when multiple recommendation are presented in a ranked order.

Lemma 5. *When the retailer uses the optimal recommendation strategy in the multiple recommendations case, the equilibrium prices, demands, manufacturer profits, retailer profit, consumer surplus, and social welfare are as follows:*

(a) Weight assigned to retailer profit

$$w^* = \frac{\beta-\lambda\beta(\theta+\theta_b)+\lambda(1-\theta-\theta_b)}{\alpha\lambda(1-\theta-\theta_b)}$$

Prices

$$p_A^* = p_B^* = \frac{1+2h}{4\tau}$$

(b) Demand:

$$D_A^* = D_B^* = \frac{1}{4} + \frac{h}{2} \tag{2.47}$$

(c) Manufacturer profit:

$$\pi_A^* = \pi_B^* = (1-\alpha)\frac{(1+2h)^2}{16\tau} \tag{2.48}$$

(d) Retailer profit:

$$\pi_R^* = \alpha\frac{(1+2h)^2}{8\tau} \tag{2.49}$$

(e) Consumer surplus:

$$CS = v_S - \frac{t_S[1+(1-\beta)(1-\theta-\theta_b)\lambda]}{4} + \frac{4h^2-12h-3}{16\tau} \tag{2.50}$$

(f) Social welfare:

$$W = v - \frac{t_S}{4}\left[1+(1-\beta)(1-\theta-\theta_b)\lambda\right] + \frac{(2h-1)(10h+3)}{16\tau} \tag{2.51}$$

One of the key differences between Lemma 5 and Lemma 3 is that the optimal recommendation strategy is more profit oriented in the case with multiple recommendations compared to the case with single recommendation. Intuitively, when multiple recommendations are provided, more consumers become fully informed which tends to increase price competition between manufacturers. Therefore, the retailer increases the weight on price to compensate for the increased incentives of manufacturers to cut prices in the multiple recommendations case. The equilibrium price, and seller profits remain identical in the single recommendation and multiple recommendations cases as the demand function from the retailer's perspective is unaffected by the number of recommendations. However, the consumer surplus and social welfare are higher in the multiple recommendations case than in the single recommendations because the higher number of fully informed shoppers reduces the misfit costs in the multiple recommendations case. Finally, a comparison of Lemma 5 and Lemma 3 also reveals that the impacts of the recommender system on the different players under the optimal recommendation strategy are qualitatively similar in the single recommendation and multiple recommendations cases.

In summary, the results of this section show that the impacts of recommender system remain qualitatively the same whether the retailer recommends only the product with the highest recommendation score or both products in the order of recommendation score.

2.8 Discussion and Conclusions

We examine the effect of recommender systems in a channel structure with a retail platform and two competing manufacturers selling substitute products. Manufacturers set the prices for their products and the retail platform takes a fraction of the sales price as a commission for each transaction. The recommendations have an informative role and increase the consumer awareness of products. The retail platform recommends the product based on a predetermined strategy: platform assigns a weight on the retailer profit factor and a weight on the consumer value factor. We identify two effects of recommender system: demand effect and substitution effect. The demand effect of the recommender system increases the proportion of consumers that are aware of at least one of the products, and alters the overall market size. The substitution effect of the recommender system changes the price competition between two manufacturers. The recommender system may benefit or hurt the retailer and the manufacturers depending on the relative magnitudes of these two effects of the recommender system. We find that the retail platform and the manufacturers do not always benefit from the recommender system. The benefit of a recommender system depends critically on the type of recommender system employed—profit oriented or consumer oriented—, the recommender system precision, and consumers' awareness about products. However, it is optimal for the retailer will choose a recommendation strategy that is mildly profit oriented, and under this optimal recommendation strategy, the retailer and manufacturers always benefit from the recommender system but the consumers and the society may not.

The findings have several implications for electronic marketplaces that deploy recommender systems.

(i) Focusing solely on the additional sales or demand created by recommender systems and ignoring possible strategic price responses from manufacturers who sell in the marketplace can hurt the retail platform that implements the recommender system. Furthermore, when the cost of developing recommender systems is also accounted for, the value of these systems diminishes further.

(ii) Since recommender systems have an advertising role, the manufacturers enjoy "free" advertising provided by the retail platform when it deploys a recommender system. A retail platform may have to reconsider its contract with the manufacturers in light of this advertising service it provides them and to benefit from recommender systems.

(iii) Improving recommendation precision is not always beneficial for the platforms. Under some recommender system designs, improving precision would lead to a decrease in sellers' profit.

(iv) Neither a consumer-oriented recommender system always hurts the sellers nor a profit-oriented recommender system always benefits the sellers. The optimal recommender system is one that is mildly profit oriented.

(v) As more consumers become better informed about competing products in the marketplace, the retailer has incentives to be more profit oriented in its recommendation strategy.

(vi) Finally, assuming that the retailer deploys a recommendation strategy that maximizes its own benefit, the consumers and the society may also benefit from the recommender system if it has a high precision. Consequently, it may be in the best interest of consumers to reveal their preferences to the retailer.

This study can be further extended in several directions. One particularly interesting extension relates to the empirical validation of the hypotheses alluded to by the implications we listed above. This being a theoretical study is limited to developing insights regarding the impacts of recommender system. Clearly, an empirical validation of the insights is critical. However, we would also point out that an empirical study is challenging because data about specific recommendation strategies used by retailers are generally private information which are difficult, if not impossible, to obtain. Despite this challenge, a follow-up empirical study will be valuable. On the theoretical side, future research can further examine the economic impact of recommender systems contexts or the manner in which recommender systems are employed. For instance, recommender systems vary in terms of when recommendations are presented to the user and the role they play (whether informative or persuasive). As a first study to examine the upstream impact of recommender systems, we have analyzed a specific context in this paper using a stylized model of two firms. Future research can extend the study by changing any of the above mentioned dimensions to enrich our understanding of recommender systems' economic impact.

CHAPTER 3

INFORMATIVE ROLE OF RECOMMENDER SYSTEMS IN ELECTRONIC MARKETPLACES: IS IT A BOON OR A BANE FOR COMPETING SELLERS?

3.1 Synopsis

Electronic marketplaces such as Amazon Marketplace and Ebay deploy recommender systems as sales support tools to help buyers find their "ideal" product in the vast variety of products sold by those marketplaces (Hennig-Thurau et al., 2012). Recommender systems have been reported to increase sales on these marketplaces—over 35% of sales on Amazon.com and more than 60% of the rentals on Netflix result from recommendations (Fleder and Hosanagar, 2009). A typical recommender system recommends the product the consumer is most likely to buy or prefer (Resnick and Varian, 1997). Recommender systems use different information sources to predict a consumer's preference, including item's content, consumer's behavior history, and consumer's demographic information.

Research indicates that a recommender system affects consumer decision making by informing consumers about products which they may not be aware (Tam and Ho, 2006).[1] In this sense, a recommender system deployed by an electronic marketplace functions as a medium for targeted advertising for sellers, analogous to traditional advertising media such as TV, newspaper, and, recently, the Internet. However, some key differences exist between advertising in the traditional media and advertising offered by the recommender system. The sellers do not "buy" or explicitly pay for recommendations and therefore recommendations are "free," whereas traditional advertising is costly for sellers. On the other hand, while the traditional advertising strategy, including the budget or advertising intensity, is under the

[1]This is referred to as the *informative role* of recommender systems. Recommender systems may also have a *persuasive role* which refers to increasing the likelihood of consumer's purchase of an already known product. We focus on the informative role in this paper.

direct control of the advertisers or sellers, the recommendation strategy is under the control of the electronic marketplace that deploys the recommender system.

When the purpose of recommendations as well as seller advertising is creating consumer awareness about products, the presence of the recommender system poses new challenges to sellers in electronic marketplaces regarding their advertising and pricing decisions. One could argue that recommender systems and traditional media are substitutes, and therefore sellers would decrease their traditional advertising in the presence of the recommender system. However, one could also make the argument that the two are complements in the sense that a seller would increase his advertising in the presence of the recommender system in order to mitigate the potential disadvantage it may suffer if the recommender system recommends the competitor's product. Furthermore, when the recommender system factors in price while choosing the recommendation—as when it chooses to recommend the product the consumer is most likely to buy—, the sellers' decisions become even more complicated because sellers will have to consider the interaction between price and advertising while making decisions. This paper examines the intricate interaction between competing sellers' advertising and pricing strategies in the presence of a recommender system in an electronic marketplace.

We develop an analytical model in which two competing sellers sell their products through a common electronic marketplace. Such a two level channel structure with one dominant e-commerce platform is commonly observed in practice (e.g., Amazon's marketplace). Consumers are heterogeneous with respect to their preferences for the two products. The marketplace deploys a recommender system that recommends one product to every consumer. Consistent with the broad literature in recommender systems, we model that recommender system would choose to recommend the product that offers the highest expected net utility to the consumer. Sellers can also advertise their products in traditional advertising channels. Both traditional advertising and the recommender system have an informative role in the sense both influence only consumers' awareness of the products.

Our analysis reveals that the overall impacts of of the recommender system are governed by the subtle interaction between two effects: *advertising effect* and *competition effect*. The advertising effect causes firms to advertise less and the competition effect causes firms to decrease prices in the presence of the recommender system. More importantly, we find that while a seller's own strategies related to advertising intensity and price are strategic substitutes in the absence of the recommender system, they are strategic complements and hence reinforce each other in the presence of the recommender system. As a result of these two effects, though the advertising provided by the recommender system is free, sellers may be better off or worse off in the presence of the recommender system than in the absence. The magnitudes of the competition and advertising effects depend on the recommender system precision and the advertising cost. The sellers are better off in the presence of the recommender system than in the absence when the recommender system precision is sufficiently high and the advertising cost is sufficiently low. On the other hand, consumers always benefit when the marketplace deploys the recommender system not only because of the increase in the awareness and hence the total demand created by the recommender system but also because of the competition effect induced by the recommender system.

Since recommender systems have become ubiquitous in electronic marketplaces and precisions of these systems have been continuously improving, a pertinent question relates to how recommender system precision affects the sellers and consumers in marketplaces with the recommender system. We find that an increase in the recommender system precision softens the price competition. This result arises because an improved precision reduces the relative role of prices vis-a-vis consumer preference in the recommender system's determination of the recommendation. As a result of the softening of the price competition, sellers benefit and consumers lose when precision improves. The mitigation of price competition also provides an incentive for sellers to advertise more when recommender system precision improves. On the other hand, increased recommender system precision implies that consumers are more likely to buy the product suggested by the recommender system, thus reducing the marginal benefit from sellers' advertising and limiting their incentives to advertise. Consequently, sellers' advertising level follows an inverse U-shape with recommender system precision.

The results regarding the impacts of the recommender system and the recommender system precision on sellers and consumers do not change qualitatively whether sellers use targeted advertising or uniform advertising. The key additional results when sellers adopt targeted advertising relate to how the sellers' targeting precision affects these impacts. An interesting finding is that a recommender system that benefits sellers when sellers do not employ targeted advertising may actually hurt sellers when sellers adopt targeted advertising with a high precision. On the other hand, in electronic marketplaces with the recommender system, an increase in sellers' targeting precision beyond a threshold softens price competition, increases seller profits, and reduces consumer surplus.

The rest of this paper is organized as follows. In the next section, we review the related literature. In Section 3.2, we develop an analytical framework to model the impact of a recommender system in a channel structure where two competing sellers sell on a common platform. Section 3.3 examines the implication of the recommender system by comparing the scenario in which consumers purchase without recommendations with the scenario with recommendation. In Section 3.4, we examine the case in which sellers can target consumers and show how targeted advertising by sellers would affect the implications of the recommender system. Section 3.5 concludes the paper with discussion on managerial implications.

3.2 Related Literature

The recommender system literature has examined technical and economic aspects of recommender systems. Adomavicius and Tuzhilin (2005) provide a review of work related to recommender system algorithms. Various metrics to evaluate recommendation algorithms such as prediction accuracy, recommendation diversity, consumer trust in recommendations, and seller profitability have been proposed (Shani and Gunawardana, 2011; Chen et al., 2008; Bodapati, 2008). In this study, we take the recommendation algorithm as exogenously given and focus on the economic impact of recommender systems on sellers. In particular, we assume that the recommender system recommends the product that offers the highest expected net utility to the consumer, consistent with the notion that the primary goal of recommendations is to match the consumer with her preferred product. Moreover, Wang and Zhang (2011) develop an algorithm which uses consumer's net utility, and show that their algorithm outperforms the widely used collaborative filtering-based recommender.

Research on the economics aspects of recommender systems has focused on their impacts on product sales. Fleder and Hosanagar (2009) show that some recommender system designs reinforce the popularity of already popular products. Hosanagar et al. (2013) show recommender systems can lead to consumers purchasing more similar items. Oestreicher-Singer and Sundararajan (2012a) show that recommender system can flatten the demand distributions. Oestreicher-Singer and Sundararajan (2012b) show that, on average, the explicit visibility of a co-purchase relationship can amplify the influence of complementary products on each other's demands. Pathak et al. (2010) show that the strength of recommendations has a positive effect on sales and prices and that this effect is moderated by the recency effect.

Jabr and Zheng (2014) analyze the effect of recommendations and word-of-moth reviews on product sales in a competitive environment and show that higher referral centrality of competing products is associated with lower product sales.

Research that uses an analytical model to examine the impact of recommender systems is limited. Hervas-Drane (2009) show that recommender systems based on consumer taste enhance consumers' interest in niche products and decrease market concentration. Bergemann and Ozmen (2006) use a two-stage game to examine how a firm can strategically choose its first-stage price to generate recommendations in the second stage. Our study differs from these in that we examine the effect of recommender systems in the setting of competing sellers in a channel structure, while previous analytical studies consider a single seller and ignore the strategic interactions between sellers.

Our study is also related to the marketing literature on informative advertising and competition. Bester and Petrakis (1995) and Grossman and Shapiro (1984) predict an inverse relationship between advertising level and prices in a differentiated product market when advertising provides uninformed consumers with price information. Soberman (2004) extends Grossman and Shapiro (1984) to show that informative advertising alone can lead to either higher or lower prices depending on the level of differentiation between competing firms. While these studies consider uniform advertising, another stream of literature considers targeted advertising. Iyer et al. (2005) investigate how competing firms in a horizontally differentiated market choose the advertising strategy when they can target consumer segments according to their preferences. Gal-Or et al. (2006) examine how an advertiser should allocate resources to increase the quality of targeting. Gal-Or and Gal-Or (2005) study firms' advertising strategy when they use a single media distributor such as television cable company as the channel for advertising. Different from these studies, our focus is on the interaction between seller advertising and recommender systems.

3.3 Model

We consider a channel structure with two competing sellers (A and B), an electronic marketplace (R), and a continuum of consumers with heterogeneous preferences. Seller $A(B)$ sells product $A(B)$ and both sell their products via R. Consumers can buy these products only through R. Each seller sets the price of its product, and the marketplace charges the sellers a commission equal to α fraction of the price on each sale. The two products are horizontally differentiated and have different degrees of misfit to different consumers. In particular, we assume that the products are located at the two end points of a Hotelling line of unit length, with product A being at 0 and product B being at 1. Consumers are uniformly distributed along the line. The distance between a product and a consumer measures the degree of misfit of the product to the consumer. The misfit cost is the degree of misfit times a unit misfit cost t. A consumer's net utility for product i, $i \in \{A, B\}$, is equal to the value of the products v net the misfit cost and product price p_i. Specifically, for a consumer located at z, the net utility from buying product A is $U_A = v - zt - p_A$ and from buying product B is $U_B = v - (1 - z)t - p_B$. We refer to consumer's differentiation along the Hotelling line as *locational differentiation*.

Sellers can inform the consumers about their products and prices by advertising their products in media channels such as TV, newspaper, and the Internet. Without a recommender system, the only way a consumer becomes informed about a seller's product is by receiving its advertisement. Advertising is assumed to be informative and has no effect on customer's willingness to pay. In the baseline model, firms use uniform advertising; that is, all consumers have equal chance of receiving a given advertisement. Following the standard procedure in modeling informative advertising (e.g., Tirole, 1988), we denote ϕ_i as the probability that a consumer receives the advertisement from seller i and is aware of product i, and refer to ϕ_i as the advertising intensity of seller i. An advertising intensity of ϕ_i costs $a\phi_i^2/2$, where a measures the cost of advertising. The convexity of the cost function reflects the fact that some consumers are harder to reach than others. As a consequence of sellers' advertisements, consumers become heterogeneous in their awareness about products. In particular, three types of consumers exist in the marketplace in terms of their product awareness: fully-informed consumers, uninformed consumers, and partially-informed consumers. Fully-informed consumers are aware of both products. The size of the fully-informed consumer segment is $\phi_A\phi_B$. Uninformed consumers are aware of neither product. The size of the uninformed consumer segment is $(1-\phi_A)(1-\phi_B)$. Partially-informed consumer segment includes consumers who are only aware of product A and consumers who are only aware of product B. The size of the group aware of product A only is $\phi_A(1 - \phi_B)$ and the size of group aware of product B only is $\phi_B(1 - \phi_A)$. We refer to consumer's differentiation along the awareness dimension as *informational differentiation*.

Each consumer has a unit demand and can only purchase a product that she is aware of. We denote z_0 as the location of the marginal consumer who would be indifferent between the two products if she were fully informed. Based on the utility function, we have

$$z_0 = \frac{p_B - p_A + t}{2t} \tag{3.1}$$

Consumers located at $z < z_0$ would, if fully-informed, buy product A. On the contrary, a fully-informed consumer located at $z > z_0$ would buy product B. For partially-informed consumers, demand is determined by individual rationality; that is, all consumers who are only aware of product i buy from firm i as long as they obtain positive surplus from product i. Consistent with the extant literature (e.g., Tirole, 1988; Grossman and Shapiro, 1984), we assume that the value of a product to a consumer v is large enough such that both products provide all consumers in the market with a positive surplus; that is, all partially-informed consumers make a purchase in the equilibrium.

The marketplace deploys a recommender system. The recommender system is consumer-centric in that the primary goal of the recommender system is to help a consumer find her best product. In particular, we assume that the recommender system recommends the product that offers a greater expected net utility to a consumer based on the information it has about her (e.g, consumer's rating information). This assumption is consistent with the finding of Li and Hitt (2010) that empirically demonstrates that consumer rating is based on consumer's net utility. The recommender system may be imperfect in estimating a consumer's preference, which is modeled as a location in the Hotelling line. Following a commonly used approach to model this estimation (e.g., Lewis and Sappington 1994; Johnson and Myatt 2006), we assume that the recommender system observes a signal s

regarding consumer's preference (location) based on available information. The signal equals the consumer's true location with probability β, and with probability $(1-\beta)$ the signal is uninformative and follows the distribution of consumer locations. That is, $P(z = y|s = y) = \beta$ and $P(z \neq y|s = y) = 1 - \beta$, where $y \in [0, 1]$. The model indicates that the signal is informative (i.e, provides useful information for the recommender system to estimate the consumer's preference) but noisy (i.e, does not perfectly reveal the true preference). We refer to β as the precision of the recommender system.

The sequence of events is as follows. In stage 1, sellers set prices p_i and advertising intensities ϕ_i simultaneously. In stage 2, consumers visit the marketplace and make their purchase decisions. Two scenarios are considered: one without the recommender system and the other with the recommender system. We use the scenario without the recommender system as the benchmark to analyze the impacts of the recommender system. In the scenario without the recommender system, the purchase decisions are made solely based on consumers' awareness about products resulting from sellers' advertising. In the scenario with the recommender system, the recommender system recommends one product to each consumer, the consumer observes the recommendation and then makes her purchase decision. A consumer's awareness and her preference are private information.

All other model parameters are common knowledge. All players are risk neutral.

We focus on the interesting case where the competition between sellers plays a role in the equilibrium, and, accordingly, make the following technical assumptions for our analysis. We denote $\tau = \frac{a}{1-\alpha}$.

Assumption 1: $\max\{\frac{3t^2-4tv+\sqrt{(v-5t)(v-t)^3}+v^2}{4t}, \frac{t}{2}\} < \tau < \frac{(v-t)^2}{2t}$

Assumption 2:
$$\begin{aligned}&16(1-\beta)^6t^2\tau\left(\beta(1-\beta)^2t+6\tau-6\sqrt{\tau\left(\tau-(1-\beta)^2\beta t\right)}\right)\\&\quad>\left((1-\beta)^2t+2\tau-2\sqrt{\tau\left(\tau-(1-\beta)^2\beta t\right)}\right)^4\end{aligned}$$

In Assumption 1, $\tau > t/2$ guarantees that the advertising cost is not too low such that the advertising intensities for both sellers are less than 1. $\tau > \frac{3t^2-4tv+\sqrt{(v-5t)(v-t)^3}+v^2}{4t}$ is to rule out the trivial case in which sellers only serve the partially-informed consumers and ignore the fully-informed consumers. $\tau < (v-t)^2/2t$ guarantees that all partially informed consumers make a purchase in the equilibrium (i.e., $p^* < v - t$). Assumption 2 guarantees a pure strategy equilibrium for the game when the recommender system is in place. The condition in Assumption 2 is always satisfied when $\beta > \frac{1}{25}$.

3.4 Impacts of Recommender System

In this section, we first derive the sub-game perfect equilibrium for the scenario without the recommender system and for the scenario with the recommender system using the backward induction approach. Then, we analyze the impact of the recommender system by comparing the equilibria in the two scenarios.

3.4.1 Benchmark (No Recommender System)

Without the recommender system, a consumer who is only aware of product i would purchase product i, $i \in \{A, B\}$. A consumer who is aware of both products would purchase the product that yields a higher net utility. A consumer who is aware of neither product would not buy any product. Therefore, the demand functions for the two products can be formulated as:

$$\begin{aligned} D_A =& \phi_A(1-\phi_B) + \phi_A\phi_B\frac{p_B - p_A + t}{2t} \\ D_B =& (1-\phi_A)\phi_B + \phi_A\phi_B\frac{p_A - p_B + t}{2t} \end{aligned} \tag{3.2}$$

Sellers maximize their profits by choosing their optimal prices and advertising levels:

$$\max_{p_i,\phi_i} \pi_i = (1-\alpha)p_i D_i - \frac{a\phi_i^2}{2} \tag{3.3}$$

Based on the first-order conditions, we obtain the equilibrium price and advertising level for each seller. The following lemma summarizes the equilibrium outcome.

Lemma 6. *When the marketplace does not use the recommender system, the equilibrium prices, advertising levels, seller profits, and consumer surplus are as follows:*

(a) Prices:

$$p_A^* = p_B^* = \sqrt{2\tau t} \tag{3.4}$$

(b) Advertising intensities:

$$\phi_A^* = \phi_B^* = \frac{2t}{\sqrt{2\tau t}+t} \tag{3.5}$$

(c) Seller profits:

$$\pi_A^* = \pi_B^* = (1-\alpha)\frac{2\tau t}{\left(\sqrt{2\tau}+\sqrt{t}\right)^2} \tag{3.6}$$

(d) Consumer Surplus:

$$CS = \frac{4\sqrt{2\tau t}}{\left(\sqrt{2\tau}+\sqrt{t}\right)^2}v + \frac{t^2-2t\left(\sqrt{2t\tau}+4\tau\right)}{\left(\sqrt{2\tau}+\sqrt{t}\right)^2} \tag{3.7}$$

Proof. All proofs are in the appendix. □

Examining sellers' best response functions reveals the following insight that drives the results shown in the Lemma: $p_i^* = t\left(\frac{2}{\phi_i^*} - 1\right)$. In other words, p_i^* and ϕ_i^* of seller i's own equilibrium strategies are strategic substitutes in the absence of the recommender system. An increase in ϕ has two effects on consumers: it decreases the informational differentiation among those that are partially informed, and it decreases the number of uninformed consumers. While the former effect intensifies competition between sellers by enlarging the "common turf" of consumers that sellers compete for, the latter increases the "monopoly turf" of each seller. We find that the former effect dominates the latter in the equilibrium, leading to a decrease in price when ϕ increases. Consequently, in the absence of the recommender system, when advertising cost a increases (which implies τ increases), while the advertising intensities decrease, the product prices increase. Interestingly, when the advertising cost increases, the increase in prices offsets the decrease in demands caused by a decrease in advertising intensities and increases seller profits. On the other hand, consumer surplus is decreasing in the advertising cost because consumers are hurt by the price increase and the demand decrease.

3.4.2 With Recommender System

When the recommender system is in place, the marketplace observes a signal s regarding the consumer's preference. Using Bayesian updating, we can derive the marketplace's expected location for the consumer conditional on the signal being equal to y to be:

$$\mathbb{E}(z|s = y) = \beta y + \tfrac{1-\beta}{2} \tag{3.8}$$

Given the signal, the expected consumer net utility from product A is $U_A = v - t\mathbb{E}(z|s = y) - p_A$ and expected consumer net utility from product B is $U_B = v - t\left(1 - \mathbb{E}(z|s = y)\right) - p_B$. We denote y_0 as the marginal signal for which the expected consumer net utility is the same for both products. Using Equation (3.8), we can derive the marginal signal y_0 as follows:

$$y_0 = \tfrac{p_B - p_A}{2\beta t} + \tfrac{1}{2} \tag{3.9}$$

If the signal is less than y_0, the recommender system recommends product A; otherwise, it recommends product B.

We next formulate the demand functions for A and B. Suppose $p_A \leq p_B$, which implies $z_0 \geq \frac{1}{2}$ and $y_0 \geq z_0$. A consumer located between 0 and z_0 purchases A if she gets an advertisement from A or she is recommended A. The probability that this consumer receives an advertisement from A is ϕ_A. The probability that this consumer does not receive an advertisement about A but is recommended A is $(1 - \phi_A)\left[\beta + (1 - \beta)y_0\right]$. Therefore, the demand for A from all consumers located between 0 and z_0 is $\int_0^{z_0} \left[\phi_A + (1 - \phi_A)\left(\beta + (1 - \beta)y_0\right)\right] \mathrm{d}z$. Similarly, a consumer located between z_0 and y_0 purchases A if and only if she does not receive an advertisement from B and is recommended A, which has the probability $(1 - \phi_B)\left[\beta + (1 - \beta)y_0\right]$. A consumer located between y_0 and 1 purchases A if and only if she does not receive an advertisement from B and is recommended A, which has the probability $(1 - \phi_B)(1 - \beta)y_0$. Aggregating the demand of each consumer segment, we can derive the demand function of product A as follows:

$$D_A = z_0\left[(1 - \beta)\left(1 - y_0\right)\phi_A + \phi_B\left(\beta + (1 - \beta)y_0\right)\right] + y_0\left(1 - \phi_B\right) \tag{3.10}$$

By using a similar logic, we can derive the demand function of product B when $p_A \leq p_B$ as follows:

$$D_B = (1 - z_0)\left[\phi_B + (1-\beta)(1-y_0)\left(\phi_A - \phi_B\right)\right] + (1 - y_0)\left[1 - \phi_A + \beta(\phi_A - \phi_B)\right] \tag{3.11}$$

We can get the demand function when $p_A > p_B$ using derivation steps analogous to those discussed for the case $p_A \leq p_B$. After substituting z_0 from Equation (3.1) and y_0 from Equation (3.9), we get:

$$D_A = \begin{cases} \frac{(1-\beta)(\phi_B-\phi_A)(p_B-p_A)^2+t(p_B-p_A)\left[2-\left(1-\beta^2\right)\phi_B-(1-\beta)^2\phi_A\right]+\beta t^2\left[2-(1-\beta)(\phi_B-\phi_A)\right]}{4\beta t^2} & \text{if } p_A \leq p_B \\ \frac{(1-\beta)(\phi_B-\phi_A)(p_B-p_A)^2+t(p_B-p_A)\left[2-\left(1-\beta^2\right)\phi_A-(1-\beta)^2\phi_B\right]+\beta t^2\left[2-(1-\beta)(\phi_B-\phi_A)\right]}{4\beta t^2} & \text{if } p_A > p_B \end{cases} \tag{3.12}$$

$$D_B = \begin{cases} \frac{(1-\beta)(\phi_A-\phi_B)(p_A-p_B)^2+t(p_A-p_B)\left[2-\left(1-\beta^2\right)\phi_B-(1-\beta)^2\phi_A\right]+\beta t^2\left[2-(1-\beta)(\phi_A-\phi_B)\right]}{4\beta t^2} & \text{if } p_A \leq p_B \\ \frac{(1-\beta)(\phi_A-\phi_B)(p_A-p_B)^2+t(p_A-p_B)\left[2-\left(1-\beta^2\right)\phi_A-(1-\beta)^2\phi_B\right]+\beta t^2\left[2-(1-\beta)(\phi_A-\phi_B)\right]}{4\beta t^2} & \text{if } p_A > p_B \end{cases} \tag{3.13}$$

Sellers maximize their profits by choosing their optimal prices and advertising intensities. The following lemma characterizes the equilibrium when the marketplace uses the recommender system.

Lemma 7. *When the marketplace uses the recommender system, the equilibrium prices, advertising intensities, seller profits, and consumer surplus are as follows:*

(a) Prices:

$$\hat{p}_A^* = \hat{p}_B^* = \frac{2\left[\tau - \sqrt{\tau(\tau-\beta(1-\beta)^2 t)}\right]}{(1-\beta)^2} \tag{3.14}$$

(b) Advertising intensities:

$$\hat{\phi}_A^* = \hat{\phi}_B^* = \frac{1}{2(1-\beta)} - \frac{\sqrt{\tau(\tau-\beta(1-\beta)^2 t)}}{2\tau(1-\beta)} \tag{3.15}$$

(c) Seller profits:

$$\hat{\pi}_A^* = \hat{\pi}_B^* = (1-\alpha)\frac{6\tau+\beta(1-\beta)^2 t - 6\sqrt{\tau(\tau-\beta(1-\beta)^2 t)}}{8(1-\beta)^2} \tag{3.16}$$

(d) Consumer surplus:

$$\hat{CS} = v - \frac{2\left[\tau - \sqrt{\tau(\tau-\beta(1-\beta)^2 t)}\right]}{(1-\beta)^2} - \frac{t}{4}\left[\frac{3}{2} - \beta + \frac{\sqrt{\tau(\tau-\beta(1-\beta)^2 t)}}{2\tau}\right] \tag{3.17}$$

Examining the sellers' best response functions when the marketplace uses the recommender system reveals a sharp contrast in how the equilibrium price and advertising intensity of a firm are related to each other in the scenario with the recommender system and the one without. When the recommender system is present, we find that $\hat{p_i}^* = \frac{\beta t}{1-(1-\beta)\hat{\phi_i}^*}$. That is, $\hat{p_i}^*$ and $\hat{\phi_i}^*$ are strategic complements and hence the presence of the recommender system alters the relationship between price and advertising intensity fundamentally, from strategic substitutes to strategic complements. The reason for this switch is as follows. In the presence of the recommender system, price, in addition to advertising, plays a role in determining the extent of informational differentiation that exists among consumers—a price decrease by a seller generates more recommendations in his favor and hence an informational advantage for him among uninformed consumers and partially informed consumers that were aware of his product, ceteris paribus. On the other hand, recommendations and hence prices do not alter the informational differentiation among fully informed consumers. An increase in ϕ increases the size of the fully informed consumer segment, and reduces the total size of the uniformed consumer segment and the partially informed segment that is aware of only one product. Consequently, the marginal impact of price on informational differentiation through recommendations decreases when ϕ increases. Therefore, an increase in ϕ leads to an increase in p. Because of the strategic complementarity between price and advertising intensity, when advertising cost a increases, the advertising levels, product prices and seller profits decrease and consumer surplus increases in the presence of the recommender system.

3.4.3 Impacts of the Recommender System

We can assert the impacts of the recommender system by comparing equilibrium quantities in the scenarios with and without the recommender system.

Proposition 7. *Compared to the scenario without the recommender system, in the presence of the recommender system:*

(a) sellers' advertising intensity is lower; that is, $\hat{\phi}_i^* \leq \phi_i^*$*;*

(b) product price is lower; that is, $\hat{p}_i^* \leq p_i^*$*;*

(c) the sellers are better off (i.e.,$\hat{\pi}_i^* > \pi_i^*$*) if and only if* $a \leq \frac{\left(2\sqrt{2(1-\alpha)}+3-2\alpha\right)(1-\alpha)t}{2(1-2\alpha)^2}$ *and* $\frac{6\tau+\beta(1-\beta)^2-6\sqrt{\tau(\tau-\beta(1-\beta)^2)}}{8(1-\beta)^2} > \frac{2t\tau}{\left(\sqrt{2\tau}+\sqrt{t}\right)^2}$;

(d) consumer surplus is higher.

Proposition 7(a) shows that sellers would reduce advertising intensity in the presence of the recommender system. This finding supports the notion that the recommender system functions as a substitute to traditional advertising. This result suggests that using traditional advertising to mitigate the potential informational disadvantage resulting from recommendations received by the competitor is likely to be too costly for sellers. We denote the impact of recommender system on sellers' advertising as the *advertising effect*. Proposition 7(b) shows that firms would lower product prices in the presence of recommender system, indicating that recommender system intensifies the competition between sellers. We call this as the *competition effect* of the recommender system. The competition effect stems from two driver forces. The direct driver is that firms have incentive to use price as a lever to attract more recommendations and increase the informational advantage. This incentive tends to increase the competition. Furthermore, because the recommender system alters the relationship between equilibrium price and equilibrium advertising level from strategic substitutability to strategic complementarity, the advertising effect of the recommender system indirectly contributes to the competition effect as well.

Proposition 7(c) reveals that the recommender system does not always benefit sellers. While the competition effect hurts sellers, the advertising effect resulting from the "free" advertising provided by the recommender system reduces sellers' advertising cost and benefits sellers. The tradeoff between these two determines whether sellers benefit from the recommender system. Figure 3.1 illustrates Proposition 7(c), and it shows that the recommender system is more likely to benefit sellers when the recommender system precision is high and,

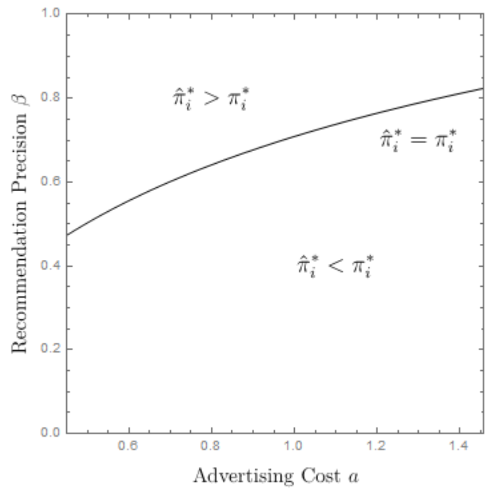


Figure 3.1. Impact of Recommender System on Profits in Uniform Advertising Case ($v = 2.8$, $t = 1$, and $\alpha = 0.1$)

surprisingly, the advertising cost is relatively low. One would intuitively expect that the free advertising offered by the recommender system will benefit sellers when the cost of traditional advertising is high. The explanation for this counter-intuitive finding is the alteration of the relationship between advertising intensity and price by the recommender system as we explained earlier—as a result of the strategic substitution relationship, an increase in advertising cost increases sellers' profit in the no-recommender system scenario but decreases the sellers' profit in the recommender system scenario, thus reducing the benefit offered by the recommender system to sellers.

Finally, the recommender system always benefits consumers. Three factors contribute to this result. First, the recommender system expands the market by increasing consumer awareness, and more consumers make a purchase in the presence of recommender system. Second, the competition between two sellers increases in the presence of recommender system, which drives the prices down. Third, by recommending the product which offers a higher expected utility for the consumer, the recommender system reduces consumers' misfits costs.

Clearly, the recommender system precision plays an important role in determining whether sellers and the marketplace benefit from the recommender system. We examine the role of recommender system precision next.

Proposition 8. *The impact of recommendation precision β is as follows:*

(a) $\hat{p}_i^*$ is increasing in β.

(b) $\hat{\phi}_i^*$ follows an inverse U-shape with β, that is, $\frac{\partial \hat{\phi}_i^*}{\partial \beta} > 0$ when $\frac{4\tau}{t}(1-2\beta) > (1-\beta)^4$.

(c) $\hat{\pi}_i^*$ is increasing in β.

(d) $\hat{CS}$ is decreasing in β.

Proposition 8(a) shows that an increase in the recommender system precision softens the price competition between sellers. The intuition for this result is the following. Regardless of the recommender system precision, a decrease in the price of a product increases the likelihood of that product being recommended, *ceteris paribus*. However, the marginal increase in this likelihood is greater when the recommender system precision is low than when it is high. When the precision is low, from the recommender system's perspective, most consumers are concentrated in the middle of the Hotelling line (as reflected by the expected location in Equation (3.8)), and therefore a small decrease in the price of a product will induce recommendation of that product to a large number of consumers (as seen in the marginal signal in Equation (3.9)). Consequently, sellers' incentives to compete on price are higher when recommender system precision is low than when it is high.

Proposition 8(b) indicates that firm's advertising intensity follows an inverse U-shape with recommender system precision as shown in Figure 3.2. A increase in β has two implications for the sellers' advertising choices. First, it increases the price as suggested by proposition 8(a), and thus sellers can get more profit from each additional consumer they get, which induces sellers to advertise more to get more demand. Second, an increase in β decreases the effectiveness of seller's advertising. That is, the marginal demand each seller gets from

Figure 3.2. Effect of Recommendation Precision ($t = 1$ and $\tau = 0.6$)

additional advertising decreases in β. For instance, consider the extreme situation when recommender system has perfect precision (i.e., $\beta = 1$). In this case, because consumers will buy only the product recommended to them, a seller's advertising has no impact on his demand. This implication would discourage sellers from advertising. The relationship between $\hat{\phi}_i^*$ and β varies depending on which implication dominates. From proposition 8(b), the first (second) implication dominates the second (first) implication when precision is low (high).

Proposition 8(c) and (d) imply that sellers benefit from but consumers are hurt from an improvement in recommender precision. The primary driver of these two results is the softening of the price competition induced by an increase in recommender system precision.

3.5 Targeted Advertising

In the previous section, we examined the case in which sellers use uniform advertising and do not target consumers; that is, every consumer has the same probability of receiving the advertisement from a seller. Over years, sellers are increasingly able to target advertising to specific consumers. In this section, we investigate how targeted advertising by sellers would affect the implications of the recommender system.

The approach we use to model the targeting ability of sellers is similar to that we use for the recommender system. In particular, for a consumer located at z, seller i gets signal η_i

about z. The probability that the signal reveals the consumer's true location is γ; that is, $P(z = y|\eta_i = y) = \gamma$; otherwise, the signal is uninformative and follows the prior distribution of y. A larger γ indicates that firms have a better targeting ability. We assume that both sellers have the same targeting precision and that the signals are conditionally independent given the true location. We assume that the sellers use the signal only to target their advertising, but they do not engage in price discrimination; that is, each charges the same price for all consumers.

The timeline for the game under targeted advertising is as follows. In stage 1, sellers set prices and advertising strategies simultaneously. In stage 2, sellers observe private signals about consumer locations, and follows the advertising strategies made in stage 1. In stage 3, consumers visit the marketplace. If the marketplace employs a recommender system, then the recommender system makes the recommendation following the strategy discussed in Section 3.3. Finally, consumers purchase the product based on their awareness and preferences. We use Bayesian Nash Equilibrium (BNE) as the solution concept.

We make the following technical assumption in order to get an interior equilibrium in both no-recommendation and with recommendation scenarios.

Assumption 3: $2t \leq \tau$

3.5.1 No Recommender System

Following the standard procedure for determining a BNE, we first propose each seller's set of beliefs, derive rational strategies for sellers under this belief set, and show that their strategies and beliefs are consistent with each other (e.g., Vives, 1984). We consider the following belief about seller B's strategy as function of its signal η_B.

$$\phi_B(\eta_B) = \begin{cases} \phi_B^L & \text{if } \eta_B \leq z_0 \\ \phi_B^H & \text{if } \eta_B > z_0 \end{cases} \tag{3.18}$$

where ϕ_B^L and ϕ_B^H are constants. Without loss of generality, we consider seller A's optimization problem in stage 1. We denote $\mathbb{E}[\pi_A|\eta_A]$ as the expected profit function of seller A conditional on its own signal η_A, which can be formulated as follows:

$$\mathbb{E}[\pi_A|\eta_A] = (1-\alpha)p_A\phi_A\left[\int_0^{z_0} f(z|\eta_A)\mathrm{d}z + \int_{z_0}^1 \left(1-\mathbb{E}[\phi_B(\eta_B)|z]\right) f(z|\eta_A)\mathrm{d}z\right] - \tfrac{a}{2}\phi_A^2 \quad (3.19)$$

where $f(z|\eta_A)$ is the probability density function of location z given signal η_A. Under A's belief about B's strategy, the expected advertising intensity of seller B is

$$\mathbb{E}[\phi_B(\eta_B)|z] = \phi_B^L(1-\gamma)z_0 + \phi_B^H\left[\gamma + (1-\gamma)(1-z_0)\right] \quad (3.20)$$

when $z_0 < z < 1$. Given a η_A, seller A choose advertising intensity ϕ_A to maximize $\mathbb{E}[\pi_A|\eta_A]$. Substituting Equation (3.20) into A's profit function and maximizing it, we get the following optimal advertising strategy for seller A:

$$\phi_A^*(\eta_A) = \begin{cases} \frac{p_A}{4t^2\tau}\left[4t^2 + (1-\gamma)(p_B - p_A - t)\left((\phi_B^L - \phi_B^H)(1-\gamma)(p_B - p_A + t) + 2t\phi_B^H\right)\right], & \text{If } \eta_A \le z_0 \\ \frac{p_A}{4t^2\tau}\left[4t^2 + ((1-\gamma)(p_B - p_A - t) - 2\gamma t)\left((\phi_B^L - \phi_B^H)(1-\gamma)(p_B - p_A + t) + 2t\phi_B^H\right)\right], & \text{If } \eta_A > z_0 \end{cases} \quad (3.21)$$

Denoting A's advertising intensity when $\eta_A \le z_0$ as ϕ_A^H and A's advertising intensity when $\eta_A > z_0$ as ϕ_A^L, we verify that the sellers' strategies and belief sets are consistent with each other. In stage 1, each seller sets product price by maximizing the overall expected profit function:

$$\underset{p_i}{max}\mathbb{E}(\pi_i) = \int_0^1 \mathbb{E}(\pi_i|\eta_i)f(\eta_i)\mathrm{d}\eta_i \quad (3.22)$$

We summarize the symmetric BNE as the following lemma.

Lemma 8. *In the absence of the recommender system, the BNE prices, advertising intensities , seller profits, and consumer surplus when sellers use targeted advertising are as follows:*

(a) Prices:

$$p_A^* = p_B^* = p^*$$

where p^* is the real root of

$$p^4(\gamma^4 - \gamma^3) + p^3\tau\gamma(1 - 2\gamma) + p^2\tau(2\tau - \gamma^4 t - \gamma^2 t) + 4pt\tau^2\gamma^2 - 4t\tau^3 = 0 \tag{3.23}$$

(b) Advertising intensities:

$$\phi_A^{H*} = \phi_B^{H*} = \phi^{H*} = \tfrac{p^*[2\tau+(1-\gamma)\gamma p^*]}{\tau[2\tau+(1-\gamma^2)p^*]} \tag{3.24}$$

$$\phi_A^{L*} = \phi_B^{L*} = \phi^{L*} = \tfrac{p^*[2\tau-(1+\gamma)\gamma p^*]}{\tau[2\tau+(1-\gamma^2)p^*]} \tag{3.25}$$

(c) Seller profits:

$$\pi_A^* = \pi_B^* = (1-\alpha)\tfrac{(p^*)^2\left[(\gamma^4+\gamma^2)(p^*)^2-4\gamma^2(p^*)\tau+4\tau^2\right]}{2\tau((1-\gamma^2)p^*+2\tau)^2} \tag{3.26}$$

(d) Consumer surplus:

$$CS = p^*\tfrac{4(v-p^*)\left[(\gamma^4+\gamma^2)(p^*)^2-4\gamma^2\tau p^*+4\tau^2\right]-t\left[3(\gamma^4+\gamma^2)(p^*)^2-2p^*(5\gamma^2\mathcal{T}+\mathcal{T}\right)+8\tau^2\right]}{2\tau((1-\gamma^2)p^*+2\tau)^2} \tag{3.27}$$

When $\gamma = 0$, the targeted advertising case becomes equivalent to the uniform advertising case where sellers set the same adverting intensity for all consumers. It is easy to verify that $\phi^{H*} \geq \phi^{L*}$, which implies that a seller advertises more to consumers that are located closer to him as indicated by the signal. Furthermore, as targeting precision improves, sellers increase the differentiation in advertising intensity between consumers located closer to them vis-a-vis those located closer to their competitor. Though an increase in precision enables the sellers to target consumers more effectively, we find that it does not always increase their profits. In particular, improving targeting precision hurts sellers if the precision is low. Two drivers contribute to the impact of targeting precision on seller profits. One, an increase in targeting precision induces sellers to increase their overall advertising and hence the advertising costs. Two, sellers may engage in more intense competition to attract marginal consumers that are located closer to their competitor in response to a reduction in their own advertising and an increase in competitor's advertising to those consumers. The second driver is particularly strong when the targeting precision is low or the cost of advertising is low. On the other hand, consumers always benefit with better targeting by sellers because more consumers become aware of at least one product when sellers increase their overall advertising. The implication of advertising cost when sellers can target consumers is the same as in the uniform advertising—an increase in advertising cost decreases advertising intensity to all consumers, increases product prices and seller profits, and decreases consumer surplus.

3.5.2 With Recommender System

We find that when the recommender system is in place, sellers use a similar strategy as in the no recommender system scenario under targeted advertising. We present the detailed derivation in the appendix and summarize the equilibrium for the scenario with the recommender system in the following lemma.

Lemma 9. *When the marketplace uses the recommender system, the BNE prices, advertising intensities, seller profits and consumer surplus in the targeted advertising case are as follows:*

(a) Prices:

$$\hat{p}_A^* = \hat{p}_B^* = \frac{2\tau - 2\sqrt{\tau^2 - t\tau\beta(1-\beta)^2\left[(1+\beta)\gamma^2 + 1 - \frac{\beta\gamma}{2}\right]}}{(1-\beta)^2\left[(1+\beta)\gamma^2 + 1 - \frac{\beta\gamma}{2}\right]} \tag{3.28}$$

(b) Advertising intensities

$$\hat{\phi}_A^{H*} = \hat{\phi}_B^{H*} = \hat{\phi}^{H*} = (1+\gamma)\frac{\tau - \sqrt{\tau^2 - t\tau\beta(1-\beta)^2\left[(1+\beta)\gamma^2 + 1 - \frac{\beta\gamma}{2}\right]}}{2\tau(1-\beta)\left[(1+\beta)\gamma^2 + 1 - \frac{\beta\gamma}{2}\right]} \tag{3.29}$$

$$\hat{\phi}_A^{L*} = \hat{\phi}_B^{L*} = \hat{\phi}^{L*} = (1-\gamma)\frac{\tau - \sqrt{\tau^2 - t\tau\beta(1-\beta)^2\left[(1+\beta)\gamma^2 + 1 - \frac{\beta\gamma}{2}\right]}}{2\tau(1-\beta)\left[(1+\beta)\gamma^2 + 1 - \frac{\beta\gamma}{2}\right]} \tag{3.30}$$

(c) Seller profits:

$$\hat{\pi}_A^* = \hat{\pi}_B^* = \beta t\tau(1-\alpha)\frac{8\tau + 8\sqrt{\tau^2 - t\tau\beta(1-\beta)^2\left[(1+\beta)\gamma^2 + 1 - \frac{\beta\gamma}{2}\right]} - \beta t(1-\beta)^2\left(1+\gamma^2\right)}{8\left(\sqrt{\tau^2 - t\tau\beta(1-\beta)^2\left[(1+\beta)\gamma^2 + 1 - \frac{\beta\gamma}{2}\right]} + \tau\right)^2} \tag{3.31}$$

(d) Consumer surplus:

$$\hat{CS} = v - \frac{2\tau - 2\sqrt{\tau^2 - t\tau\beta(1-\beta)^2\left[(1+\beta)\gamma^2 + 1 - \frac{\beta\gamma}{2}\right]}}{(1-\beta)^2\left[(1+\beta)\gamma^2 + 1 - \frac{\beta\gamma}{2}\right]} - \frac{t(2-\beta)}{4} + \frac{t(1+\gamma^2)\left[2\tau - 2\sqrt{\tau^2 - t\tau\beta(1-\beta)^2\left[(1+\beta)\gamma^2 + 1 - \frac{\beta\gamma}{2}\right]}\right]}{16\tau\left[(1+\beta)\gamma^2 + 1 - \frac{\beta\gamma}{2}\right]} \tag{3.32}$$

While the expressions shown in the lemma are complex, the qualitative insights revealed by them are similar to those found for the recommender system scenario in the uniform advertising case. When $\gamma = 0$, the targeted advertising case becomes equivalent to the uniform advertising case. In particular, advertising cost a has a similar effect as in the uniform advertising—when advertising cost increases, firms decrease the advertising level to consumers, decrease prices, and decrease seller profits. Furthermore, as in the no-recommender system scenario with targeted advertising, sellers advertise more to consumers that are located closer to them in the presence of the recommender system.

3.5.3 Impacts of Recommender System

In this section, we compare the equilibrium quantities provided in Lemmas 8 and 9 to get the implications of the recommender system when sellers use targeted advertising.

Proposition 9. *When sellers use targeted advertising, compared to the scenario without the recommender system, in the presence of the recommender system:*

(a) Each product's price is lower; that is, $\hat{p}_i^* \leq p_i^*$, $i \in \{A, B\}$.

(b) Each product's advertising intensity is lower; that is, $\hat{\phi}^{H*}$ is lower than ϕ^{H*} and $\hat{\phi}^{L*}$ is lower than ϕ^{L*}.

(c) The firms can either be better off or worse off.

(d) Consumer surplus is higher.

We find that the impacts of the recommender system on price competition and advertising intensity are qualitatively identical in the targeted and uniform advertising cases—namely, the recommender system induces both competition effect and advertising effect on sellers in both cases. The impact of the recommender system on seller profit is also similar in both cases except that the sellers' targeting precision also plays a role in determining whether sellers benefit or are hurt by the recommender system in the targeted advertising case.

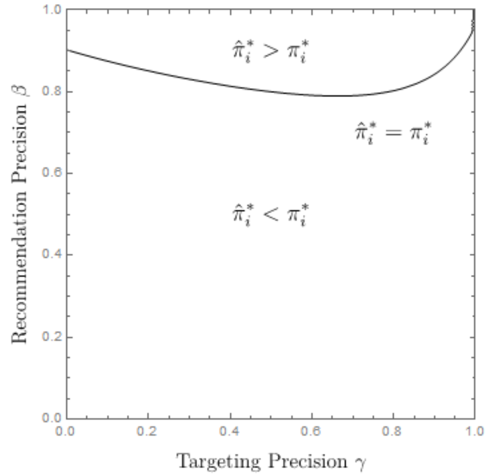


Figure 3.3. Impact of Recommender System on Profits in Targeted Advertising Case ($t = 1$ and $\tau = 2$)

Proposition 9(c) is illustrated by Figure 3.3. For any fixed targeting precision, the recommender system would make sellers better off only when recommender system has a high precision, and this result is similar to that we find in the uniform advertising case. An improvement in targeting precision first increases the area in the parameter space where the recommender system benefits sellers, but when targeting precision is sufficiently high, improving targeting precision decreases the area where the recommender system benefits sellers. This finding reveals that when the marketplace deploys a recommender system—a common current business practice—, improving their own targeting precision may not necessarily offset the negative impacts of the recommender system on sellers. In contrast, this strategy can worsen the impact of the recommender system.

Clearly, the value of the recommender system to sellers depends critically on the recommender system precision and the sellers' targeting precision. We examine the impacts of these two precisions under targeted advertising next.

Proposition 10. *When sellers use targeted advertising, in the presence of the recommender system,*

(a) $\hat{p}_i^*$ *increases in* β*.*

(b) $\hat{\phi}^{H*}$ and $\hat{\phi}^{L*}$ first increases and then decreases in β; that is, $\frac{\partial \phi}{\partial \beta} > 0$ if and only if $\frac{(1-\beta)^4}{(1-2\beta)(1+2\beta\gamma^2+\frac{\gamma}{2}-\beta\gamma)} < \frac{4\tau}{t(\gamma^2+1)^2}$.

(c) $\hat{\pi}_i^*$ increases in β.

(d) $\hat{CS}$ decreases in β.

Comparing Proposition 10 with Proposition 8, we find that the recommender system precision has the same qualitative impact on sellers and consumers no matter whether sellers use targeted advertising.

Proposition 11. *When sellers use targeted advertising, in the presence of the recommender system,*

(a) $\hat{p}_i^*$ first decreases and then increases in γ; that is, $\frac{\partial \hat{p}_i^*}{\partial \gamma} < 0$ if and only if $\gamma < \frac{\beta}{4(1+\beta)}$;

(b) $\hat{\phi}^{H*}$ increases in γ and $\hat{\phi}^{L*}$ decreases in γ;

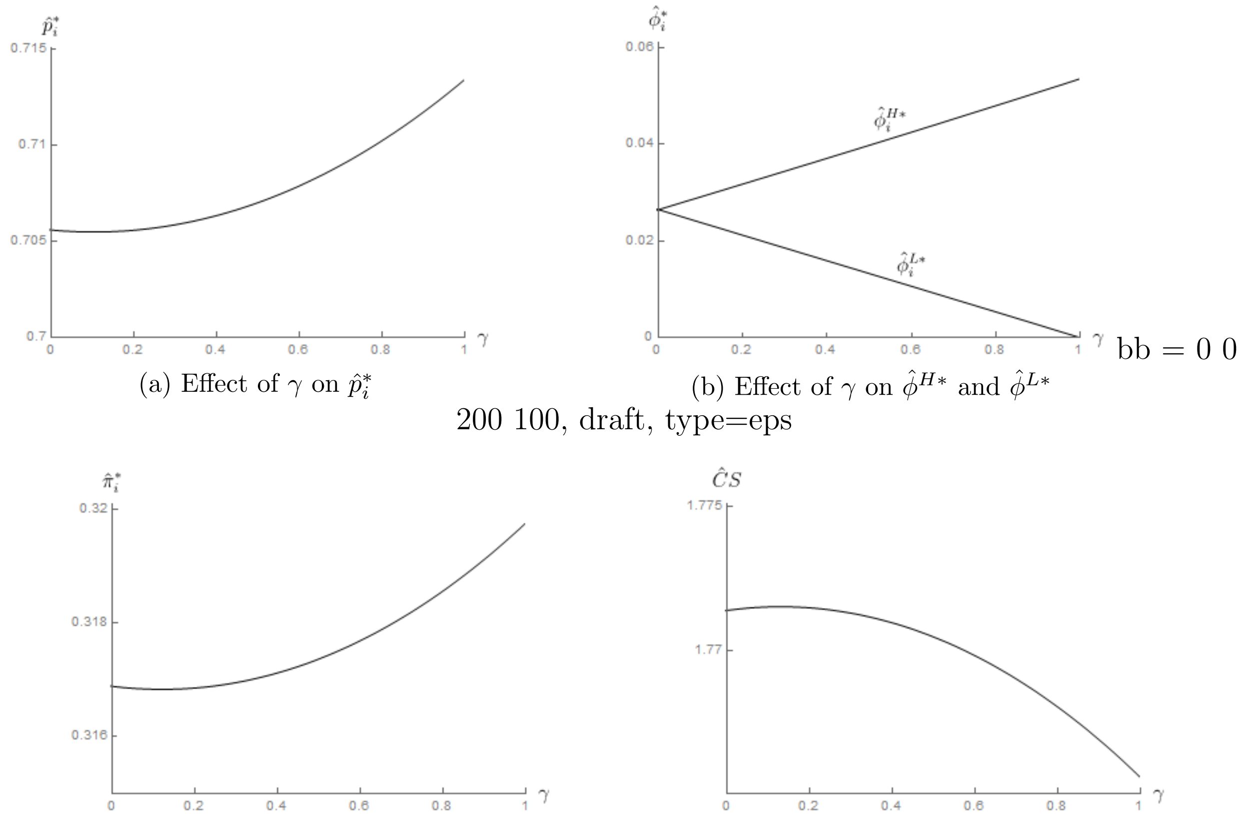
(c) $\hat{\pi}_i^*$ first decreases and then increases in γ; that is, $\frac{\partial \hat{\pi}_i^*}{\partial \gamma} < 0$ if and only if $\gamma < \tilde{\gamma}$, where $\tilde{\gamma}$ is the root of

$$4(4\beta\gamma - \beta + 3\gamma)\left[\sqrt{\tau^2 - t\tau\beta(1-\beta)^2[(1+\beta)\gamma^2 + 1 - \tfrac{\beta\gamma}{2}]} + \tau\right] - (1-\beta)^2\beta^2 t(\gamma^2 + 4\gamma - 1) = 0$$

(d) CS increases in γ if and only if

$$8\gamma\sqrt{\tau^2 - t\tau\beta(1-\beta)^2[(1+\beta)\gamma^2 + 1 - \tfrac{\beta\gamma}{2}]} + 8\tau\left(2\beta^2 - 8\beta(1+\beta)\gamma + \gamma\right)$$
$$> \beta(1-\beta)^2 t\left(4\beta\gamma^3 - 3\beta\gamma^2 - 4\beta\gamma + \beta + 4\gamma^3 + 4\gamma\right)$$

Figure 3.4 illustrates Proposition 11. It is intuitive that an improvement in sellers' targeting precision increases the differentiation in sellers' advertising intensities for consumers located closer to them vis-a-vis those located closer to the competitor. While an improvement in targeting precision intensifies the price competition when the targeting precision is low, it softens the price competition beyond a threshold value. Consequently, sellers start to benefit from improved targeting precision when precision is sufficiently high. While an increase in sellers' targeting precision benefits consumers when there is no recommender system, it



(a) Effect of γ on $\hat{p}_i^*$

(b) Effect of γ on $\hat{\phi}^{H*}$ and $\hat{\phi}^{L*}$

(c) Effect of γ on $\hat{\pi}_i^*$

(d) Effect of γ on $\hat{CS}$

Figure 3.4. Effect of Targeting Precision ($t = 1$, $\alpha = 0.1$, $\tau = 2$, $\beta = 0.7$, and $v = 2.8$)

can hurt consumers when the recommender system is present. The primary reason for this difference is that all consumers are aware of at least one product when the marketplace deploys a recommender system and hence an increase in targeting precision does not increase the total demand—the key reason for the higher consumer surplus under a higher targeting precision when there is no recommender system. Taken together, Propositions 7 and 9 show that in electronic marketplaces that deploy recommender systems, the combination of a high recommender system precision and a high seller's targeting precision indeed benefits sellers at the expense of consumers.

3.6 Discussion and Conclusions

We show in this paper that the impacts of the recommender system on competing sellers in an electronic marketplace are the result of a subtle interaction between *advertising effect* and *competition effect*. The advertising effect causes firms to advertise less and the competition effect causes firms to decrease prices in the presence of a recommender system. The recommender system alters the seller's own strategies related to advertising intensity and price from being strategic substitutes in the absence of the recommender system to being strategic complements in the presence of the recommender system, as a result of these two effects. Though the advertising provided by the recommender system is free, sellers may be better off or worse off in the presence of the recommender system than in the absence—sellers are more likely to benefit from recommender system only when it has a high precision. The results do not change qualitatively whether sellers use targeted advertising or uniform advertising. However, when sellers are able to target consumers with a high degree of precision, even a high recommender system precision may not benefit sellers.

Our results have significant implications for electronic marketplaces and offer a few testable hypotheses. The implications fall under two categories. The first set of implications relates to whether sellers and consumers benefit when electronic marketplaces deploy recommender systems as compared to when marketplaces do not deploy these systems. Clearly, the overall awareness of consumers about products, and thus overall demand (in terms of sales units) is higher in the marketplace with recommender systems. However, whether the sellers' revenue and profit are higher in marketplaces with recommender systems compared to those without recommender systems is unclear. While existing empirical research tout the increased sales contributed by recommender systems, these studies use unit sales or its proxies. Our results suggest that empirical studies that examine the seller revenue or profit will provide a more complete understanding of the recommender systems' impact. Another direct implication is that the overall advertisement levels from sellers in electronic marketplaces with recommender systems are likely be lower than in marketplaces without recommender systems. The lower demand for traditional advertising may lead to a decrease in the price of traditional advertising, contributing to a further decline in revenue for traditional advertisers. A worthwhile empirical study is to examine the trend of total revenue of the advertising industry and isolate the role played by marketplaces with recommender systems in this revenue.

The second set of implications relates to the scenario in which recommender systems are inevitable. Regardless of whether recommender systems benefit or hurt the sellers and the marketplace, if recommender systems are indeed here to stay, an improvement in precision benefits sellers and the marketplace. This result provides partial support to marketplaces' continuing efforts to improve the accuracy of recommender system in predicting consumer preference. On the other hand, the findings also suggest that while consumers can expect better recommendations from the marketplaces, consumers may end up paying more than the value of such recommendations in the form of increased prices. Not only the marketplaces, but also the sellers have an incentive to adopt targeted advertising with a high degree of precision. A valuable extension of this research is to empirically examine the relationship between the trends of recommender system precision and sellers' adoption of targeted advertising.

CHAPTER 4

RECOMMENDER SYSTEM INDUCED BUNDLING OF COMPLEMENTARY PRODUCTS: ROLES OF PRODUCT COMPATIBILITY AND COMPETITION

4.1 Synopsis

Recommender systems are used extensively in online marketplaces. One common type of recommender systems is when consumers are browsing a focal product page, hyperlinks to related products are displayed as a recommendation list in that page. For example, Amazon.com uses a recommendation feature labeled "customers who bought this item also bought" these other items. As the label indicates, this feature lists the products that customers purchased as well either simultaneously with the focal product or during other sessions. Due to the visibility of associated products, consumers can broaden their search space and become aware of products they may not be aware of otherwise (Tam and Ho, 2006). As a result, recommender system can affect the magnitude and the nature of influence that products have on each other's demand levels (Oestreicher-Singer and Sundararajan, 2012b).

While recommendations of either competing products or complementary products increase consumer awareness, these two types of recommendations serve different purposes. By recommending competing products, retailer is aim to help buyers find their "ideal" product. By recommending complementary products, the retailer aims to cross sell other products and increase total sales. In the literature, most studies either focus on recommender systems that recommend competing products or recommender systems that recommend complementary products. For example, Fleder and Hosanagar (2009) examine the effect of a recommender system that recommends competing products and show that such recommender systems can reinforce the popularity of already popular products. Jabr and Zheng (2013) find that higher referral centrality of competing products is associated with lower product sales indicating recommender systems intensify competition among products. Hinz and Eckert (2010) show that recommender systems that recommend competing products yield a substitution effect from popular products to niche products. Oestreicher-Singer and Sundararajan (2012b) find that such recommending complementary products can lead to up to an average threefold amplification of the influence that products have on each other's demand.

To the best of our knowledge, few studies have evaluated the overall effect of recommender systems when both competing and complementary products are included, even though such mixed recommender systems are almost universal in electronic marketplaces. More importantly, such mixed recommender systems affect the competition between products at various levels such as the competition between the focal product and competing products in the same category, the competition between the products that are complementary to the focal product, as well as the interaction between the competing products and complementary products. These aspects are especially relevant in an electronic marketplace because sellers and the marketplace that provides the recommendations are independent parties. We develop an game-theoretic model to derive insights regarding how mixed recommender systems affect the various parties in an electronic marketplace.

Our study is also related to the stream of literature that analyzes bundling of complementary products. In the literature, most studies tend to focus on duopoly setting where each firm produces and sells two complementary products. Matutes and Regibeau (1992) consider a model in which all consumers must purchase both complementary products to derive any positive utility, they showed how firms determine the optimal bundle and pricing strategy. Sinitsyn (2016) analyzed the case where firms can choose the degree of complementary between their complementary products before pricing their products. They found that both firms get the highest profits when both firms do not employ complementary enhancing strategies. In their model, the additional value for purchasing two complementary products are constant for all consumers, while in our model, the additional value for purchasing two complementary products depends on the consumer's preferences. There are also some studies that examine the competition among three or four firms. Denicolo (2000) considers three firms where one generalist firm offers two complementary products and the other two specialist firms each produce one product only, they show that the generalist firm will pursue pure bundling when one product is less differentiated than the other. Gans and King (2006) considers the interaction between four firms that produce and sell two products. They examine the case when two firms can cooperate to offer a price discount to consumers who buy their products as a bundle. Our study differs from them in that firms can not decide whether to "bundle" or not, the "bundle" is cased by recommendations.

4.2 Model

We consider a channel structure with four firms that sell in two categories (X and Y), an electronic marketplace (R), and a continuum of consumers with heterogeneous preferences. In category X, there are two competing firms A and B that each sells one product. In category Y, there are two competing firms C and D that each sells one product. All the products are sold via R, each seller sets the price of its product, and the marketplace charges the sellers a commission equal to γ fraction of the price on each sale.

Consumer preferences: Consumers have heterogeneous preferences in both categories. In particular, we model that consumers are uniformly distributed on the unit square. With regards to category X, consumers can be viewed as arrayed along the horizontal unit interval. A consumer's location on the line is denoted by x. Firm A is located at $x = 0$ and firm B is located as $x = 1$. If a consumer located at x purchases product A, then the consumer get net utility, $v - t_1 x - p_A$, where v is the consumer's base value and t_1 is the unit misfit cost in category X. If the consumer purchases product B, then the consumer gains net utility $v - t_1(1-x) - p_B$. With regards to category Y, we use an analogous structure. A consumer's location is denoted by y along the vertical unit interval with firm C located at $y = 0$ and firm

D located at $y = 1$. If a consumer located at y purchases product C, then the consumer get net utility, $w - t_2 y - p_c$, where w is the consumer's base value of the product and t_2 is the unit misfit cost in category Y. If the consumer purchases product D, then the consumer gains net utility $w - t_2(1-y) - p_D$. Consumers purchase at most one unit of product from each category, but consumer may choose to purchase both complementary products. Specifically, there are four possible bundles: AC, AD, BC and BD.

Degree of complementarity: Products in category X and in category Y are complementary products. For instance, if consumers purchase product A in category X and product C in category Y together, consumers gain a higher utility than the additive sum of utility from purchasing A alone and utility from purchasing C alone. Following the literature (Venkatesh and Kamakura, 2003; Subramaniam and Venkatesh, 2009), we define the degree of complementarity ρ between two products, i and j as follows:

$$\rho = \frac{\text{Utility for bundle } ij}{\text{Utility for } i + \text{Utility for } j}$$

We denote the degree of complementarity between product A and product C as ρ_1, thus the utility for purchasing A and C together is $(1+\rho_1)(v + w - t_1 x - t_2 y)$, and the net utility is $(1+\rho_1)(v + w - t_1 x - t_2 y) - p_A - p_C$. We assume product C has a better match with product A than with product B (degree of complementarity between A and C is ρ_1 and degree of complementary between B and C is ρ_2, where $\rho_2 \leq \rho_1$). Similarly, product D has a better match with product B than with product A (degree of complementarity between B and D is ρ_1 and degree of complementary between A and D is ρ_2).

The sequence of events is as follows. In Stage 1, sellers set prices p_A, p_B, p_C, and p_D simultaneously. In Stage 2, consumers make their purchase decisions. Two scenarios are considered: (i) without the recommender system and (ii) with the recommender system.

4.3 Impacts of Recommender System

In this section, using backward induction, we first derive the subgame perfect equilibrium for the case with the recommender system and for the case without the recommender system. We then analyze the implications of the recommender system.

4.3.1 With Recommendation

In the presence of recommender system, consumers get recommendations for the competing product as well as the complementary products. For example, a consumer who is browsing product A will be recommended the competing product B and complementary products C and D. Therefore, consumers are fully informed of four products in the presence of recommender system. Consumers can mix and match so that four bundles are available: AC, AD, BC and BD. When $\rho_1 - \rho_2$ is relatively small, consumers are divided as in Figure 4.1. When $\rho_1 - \rho_2$ is large, consumers would only purchase the highly matched bundles, that is, AC or BD. We focus on the first case where $\rho_1 - \rho_2$ is relatively small, because this case is more interesting and leads to a richer pattern of consumers demands with positive demand for all four bundles. For ease of exposition, we denote $u = v + w$. The demand functions for the four products can be formulated as follows:

$$D_A = \tfrac{1}{2} + \tfrac{4t_2(1+\rho_1)(1+\rho_2)-(2u-t_1-t_2)(\rho_1-\rho_2)^2}{4t_1t_2(1+\rho_1)(1+\rho_2)(2+\rho_1+\rho_2)}(p_B - p_A) + \tfrac{(\rho_1-\rho_2)(2u-t_1-t_2)}{4t_1t_2(1+\rho_1)(1+\rho_2)}(p_D - p_C)$$

$$D_B = \tfrac{1}{2} + \tfrac{4t_2(1+\rho_1)(1+\rho_2)-(2u-t_1-t_2)(\rho_1-\rho_2)^2}{4t_1t_2(1+\rho_1)(1+\rho_2)(2+\rho_1+\rho_2)}(p_A - p_B) + \tfrac{(\rho_1-\rho_2)(2u-t_1-t_2)}{4t_1t_2(1+\rho_1)(1+\rho_2)}(p_C - p_D)$$

$$D_C = \tfrac{1}{2} + \tfrac{4t_1(1+\rho_1)(1+\rho_2)-(2u-t_1-t_2)(\rho_1-\rho_2)^2}{4t_1t_2(1+\rho_1)(1+\rho_2)(2+\rho_1+\rho_2)}(p_D - p_C) + \tfrac{(\rho_1-\rho_2)(2u-t_1-t_2)}{4t_1t_2(1+\rho_1)(1+\rho_2)}(p_B - p_A)$$

$$D_D = \tfrac{1}{2} + \tfrac{4t_1(1+\rho_1)(1+\rho_2)-(2u-t_1-t_2)(\rho_1-\rho_2)^2}{4t_1t_2(1+\rho_1)(1+\rho_2)(2+\rho_1+\rho_2)}(p_C - p_D) + \tfrac{(\rho_1-\rho_2)(2u-t_1-t_2)}{4t_1t_2(1+\rho_1)(1+\rho_2)}(p_B - p_A)$$

The sellers maximize their profits by choosing their optimal prices:

$$\max_{p_i} \pi_i = (1-\gamma)p_i D_i \tag{4.1}$$

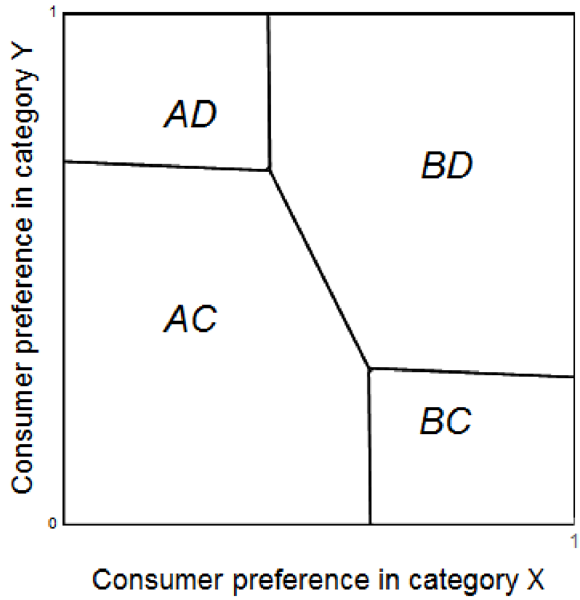


Figure 4.1. Demand Division with Recommender System

where $i \in \{A, B, C, D\}$ and γ is the commission charged by the marketplace. Based on their best response to each other, we obtain the equilibrium price and profit for each seller. The following lemma summarizes the equilibrium outcomes.

Lemma 10. *When $\rho_1 - \rho_2 < \frac{t_2(1+\rho_1)}{u}$, in the presence of recommender system, equilibrium outcomes are as follows:*

(a) prices:

$$p_A^* = p_B^* = \frac{2t_1t_2(1+\rho_1)(1+\rho_2)(2+\rho_1+\rho_2)}{4t_2(1+\rho_1)(1+\rho_2)-(\rho_1-\rho_2)^2(2u-t_1-t_2)} \tag{4.2}$$

$$p_C^* = p_D^* = \frac{2t_1t_2(1+\rho_1)(1+\rho_2)(2+\rho_1+\rho_2)}{4t_1(1+\rho_1)(1+\rho_2)-(\rho_1-\rho_2)^2(2u-t_1-t_2)} \tag{4.3}$$

(b) seller profits:

$$\pi_A^* = \pi_B^* = \frac{t_1t_2(1+\rho_1)(1+\rho_2)(2+\rho_1+\rho_2)}{4t_1(1+\rho_1)(1+\rho_2)-(\rho_1-\rho_2)^2(2u-t_1-t_2)}(1-\gamma) \tag{4.4}$$

$$\pi_C^* = \pi_D^* = \frac{t_1t_2(1+\rho_1)(1+\rho_2)(2+\rho_1+\rho_2)}{4t_2(1+\rho_1)(1+\rho_2)-(\rho_1-\rho_2)^2(2u-t_1-t_2)}(1-\gamma) \tag{4.5}$$

(c) retailer profit:

$$\pi_R^* = \gamma\frac{2t_1t_2(1+\rho_1)(1+\rho_2)(2+\rho_1+\rho_2)}{4t_1(1+\rho_1)(1+\rho_2)-(\rho_1-\rho_2)^2(2u-t_1-t_2)} + \gamma\frac{2t_1t_2(1+\rho_1)(1+\rho_2)(2+\rho_1+\rho_2)}{4t_2(1+\rho_1)(1+\rho_2)-(\rho_1-\rho_2)^2(2u-t_1-t_2)} \tag{4.6}$$

From the above lemma, the competition in each category is affected by two important factors. One is the category level competition (represented by t_1 and t_2), the other one is the degree of complementarity (represented by ρ_1 and ρ_2). Next corollary summarized the effect of the category level competition.

Corollary 3. *When $\rho_1 \neq \rho_2$, $p_A^*(p_B^*)$ decreases in t_2; $p_C^*(p_D^*)$ decreases in t_1; When $\rho_1 = \rho_2$, product differentiation in one category does not affect the other category.*

As Corollary 3 shows, keeping everything else constant, the lower the product differentiation in the complimentary category, the higher the price in the focal category. Intuitively, low product differentiation indicates that the intrinsic competition in that category is high, sellers in one category can benefit from the high competition among sellers in the other category. Next Proposition summarizes the effects of the degree of complementarity between products.

Proposition 12. *(a) In symmetric case ($\rho_1 = \rho_2 = \rho$), $p_A^*(p_B^*)$ and $p_C^*(p_D^*)$ are always higher than that in asymmetric case ($\rho_1 = \rho$ and $\rho_2 = 0$).*

(b) When ρ_1 increases, $p_A^*(p_B^*)$ and $p_C^*(p_D^*)$ always increase.

(c) When ρ_2 increases from 0 to ρ_1, $p_A^*(p_B^*)$ and $p_C^*(p_D^*)$ may first decrease then increase. There are three subcases:

(c.1) when $\rho_1 < 2\sqrt{\frac{2u-t_1}{2u-t_1-t_2}} - 2$, $p_A^*(p_B^*)$ and $p_C^*(p_D^*)$ always increase in ρ_2;

(c.2) when $\rho_1 > 2\sqrt{\frac{2u-t_2}{2u-t_1-t_2}} - 2$, $p_A^*(p_B^*)$ and $p_C^*(p_D^*)$ first decrease then increase in ρ_2;

(c.3) when $2\sqrt{\frac{2u-t_1}{2u-t_1-t_2}} - 2 < \rho_1 < 2\sqrt{\frac{u-t_2}{2u-t_1-t_2}} - 2$, $p_A^*(p_B^*)$ first decrease then increase in ρ_2, $p_C^*(p_D^*)$ always increase.

Interestingly, we find that all sellers are better off when products have the same level of match (symmetric case) than the case when products have different level of match (asymmetric case). While increasing the level of match of the high matched bundle (e.g., AC and BD) always benefits sellers, it is not necessarily the case with regard to the level of match of the low matched bundle (e.g., AD and BC). As Proposition 12(c) shows, when the level of match of the low matched bundle increases from 0, prices may decrease.

4.3.2 No Recommendation

Without recommendations, consumers make their purchases according to their initial awareness of the four products. We assume α fraction of consumers are aware of both products A and B and $\frac{1-\alpha}{2}$ fraction of consumers are aware of only either product A or B. With probability β, a consumer knows the complementary product, either C or D. Therefore, with probability $\beta(1-\beta)$, a consumer only knows C or only knows D; with probability β^2, a consumer knows both C and D; with $(1-\beta)^2$, a consumer does not know any complementary products. We derive the demand functions for each seller in the absence of recommendation. Using backward induction, we derive the subgame perfect equilibrium summarized as follows:

Lemma 11. *When $\rho_1 - \rho_2 \leq \frac{t_2(1+\rho_1)}{u}$, in the absence of recommender system, equilibrium outcomes are as follows:*

(a) prices:

$$\bar{p}_A = \bar{p}_B = \frac{1}{\alpha} \frac{2t_1 t_2(1+\rho_1)(1+\rho_2)(2+\rho_1+\rho_2)}{4t_2(1+\rho_1)(1+\rho_2)\left(1+\frac{(1-\beta)^2(\rho_1+\rho_2)}{2}\right)-\beta^2(\rho_1-\rho_2)^2(2u-t_1-t_2)} \tag{4.7}$$

$$\bar{p}_C = \bar{p}_D = \frac{(2-\beta)}{\beta} \frac{2t_1 t_2(1+\rho_1)(1+\rho_2)(2+\rho_1+\rho_2)}{4t_1(1+\rho_1)(1+\rho_2)-\alpha(\rho_1-\rho_2)^2(2u-t_1-t_2)} \tag{4.8}$$

(b) seller profits:

$$\bar{\pi}_A = \bar{\pi}_B = \frac{(1-\gamma)\bar{p}_A}{2}$$
$$\bar{\pi}_C = \bar{\pi}_D = \frac{\beta(2-\beta)(1-\gamma)\bar{p}_C}{2}$$

(c) retailer profit:

$$\bar{\pi}_R = \gamma\bar{p}_A + \gamma\beta(2-\beta)\bar{p}_C$$

The equilibrium prices and profits depend on customer initial awareness (α and β). If all consumers are fully informed of all the products even without recommendation, the results are equivalent to the case with recommender system. In next proposition, we show the effect of consumers awareness.

Proposition 13. *(a)* $\bar{p}_A(\bar{p}_B)$ *decreases in* α *and increases in* β*;* $\bar{p}_C(\bar{p}_D)$ *increases in* α *and decreases in* β*.*

(b) $\bar{\pi}_A(\bar{\pi}_B)$ *decreases in* α *and increases in* β*;* $\bar{\pi}_C(\bar{\pi}_D)$ *increases in* α *and decreases in* β*.*

(c) when $\rho_1 = \rho_2$*, marketplace profit is higher in the presence of recommender system than in the absence of recommender system if and only if* $\alpha > \frac{t_1}{(1+(1-\beta)^2\rho_1)(t_1-t_2(3-\beta)(1-\beta))}$*.*

On one hand, the price in category X decreases in α but increases in β. On the other hand, the price in category Y increases in α but decreases in β. Thus the overall profit of the marketplace from category X and Y does not necessarily increase in α and β. In other words, recommender system can either increase or decrease the retailer profit. As proposition 13(c) shows, in the symmetric case where all products have the same level of complementarity, recommender system does not necessarily benefit the marketplace. As proposition 13(C) shows, in the symmetric case where all products have the same level of complementarity, recommender system benefits the marketplace if and only if $\alpha > \frac{t_1}{(1+(1-\beta)^2\rho_1)(t_1-t_2(3-\beta)(1-\beta))}$.

4.4 Discussion and Conclusions

In this study, we examine the economic impact of recommender system that recommends both competing products and complementary products. The system is applied in a competitive environment where independent sellers set their prices respectively. In particular, we choose a channel structure with a common e-commerce platform and four sellers selling products in two categories. Our initial results indicate that such recommender system does not necessarily benefit the marketplace, on one side, recommender system increases the total sales, on the other hand, recommender system alters the competition in each category. In the presence of recommender system, the price and profit of each seller critically depends on the degree of complementary among products. Surprisingly, sellers are better off when products have the same level of match (symmetric case) than the case when products have different level of match (asymmetric case).

APPENDIX

Proof of Lemma 1

Proof. (a) Each manufacturer's best response to its competitor in stage 1 is characterized by the first-order conditions of Equation (2.6):

$$\begin{aligned}\frac{\partial \pi_A}{\partial p_A} &= (1-\alpha)\left(\theta + \frac{p_B - 2p_A + t_S}{2t_S}\theta_b + \gamma\frac{v_L}{t_L} - \frac{2\gamma}{t_L}p_A\right) = 0 \\ \frac{\partial \pi_B}{\partial p_B} &= (1-\alpha)\left(\theta + \frac{p_A - 2p_B + t_S}{2t_S}\theta_b + \gamma\frac{v_L}{t_L} - \frac{2\gamma}{t_L}p_B\right) = 0\end{aligned}$$

Based on these equations, we can derive the manufacturer equilibrium prices as in Equation (2.7).

(b) Substituting the equilibrium prices into Equation (2.5), we can derive the equilibrium demands as in Equation (2.8).

(c) Substituting the equilibrium prices into Equation (2.6), we can derive the equilibrium profits as in Equation (2.9). Assumption 1, $v_L > \frac{(2\theta+\theta_b)t_S}{(\theta_b + 2\gamma t_S/t_L)}$ can guarantee $\gamma(v_L - \bar{p}_i^*)/t_L > 0$, hence some loyal consumers would purchase in the equilibrium. Assumption 2, $v_S > t_S\frac{2(\theta+\theta_b)+2\gamma(2t_S+v_L)/t_L}{\theta_b+4\gamma t_S/t_L}$ can guarantee $v_S - \bar{p}_i^* - t_S > 0$, hence even the shopper with the largest misfit cost has incentive to purchase if she is aware of the product.

(d) According the commission scheme, we have $\frac{\pi_R}{\pi_A+\pi_B} = \frac{\alpha}{1-\alpha}$. Based on π_i^*, we can derive π_R^* as in Equation (2.10).

(e) The consumer surplus from the loyal consumers is

$$CS_L = 2\gamma\int_0^{\frac{v_L-\bar{p}^*}{t_L}}(v_L - \bar{p}^* - zt_L)dz = \gamma\frac{(v_L-\bar{p}^*)^2}{t_L} \tag{A.1}$$

Next we derive the consumer surplus for shoppers. The consumer surplus from each partially informed segment is

$$CS_S^p = \theta\int_0^1(v_S - \bar{p}^* - zt_S)dz = \theta(v_S - \bar{p}^* - \tfrac{t_S}{2}) \tag{A.2}$$

and from the fully informed segment is

$$CS_S^f = \theta_b\left[\int_0^{\frac{1}{2}}(v_S-\bar{p}^*-zt_S)dz + \int_{\frac{1}{2}}^1[v_S - \bar{p}^* - (1-z)t_S]\,dz\right] = \theta_b\left(v_S - \bar{p}^* - \tfrac{t_S}{4}\right) \tag{A.3}$$

The consumer surplus from the uninformed segment is zero. Aggregating the consumer surplus from the loyal consumers, the two partially informed shopper segments and one fully informed shopper segment, we have

$$CS = CS_L + CS_S^p + CS_S^f = \gamma \frac{(v_L - \bar{p}^*)^2}{t_L} + (2\theta + \theta_b)(v_S - \bar{p}^*) - t_S(\theta + \frac{\theta_b}{4}) \tag{A.4}$$

Substituting $\bar{p}^*$ in Equation (2.5), we derive the total surplus as in Equation (2.11).

(f) Social welfare is the sum of consumer surplus, manufacturer's profits, and retailer's profit; that is, $W = CS + \bar{\pi}_A^* + \bar{\pi}_B^* + \bar{\pi}_R^*$, which can be derived as in Equation (2.12). □

Proof of conditional expectation of misfit

Proof. The cumulative density function of s, conditional on the consumer's true location λ being z, can be formulated as:

$$P(s \leqslant y | \lambda = z) = (1 - \beta)y + \beta H(y - z) \tag{A.5}$$

where $H(\cdot)$ is the Heaviside step function that evaluates to zero if the argument is negative, and to one otherwise. The corresponding probability density function is:

$$P(s = y | \lambda = z) = (1 - \beta) + \beta\delta(y - z) \tag{A.6}$$

where $\delta(x)$ is the Dirac delta distribution that satisfies $\int_{-\infty}^{\infty} \delta(x)dx = 1$ and $\delta(x) = 0$ if $x \neq 0$, $\delta(x) = \infty$ if $x = 0$.

Using the Bayes Law,

$$P(\lambda = z | s = y) = \frac{P(s = y | \lambda = z)P(\lambda = z)}{P(s = y)} = (1 - \beta) + \beta\delta(y - z) \tag{A.7}$$

and the conditional expectation is:

$$\mathbb{E}(\lambda = z | s = y) = \frac{1 - \beta}{2} + \beta y \tag{A.8}$$

□

Proof of Lemma 2

Proof. The manufacturers maximize their profits by choosing their optimal prices; that is,

$$\max_{p_i} \pi_i = (1-\alpha)p_i(D_{iL}+D_{iS}) \tag{A.9}$$

where D_{iS} is defined in Equation (2.17) and D_{iL} is defined in Equation (2.3).

(a) Each manufacturer's best response to its competitor in stage 1 is characterized by the first-order conditions of Equation (A.9)

$$\begin{aligned}\frac{\partial \pi_A}{\partial p_A} &= (1-\alpha)\left[\frac{1}{2}+\frac{\beta(\theta+\theta_b)+(1-\theta-\theta_b)(1-\alpha w)}{2\beta t_S}\left(p_B-2p_A\right)+\gamma\frac{v_L}{t_L}-\frac{2\gamma}{t_L}p_A\right]=0\\ \frac{\partial \pi_B}{\partial p_B} &= (1-\alpha)\left[\frac{1}{2}+\frac{\beta(\theta+\theta_b)+(1-\theta-\theta_b)(1-\alpha w)}{2\beta t_S}\left(p_A-2p_B\right)+\gamma\frac{v_L}{t_L}-\frac{2\gamma}{t_L}p_A\right]=0\end{aligned}$$

Based on these equations, we can derive the manufacturer equilibrium prices as in Equation (2.18).

We next check the second-order derivatives,

$$\begin{aligned}\frac{\partial^2 \pi_A}{\partial p_A^2} &= -\left(1-\alpha\right)\frac{\beta(\theta+\theta_b)+(1-\theta-\theta_b)(1-\alpha w)+2\beta\gamma t_S/t_L}{t_S\beta}\\ \frac{\partial^2 \pi_B}{\partial p_B^2} &= -\left(1-\alpha\right)\frac{\beta(\theta+\theta_b)+(1-\theta-\theta_b)(1-\alpha w)+2\beta\gamma t_S/t_L}{t_S\beta}\end{aligned}$$

Assumption 3, $\beta > \frac{(1-\theta-\theta_b)(\alpha w-1)}{\theta+\theta_b+2\gamma t_S/t_L}$ can guarantee that the profit function is concave and the maximization problem is well behaved.

(b) Part (b) follows by substituting the equilibrium prices into Equation (2.17) and Equation (2.3).

(c) Part (c) follows by substituting the equilibrium prices into Equation (A.9). Assumption 1, $v_L > \frac{\beta t_S}{[(1-\alpha w)(1-\theta-\theta_b)+\beta(\theta+\theta_b)+2\beta\gamma t_S/t_L]}$ can guarantee $\gamma(v_L-p_i^*)/t_L > 0$, hence some loyal consumers would purchase in the equilibrium. Assumption 2, $v_S > t_S\frac{(1-\alpha w)(1-\theta-\theta_b)+\beta(1+\theta+\theta_b)+2\beta\gamma(2t_S+v_L)/t_L}{(1-\alpha w)(1-\theta-\theta_b)+\beta(\theta+\theta_b)+4\beta\gamma t_S/t_L}$ can guarantee $v_S-p_i^*-t_S>0$, hence even the shopper with the largest misfit cost has incentive to purchase if she is aware of the product.

(d) According the commission scheme, we have $\frac{\pi_R}{\pi_A+\pi_B}=\frac{\alpha}{1-\alpha}$. Based on π_i^*, we can derive π_R^* as in Equation (2.21).

(e) The consumer surplus from the loyal consumers for manufacturer i is similar as Equation (A.1); that is, $CS_L = \gamma\frac{(v_L-p^*)^2}{t_L}$. The consumer surplus from each partially-informed segment of shoppers is:

$$\begin{aligned} CS_S^p =& \theta\left[\int_0^{\frac{1}{2}}(v_S-p^*-zt_S)dz+\tfrac{1-\beta}{2}\int_{\frac{1}{2}}^1(v_S-zt_S-p^*)dz+\tfrac{1+\beta}{2}\int_{\frac{1}{2}}^1[v_S-(1-z)t_S-p^*]dz\right] \\ =& \theta\left[v_S-p^*-\frac{t_S}{2}+\frac{(1+\beta)t_S}{8}\right] \end{aligned} \tag{A.10}$$

The consumer surplus from the fully-informed segment of shoppers is

$$\begin{aligned} CS_S^f =& \theta_b\left[\int_0^{\frac{1}{2}}(v_S-p^*-zt_S)dz+\int_{\frac{1}{2}}^1\left[v_S-p^*-(1-z)t_S\right]dz\right] \\ =& \theta_b(v_S-p^*-\frac{t_S}{4}) \end{aligned} \tag{A.11}$$

The consumer surplus from the uninformed segment of shoppers is

$$\begin{aligned} CS_S^u =& \; 2(1-2\theta-\theta_b)\left[\tfrac{1+\beta}{2}\int_0^{\frac{1}{2}}(v_S-p^*-zt_S)dz+\tfrac{1-\beta}{2}\int_0^{\frac{1}{2}}[v_S-p^*-(1-z)t_S]dz\right] \\ =& \; (1-2\theta-\theta_b)\left[v_S-p^*-\tfrac{t}{4}-\tfrac{(1-\beta)t_S}{4}\right] \end{aligned} \tag{A.12}$$

Aggregating the consumer surplus from the loyal consumers and the shoppers, we can derive the total surplus as follows:

$$CS = CS_L + CS_S^p + CS_S^f = v_S - \tfrac{t_S}{4}\left[1+(1-\beta)(1-\theta-\theta_b)\right] - p^* + \gamma\tfrac{(v_L-p^*)^2}{t_L} \tag{A.13}$$

By Substituting the equilibrium price p^* in Equation (2.18), we derive the total surplus as in Equation (2.22).

(f) Social welfare is the sum of consumer surplus, manufacturer's profits, and retailer's profit; that is, $W = CS + \pi_A^* + \pi_B^* + \pi_R^*$, which can be derived as in Equation (2.23). □

Proof of Proposition 1

Proof. (a) For ease of exposition, we denote $M \equiv (1-\alpha w)(1-\theta-\theta_b)+\beta(\theta+\theta_b)+4\beta\tau t_S$, note that

$$p_i^* - \bar{p}_i^* = t_S\frac{\beta[\theta_b-(2\theta+\theta_b)(\theta_b+\theta)+4t_S\tau(1-2\theta-\theta_b)-2\theta h]-(1-\alpha w)(1-\theta-\theta_b)[2\theta+\theta_b+2h]}{M(\theta_b+4t_S\tau)}$$

p_i^* is less than $\bar{p}_i^*$ if the numerator is negative, which leads to the condition in Part (a).

(b)Note that

$$\begin{aligned} D_i^* - \bar{D}_i^* &= \left(\tfrac{1}{2}+h\right)\left(1-\tfrac{2\beta t_S\tau}{M}\right) - \left(\tfrac{2\theta+\theta_b}{2}+h\right)\left(1-\tfrac{2t_S\tau}{\theta_b+4t_S\tau}\right) \\ &= \tfrac{1-2\theta-\theta_b}{2} + t_S\tau\left(\tfrac{2\theta+\theta_b+2h}{\theta_b+4t_S\tau} - \tfrac{(1+2h)\beta}{M}\right) \end{aligned}$$

Hence, $D_i^* > \bar{D}_i^*$ under the condition in Part (b).

(c) Note that

$$\begin{aligned} \pi_i^* - \bar{\pi}_i^* =& (1-\alpha)\beta t_S\left(\tfrac{1+2h}{M}\right)^2\left(\tfrac{(1-\theta-\theta_b)(1-\alpha w)+(\theta+\theta_b)\beta}{2}+\beta t_S\tau\right) - (1-\alpha)t_S\left(\tfrac{2\theta+\theta_b+2h}{\theta_b+4t_S\tau}\right)^2\left(\tfrac{\theta_b}{2}+t_S\tau\right) \\ =& -(1-\alpha)\tfrac{t_S}{M^2}\left[\left(\tfrac{2\theta+\theta_b+2h}{\theta_b+4t_S\tau}\right)^2\left(\tfrac{\theta_b}{2}+t_S\tau\right)M^2 - (1+2h)^2\tfrac{\beta}{2}M + (1+2h)^2\beta^2 t_S\tau\right] \end{aligned}$$

$\pi_i^* - \bar{\pi}_i^* > 0$ when

$$\left(\tfrac{2\theta+\theta_b+2h}{\theta_b+4t_S\tau}\right)^2\left(\tfrac{\theta_b}{2}+t_S\tau\right)M^2 - (1+2h)^2\tfrac{\beta}{2}M + (1+2h)^2\beta^2 t_S\tau < 0$$

By solving the above inequality, we can get the condition in Part (c).

(d) From Equation (A.13) and Equation (A.4), we get:

$$\begin{aligned} CS - \bar{CS} =& v_S - \tfrac{t_S}{4}\left[1+(1-\beta)(1-\theta-\theta_b)\right] - p_i^* + \gamma\tfrac{(v_l-p^*)^2}{t_L} - \left[(2\theta+\theta_b)(v_S-\bar{p}_i^*) - t_S(\theta+\tfrac{\theta_b}{4}) + \gamma\tfrac{(v_l-\bar{p}_i^*)^2}{t_L}\right] \\ =& (1-2\theta-\theta_b)\left[v_S - \tfrac{t_S}{4}(2-\beta) - p_i^*\right] + \tfrac{t_S}{4}\theta(1+\beta) + (\bar{p}_i^* - p_i^*)\left[2\theta+\theta_b+\tfrac{\gamma}{t_L}(2v_L-\bar{p}_i^*-p_i^*)\right] \end{aligned} \tag{A.14}$$

From above equation, we can get $\frac{\partial\left(CS-\bar{CS}\right)}{\partial w} == -\frac{\partial p_i^*}{\partial w}\left[1+\frac{2\gamma v_L}{t_L}-\frac{2\gamma}{t_L}p_i^*\right]$. According to Proposition 3 (a), $\frac{\partial p_i^*}{\partial w} > 0$, so we can $\frac{\partial\left(CS-\bar{CS}\right)}{\partial w} < 0$. $CS - \bar{CS} > 0$ under the condition in Part (d).

(e) From Equation (A.13) and Equation (A.4), we get:

$$\begin{aligned} W - \bar{W} =& v_S - \tfrac{t_S}{4}\left[1+(1-\beta)(1-\theta-\theta_b)\right] - p_i^* + \gamma\tfrac{(v_l-p^*)^2}{t_L} - \left[(2\theta+\theta_b)(v_S-\bar{p}_i^*) - t_S(\theta+\tfrac{\theta_b}{4}) + \gamma\tfrac{(v_l-\bar{p}_i^*)^2}{t_L}\right] \\ =& (1-2\theta-\theta_b)\left[v_S - \tfrac{t_S}{4}(2-\beta) - p_i^*\right] + \tfrac{t_S}{4}\theta(1+\beta) + (\bar{p}_i^* - p_i^*)\left[\tfrac{3\gamma}{t_L}(\bar{p}^*-p_i^*) - \tfrac{2\gamma v_L}{t_L}\right] \end{aligned} \tag{A.15}$$

From above equation, we can get $\frac{\partial\left(W-\bar{W}\right)}{\partial w} == 2\frac{\partial p_i^*}{\partial w}\left[\frac{2\gamma v_L}{t_L}-\frac{3\gamma}{t_L}p_i^*\right]$. Under the condition $\frac{1-\theta-\theta_b}{\beta} + (\theta+\theta_b) < (2+\frac{3t_L}{\gamma v_L})\frac{\gamma}{t_L}t_S$, we have $\frac{2\gamma v_L}{t_L} - \frac{3\gamma}{t_L}p_i^* < 0$, so $\frac{\partial\left(CS-\bar{CS}\right)}{\partial w} < 0$. $W - \bar{W} > 0$ under the condition in Part (e). + $\square$

Proof of Corollary 1

Proof. (i) When the recommender system is consumer oriented (i.e.,$w \leq \frac{1}{\alpha}$), by Inequality (2.25), p_i^* is always less than $\bar{p}_i^*$ if $\theta_b - (2\theta + \theta_b)(\theta_b + \theta) - 2\theta h + 4\tau(1 - 2\theta - \theta_b)t_S < 0$, or, equivalently, if $\theta > \frac{\sqrt{(3\theta_b + 8t_S\tau + 2h)^2 + 8(4t_S\tau + \theta_b)(1-\theta_b)} - 3\theta_b - 8t_S\tau - 2h}{4}$. Otherwise, $p_i^* \leq \bar{p}_i^*$ if $\beta < \frac{(1-\theta-\theta_b)(1-\alpha w)(2\theta+\theta_b+2h)}{\theta_b-(2\theta+\theta_b)(\theta+\theta_b)+4t_S\tau(1-2\theta-\theta_b)-2\theta h}$ (by solving Inequality (2.25)).

(ii) When the recommender system is profit oriented (i.e.,$w > \frac{1}{\alpha}$), by Inequality (2.25), p_i^* is always greater than $\bar{p}_i^*$ if $\theta_b - (2\theta + \theta_b)(\theta_b + \theta) - 2\theta h + 4\tau(1 - 2\theta - \theta_b)t_S > 0$. Therefore, $\hat{p}_i < p_i$ requires $\theta > \frac{\sqrt{(3\theta_b + 8t_S\tau + 2h)^2 + 8(4t_S\tau + \theta_b)(1-\theta_b)} - 3\theta_b - 8t_S\tau - 2h}{4}$ and $\beta > \frac{(1-\theta-\theta_b)(1-\alpha w)(2\theta+\theta_b+2h)}{\theta_b-(2\theta+\theta_b)(\theta+\theta_b)+4t_S\tau(1-2\theta-\theta_b)-2\theta h}$ (by solving Inequality (2.25)). □

Proof of Proposition 2

Proof. (a) Because $\triangle p_i = p_i^* - \bar{p}_i^*$ and β does not affect $\bar{p}_i^*$,

$$\frac{\partial \triangle p_i}{\partial \beta} = \frac{\partial p_i^*}{\partial \beta} = \frac{t_S(1-\alpha w)(1-\theta-\theta_b)(1+2h)}{[(1-\alpha w)(1-\theta-\theta_b)+\beta(\theta+\theta_b)+4\beta t_S\tau]^2}$$

Because all the terms are always positive except $(1 - \alpha w)$, $\frac{\partial \triangle p_i}{\partial \beta} > 0$ if and only if $w < \frac{1}{\alpha}$.

(b) Because $\triangle D_i = D_i^* - \bar{D}_i^*$ and β does not affect $\bar{D}_i^*$,

$$\frac{\partial \triangle D_i}{\partial \beta} = \frac{\partial D_i^*}{\partial \beta} = \frac{t_S(\alpha w-1)(1-\theta-\theta_b)(1+2h)}{[(1-\alpha w)(1-\theta-\theta_b)+\beta(\theta+\theta_b)+4\beta t_S\tau]^2}$$

Because all the terms are always positive except $(\alpha w - 1)$, $\frac{\partial \triangle D_i}{\partial \beta} > 0$ if and only if $w > \frac{1}{\alpha}$.

(c) Because $\triangle \pi_i = \pi_i^* - \bar{\pi}_i^*$ and β does not affect $\bar{\pi}_i^*$,

$$\frac{\partial \triangle \pi_i}{\partial \beta} = \frac{\partial \pi_i^*}{\partial \beta} = \frac{t_S(1+2h)^2(1-\alpha)(1+2h)(1-\theta-\theta_b)(1-\alpha w)[(1-\alpha w)(1-\theta-\theta_b)+\beta(\theta+\theta_b)]}{2[(1-\alpha w)(1-\theta-\theta_b)+\beta(\theta+\theta_b)+4\beta t_S\tau]^3}$$

Because all the terms are always positive except $(1 - \alpha w)\left[(1 - \alpha w)(1 - \theta - \theta_b) + \beta(\theta + \theta_b)\right]$, Hence $\frac{\partial \triangle \pi_i}{\partial \beta} > 0$ if $w < \frac{1}{\alpha}$, or $w > \frac{1}{\alpha}$ and $w > \frac{\beta(\theta+\theta_b)+(1-\theta-\theta_b)}{\alpha(1-\theta-\theta_b)}$.

(d) Note that

$$\frac{\partial \triangle CS}{\partial \beta} = -\frac{t_S(1-\theta-\theta_b)}{4M^3}\left[4(2h+1)^2(1-\alpha w)\left[(1-\alpha w)(1-\theta-\theta_b) + \beta(\theta+\theta_b+2\tau t_S)\right] - M^3\right]$$

$\frac{\partial \triangle CS}{\partial \beta} < 0$ when $4(2h+1)^2(1-\alpha w)\left[(1-\alpha w)(1-\theta-\theta_b)+\beta(\theta+\theta_b+2\tau t_S)\right]-M^3>0$. By solving the above inequality, we can get the the condition in Part (d).

(e) Note that

$$\frac{\partial \triangle W}{\partial \beta} = -\frac{t_S(1-\theta-\theta_b)}{4M^3}\left[8(1+2h)(1-\alpha w)\left(3\beta t_S\tau - h\left[(1-\alpha w)(1-\theta-\theta_b)+\beta(\theta+\theta_b-2t_s\tau)\right]\right)-M^3\right]$$

By solving $\frac{\partial \triangle W}{\partial \beta} < 0$, we get the condition in part(e). □

Proof of Proposition 3

Proof. (a) Part (a) follows from

$$\frac{\partial \triangle p_i}{\partial w} = \frac{\partial p_i^*}{\partial w} = \frac{\alpha\beta t_S(1-\theta-\theta_b)(1+2h)}{\left[(1-\alpha w)(1-\theta-\theta_b)+\beta(\theta+\theta_b)+4\beta t_S\tau\right]^2} > 0$$

(b) Part (b) follows from

$$\frac{\partial \triangle D_i}{\partial w} = \frac{\partial D_i^*}{\partial w} = -\frac{\alpha\beta t_S\tau(1-\theta-\theta_b)(1+2h)}{\left[(1-\alpha w)(1-\theta-\theta_b)+\beta(\theta+\theta_b)+4\beta t_S\tau\right]^2} < 0$$

(c)Part (c) follows from

$$\frac{\partial \triangle \pi_i}{\partial w} = \frac{\partial \pi_i^*}{\partial w} = \frac{\alpha(1-\alpha)\beta t_S(1+2h)^2(1-\theta-\theta_b)\left[(1-\alpha w)(1-\theta-\theta_b)+\beta(\theta+\theta_b)\right]}{\left[(1-\alpha w)(1-\theta-\theta_b)+\beta(\theta+\theta_b)+4\beta t_S\tau\right]^3}$$

Hence, $\frac{\partial \triangle \pi_i}{\partial w}$ increases if and only if $(1-\alpha w)(1-\theta-\theta_b)+\beta(\theta+\theta_b)>0$ or $w<\frac{1}{\alpha}+\frac{\beta(\theta+\theta_b)}{\alpha(1-\theta-\theta_b)}$.

(d) Part (d) follows from $\frac{\partial\left(CS-\bar{CS}\right)}{\partial w} = -\frac{\partial p_i^*}{\partial w}\left[1+\frac{2\gamma v_L}{t_L}-\frac{2\gamma}{t_L}p_i^*\right] > 0$.

(e) Part (e) follows from $\frac{\partial\left(W-\bar{W}\right)}{\partial w} = 2\frac{\partial p_i^*}{\partial w}\left[\frac{2\gamma v_L}{t_L}-\frac{3\gamma}{t_L}p_i^*\right]$. Under the condition $\frac{1-\theta-\theta_b}{\beta}+(\theta+\theta_b)<(2+\frac{3t_L}{\gamma v_L})\frac{\gamma}{t_L}t_S$, we get $\frac{2\gamma v_L}{t_L}-\frac{3\gamma}{t_L}p_i^*<0$ and $\frac{\partial\left(W-\bar{W}\right)}{\partial w}<0$. □

Proof of Proposition 4

Proof. (a) Notice that

$$\begin{aligned}\frac{\partial \triangle p_i}{\partial \theta} &= \frac{\beta t_S(1+2h)(1-w\alpha-\beta)}{\left[(1-\alpha w)(1-\theta-\theta_b)+\beta(\theta+\theta_b)+4\beta t_S\tau\right]^2} - \frac{2t_S}{\theta_b+4t_S\tau}\\ &= \frac{\beta t_S(1+2h)(1-w\alpha-\beta)(\theta_b+4t_S\tau)-2t_S M^2}{M^2(\theta_b+4t\tau)}\end{aligned}$$

$\frac{\partial \triangle p_i}{\partial \theta} > 0$ requires $\beta t_S(1+2h)(1-w\alpha-\beta)\left(\theta_b + 4t_S\tau\right) > 2t_S M^2$. Therefore, when $1-w\alpha-\beta <$ 0, the first fraction on the right-hand side is negative and thus $\frac{\partial \triangle p_i}{\partial \theta} < 0$. When $1-w\alpha-\beta \geq 0$, or, equivalently, when $w \leq \frac{1-\beta}{\alpha}$, solving $\frac{\partial \triangle p_i}{\partial \theta} \geq 0$ leads to the condition on θ in Part (a).

(b) Notice that

$$\begin{aligned}\frac{\partial \triangle D_i}{\partial \theta} &= -\frac{\beta t_S\tau(1+2h)(1-w\alpha-\beta)}{[(1-\alpha w)(1-\theta-\theta_b)+\beta(\theta+\theta_b)+4\beta t_S\tau]^2} - \frac{\theta_b+2t_S\tau}{\theta_b+4t_S\tau}\\ &= -\frac{\beta t_S\tau(1+2h)(1-w\alpha-\beta)(\theta_b+4t_S\tau)+(\theta_b+2t_S\tau)M^2}{M^2(\theta_b+4t_S\tau)}\end{aligned}$$

$\frac{\partial \triangle D_i}{\partial \theta} > 0$ requires $\beta t_S\tau(1+2h)(1-w\alpha-\beta)\left(\theta_b+4t_S\tau\right) + \left(\theta_b+2t_S\tau\right)M^2 < 0$. Therefore, when $1-w\alpha-\beta > 0$, the first fraction on the right-hand side is negative and thus $\frac{\partial \triangle D_i}{\partial \theta} < 0$. When $1-w\alpha-\beta < 0$, or, equivalently, when $w > \frac{1-\beta}{\alpha}$, solving $\frac{\partial \triangle D_i}{\partial \theta} \geq 0$ leads to the condition on θ in Part (b).

(c) Notice that

$$\frac{\partial \triangle \pi_i}{\partial \theta} = \frac{t_S(1-\alpha)}{2}\left(\frac{(1+2h)^2\beta(1-\alpha w-\beta)[(1-\alpha w)(1-\theta-\theta_b)+\beta(\theta+\theta_b)]}{[(1-\alpha w)(1-\theta-\theta_b)+\beta(\theta+\theta_b)+4\beta t_S\tau]^3} - \frac{4(2t_S\tau+\theta_b)(2\theta+\theta_b+2h)}{(4t_S\tau+\theta_b)^2}\right)$$

When $(1-\alpha w-\beta)\left[(1-\alpha w)\left(1-\theta-\theta_b\right)+\beta(\theta+\theta_b)\right] < 0$, the first term in the bracket is negative, we get $\frac{\partial \triangle \pi_i}{\partial \theta} < 0$. Thus, $\frac{\partial \triangle \pi_i}{\partial \theta} > 0$ if and only if $(1-\alpha w-\beta)\left[(1-\alpha w)\left(1-\theta-\theta_b\right)+\beta(\theta+\theta_b)\right] > 0$ and $\frac{(1+2h)^2\beta(1-\alpha w-\beta)[(1-\alpha w)(1-\theta-\theta_b)+\beta(\theta+\theta_b)]}{[(1-\alpha w)(1-\theta-\theta_b)+\beta(\theta+\theta_b)+4\beta t\tau]^3} > \frac{4(2t\tau+\theta_b)(2\theta+\theta_b+2h)}{(4t\tau+\theta_b)^2}$.

(d) Note that

$$\frac{\partial \triangle CS}{\partial \theta} = -2v_S + \frac{(5-\beta)t_S}{4} + 4t_S\frac{(2\theta+\theta_b+2h)(\theta_b+3t_S\tau)}{(4t_S\tau+\theta_b)^2} - \beta t_S(1+2h)^2(1-\alpha w-\beta)\frac{(1-\alpha w)(1-\theta-\theta_b)+\beta(\theta+\theta_b)+2\beta t_S\tau}{[(1-\alpha w)(1-\theta-\theta_b)+\beta(\theta+\theta_b)+4\beta t_S\tau]^3}$$

Solving $\frac{\partial \triangle CS}{\partial \theta} > 0$ leads to the condition in Part (d).

(e) Part (e) follows from that

$$\frac{\partial \triangle W}{\partial \theta} = -2v_S + \frac{(5-\beta)t_S}{4} + 4t_S\frac{\tau t_S(6\theta+3\theta_b+2h)-\theta_b h}{(4\tau t_S+\theta_b)^2} + 2t_S\beta(1+2h)(1-\alpha w-\beta)\frac{(3+2h)\beta t_S\tau - h[(1-\alpha w)(1-\theta-\theta_b)+\beta(\theta+\theta_b)]}{[(1-\alpha w)(1-\theta-\theta_b)+\beta(\theta+\theta_b)+4\beta\tau t_S]^3}$$

Solving $\frac{\partial \triangle CS}{\partial \theta} \geq 0$ leads to the condition in Part (e). $\square$

Proof of Lemma 3

Proof. In the second stage, given w, each manufacturer's best response to its competitor is the same as those in lemma 2 as long as $\beta > \frac{(1-\theta-\theta_b)(\alpha w-1)}{\theta+\theta_b+2\gamma t_S/t_L}$, or $w < \frac{1}{\alpha} + \frac{\beta(\theta+\theta_b+2\gamma t_S/t_L)}{\alpha(1-\theta-\theta_b)}$. In the first stage, the retailer maximize their profits by choosing the optimal w; that is,

$$\max_{w} \pi_R = (1-\alpha)(p_A D_A + p_B D_B) \tag{A.16}$$

where D_A and D_B is defined in Equation (2.19) and p_A and p_B is defined in (2.18).

(a) The retailer's best response in stage 1 is characterized by the first-order conditions of Equation (A.16)

$$\frac{\partial \pi_R}{\partial w} = \frac{\beta t_s \alpha^2(1+2h)^2(1-\theta-\theta_b)[(1-\alpha w)(1-\theta-\theta_b)+\beta(\theta+\theta_b)]}{[(1-\alpha w)(1-\theta-\theta_b)+\beta(\theta+\theta_b)+4\beta t_S\tau]^3}$$

Based on these equations, we can derive the optimal w as in Equation (2.30). We can verify that the optimal w in Equation (2.30) is less than $\frac{1}{\alpha} + \frac{\beta(\theta+\theta_b+2\gamma t_S/t_L)}{\alpha(1-\theta-\theta_b)}$. The retailer would not choose any w greater than $\frac{1}{\alpha} + \frac{\beta(\theta+\theta_b+2\gamma t_S/t_L)}{\alpha(1-\theta-\theta_b)}$ because when w is very high, manufacturers already have incentives to increase prices to get more recommendations and shoppers. Eventually, no loyal consumers would make a purchase and the retailer's profit is lower than the optimal profit in Equation (2.34).

(b)-(g) By substituting the optimal w in Equation (2.30) into the equations in Lemma (2), we get the equilibrium prices, demands and profits. □

Proof of Proposition 5

Proof. (a) Part (a) follows from

$$p_i^* - \bar{p}_i^* = \frac{1+2h}{4\tau} - \frac{t_S\left[(2\theta+\theta_b)+2h\right]}{\theta_b+4t_S\tau} = \frac{(1+2h)\theta_b + 4t_S\tau(1-2\theta-\theta_b)}{4\tau(\theta_b+4t_S\tau)} > 0$$

(b)Note that

$$\begin{aligned} D_i^* - \bar{D}_i^* &= \left(\frac{1}{2}+h\right)\frac{1}{2} - \left(\frac{2\theta+\theta_b}{2}+h\right)\left(1-\frac{2t_S\tau}{\theta_b+4t_S\tau}\right) \\ &= \frac{1}{4}\left(1-2\theta-\theta_b - \frac{\theta_b(2h+2\theta+\theta_b)}{\theta_b+4t_S\tau}\right) \end{aligned}$$

Solving $\triangle D_i^* - \bar{D}_i^* > 0$ leads to the condition in Part (b).

(c) Note that

$$\pi_i^* - \bar{\pi}_i^* = \frac{(1-\alpha)}{16\tau}\left((1+2h)^2 - 8t_S\tau(\theta_b + 2t_S\tau)\left(\frac{2\theta+\theta_b+2h}{\theta_b+4t_S\tau}\right)^2\right) > 0$$

(d)Part (d) follows from that

$$CS - \bar{CS} = \beta\frac{t_S(1-\theta-\theta_b)}{4} + \frac{\left(4v_S-2t_S-\frac{1+2h}{\tau}\right)(1-2\theta-\theta_b)}{4} + \frac{t_S\theta}{4} - \frac{\left[(1+2h)\theta_b+4\tau t_S(1-2\theta-\theta_b)\right]\left[\left(6h-1+8\theta+4\theta_b\right)\theta_b+4t_s\tau(4h-1+6\theta+3\theta_b)\right]}{16\tau(\theta_b+4\tau t_S)^2}$$

Solving $CS - \bar{CS} > 0$ leads to the condition in Part (d).

(e) Part (e) follows from that

$$W - \bar{W} = \left[v_S - \frac{t_S(2-\beta)}{4}\right](1-2\theta-\theta_b) + \frac{t_S(1+\beta)\theta}{4} - \frac{\left[(1+2h)\theta_b+4\tau t_S(1-2\theta-\theta_b)\right][(3-2h)\theta_b+4t_S\tau(3+6\theta+3\theta_b+4h)]}{16\tau(\theta_b+4\tau t_S)^2}$$

Solving $W - \bar{W} > 0$ leads to the condition in Part (e). $\square$

Proof of Lemma 4

Proof. The manufacturers maximize their profits by choosing their optimal prices; that is,

$$\max_{p_i} \pi_i = (1-\alpha)p_i D_i \tag{A.17}$$

where D_i is defined in Equation (2.37).

(a) Each manufacturer's best response to its competitor in stage 1 is characterized by the first-order conditions of Equation (A.17):

$$\frac{\partial \pi_A}{\partial p_A} = (1-\alpha)\left[\frac{1}{2} + \frac{\beta(1-w)[1-\lambda(1-\theta-\theta_b)]+(1-\theta-\theta_b)(1-\alpha w)}{2\beta t_S}\left(p_B - 2p_A\right) + \gamma\frac{v_L}{t_L} - \frac{2\gamma}{t_L}p_A\right] = 0$$
$$\frac{\partial \pi_B}{\partial p_B} = (1-\alpha)\left[\frac{1}{2} + \frac{\beta(1-w)[1-\lambda(1-\theta-\theta_b)]+(1-\theta-\theta_b)(1-\alpha w)}{2\beta t_S}\left(p_A - 2p_B\right) + \gamma\frac{v_L}{t_L} - \frac{2\gamma}{t_L}p_B\right] = 0$$

Similar to the base model, we assume that $\beta \geq \frac{\lambda(1-\theta-\theta_b)(\alpha w-1)}{[1-\lambda(1-\theta-\theta_b)]++2\gamma t_S/t_L}$ to guarantee that the profit function is concave. Based on the above first-order conditions, we can derive the manufacturer equilibrium prices as in Equation (2.38).

(b) and (c): By substituting the equilibrium prices into demand functions in Equation (2.37) and profit functions in Equation (A.17), we can get the equilibrium demands and equilibrium profits.

(d) According the commission scheme, we have $\frac{\pi_R}{\pi_A+\pi_B} = \frac{\alpha}{1-\alpha}$. Based on π_i^*, we can derive π_R^* as in Equation (2.41).

(e) First we derive the consumer surplus for low search cost shoppers. Because these consumers check both recommendations and purchase the product that offers them higher net utility, the consumer surplus is

$$CS_s^l = (1-\lambda)\left[\int_0^{z_0}(v_S - p_A - zt_S)dz + \int_{z_0}^1 \left[v_S - p_B - (1-z)t_S\right]dz\right] = (1-\lambda)(v_S - \frac{t_S}{4} - p^*) \tag{A.18}$$

Next, we derive the consumer surplus for high search cost shoppers. Because there consumers only check the first recommendation, their purchase decisions are similar as in the baseline model. From Equation (A.13), the consumer surplus is

$$CS_S^h = \lambda\left[v_S - \frac{t_S}{4}\left[1 + (1-\beta)(1-\theta-\theta_b)\right] - p^*\right] \tag{A.19}$$

By aggregating the consumer surplus from low search cost shoppers, high search cost shoppers and loyal consumers, we can derive the total surplus as the following:

$$CS \;= CS_s^l + CS_S^h + CS_L = \;\; v_S - \frac{t_S}{4} - \lambda(1-\beta)(1-\theta-\theta_b) - p^* + \gamma\frac{(v_L - p^*)^2}{t_L} \tag{A.20}$$

By substituting p^* in Equation (2.38) into above equation, we can derive the consumer surplus in Equation (2.42).

(e) Social welfare is the sum of consumer surplus, manufacturer's profits, and retailer's profit; that is, $W = CS + \pi_A^* + \pi_B^* + \pi_R^* = v_S - \frac{t_S}{4}\left[1 + \lambda(1-\beta)(1-\theta-\theta_b)\right] + \gamma\frac{(v_L-p^*)(v_L+3p^*)}{t_L}$.

□

Proof of Proposition 6

Proof. (a) For ease of exposition, we denote $\tilde{M} \equiv (1-\alpha w)\,\lambda(1-\theta-\theta_b)+\beta\left[1-\lambda(1-\theta-\theta_b)\right]+4\beta\tau t_S$, note that

$$p_i^* - \bar{p}_i^* = t_S \tfrac{\beta(\theta_b-(2\theta+\theta_b)[1-\lambda(1-\theta-\theta_b)]+4t_S\tau(1-2\theta-\theta_b)-2h[1-\theta_b-\lambda(1-\theta-\theta_b)])-(1-\alpha w)\lambda(1-\theta-\theta_b)[2\theta+\theta_b+2h]}{\tilde{M}(\theta_b+4t_S\tau)}$$

p_i^* is less than $\bar{p}_i^*$ if the numerator is negative, which leads to the condition in Part (a).

(b)Note that

$$D_i^* - \bar{D}_i^* = \tfrac{1-2\theta-\theta_b}{2} + t_S\tau\left(\tfrac{2\theta+\theta_b+2h}{\theta_b+4t_S\tau} - \tfrac{(1+2h)\beta}{\tilde{M}}\right)$$

Hence, $D_i^* > \bar{D}_i^*$ under the condition in Part (b).

(c) Note that

$$\pi_i^* - \bar{\pi}_i^* = -(1-\alpha)\tfrac{t_S}{M^2}\left[\left(\tfrac{2\theta+\theta_b+2h}{\theta_b+4t_S\tau}\right)^2\left(\tfrac{\theta_b+2t_S\tau}{2}\right)M^2 - \tfrac{\beta(1+2h)^2}{2}M + (1+2h)^2\beta^2 t_S\tau\right]$$

$\pi_i^* - \bar{\pi}_i^* > 0$ when

$$\left(\tfrac{2\theta+\theta_b+2h}{\theta_b+4t_S\tau}\right)^2\left(\tfrac{\theta_b+2t_S\tau}{2}\right)M^2 - \tfrac{\beta(1+2h)^2}{2}M + (1+2h)^2\beta^2 t_S\tau < 0$$

By solving the above inequality, we can get the condition in Part (c).

(d) From Equation (2.42) and Equation (A.4), we get:

$$\begin{aligned} CS - \bar{CS} =& (1-2\theta-\theta_b)\left[v_S - \frac{t_S(1+\lambda-\lambda\beta)}{4} - p_i^*\right] + \frac{\theta(2-\lambda+\lambda\beta)t_S}{4} \\ &+ (\bar{p}_i^* - p_i^*)\left[2\theta+\theta_b+\frac{\gamma}{t_L}(2v_L - \bar{p}_i^* - p_i^*)\right] \end{aligned}$$

From above equation, we can get $\frac{\partial\left(CS-\bar{CS}\right)}{\partial w} = -\frac{\partial p_i^*}{\partial w}\left[1+\frac{2\gamma v_L}{t_L} - \frac{2\gamma}{t_L}p_i^*\right] > 0$. Hence, $CS - \bar{CS} > 0$ under the condition in Part (d).

(e) From Equation (2.43) and Equation (A.4), we get:

$$W - \bar{W} = \;(1-2\theta-\theta_b)\left[v_S - \tfrac{t_S(1+\lambda-\lambda\beta)}{4}\right] + \tfrac{\theta(2-\lambda+\lambda\beta)t_S}{4} + (\bar{p}_i^* - p_i^*)\left[\tfrac{3\gamma}{t_L}(\bar{p}^* - p_i^*) - \tfrac{2\gamma v_L}{t_L}\right]$$

From above equation, we can get $\frac{\partial\left(W-\bar{W}\right)}{\partial w} = 2\frac{\partial p_i^*}{\partial w}\left[\frac{2\gamma v_L}{t_L} - \frac{3\gamma}{t_L}p_i^*\right]$. Under the condition $\frac{(1-\theta-\theta_b)(1-\beta)\lambda}{\beta} + 1 < (2+\frac{3t_L}{\gamma v_L})\frac{\gamma}{t_L}t_S$, we have $\frac{2\gamma v_L}{t_L} - \frac{3\gamma}{t_L}p_i^* < 0$, so $\frac{\partial\left(CS-\bar{CS}\right)}{\partial w} < 0$. $W - \bar{W} > 0$ under the condition in Part (e). $\square$

Proof of Lemma 5

Proof. In the second stage, given w, each manufacturer's best response to its competitor is the same as those in lemma 4 as long as $\beta \geq \frac{\lambda(1-\theta-\theta_b)(\alpha w-1)}{[1-\lambda(1-\theta-\theta_b)]++2\gamma t_S/t_L}$, or $w < \frac{1}{\alpha} + \frac{\beta([1-\lambda(1-\theta-\theta_b)]+2\gamma t_S/t_L)}{\alpha\lambda(1-\theta-\theta_b)}$. In the first stage, the retailer maximize their profits by choosing the optimal w; that is,

$$\max_{w} \pi_R = (1-\alpha)(p_A D_A + p_B D_B) \tag{A.21}$$

where D_A and D_B is defined in Equation (2.39) and p_A and p_B is defined in (2.38).

(a) The retailer's best response in stage 1 is characterized by the first-order conditions of Equation (A.21)

$$\frac{\partial \pi_R}{\partial w} = \frac{\beta t_s \alpha^2(1+2h)^2\lambda(1-\theta-\theta_b)((1-\alpha w)\lambda(1-\theta-\theta_b)+\beta[1-\lambda(1-\theta-\theta_b)])}{((1-\alpha w)\lambda(1-\theta-\theta_b)+\beta[1-\lambda(1-\theta-\theta_b)]+4\beta t_S \tau)^3}$$

Based on these equations, we can derive the optimal w as in Equation (**??**). We can verify that the optimal w in Equation (5) is less than $w < \frac{1}{\alpha} + \frac{\beta([1-\lambda(1-\theta-\theta_b)]+2\gamma t_S/t_L)}{\alpha\lambda(1-\theta-\theta_b)}$. The retailer would not choose any w greater than $w < \frac{1}{\alpha} + \frac{\beta([1-\lambda(1-\theta-\theta_b)]+2\gamma t_S/t_L)}{\alpha\lambda(1-\theta-\theta_b)}$ because when w is very high, manufacturers already have incentives to increase prices to get more recommendations and shoppers. Eventually, no loyal consumers would make a purchase and the retailer's profit is lower than the optimal profit in Equation (2.49).

(b)-(g) By substituting the optimal w in Equation (5) into the equations in Lemma (4), we get the equilibrium prices, demands and profits. □

Proof of Lemma 6

Proof. (a) and (b) Firm's optimization problem in stage 1 is characterized by the first-order conditions of Equation (3.3):

$$\begin{aligned}
\frac{\partial \pi_A}{\partial p_A} &= (1-\alpha)\phi_A\left(1-\phi_B+\phi_B\frac{p_B-2p_A+t}{2t}\right)=0 \\
\frac{\partial \pi_A}{\partial \phi_A} &= (1-\alpha)p_A\left(1-\phi_B+\phi_B\frac{p_B-p_A+t}{2t}\right)-a\phi_A=0 \\
\frac{\partial \pi_B}{\partial p_B} &= (1-\alpha)\phi_B\left(1-\phi_A+\phi_A\frac{p_A-2p_B+t}{2t}\right)=0 \\
\frac{\partial \pi_B}{\partial \phi_B} &= (1-\alpha)p_B\left(1-\phi_A+\phi_A\frac{p_A-p_B+t}{2t}\right)-a\phi_B=0
\end{aligned} \tag{A.22}$$

Based on these equations, we can derive the symmetric equilibrium as follows:

$$p_A^* = p_B^* = \sqrt{\frac{2at}{1-\alpha}} \tag{A.23}$$

$$\phi_A^* = \phi_B^* = \frac{2t}{\sqrt{\frac{2at}{1-\alpha}}+t} \tag{A.24}$$

By replacing $\tau = \frac{a}{1-\alpha}$, we get the equilibrium prices and advertising intensities in Equation (3.4) and Equation (3.5). Under the assumption $\frac{t}{2} < \tau < \frac{(v-t)^2}{2t}$, we can verify that the equilibrium advertising level $\phi_i^* \in [0,1]$ and equilibrium price $p_i^* \in [0, v-t]$.

(c) Substituting the equilibrium prices and advertising intensities into Equation (3.3), we can derive the equilibrium profits as in Equation (3.6). Next, we show there are no profitable deviations from the equilibrium under the assumption. One possible deviation is that one firm (for example, firm A) increases price to $v-t$; that is, firm A only serves partially informed customers. In this case, the demand function becomes $D_A = \phi_A(1-\phi^*)$ and the profit function becomes $\pi_A = \phi_A(1-\phi^*)p_A(1-\alpha) - \frac{a}{2}\phi_A^2$. The best response of A is $p_A^{dev} = v-t$ and $\phi_A^{dev} = \frac{\left(\sqrt{2t\tau}-t\right)(v-t)}{\tau\left(\sqrt{2t\tau}+t\right)}$, and the optimal deviation profit is $\pi_A^{dev} = \frac{(1-\alpha)\left(\sqrt{2t\tau}-t\right)^2(v-t)^2}{2\tau\left(\sqrt{2t\tau}+t\right)^2}$. If $\pi_A^{dev} < \pi_A^*$, firm A would not deviate from the equilibrium. When $v < 5t$, π_A^{dev} is always less than π_A^*, and there is no profitable deviation. When $v > 5t$, from $\pi_A^{dev} < \pi_A^*$, we derive $\frac{3t^2-4tv+\sqrt{(v-5t)(v-t)^3}+v^2}{4t} < \tau$. Under this condition, firms have no incentive to deviate from a pure price strategy to the reservation price for partially informed consumers. We can verify other possible deviations always lead to lower profits.

(d) The consumer surplus from consumers who only receive advertising from one seller is:

$$CS_p = 2\phi^*(1-\phi^*)\int_0^1 (v - p^* - zt)\,\mathrm{d}z = 2\phi^*(1-\phi^*)\left(v - p^* - \tfrac{t}{2}\right)$$

The consumer surplus from consumers who receive advertising from both sellers is:

$$CS_f = (\phi^*)^2\Big[\int_0^{\frac{1}{2}} (v - p^* - zt)\,\mathrm{d}z + \int_{\frac{1}{2}}^1 (v - p^* - (1-z)t)\,\mathrm{d}z\Big] = (\phi^*)^2(v - p^* - \tfrac{t}{4})$$

Aggregating consumers surplus from these two segments, we get

$$CS = CS_p + CS_f = \phi^*(2-\phi^*)(v - p^* - \frac{t}{4}) - \phi^*(1-\phi^*)\frac{t}{2} \tag{A.25}$$

Substituting p^* and ϕ^* with the equilibrium as showed in Equation (3.4) and Equation (3.5) and aggregating the consumer surplus above, we can derive the total surplus as in Equation (3.7). □

Proof of Lemma 7

Proof. (a) and (b) Firm's optimization problem in stage 1 is characterized by the first-order conditions of Equation (3.3) with D_A and D_B specified in Equation (3.12) and Equation (3.13):

$$\frac{\partial \pi_A}{\partial p_A} = \frac{(1-\alpha)(1-\beta)(p_B-p_A)(p_B-3p_A)(\phi_B-\phi_A)}{4\beta t^2} + \frac{(1-\alpha)(p_B-2p_A)\left[2-\left(1-\beta^2\right)\phi_B-(1-\beta)^2\phi_A\right]}{4\beta t} + \frac{(1-\alpha)\left[(1-\beta)(\phi_A-\phi_B)+2\right]}{4} = 0 \tag{A.26}$$

$$\frac{\partial \pi_A}{\partial \phi_A} = -a\phi_A + p_A\frac{(1-\alpha)(1-\beta)(p_B-p_A+t)(p_A-p_B+\beta t)}{4\beta t^2} = 0 \tag{A.27}$$

$$\frac{\partial \pi_B}{\partial p_B} = \frac{(1-\alpha)(1-\beta)(p_A-p_B)(p_A-3p_B)(\phi_A-\phi_B)}{4\beta t^2} + \frac{(1-\alpha)(p_A-2p_B)\left[2-\left(1-\beta^2\right)\phi_B-(1-\beta)^2\phi_A\right]}{4\beta t} + \frac{(1-\alpha)\left[(1-\beta)(\phi_B-\phi_A)+2\right]}{4} = 0 \tag{A.28}$$

$$\frac{\partial \pi_B}{\partial \phi_B} = -a\phi_B + p_B\frac{(1-\alpha)(1-\beta)[(p_A-p_B+t)(p_B-p_A+\beta t)-2t\beta(p_A-p_B)]}{4\beta t^2} = 0 \tag{A.29}$$

By solving the above equations, we can derive equilibrium prices as in Equation (3.14) and advertising intensities as in Equation (3.15). Under the assumption $\frac{t}{2} < \tau < \frac{(v-t)^2}{2t}$, the equilibrium advertising intensity $\hat{\phi}_i^* \in [0,1]$ and equilibrium price $\hat{p}_i^* \in [0, v-t]$.

(c) Substituting the equilibrium prices and advertising intensities into Equation (3.3), we can derive the equilibrium profits as in Equation (3.16). Next, we show there are no profitable deviations from the equilibrium under Assumption 2. One possible deviation is firm A increases price from the equilibrium such that recommender system always recommends B. In this case, the demand for product A is $\frac{\phi_A(\hat{p}_B^* - p_A + t)}{2t}$ and the profit is $\frac{\phi_A(\hat{p}_B^* - p_A + t)}{2t} p_A(1-\alpha) - \frac{a\phi_A^2}{2}$. The best response of A is $p_A^{dev} = \frac{\hat{p}_B^* + t}{2}$ and $\phi_A^{dev} = \frac{(\hat{p}_B^* + t)^2}{8\tau t}$, and the optimal deviation profit is $\pi_A^{dev} = \frac{(1-\alpha)(\hat{p}_B^* + t)^4}{128 t^2 \tau}$. $\pi_A^{dev} < \hat{\pi}_A^*$ requires

$$\begin{aligned} K \equiv 16(1-\beta)^6 t^2 \tau \left[\beta(1-\beta)^2 t + 6\tau - 6\sqrt{\tau\left(\tau - (1-\beta)^2 \beta t\right)}\right] \\ - \left((1-\beta)^2 t + 2\tau - 2\sqrt{\tau\left(\tau - (1-\beta)^2\beta t\right)}\right)^4 > 0 \end{aligned} \tag{A.30}$$

Because K is increasing in τ and $\tau \geq \frac{t}{2}$, the minimum value of K is

$$K|_{\tau=\frac{t}{2}} = 16(1-\beta)^6 t[\beta(1-\beta)^2 - 3\sqrt{1-2(1-\beta)^2\beta} + 3] + 2t[(2-\beta)\beta + \sqrt{1-2(1-\beta)^2\beta} + 2]$$

which is always positive when $\beta > \frac{1}{25}$.

(d) The consumer surplus from consumers who only receive advertising from one seller is:

$$\begin{aligned} \hat{CS}_p = \ & 2\phi^*(1-\phi^*) \int_0^{\frac{1}{2}} (v - p^* - zt)\mathrm{d}z + 2\phi^*(1-\phi^*)\tfrac{1-\beta}{2} \int_{\frac{1}{2}}^1 (v - zt - p^*)\mathrm{d}z \\ & +2\phi^*(1-\phi^*)\tfrac{1+\beta}{2} \int_{\frac{1}{2}}^1 \left(v - (1-z)t - p^*\right)\mathrm{d}z \\ = \ & 2\phi^*(1-\phi^*)\left[v - p^* - \tfrac{t}{4} - \tfrac{(1-\beta)t}{8}\right] \end{aligned}$$

The consumer surplus from consumers who receive advertising from both sellers is:

$$\hat{CS}_f = (\phi^*)^2 \left[\int_0^{\frac{1}{2}} (v - p^* - zt)\mathrm{d}z + \int_{\frac{1}{2}}^1 \left(v - p^* - (1-z)t\right)\mathrm{d}z\right] = (\phi^*)^2\left(v - p^* - \tfrac{t}{4}\right)$$

Consumer surplus from consumers who do not receive any advertising is:

$$\hat{CS}_u = 2\left(1-\phi^*\right)^2\left[\tfrac{1+\beta}{2}\int_0^1 (v-p^*-zt)\mathrm{d}z - \beta\int_{\frac{1}{2}}^1 (v-p^*-zt)\,\mathrm{d}z\right] = (1-\phi^*)^2\left[v-p^*-\tfrac{(2-\beta)t}{4}\right]$$

Aggregating the consumer surplus above, the total surplus is:

$$\hat{CS} = \hat{CS}_p + \hat{CS}_f + \hat{CS}_u = v - p^* - \tfrac{t}{4} - (1-\phi^*)\tfrac{1-\beta}{4}t \tag{A.31}$$

Substituting p^* and ϕ^* with the equilibrium as showed in Equation (3.14) and Equation (3.15) and aggregating the consumer surplus above, we can derive the total surplus as in Equation (3.17). □

Proof of Proposition 7

Proof. (a) Because $\hat{p}_i^*$ is increasing in β according to Proposition 8(a), its maximum value is $\hat{p}_i^*|_{\beta=1}= t$, which is less than $p_i^* = \sqrt{2\tau t}$. Therefore, $\hat{p}_i^* \leq p_i^*$.

(b) Note that

$$\hat{\phi}_i^* - \phi_i^* = \tfrac{1}{2(1-\beta)} - \tfrac{\sqrt{\tau[\tau-\beta(1-\beta)^2 t]}}{2\tau(1-\beta)} - \tfrac{2t}{\sqrt{2\tau t}+t} \tag{A.32}$$

$\hat{\phi}_i^* - \phi_i^* < 0$ requires $\frac{1}{2(1-\beta)} - \frac{\sqrt{\tau[\tau-\beta(1-\beta)^2 t]}}{2\tau(1-\beta)} - \frac{2t}{\sqrt{2\tau t}+t} < 0$, which is equivalent to $16t\tau(1-\beta) - 8\tau(\sqrt{2\tau t}+t) < (\sqrt{2\tau t}+t)^2\beta(1-\beta)$. Because $\tau \geq \frac{t}{2}$, we have $8\tau(\sqrt{2\tau t}+t) > 16\tau t > 16t\tau(1-\beta)$. Therefore, $16t\tau(1-\beta) - 8\tau(\sqrt{2\tau t}+t) < 0 < (\sqrt{2\tau t}+t)^2\beta(1-\beta)$, and we get $\hat{\phi}_i^* < \phi_i^*$.

(c) Because $\hat{\pi}_i^*$ is increasing in β, its maximum value is $\hat{\pi}_i^*|_{\beta=1}= \frac{(1-\alpha)t}{2}$. If the maximum value $\frac{(1-\alpha)t}{2} < \pi_i^*$, or, equivalently, if $4\alpha\tau + 2\sqrt{2t\tau} + t < 2\tau$, $\hat{\pi}_i^*$ is always less than π_i^* for all values of β. Otherwise, $\hat{\pi}_i^* > \pi_i^*$ requires:

$$\frac{2t\tau}{\left(\sqrt{t}+\sqrt{2\tau}\right)^2} < \frac{\left(-6\sqrt{\tau^2-\beta(1-\beta)^2 t\tau}+\beta(1-\beta)^2 t+6\tau\right)}{8(1-\beta)^2} \tag{A.33}$$

(d) From Equation (A.31) and Equation (A.25) we get

$$\begin{aligned}\hat{CS} - CS =\ & v - \hat{p}_i^* - \tfrac{t}{4} - (1-\hat{\phi}_i^*)\tfrac{1-\beta}{4}t - \left(\phi_i^*(2-\phi_i^*)(v-p_i^*-\tfrac{t}{4}) - \phi_i^*(1-\phi_i^*)\tfrac{t}{2}\right)\\ =&(1-\phi_i^*)^2(v-\hat{p}_i^*-\frac{t}{4}) + \phi_i^*(1-\phi_i^*)\frac{t}{2} + \phi_i^*(2-\phi_i^*)(p_i^*-\hat{p}_i^*) - (1-\hat{\phi}_i^*)\frac{1-\beta}{4}t\end{aligned}$$

The first term, $(1-\phi_i^*)^2(v-\hat{p}_i^*-\frac{t}{4})$, is positive. The second term, $\phi_i^*(1-\phi_i^*)\frac{t}{2}$, is positive. Because $\phi_i^*(2-\phi_i^*)(p_i^*-\hat{p}_i^*)-(1-\hat{\phi}_i^*)\frac{1-\beta}{4}t > \phi_i^*(p_i^*-\hat{p}_i^*)-\frac{1-\beta}{4}t$, as long as $\phi_i^*(p_i^*-\hat{p}_i^*) > \frac{1-\beta}{4}t$, we can get $\hat{CS} > CS$. By substituting p_i^* in Equation (3.4) and $\hat{p}_i^*$ in Equation (3.14) into inequality $\phi_i^*(p_i^*-\hat{p}_i^*) > \frac{1-\beta}{4}t$, we have

$$\frac{2t}{\sqrt{2\tau t}+t}\left(\sqrt{2\tau t}-\frac{2\left[\tau-\sqrt{\tau^2-\beta(1-\beta)^2 t\tau}\right]}{(1-\beta)^2}\right) > \frac{1-\beta}{4}t$$

By simplifying the above inequality, we get

$$X \equiv 8(1-\beta)^2\sqrt{2\tau t}-16\left(\tau-\sqrt{\tau^2-\beta(1-\beta)^2 t\tau}\right)-(1-\beta)^3\left(\sqrt{2\tau t}+t\right) > 0$$

We can verify that $\frac{\partial X}{\partial \beta} < 0$, and thus the minimum value of X is $X|_{\beta=1} = 0$. Therefore, $X \geq 0$ and $\hat{CS} > CS$. $\square$

Proof of Proposition 8

Proof. (a) Note that

$$\frac{\partial \hat{p}_i^*}{\partial \beta} = \frac{\tau}{(1-\beta)^3\sqrt{\tau^2-(1-\beta)^2\beta t\tau}}\left[4\sqrt{\tau^2-(1-\beta)^2\beta t\tau}+(1+\beta)(1-\beta)^2 t-4\tau\right]$$

$\frac{\partial \hat{p}_i^*}{\partial \beta} > 0$ requires $4\sqrt{\tau^2-(1-\beta)^2\beta t\tau} > 4\tau-(1+\beta)(1-\beta)^2 t$, which is equivalent to condition $\tau > \frac{(1+\beta)^2(1-\beta)}{8}t$. This condition always holds because $\tau > \frac{t}{2} > \frac{(1+\beta)^2(1-\beta)}{8}t$.

(b) Note that

$$\frac{\partial \hat{\phi}_i^*}{\partial \beta} = \frac{2\sqrt{\tau^2-\beta(1-\beta)^2 t\tau}+(1-\beta)^3 t-2\tau}{4(1-\beta)^2\sqrt{\tau^2-\beta(1-\beta)^2 t\tau}}$$

$\frac{\partial \hat{\phi}_i^*}{\partial \beta} > 0$ requires $2\sqrt{\tau^2-(1-\beta)^2\beta t\tau}+(1-\beta)^3 t-2\tau > 0$, which leads to the condition $\frac{4\tau}{t}(1-2\beta) > (1-\beta)^4$.

(c) Note that

$$\frac{\partial \hat{\pi}_i^*}{\partial \beta} = (1-\alpha)\frac{\left((1-\beta)^3 t+12\tau\right)\sqrt{\tau^2-\beta(1-\beta)^2 t\tau}+3(1+\beta)(1-\beta)^2 t\tau-12\tau^2}{8(1-\beta)^3\sqrt{\tau^2-\beta(1-\beta)^2 t\tau}}$$

In part (a), we showed that $4\sqrt{\tau^2-(1-\beta)^2\beta t\tau} > 4\tau-(1+\beta)(1-\beta)^2 t$. Therefore, $12\tau\sqrt{\tau^2-\beta(1-\beta)^2 t\tau}+3(1+\beta)(1-\beta)^2 t\tau-12\tau^2 > 0$, and we get $\frac{\partial \hat{\pi}_i^*}{\partial \beta} > 0$.

(d) Note that

$$\frac{\partial \hat{CS}}{\partial \beta} = -\frac{\left(-4(1-\beta)^3 t+64\tau\right)\sqrt{\tau^2-\beta(1-\beta)^2 t\tau}+16(1+\beta)(1-\beta)^2 t\tau-64\tau^2-(1-3\beta)(1-\beta)^4 t^2}{16(1-\beta)^3\sqrt{\tau^2-\beta(1-\beta)^2 t\tau}}$$

$\frac{\partial \hat{CS}}{\partial \beta} < 0$ requires $Y \equiv (-4(1-\beta)^3 t + 64\tau)\sqrt{\tau^2 - \beta(1-\beta)^2 t\tau} + 16(1+\beta)(1-\beta)^2 t\tau - 64\tau^2 - (1-3\beta)(1-\beta)^4 t^2 > 0$. We can verify that $\frac{\partial Y}{\partial \beta} < 0$, and the minimum value of Y is $Y|_{\beta=1} = 0$. Therefore, $Y > 0$ and $\frac{\partial \hat{CS}}{\partial \beta} < 0$. □

Proof of Lemma 8

Proof. (a) and (b): Firm i's optimal advertising strategy is as in Equation (3.21) and optimal price is characterized by the first-order condition of Equation (3.22). In a symmetric equilibrium, firms use the same price and advertising intensity; that is, $p_A = p_B = p^*$, $\phi_A^H = \phi_B^H = \phi^{H*}$, and $\phi_A^L = \phi_B^L = \phi^{L*}$. By substituting the prices and advertising intensities into Equation (3.21) and the first-order conditions of Equation (3.22), we have

$$\phi^{H*} = \frac{p^*}{4t^2\tau}\left[4t^2 - (1-\gamma)t\left((\phi^{L*} - \phi^{H*})(1-\gamma)t + 2t\phi^{H*}\right)\right] \tag{A.34}$$

$$\phi^{L*} = \frac{p^*}{4t^2\tau}\left[4t^2 - (t(1-\gamma) + 2\gamma t)\left((\phi^{L*} - \phi^{H*})(1-\gamma)t + 2t\phi^{H*}\right)\right] \tag{A.35}$$

$$\begin{aligned}\frac{\partial \mathbb{E}(\pi_i)}{\partial p_i} = \frac{1-\alpha}{8t}\Big[&(\phi^{H*})^2\left(2\tau - (1-\gamma)^2 p^* - (1-\gamma^2)t\right) - (\phi^{L*})^2\left(2\tau + (1-\gamma)^2 p^* + (1-\gamma^2)t\right)\\ &-2\phi^{H*}\phi^{L*}(2\gamma p^* + p^* - \gamma^2 p^* + \gamma^2 t + t) - 4\phi^{H*}(p^* - t) + 4\phi^L(p^* + t)\Big] = 0\end{aligned} \tag{A.36}$$

By solving the above equations, we derive the equilibrium advertising intensities in Equations (3.24) and (3.25), and the equilibrium price in Equation (3.23). Denote $F \equiv (p^*)^4(\gamma^4 - \gamma^3) + (p^*)^3\tau(\gamma - 2\gamma^2) + (p^*)^2\tau(2\tau - \gamma^4 t - \gamma^2 t) + 4p^* t\gamma^2\tau^2 - 4t\tau^3$. Under the assumption $\tau > 2t$, $F|_{p=0} < 0$ and $F|_{p=\tau} > 0$. Therefore, there is a solution $p^* \in [0, \tau]$, $\phi^{H*} \in [0,1]$ and $\phi^{L*} \in [0,1]$.

(c) By substituting ϕ^{H*} in Equation (3.24) and ϕ^{L*} in Equation (3.25) into profit function in Equation (3.22), we get the equilibrium profits in Equation (3.26).

(d) For a consumer whose true location is between 0 and $\frac{1}{2}$, with probability $\frac{1+\gamma}{2}\phi^{H*} + \frac{1-\gamma}{2}\phi^{L*}$, the consumer receives an advertisement from A and becomes aware of product

A. With probability $\frac{1-\gamma}{2}\phi^{H*} + \frac{1+\gamma}{2}\phi^{L*}$, the consumer receives an advertisement from B and becomes aware of product B. Therefore, consumer surplus for consumers whose true locations are between 0 and $\frac{1}{2}$ is as follows:

$$\begin{aligned} CS = &\int_0^{\frac{1}{2}} (v - p^* - zt) \left(\tfrac{(1+\gamma)\phi^{H*}}{2} + \tfrac{(1-\gamma)\phi^{L*}}{2} \right) \mathrm{d}z \\ &+ \int_0^{\frac{1}{2}} (1 - \tfrac{(1+\gamma)\phi^{H*}}{2} - \tfrac{(1-\gamma)\phi^{L*}}{2}) \left(\tfrac{(1+\gamma)\phi^{L*}}{2} + \tfrac{(1-\gamma)\phi^{H*}}{2} \right) (v - p^* - (1-z)t) \, \mathrm{d}z \end{aligned}$$

Because the two firms and consumers are symmetric, the total consumer surplus is $2CS$. Substituting ϕ^{H*} and ϕ^{L*} with the equilibrium as showed in Equations (3.24) and (3.25), we derive the total consumer surplus as in Equation (3.27). □

Proof of Lemma 9

Proof. In the presence of recommender system, we can show that sellers use similar advertising strategies as in the no recommendation scenario under targeted advertising. Without loss of generality, we assume $p_A \leq p_B$. We first consider seller A's advertising strategy in stage 1 given firm B's advertising strategy as in Equation (3.18). Denoting the true location as z and the signal received by recommender system as s, we can formulate the demand function and profit function of seller A conditional on its own signal η_A as follows:

$$\mathbb{E}[D_A|\eta_A] = \int_0^{z_0} (\phi_A + (1-\phi_A)P(s \leq y_0|z))\, f(z|\eta_A)\mathrm{d}z + \int_{z_0}^{1} (1 - \mathbb{E}[\phi_B(\eta_B)|z])\, P(s \leq y_0|z) f(z|\eta_A)\mathrm{d}z \tag{A.37}$$

$$\mathbb{E}[\pi_A|\eta_A] = (1-\alpha)p_A \mathbb{E}[D_A|\eta_A] - \frac{a}{2}\phi_A^2 \tag{A.38}$$

where $f(z|\eta_A)$ is the probability density function of location given signal η_A, and $P(s \leq y_0|z)$ is the probability that the signal s received by recommender system is less than y_0 given the location z. Given a signal η_A, firm A maximizes its expected profit by choosing the optimal advertising level. Firm A's advertising strategy is summarized as follows:

$$\phi_A^*(\eta_A) = \begin{cases} \frac{(1-\beta)p_A}{4\beta t^2 \tau} \left[(p_A - p_B + \beta t) \left((1-\gamma)(p_B - p_A) + t(1+\gamma) \right) \right] & \eta_A \leq z_0 \\ \frac{(1-\beta)p_A}{4\beta t^2 \tau} \left[(1-\gamma)(p_A - p_B + \beta t)(p_B - p_A + t) \right] & \eta_A > z_0 \end{cases} \tag{A.39}$$

Denoting A's advertising intensity when $\eta_A \leq z_0$ as ϕ_A^H, A's advertising intensity when $\eta_A > z_0$ as ϕ_A^L, we verify that the sellers' strategies and belief sets are consistent with each other. In stage 1, each seller chooses price by maximizing the overall expected profit:

$$\max_{p_i} \mathbb{E}(\pi_i) = \int_0^1 \mathbb{E}(\pi_i|\eta_i) f(\eta_i) \mathrm{d}\eta_i \tag{A.40}$$

Firm's optimization problem in stage 1 is characterized by Equation (A.39) and the first-order conditions of Equation (A.40). In the symmetric equilibrium, firms will set the same price and advertising intensify; that is, $p_A = p_B = \hat{p}^*$, $\phi_A^H = \phi_B^H = \hat{\phi}^{H*}$, and $\phi_A^L = \phi_B^L = \hat{\phi}^{L*}$. By substituting the price and advertising intensity into Equation (A.39) and the first-order conditions of Equation (A.40), we get the following equations:

$$\hat{\phi}^{H*} = \hat{p}^* \frac{(1-\beta)(1+\gamma)}{4\tau} \tag{A.41}$$

$$\hat{\phi}^{L*} = \hat{p}^* \frac{(1-\beta)(1-\gamma)}{4\tau} \tag{A.42}$$

$$\begin{aligned} \tfrac{\partial \mathbb{E}(\pi_i)}{\partial p_i} = \tfrac{(1-\alpha)}{4\beta t} \Bigg[&(\hat{\phi}^{H*})^2 \beta\tau - (\hat{\phi}^{L*})^2 \beta\tau + 2\beta t - 2\hat{p}^* + (1-\beta)\hat{p}^* \gamma(1+\beta)(\hat{\phi}^{H*} - \hat{\phi}^{L*}) \\ &+ (1-\beta)\hat{p}^* \left(\hat{\phi}^{H*}(1-\beta) + \hat{\phi}^{L*}(1+\beta) \right) \Bigg] = 0 \end{aligned} \tag{A.43}$$

Solving the above equations simultaneously, we get the equilibrium product prices and advertising intensities as in Lemma 9. Under the assumption $2t \leq \tau$, it is easy to verify that the equilibrium advertising level $\hat{\phi}^{H*} \in [0,1]$ and $\hat{\phi}^{H*} \in [0,1]$.

(d) Next we derive consumer surplus in the presence of recommender system. For a consumer whose true location is between 0 and $\frac{1}{2}$, with probability $\frac{1+\beta}{2}$, recommender system receives a signal $s \leq \frac{1}{2}$ and recommends A. Otherwise, recommender system receives a signal $s > \frac{1}{2}$ and recommends B. Therefore, consumer surplus for consumers between 0 and $\frac{1}{2}$ is as follows:

$$\begin{aligned} \hat{CS} = &\textstyle\int_0^{\frac{1}{2}} (v - \hat{p}^* - zt) \left(\frac{1+\gamma}{2}\hat{\phi}^{H*} + \frac{1-\gamma}{2}\hat{\phi}^{L*} + (1 - \frac{1+\gamma}{2}\hat{\phi}^{H*} - \frac{1-\gamma}{2}\hat{\phi}^{L*})\frac{1+\beta}{2} \right) dz \\ &+ \textstyle\int_0^{\frac{1}{2}} (1 - \frac{1+\gamma}{2}\hat{\phi}^{H*} - \frac{1-\gamma}{2}\hat{\phi}^{L*})\frac{1-\beta}{2} (v - \hat{p}^* - (1-z)t)\, dz = \frac{v-\hat{p}^*}{2} - \frac{t(2-\beta)}{8} + \frac{(1-\beta)^2(1+\gamma^2)t}{32\tau}\hat{p}^* \end{aligned}$$

Because the two firms and consumers are symmetric,the total consumer surplus is $2\hat{CS}$. Substituting $\hat{p}^*$ with the equilibrium as showed in Equation (3.28), we derive the total consumer surplus as in Equation (3.32). $\square$

Proof of Proposition 9

Proof. (a) Because $\hat{p}^*$ is increasing with β, the maximum value of $\hat{p}^*$ is $\hat{p}^*|_{\beta=1} = t$. Next we show that $p^* > t$. Because p^* is the root of Equation (3.23), $p^* > t$ is equivalent to $\frac{\left(\gamma^4+\gamma^2\right)(p^*)^2-4\gamma^2 p^*\tau+4\tau^2}{-(1-\gamma)\gamma^3(p^*)^2+(1-2\gamma)\gamma p^*\tau+2\tau^2} > \frac{p^*}{\tau}$. The last inequality is true because $p^* < \tau$ and $0 < \gamma < 1$. Therefore, we get $p^* > t > \hat{p}^*$.

(b) $\phi^{H*} > \hat{\phi}^{H*}$ is equivalent to $\frac{2\tau p^*+(1-\gamma)\gamma(p^*)^2}{2\tau^2+(1-\gamma^2)\tau p^*} > \frac{(1-\beta)(1+\gamma)\hat{p}^*}{4\tau}$. We already showed that $p^* > \hat{p}^*$ in part (a). Therefore, $\phi^{H*} > \hat{\phi}^{H*}$ is true as long as $\frac{2\tau+(1-\gamma)\gamma p^*}{2\tau+(1-\gamma^2)p^*} > \frac{(1+\gamma)}{4}$. Because $p^* < \tau$ and $0 < \gamma < 1$, the last inequality is true. Similarly, $\phi^{L*} > \hat{\phi}^{L*}$ is equivalent to $\frac{2\tau p^*-(1-\gamma)\gamma(p^*)^2}{2\tau^2+(1-\gamma^2)\tau p^*} > \frac{(1-\beta)(1-\gamma)\hat{p}^*}{4\tau}$. Because $\tau > p^* > \hat{p}^*$ and $0 < \gamma < 1$, we can verify that this inequality always holds true.

(c) Because $\hat{\pi}^*$ is increasing in β, the maximum value of $\hat{\pi}^*|_{\beta=1} = \frac{(1-\alpha)t}{2}$. If $\pi^* > \frac{(1-\alpha)t}{2}$, or, equivalently, $(p^*)^2\left[(\gamma^4+\gamma^2)(p^*)^2 - 4\gamma^2(p^*)\tau + 4\tau^2\right] > t\tau\left((1-\gamma^2)p^*+2\tau\right)^2$, π^* is always greater than $\hat{\pi}^*$. Otherwise, π^* can be less than $\hat{\pi}^*$ when β is high.

(d) Because $\hat{CS}$ is decreasing in β, the minimum value is $\hat{CS}|_{\beta=1} = v - \frac{5}{4}t$. Because $p^* < \tau$ and $v > 2t$, we can verify that

$$CS = p^* \frac{4(v-p^*)\left[(\gamma^4+\gamma^2)(p^*)^2-4\gamma^2\tau p^*+4\tau^2\right]-t\left[3(\gamma^4+\gamma^2)(p^*)^2-2p^*(5\gamma^2\mathcal{T}+\mathcal{T}\right)+8\tau^2\right]}{2\tau\left((1-\gamma^2)p^*+2\tau\right)^2} < v - \frac{5t}{4} < \hat{CS}$$

$\square$

Proof of Proposition 10

Proof. For ease of exposition, we denote $K \equiv \sqrt{\tau^2 - t\tau\beta(1-\beta)^2[(1+\beta)\gamma^2+1-\frac{\beta\gamma}{2}]}$.

(a) Note that

$$\frac{\partial \hat{p}^*}{\partial \beta} = \tau^2 t \frac{2K+2\tau-(1-\beta)\beta t\left[\left(2\beta^2+\beta+1\right)\gamma^2+1+\beta-\gamma\beta^2\right]}{K(K+\tau)^2}$$

The third term in the numerator, $(1-\beta)\beta t\left[\left(2\beta^2+\beta+1\right)\gamma^2+1+\beta-\gamma\beta^2\right]$, is increasing in γ. When $\gamma = 1$, the maximum value is $t\beta\left(2-\beta^3+\beta^2\right) < 2\tau$. Therefore, the numerator is positive and $\frac{\partial \hat{p}^*}{\partial \beta} > 0$.

(b) Note that

$$\begin{aligned} \frac{\partial \hat{\phi}^H}{\partial \beta} &= t\tau(1+\gamma)\frac{2(1-2\beta)(K+\tau)-(1-\beta)^3\beta\left(\gamma^2+1\right)t}{4K(K+\tau)^2} \\ \frac{\partial \hat{\phi}^L}{\partial \beta} &= t\tau(1-\gamma)\frac{2(1-2\beta)(K+\tau)-(1-\beta)^3\beta\left(\gamma^2+1\right)t}{4K(K+\tau)^2} \end{aligned}$$

The denominators in both equations are positive. The numerators are the same. Therefore, as long as

$$2(1-2\beta)\left(\sqrt{\tau^2-t\tau\beta(1-\beta)^2[(1+\beta)\gamma^2+1-\frac{\beta\gamma}{2}]}+\tau\right) > (1-\beta)^3\beta\left(\gamma^2+1\right)t$$

$\frac{\partial \hat{\phi}_i^H}{\partial \beta} > 0$ and $\frac{\partial \hat{\phi}_i^L}{\partial \beta} > 0$. By simplifying the above inequality, we get the condition $\frac{(1-\beta)^4}{(1-2\beta)(1+2\beta\gamma^2+\frac{\gamma}{2}-\beta\gamma)} < \frac{4\tau}{t(\gamma^2+1)^2}$. Otherwise, $\frac{\partial \hat{\phi}_i^H}{\partial \beta} < 0$ and $\frac{\partial \hat{\phi}_i^L}{\partial \beta} < 0$.

(c) Because $\hat{\pi}^* = \hat{p}^*\frac{16\tau-(\beta-1)^2\left(\gamma^2+1\right)\hat{p}^*}{32\tau}$, we can get

$$\frac{\partial \hat{\pi}^*}{\partial \beta} = \frac{\left(8\tau-(1-\beta)^2\left(\gamma^2+1\right)\hat{p}_i^*\right)\frac{\partial \hat{p}^*}{\partial \beta}+(1-\beta)\left(\gamma^2+1\right)\hat{p}^{*2}}{16\tau}$$

Because $(1-\beta)^2\left(\gamma^2+1\right)\hat{p}^* < 2\hat{p}^* < 8\tau$ and $\frac{\partial \hat{p}^*}{\partial \beta} > 0$, we get $\frac{\partial \hat{\pi}^*}{\partial \beta} > 0$.

(d) Because $\hat{CS} = v - \hat{p}^* - \frac{t}{4}(2-\beta) + \frac{t(1-\beta)^2(1+\gamma^2)}{16\tau}\hat{p}^*$, we can get

$$\frac{\partial \hat{CS}}{\partial \beta} = \frac{t}{4} - \left(1-\frac{t(1-\beta)^2(1+\gamma^2)}{16\tau}\right)\frac{\partial \hat{p}_i^*}{\partial \beta} - \hat{p}_i^*\frac{t(1+\gamma^2)(1-\beta)}{8\tau}$$

$\frac{\partial \hat{CS}}{\partial \beta} < 0$ if $\frac{t}{4} - \left[1-\frac{t(1-\beta)^2(1+\gamma^2)}{16\tau}\right]\frac{\partial \hat{p}_i^*}{\partial \beta} < 0$, or, equivalently, $\frac{\partial \hat{p}_i^*}{\partial \beta} > \frac{4t\tau}{16\tau-(1-\beta)^2(\gamma^2+1)t}$. This inequality is true because $\frac{\partial \hat{p}_i^*}{\partial \beta} = \tau^2 t\frac{2K+2\tau-(1-\beta)\beta t\left[\left(2\beta^2+\beta+1\right)\gamma^2+1+\beta-\gamma\beta^2\right]}{K(K+\tau)^2} > \frac{2\tau^2 t}{(K+\tau)^2} > \frac{4t\tau}{16\tau-(1-\beta)^2(\gamma^2+1)t}$.

□

Proof of Proposition 10

Proof. For ease of exposition, we denote $N \equiv \sqrt{\tau^2 - t\tau\beta(1-\beta)^2[(1+\beta)\gamma^2 + 1 - \frac{\beta\gamma}{2}]}$.

(a) Note that

$$\frac{\partial \hat{p}^*}{\partial \gamma} = \frac{(1-\beta)^2\beta^2 t^2\tau^2(4(\beta+1)\gamma-\beta)}{2N(N+\tau)^2}$$

$\frac{\partial \hat{p}^*}{\partial \gamma} > 0$ requires $4(\beta+1)\gamma - \beta > 0$, which is equivalent to $\gamma > \frac{\beta}{4(1+\beta)}$. Otherwise, $\frac{\partial \hat{p}^*}{\partial \gamma} < 0$.

(b) Note that

$$\frac{\partial \hat{\phi}^{H*}}{\partial \gamma} = t\tau\beta(1-\beta)\frac{4N+4\tau-t\beta(1-\beta)^2(4+\beta-5\beta\gamma-4\gamma)}{8N(N+\tau)}$$

The third term in the numerator, $t\beta(1-\beta)^2(4+\beta-5\beta\gamma-4\gamma)$, is decreasing in γ. When $\gamma = 0$, the maximum value is $t\beta(1-\beta)^2(4+\beta) < 4\tau$. Therefore, the numerator is always positive and $\frac{\partial \hat{\phi}^H}{\partial \gamma} > 0$.

$$\frac{\partial \hat{\phi}^{L*}}{\partial \gamma} = t\tau\beta(1-\beta)\frac{t\beta(1-\beta)^2(4-\beta+3\beta\gamma+4\gamma)-4\tau-4N}{8N(N+\tau)^2}$$

The first term in the numerator, $t\beta(1-\beta)^2(4-\beta+3\beta\gamma+4\gamma)$, is increasing in γ. When $\gamma = 1$, the maximum value is $t\beta(1-\beta)^2(8-2\beta) < 4\tau$. Therefore, the numerator is always negative and $\frac{\partial \hat{\phi}^L}{\partial \gamma} < 0$.

(c) Denoting $M \equiv 4(4\beta\gamma - \beta + 3\gamma)(K+\tau) - (1-\beta)^2\beta^2 t(\gamma^2 + 4\gamma - 1)$, we get $\frac{\partial \hat{\pi}^*}{\partial \gamma} = \frac{(1-\alpha)(1-\beta)^2\beta^2t^2\tau^2}{16N(N+\tau)^3}M$. $\frac{\partial \hat{\pi}_i^*}{\partial \gamma} > 0$ requires $M > 0$. Because $\frac{\partial M}{\partial \gamma} > 0$, $M|_{\gamma=0} < 0$ and $M|_{\gamma=1} > 0$. Therefore, there exists $\tilde{\gamma}$, where $\tilde{\gamma}$ is the root of $M = 0$ such that if $\gamma < \tilde{\gamma}$, $\frac{\partial \hat{\pi}^*}{\partial \gamma} < 0$. Otherwise, $\frac{\partial \hat{\pi}^*}{\partial \gamma} > 0$.

(d) Note that

$$\frac{\partial \hat{CS}}{\partial \gamma} = \frac{(1-\beta)^2\beta t^2\tau}{32N(N+\tau)^2}\left[8\gamma N + 8\tau\left(2\beta^2 - 8\beta(1+\beta)\gamma + \gamma\right) - \beta(1-\beta)^2 t\left(4\beta\gamma^3 - 3\beta\gamma^2 - 4\beta\gamma + \beta + 4\gamma^3 + 4\gamma\right)\right]$$

By simplifying $\frac{\partial \hat{CS}}{\partial \gamma} > 0$, we derive the condition in part (d). $\square$

Proof of Lemma 10

Proof. (a) Each seller's best response to its competitor in stage 1 is characterized by the first-order conditions of Equation (4.1):

$$\frac{\partial \pi_A}{\partial p_A} = \frac{1}{2} + \frac{4t_2(1+\rho_1)(1+\rho_2)-(2u-t_1-t_2)(\rho_1-\rho_2)^2}{4t_1t_2(1+\rho_1)(1+\rho_2)(2+\rho_1+\rho_2)}(p_B - 2p_A) + \frac{(\rho_1-\rho_2)(2u-t_1-t_2)}{4t_1t_2(1+\rho_1)(1+\rho_2)}(p_D - p_C)$$

$$\frac{\partial \pi_B}{\partial p_B} = \frac{1}{2} + \frac{4t_2(1+\rho_1)(1+\rho_2)-(2u-t_1-t_2)(\rho_1-\rho_2)^2}{4t_1t_2(1+\rho_1)(1+\rho_2)(2+\rho_1+\rho_2)}(p_A - 2p_B) + \frac{(\rho_1-\rho_2)(2u-t_1-t_2)}{4t_1t_2(1+\rho_1)(1+\rho_2)}(p_C - p_D)$$

$$\frac{\partial \pi_c}{\partial p_c} = \frac{1}{2} + \frac{4t_1(1+\rho_1)(1+\rho_2)-(2u-t_1-t_2)(\rho_1-\rho_2)^2}{4t_1t_2(1+\rho_1)(1+\rho_2)(2+\rho_1+\rho_2)}(p_D - 2p_C) + \frac{(\rho_1-\rho_2)(2u-t_1-t_2)}{4t_1t_2(1+\rho_1)(1+\rho_2)}(p_B - p_A)$$

$$\frac{\partial \pi_D}{\partial p_D} = \frac{1}{2} + \frac{4t_1(1+\rho_1)(1+\rho_2)-(2u-t_1-t_2)(\rho_1-\rho_2)^2}{4t_1t_2(1+\rho_1)(1+\rho_2)(2+\rho_1+\rho_2)}(p_C - 2p_D) + \frac{(\rho_1-\rho_2)(2u-t_1-t_2)}{4t_1t_2(1+\rho_1)(1+\rho_2)}(p_B - p_A)$$

Based on these equations, we can derive the seller equilibrium prices as in Equations (4.2) and (4.3).

(b) Substituting the equilibrium prices into Equation (4.1), we can derive the equilibrium derive the equilibrium profits as in Equation (4.4) and (4.5). □

Proof of Corrollary 3

Proof. When $\rho_1 \neq \rho_2$, we can get

$$\frac{\partial p_A^*}{\partial t_2} = \frac{\partial p_B^*}{\partial t_2} = -\frac{2t_1(\rho_1+1)(\rho_2+1)(\rho_1-\rho_2)^2(\rho_1+\rho_2+2)(2u-t_1)}{\left((\rho_1-\rho_2)^2(t_1-2u)+t_2(\rho_1+\rho_2+2)^2\right)^2} < 0$$

$$\frac{\partial p_C^*}{\partial t_1} = \frac{\partial p_D^*}{\partial t_1} = -\frac{2t_2(\rho_1+1)(\rho_2+1)(\rho_1-\rho_2)^2(\rho_1+\rho_2+2)(2u-t_2)}{\left((\rho_1-\rho_2)^2(t_2-2u)+t_1(\rho_1+\rho_2+2)^2\right)^2} < 0$$

When $\rho_1 = \rho_2 = \rho$, we get $p_A^* = p_B^* = (\rho+1)t_1$ and $p_C^* = p_D^* = (\rho+1)t_2$. Therefore, product differentiation in one category does not affect the other category. □

Proof of Proposition 12

Proof. (a) In symmetric case ($\rho_1 = \rho_2 = \rho$), we get

$$p_A^* = p_B^* = p_X^s = (\rho+1)t_1$$

$$p_C^* = p_D^* = p_Y^s = (\rho+1)t_2$$

In asymmetric case ($\rho_1 = \rho$ and $\rho_2 = 0$), we get

$$p_A^* = p_B^* = p_X^a = \frac{2(\rho+1)(\rho+2)t_1t_2}{\rho^2(t_1-2u)+(\rho+2)^2t_2}$$

$$p_C^* = p_D^* = p_Y^a = \frac{2(\rho+1)(\rho+2)t_1t_2}{\rho^2(t_2-2u)+(\rho+2)^2t_1}$$

Hence, we get $p_X^s - p_X^a = \frac{\rho t_1(\rho+1)(\rho(t_1+t_2-2U)+2t_2)}{\rho^2(t_1-2u)+(\rho+2)^2t_2} > 0$ and $p_Y^s - p_Y^a = \frac{\rho t_2(\rho+1)(\rho(t_1+t_2-2U)+2t_1)}{\rho^2(t_2-2u)+(\rho+2)^2t_1} > 0$

(b) Part (b) follows from

$$\frac{\partial p_A}{\partial \rho_1} = \frac{\partial p_B}{\partial \rho_1} = \frac{2(\rho_1+1)^2t_1t_2\big(t_2(\rho_1+\rho_2+2)^2-(\rho_1-\rho_2)(3\rho_1+\rho_2+4)(t_1-2u)\big)}{\left((\rho_1-\rho_2)^2(t_1-2u)+t_2(\rho_1+\rho_2+2)^2\right)^2} > 0$$

$$\frac{\partial p_C}{\partial \rho_1} = \frac{\partial p_D}{\partial \rho_1} = \frac{2(\rho_2+1)^2t_1t_2\big(t_1(\rho_1+\rho_2+2)^2-(\rho_1-\rho_2)(3\rho_1+\rho_2+4)(t_2-2u)\big)}{\left((\rho_1-\rho_2)^2(t_2-2u)+t_1(\rho_1+\rho_2+2)^2\right)^2} > 0$$

(c) Notice that

$$\frac{\partial p_A}{\partial \rho_2} = \frac{\partial p_B}{\partial \rho_2} = \frac{2(\rho_2+1)^2t_1t_2\Big(\big(\rho_1-\rho_2\big)(\rho_1+3\rho_2+4)(t_1-2u)+t_2(\rho_1+\rho_2+2)^2\Big)}{\left((\rho_1-\rho_2)^2(t_1-2u)+t_2(\rho_1+\rho_2+2)^2\right)^2}$$

Because all the terms are always positive except $(\rho_1-\rho_2)(\rho_1+3\rho_2+4)(t_1-2u)+t_2(\rho_1+\rho_2+2)^2$, we get $\frac{\partial p_A}{\partial \rho_2} = \frac{\partial p_B}{\partial \rho_2} > 0$ under the condition in part (c). Similarly, because

$$\frac{\partial p_C}{\partial \rho_2} = \frac{\partial p_D}{\partial \rho_2} = \frac{2(\rho_2+1)^2t_1t_2\big(t_1(\rho_1+\rho_2+2)^2-(\rho_1-\rho_2)(3\rho_1+\rho_2+4)(t_2-2u)\big)}{\left((\rho_1-\rho_2)^2(t_2-2u)+t_1(\rho_1+\rho_2+2)^2\right)^2}$$

by solving $\frac{\partial p_C}{\partial \rho_2} = \frac{\partial p_B}{\partial \rho_2} > 0$, we get the condition in part (c). □

Proof of Proposition 13

Proof. (a) In symmetric case ($\rho_1 = \rho_2 = \rho$), we get

$$p_A = p_B = p_X^s = (\rho+1)t_1$$

$$p_C = p_D = p_Y^s = (\rho+1)t_2$$

In asymmetric case ($\rho_1 = \rho$ and $\rho_2 = 0$), we get

$$p_A = p_B = p_X^a = \frac{2(\rho+1)(\rho+2)t_1t_2}{\rho^2(t_1-2u)+(\rho+2)^2t_2}$$

$$p_C = p_D = p_Y^a = \frac{2(\rho+1)(\rho+2)t_1t_2}{\rho^2(t_2-2u)+(\rho+2)^2t_1}$$

Hence, we get $p_X^s - p_X^a = \frac{\rho t_1(\rho+1)(\rho(t_1+t_2-2U)+2t_2)}{\rho^2(t_1-2u)+(\rho+2)^2t_2} > 0$ and $p_Y^s - p_Y^a = \frac{\rho t_2(\rho+1)(\rho(t_1+t_2-2U)+2t_1)}{\rho^2(t_2-2u)+(\rho+2)^2t_1} > 0$

(b) Part (b) follows from

$$\frac{\partial p_A^*}{\partial \rho_1} = \frac{\partial p_B^*}{\partial \rho_1} = \frac{2(\rho_2+1)^2t_1t_2\left(t_2(\rho_1+\rho_2+2)^2-(\rho_1-\rho_2)(3\rho_1+\rho_2+4)(t_1-2u)\right)}{((\rho_1-\rho_2)^2(t_1-2u)+t_2(\rho_1+\rho_2+2)^2)^2} > 0$$

$$\frac{\partial p_C^*}{\partial \rho_1} = \frac{\partial p_D}{\partial \rho_1} = \frac{2(\rho_2+1)^2t_1t_2\left(t_1(\rho_1+\rho_2+2)^2-(\rho_1-\rho_2)(3\rho_1+\rho_2+4)(t_2-2u)\right)}{((\rho_1-\rho_2)^2(t_2-2u)+t_1(\rho_1+\rho_2+2)^2)^2} > 0$$

(c) Notice that

$$\frac{\partial p_A}{\partial \rho_2} = \frac{\partial p_B}{\partial \rho_2} = \frac{2(\rho_2+1)^2t_1t_2\left((\rho_1-\rho_2)(\rho_1+3\rho_2+4)(t_1-2u)+t_2(\rho_1+\rho_2+2)^2\right)}{((\rho_1-\rho_2)^2(t_1-2u)+t_2(\rho_1+\rho_2+2)^2)^2}$$

Because all the terms are always positive except $(\rho_1-\rho_2)(\rho_1+3\rho_2+4)(t_1-2u)+t_2(\rho_1+\rho_2+2)^2$, we get $\frac{\partial p_A}{\partial \rho_2} = \frac{\partial p_B}{\partial \rho_2} > 0$ under the condition in part (c). Similarly, because

$$\frac{\partial p_C}{\partial \rho_2} = \frac{\partial p_D}{\partial \rho_2} = \frac{2(\rho_1+1)^2t_1t_2\left((\rho_1-\rho_2)(\rho_1+3\rho_2+4)(t_2-2u)+t_1(\rho_1+\rho_2+2)^2\right)}{((\rho_1-\rho_2)^2(t_2-2u)+t_1(\rho_1+\rho_2+2)^2)^2}$$

By solving $\frac{\partial p_C}{\partial \rho_2} = \frac{\partial p_B}{\partial \rho_2} > 0$, we get the condition in part (c). ☐

Proof of Proposition 12

Proof. (a) Because

$$\frac{\partial \bar{p}_A}{\partial \alpha} = \frac{\partial \bar{p}_B}{\partial \alpha} = -\frac{2t_1t_2(\rho_1+1)(\rho_2+1)(\rho_1+\rho_2+2)}{\alpha^2\left(\beta^2(\rho_1-\rho_2)^2(t_1+t_2-2u)+4t_2(\rho_1+1)(\rho_2+1)\left(\frac{1}{2}(\beta-1)^2(\rho_1+\rho_2)+1\right)\right)} < 0$$

$$\frac{\partial \bar{p}_A}{\partial \beta} = \frac{\partial \bar{p}_B}{\partial \beta} = \frac{2t_1t_2(\rho_1+1)(\rho_2+1)(\rho_1+\rho_2+2)\left(\beta^2(\rho_1-\rho_2)^2(2u-t_1-t_2)+4t_2(1-\beta)(\rho_1+1)(\rho_2+1)(\rho_1+\rho_2+2)\right)}{\alpha\left(\beta^2(\rho_1-\rho_2)^2(t_1+t_2-2u)+4t_2(\rho_1+1)(\rho_2+1)\left(\frac{1}{2}(\beta-1)^2(\rho_1+\rho_2)+1\right)\right)^2} > 0$$

(b) Part (b) follows from

$$\frac{\partial \bar{\pi}_A}{\partial \alpha} = \frac{\partial \bar{\pi}_B}{\partial \alpha} = \frac{(1-\gamma)}{2}\frac{\partial \bar{p}_A}{\partial \alpha} < 0$$

$$\frac{\partial \bar{\pi}_A}{\partial \beta} = \frac{\partial \bar{\pi}_B}{\partial \beta} = \frac{(1-\gamma)}{2} \frac{\partial \bar{p}_A}{\partial \beta} > 0$$

(c) When $\rho_1 = \rho_2 = \rho$, marketplace profit in the presence of recommender system is

$$\pi_R^* = \gamma(t_2 + t_1)(1 + \rho)$$

, marketplace profit in the absence of recommender system is

$$\bar{\pi}_R = \gamma \frac{(\rho+1)\left(t_1 + \alpha t_2(\beta-2)^2\left((\beta-1)^2\rho+1\right)\right)}{\alpha(\beta-1)^2\rho+\alpha}$$

By solving $\pi_R^* > \bar{\pi}_R$, we get the condition in part (c). □

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Lusi Li was born in Heilongjiang, China. She received her Bachelor of Science degree in Management Information Systems in July 2008 and her Master of Science degree in Management Information Systems in July 2010 both from Harbin Institute of Technology. She worked at Alibaba Group as a Business Analyst in Taobao BI department for one year. In Fall 2011, she came to The University of Texas at Dallas (UTD) to pursue a doctoral degree in Management Science at the Naveen Jindal School of Management. At UTD, she taught an undergraduate course in Database Fundamentals in Summer 2016, Spring 2017, and taught Introduction to Management Information System in Fall 2014, Fall 2015, Spring 2016 and Fall 2016.

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