ESSAYS IN OPERATIONS MANAGEMENT

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Dedicated to God Almighty.

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by

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This dissertation consists of three main chapters, each focusing on operational problems that arise in surgical suites' operations, agricultural operations in developing economies, and operations in the on-demand economy, respectively.

In Chapter 2, we study the stochastic, single-machine earliness/tardiness problem (SET), with the sequence of processing of the jobs and their due-dates as decisions and the objective of minimizing the sum of the expected earliness and tardiness costs over all the jobs. We show that the Shortest-Variance-FIrst (SVF) rule is optimal under the assumption of *dilation ordering* of the processing durations. Since *convex ordering* implies dilation ordering (under finite means), the SVF sequence is also optimal under convex ordering of the processing durations.

In Chapter 3, we consider a governmental scheme, viz., the Guaranteed Support Price (GSP) scheme, that several developing countries have adopted to support their farmers and underprivileged population. Through this scheme, the government, operating under a budget, procures a crop from farmers at a guaranteed (and attractive) price, announced ahead of the selling season, and then distributes the procured amount to the underprivileged segment of the population at a subsidized price. We offer analytically-supported insights on fundamental operational decisions of the GSP scheme.

In Chapter 4, we explore problems related to workforce management in the emerging on-demand economy. A fundamental challenge for on-demand platforms (e.g., Uber, InstaCart) is to ensure that independent workers, who are not directly under the platform's control, are available at the right time and locations to serve consumers at short notice. We study the role of surge pricing, worker incentives and information sharing to effectively manage these platforms.

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CHAPTER 1

INTRODUCTION

This dissertation consists of three main chapters. Chapter 2 focuses on a stochastic scheduling problem with applications to appointment scheduling. Chapter 3 considers a governmental scheme, viz., the Guaranteed Support Price (GSP) scheme, that has been implemented in developing countries to support the farming population and the poor consumers. Chapter 4 explores problems in the emerging on-demand economy.

In Chapter 2, we consider the stochastic, single-machine earliness/tardiness problem (SET), with the sequence of processing of the jobs and their due-dates as decisions and the objective of minimizing the sum of the expected earliness and tardiness costs over all the jobs. In a recent paper, (Baker, 2014) shows the optimality of the Shortest-Variance-First (SVF) rule under the following two assumptions: (a) The processing duration of each job follows a normal distribution. (b) The earliness and tardiness cost parameters are the same for all the jobs. In this paper, we consider problem SET under assumption (b). We generalize Baker's result by establishing the optimality of the SVF rule for more general distributions of the processing durations and a more general objective function. Specifically, we show that the SVF rule is optimal under the assumption of *dilation ordering* of the processing durations. Since *convex ordering* implies dilation ordering (under finite means), the SVF sequence is also optimal under convex ordering of the processing durations. We also study the effect of variability of the processing durations of the jobs on the optimal cost. An application of problem SET in surgical scheduling is discussed.

In Chapter 3, we consider a governmental scheme, viz., the Guaranteed Support Price (GSP) scheme, that several developing countries have adopted to support their farmers and underprivileged population. Through this scheme, the government, operating under a budget, procures a crop from farmers at a guaranteed (and attractive) price, announced ahead of the selling season, and then distributes the procured amount to the underprivileged segment of the population at a subsidized price. The goal of this scheme is twofold: (a) as a supply-side incentive, to ensure high output from the farmers, and (b) as a demand-side provisioning tool, to subsidize the consumption needs of the poor.

We offer analytically-supported insights on several fundamental aspects of the GSP scheme by analyzing a Stackelberg game between a homogenous population of small farmers and a social planner. We explicitly model the strategic behavior of the farmers and the consuming population, characterize the equilibrium market outcome (i.e., the production decisions of the farmers and their selling decisions to the government and in the open market, the consumption decisions by the strategic Above-Poverty-Line (APL) and Below-Poverty-Line (BPL) consumers), the resulting equilibrium welfare of each segment, and compare them with that under two benchmarks: (a) the absence of any intervention, and (b) the Direct Benefit Transfer scheme, where the social planner simply distributes the budget among the BPL consumers. We find that two key economic forces – the poorness of the BPL consumers (a demand-side force), and yield uncertainty (a supply-side force) – act as impediments to high production by farmers and consumption by the BPL consumers. Seemingly disparate and acting on different segments of the population, the complex interactions of these forces leads to an interesting analysis. If the poorness of the BPL consumers is extreme, then the GSP scheme improves the production by the farmers and consumption by the BPL consumers, and leads to an increase in the social planner's surplus. If yield uncertainty is dominant, then the social planner can use the GSP scheme as a mechanism to divide his budget in any proportion to improve the surplus of the BPL consumers and the farmers; the desired split is achieved by setting an appropriate support price. We also discuss an extension where the social planner weighs the surplus of the BPL consumers and the farmers more than the APL consumers and the unused budget.

In Chapter 4, we study operational problems in the emerging on-demand economy. A fundamental challenge for on-demand platforms (e.g., Uber, InstaCart) is to ensure that independent workers, who are not directly under the platform's control, are available at the right time and locations to serve consumers at short notice. We study the role of surge pricing (dynamically raising the price at a particular location above the regular price) in managing worker availability across market locations, explicitly accounting for how workers strategically move between locations. We show that, contrary to conventional wisdom, surge pricing can be useful even at locations with excess supply of workers to improve worker availability across locations. Specifically, because workers face costs to move and competition from others who move, simply informing workers about where they are needed is not sufficient to ensure that enough workers move to that location. Surge pricing at a location with excess supply of workers lowers workers' revenue potential of serving that location, thereby inducing more workers to move to where they are needed. Even though such surge pricing reduces platform profit at the location where it is used, it can increase total platform profit across locations, and can even be more profitable than offering workers bonuses to move. Surge pricing at a location with excess supply may also serve as a credible signal of higher demand for workers to move. It may also be used to avoid too many workers from flocking to a location with shortage of workers. Our analysis offers insights for effectively managing on-demand platforms relying on independent workers.

CHAPTER 2

OPTIMAL POLICY FOR A STOCHASTIC SCHEDULING PROBLEM WITH APPLICATIONS TO SURGICAL SCHEDULING[1]

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2.1 Introduction

The single-machine earliness/tardiness problem (SET) with random processing times of the jobs is one of the fundamental problems in stochastic scheduling theory; see e.g., (Cheng, 1991), (Soroush, 1999), (Xia et al., 2008), and (Baker, 2014). This problem is defined as follows: Consider n jobs, indexed by $j = 1, 2, \ldots n$, that are all available to be processed on a single machine at time 0. The processing duration p_j of job j is a random variable with a known distribution. The processing durations are assumed to be independent of each other and job pre-emption is not allowed. We denote the current time by 0 (i.e., the earliest time a job can start processing is 0). We are to decide the *sequence* in which the jobs are to be processed and also their *due-dates*. Let $\boldsymbol{\pi} = (\pi_1, \pi_2, \ldots, \pi_n)$ denote an arbitrary sequence in which the jobs are processed; thus, π_j denotes the j^{th} job in the sequence $\boldsymbol{\pi}$. Let $\mathbf{p}^\pi = (p_{\pi_1}, p_{\pi_2}, \ldots, p_{\pi_n})$. Let d_j^π denote the due-date of job π_j in the sequence $\boldsymbol{\pi}$ and let $\mathbf{d}^\pi = (d_1^\pi, d_2^\pi, \ldots, d_n^\pi)$. The realized completion time of job π_j in the sequence $\boldsymbol{\pi}$ is $C_j^\pi = \sum_{i=1}^{j} p_{\pi_i}$. The earliness and tardiness of job π_j in the sequence $\boldsymbol{\pi}$, denoted by E_j^π and T_j^π respectively, are defined as follows: $E_j^\pi = (d_j^\pi - C_j^\pi)^+$ and $T_j^\pi = (C_j^\pi - d_j^\pi)^+$, where $x^+ = \max\{0, x\}$. Let the unit earliness and tardiness costs for job j be α_j and β_j, respectively. The objective is to minimize the sum of the total expected earliness cost and the total expected tardiness cost, with the sequence $\boldsymbol{\pi}$ and due-dates $\mathbf{d}^\pi$ as decisions. Let $F(\boldsymbol{\pi}, \mathbf{d}^\pi)$ denote the expected cost corresponding to $(\boldsymbol{\pi}, \mathbf{d}^\pi)$. Formally, problem SET is defined as follows:

$$\min_{\boldsymbol{\pi}, \mathbf{d}^\pi} \quad F(\boldsymbol{\pi}, \mathbf{d}^\pi) \quad = \quad \min_{\boldsymbol{\pi}, \mathbf{d}^\pi} \quad \sum_{j=1}^{n} \mathbb{E}[\alpha_{\pi_j} E_j^\pi + \beta_{\pi_j} T_j^\pi]. \tag{SET}$$

In a recent paper, (Baker, 2014) studied Problem SET under the assumption that the processing duration of job j, $j = 1, 2, \ldots, n$, follows a normal distribution with mean μ_j and variance σ_j^2, i.e., $p_j \sim N(\mu_j, \sigma_j^2)$, and developed a Branch-and-Bound algorithm to find an optimal solution and reported its computational performance. He also studied the special case in which all the jobs have common earliness and tardiness cost parameters, i.e., $\alpha_j = \alpha$ and $\beta_j = \beta, j = 1, 2, \ldots, n$, and showed that for this special case the following *Shortest Variance First* (SVF) rule is optimal: *Sequence the jobs in the increasing order of the variances of their processing durations.* We will henceforth refer to the "symmetric" version of problem SET, in which all jobs have common cost parameters (namely, α and β), as **Problem SSET**. Formally, problem SSET is defined as follows:

$$\min_{\boldsymbol{\pi}, \mathbf{d}^{\pi}} \quad F\big(\boldsymbol{\pi}, \mathbf{d}^{\pi}\big) \quad = \quad \min_{\boldsymbol{\pi}, \mathbf{d}^{\pi}} \quad \sum_{j=1}^{n} \mathbb{E}[\alpha E_j^{\pi} + \beta T_j^{\pi}]. \tag{SSET}$$

In this paper, we study problem SSET and establish the optimality of the SVF rule for more general distributions. Specifically, we show that the SVF sequence and appropriately chosen due-dates form an optimal solution to problem SSET under the dilation order of the random processing durations. It is easy to establish that normally distributed random variables, arranged according to the SVF rule, are in dilation order. Thus, our result generalizes the result in (Baker, 2014) for problem SSET.

Problem SSET has an interesting application in surgical scheduling, and is closely related to the classical Appointment Scheduling Problem in that domain. In Section 2.2, we discuss this application and connection. Section 2.3 summarizes the relevant literature. Section 2.4 contains our analysis of problem SSET under dilation ordering of the random processing durations and our result. Section 2.5 establishes the validity of our main result for two extensions of problem SSET. Section 2.6 analyzes the impact of the variability of the processing durations of the jobs on the optimal cost. Section 2.7 offers some useful directions for future research.

2.2 Application to Surgical Scheduling

The well-known Appointment Scheduling Problem (ASP) is one of deciding the sequence in which a set of surgeries is performed and the appointment times given to the corresponding patients, with the objective of minimizing the sum of the expected waiting costs (of the patients) and the expected idling cost (of the resources, including the operating room (OR) and the team of doctors). A standard assumption in this literature is that a surgery is not allowed to start earlier than the scheduled appointment time, while idling of the resources is permitted. We will abbreviate this problem by ASwE. A variant of this problem is when the surgeries are allowed to start earlier than the scheduled appointment time, but idling of the resources is not allowed. We will abbreviate this problem by ASwI. Both problems are extreme representations of reality and the understanding of these special cases is valuable; (Pinedo, 2009) discusses both these problems.

We now proceed to define the problems ASwE and ASwI formally. Consider n patients, indexed by $j = 1, 2, \ldots, n$, each of whom needs to undergo a surgery in an OR. The duration of each surgery is random, with a known distribution. Let Z_j denote the random duration of the surgery for patient j, $j = 1, 2, \ldots, n$. The durations of the surgeries are assumed to be independent of each other. We denote the current time by 0 (i.e., the earliest time a surgery can start is 0).

Let $\boldsymbol{\pi} = (\pi_1, \pi_2, \ldots, \pi_n)$ be the sequence in which the surgeries are performed and let $\mathbf{A}^\pi = (A_1^\pi, A_2^\pi, \ldots, A_n^\pi)$ be the corresponding appointment times. Let S_j^π denote the actual (or realized) start time of the surgery of patient π_j, $j = 1, 2, \ldots, n$. We consider the following costs:

- *Waiting cost* of a patient, denoted by c_W per unit time, incurred when the patient waits for the start of his surgery after the scheduled appointment time. The waiting cost of patient π_j is $c_W[S_j^\pi - A_j^\pi]^+$, $j = 1, 2, \ldots, n$.

- *Idling cost* of the OR, denoted by c_I per unit time, incurred due to the non-usage of the OR. The idling cost incurred between the end of the surgery of patient π_j and the start of the surgery of patient π_{j+1} is $c_I[S_{j+1}^\pi - (S_j^\pi + Z_{\pi_j})]^+$.

- *Earliness penalty* for a patient, denoted by c_E per unit time, incurred in advancing the start time of the patient's surgery. The earliness penalty incurred for patient π_j is $c_E[A_j^\pi - S_j^\pi]^+$.

The total cost corresponding to $(\boldsymbol{\pi}, \mathbf{A}^\pi)$ is the sum of the three costs mentioned above for all the patients. The goal is to obtain a sequence in which the surgeries are performed and the appointment schedule, such that the total expected cost is minimized; such a schedule is referred to as an *optimal schedule*. The corresponding sequence is an *optimal sequence*. Note that problem ASwE corresponds to the following choice of the cost parameters: $c_E = \infty$, $c_W < \infty, c_I < \infty$, while the variant ASwI corresponds to $c_E < \infty, c_W < \infty, c_I = \infty$. We formally define problems ASwE and ASwI below:

$$(\mathbf{ASwE}) \qquad \min_{\boldsymbol{\pi}, A^\pi \in \mathbb{R}_+^n} \quad \mathbb{E}\left[\sum_{j=1}^{n} \left[c_W(S_j^\pi - A_j^\pi)^+ + c_I(A_j^\pi - S_{j-1}^\pi - Z_{\pi_{j-1}})^+\right]\right], \text{ where}$$
$$S_0^\pi = Z_{\pi_0} = 0 \text{ and } S_{j+1}^\pi = \max(A_{j+1}^\pi, S_j^\pi + Z_{\pi_j}), i = 0, 1, \ldots, n-1.$$

$$(\mathbf{ASwI}) \qquad \min_{\boldsymbol{\pi}, A^\pi \in \mathbb{R}_+^n} \quad \mathbb{E}\left[\sum_{j=1}^{n} \left[c_W(S_j^\pi - A_j^\pi)^+ + c_E(A_j^\pi - S_j^\pi)^+\right]\right], \text{ where}$$
$$S_1^\pi = 0 \text{ and } S_{j+1}^\pi = S_j^\pi + Z_{\pi_j} = \textstyle\sum_{i=1}^{j} Z_{\pi_i}, j = 1, 2, \ldots, n-1.$$

An outpatient setting – where patients arrive for surgeries from outside the hospital – is the most relevant one for these two problems. It is in this setting that the notions of waiting and earliness of the patients – and their impact on customer service metrics at hospitals – become more meaningful; see, e.g., (Stempniak, 2013), (Globerman et al., 2013), (Klassen and Yoogalingam, 2009), (Pinedo, 2009). The importance of the variant ASwI is based on the fact that surgeons and operating rooms are extremely valuable resources for hospitals; see, e.g., (Marshall Steele & Associates, 2014), (Macario, 2010) and (Gupta, 2007); therefore, minimizing the idle time of these resources is a priority. Indeed, the practice of starting surgeries earlier than scheduled to avoid loss of the surgeon's time is well-accepted; we provide the following illustrative quotes:

- "*Sometimes your surgeon may request you come earlier than scheduled, so please try and be available by telephone the day of your surgery.*" – Institute for Orthopaedic Surgery, Las Vegas, NV. (Institute of Orthopaedic Surgery, 2014)

- "*We do ask patients to arrive early for their operation in order that we can carry out any checks and also because there is inevitably some uncertainty around how long each procedure will take. It may therefore be that an operation may take place earlier than scheduled, should another be completed more quickly than expected ...*" – Chris Watt, Director of Performance and Delivery, Harrogate and District Hospital, Yorkshire, UK. (Watt, 2014)

Connection to problem SSET: Consider a more generalized version of problem SSET (introduced in Section 4.1), where the objective function is a non-negative linear combination of the expected costs of all the jobs, i.e. $F(\boldsymbol{\pi}, \mathbf{d}^{\pi}) = \sum_{j=1}^{n} a_j \mathbb{E}[\alpha(d_j^{\pi} - C_j^{\pi})^{+} + \beta(C_j^{\pi} - d_j^{\pi})^{+}]$, where $a_j \geq 0$, for all j, $j = 1, 2, \ldots, n$. The special case of this problem corresponding to $a_j = 1$ for $j = 1, 2, \ldots, n-1$, and $a_n = 0$, is identical to problem ASwI. To see this, it is sufficient to make the following observations: Consider an arbitrary sequence $\boldsymbol{\pi}$ that corresponds to a sequence of patients in ASwI and a sequence of jobs in the special case defined above. Then,

- The appointment time A_{j+1}^{π} of patient π_{j+1} in ASwI corresponds to the due-date d_j^{π} of job π_j in problem SSET, i.e., $A_{j+1}^{\pi} \equiv d_j^{\pi}$.

- The random start time S_{j+1}^{π} of the surgery of patient π_{j+1} in ASwI corresponds to the random completion time C_j^{π} of job π_j in problem SSET, i.e., $S_{j+1}^{\pi} \equiv C_j^{\pi}$.

- The cost parameter c_E (resp., c_W) for earliness (resp., waiting) in problem ASwI corresponds to the cost parameter α (resp., β) for earliness (resp., tardiness) in problem SSET, i.e., $c_E \equiv \alpha$ and $c_W \equiv \beta$.

Throughout this paper, we will analyze problem SSET (as defined in Section 4.1). However, all our results remain valid for the generalized version above and, therefore, also for problem ASwI.

There exists a considerable amount of literature that investigates the optimality of the SVF rule for problem ASwE, see Section 2.3. In general, however, this problem has remained open. Our main result, Theorem 2.4.1, establishes the optimality of the SVF rule for the variant ASwI.

2.3 Related Literature

We briefly summarize the literature on problem SET. The deterministic version of this problem has received significant attention; we refer the reader to the review paper by (Baker and Scudder, 1990) and the textbook by (Pinedo, 2011). (Cheng, 1991) is one of the first to address the problem with random processing times. For each job, he models the total cost as the sum of two components: (a) a function of the length of the assigned due-date for that job and (b) a function of the deviation of the completion time of that job from its due-date. For this problem, he derives analytical expressions of the optimal due-dates for a given processing sequence and proposes a sorting-based algorithm to find the optimal job sequence under certain simplifying assumptions. (Soroush, 1999) develops two effective heuristics for problem SET and reports their computational performance. (Xia et al., 2008) propose an effective heuristic for a variant of problem SSET in which the objective includes, for each job, an additional cost penalty that is proportional to the length of its due-date. As mentioned earlier, (Baker, 2014) studies problem SET assuming normally distributed processing times and proposes a branch-and-bound algorithm to obtain optimal solutions. He also establishes the optimality of the SVF rule for the special case of problem SSET with normally distributed processing times.

We now summarize the investigations on problem ASwE. (Weiss, 1990) is arguably the first study to analyze this problem. For the special case of two patients (i.e., $n = 2$), he shows that scheduling the surgery with the lower variance first is optimal under both uniform and exponential surgery durations. This study also conjectures that the SVF rule might be optimal in general (i.e., for $n \geq 3$). (Gupta, 2007) investigates the optimal sequence of patients under the convex ordering of the random surgery durations. He proves the optimality of the SVF rule for $n = 2$, but reports that attempts to generalize the SVF rule to larger values of n have not been fruitful. In a recent paper, (Kong et al., 2016) investigate the sequencing problem under the assumption that the allowance (difference between successive appointment times) for every surgery equals its mean duration. They produce an example in which the SVF sequence is strictly sub-optimal. Moreover, they show that the SVF rule is optimal (when each allowance equals the expected duration) under a set of assumptions, which includes the assumption that the surgery durations can be ordered with respect to the likelihood ratio order. While (Kong et al., 2016) focus on the sequencing problem for a given allowance vector (equal to the mean of the surgery durations), (Robinson and Chen, 2003) focus on the problem of finding the optimal allowances for a given sequence of patients. (Mak et al., 2014) consider a robust min-max variant of the classical problem that seeks a sequence and corresponding appointment times to minimize the maximum expected value of the total waiting and overtime costs over all distributions with given first and second moments. Under some technical conditions, it is shown that the SVF rule is optimal for this variant.

2.4 Main Result: Optimality of the SVF Rule for Problem SSET under Dilation Ordering

We start by defining the well-known notions of the convex order and the dilation order of random variables, see e.g., (Shaked and Shanthikumar, 2007), and then state and prove our result.

Definition 2.4.1. (Convex Order) *A random variable X is said to be smaller than another random variable Y in the convex order, denoted by $X \leq_{cx} Y$, if $\mathbb{E}[f(X)] \leq \mathbb{E}[f(Y)]$ for all convex functions $f : \mathbb{R} \to \mathbb{R}$, provided the expectations exist.*

Definition 2.4.2. (Dilation Order) *A random variable X with a finite mean is said to be smaller than another random variable Y with a finite mean in the dilation order, denoted by $X \leq_{dil} Y$, if $\big[X - \mathbb{E}[X]\big] \leq_{cx} \big[Y - \mathbb{E}[Y]\big]$.*

Theorem 2.4.1. (Optimal Solution) *Let p_j and μ_j denote, respectively, the random processing duration and the mean processing duration of job j, $j = 1, 2, \ldots, n$. Assume that $p_1 \leq_{dil} p_2 \leq_{dil} \ldots \leq_{dil} p_n$ (i.e., $p_1 - \mu_1 \leq_{cx} p_2 - \mu_2 \leq_{cx} \ldots \leq_{cx} p_n - \mu_n$). Let $d_j^* = \Phi_j^{-1}\left(\dfrac{\beta}{\alpha + \beta}\right)$, where Φ_j is the CDF of $\sum_{k=1}^{j} p_k$. Then, the sequence $(1, 2, \ldots, n)$ and the due-dates d_j^*, $j = 1, 2, \ldots, n$, form an optimal solution to Problem SSET.*

Proof. Let $\boldsymbol{\pi} = (\pi_1, \pi_2, \ldots, \pi_n)$ denote an arbitrary sequence in which jobs are processed. Let $\mathbf{d}^\pi = (d_1^\pi, d_2^\pi, \ldots, d_n^\pi)$ denote the vector of due-dates corresponding to the jobs in the sequence $\boldsymbol{\pi}$, where d_j^π denotes the due-date of job π_j. The completion time of job π_j is $C_j^\pi = \sum_{i=1}^{j} p_{\pi_i}$.

Next, for any $x \in \mathbb{R}$, let $f(x) = \alpha(-x)^+ + \beta(x)^+$. It is easy to see that $\mathbb{E}[f(C_j^\pi - d_j^\pi)]$ is the expected cost corresponding to job π_j. For any $\mathbf{d} = (d_1, d_2, \ldots, d_n)$, let

$$F(\boldsymbol{\pi}, \mathbf{d}) = \sum_{j=1}^{n} \mathbb{E}[f(C_j^\pi - d_j)]. \tag{2.1}$$

Thus, $F(\boldsymbol{\pi}, \mathbf{d}^{\pi})$ denotes the expected cost corresponding to $(\boldsymbol{\pi}, \mathbf{d}^{\pi})$.

Let $\hat{\boldsymbol{\pi}}$ be the special sequence $(1, 2, \ldots, n)$, which is the SVF sequence. We are required to show that

$$\min_{\mathbf{d}} \quad F(\hat{\boldsymbol{\pi}}, \mathbf{d}) \quad \leq \quad \min_{\mathbf{d}} \quad F(\boldsymbol{\pi}, \mathbf{d}). \tag{2.2}$$

We will prove (2.2) by showing that for any vector $\mathbf{d}^{\pi}$ of due-dates, there exists a carefully chosen vector $\mathbf{d}^{\hat{\pi}}$ such that $F(\hat{\boldsymbol{\pi}}, \mathbf{d}^{\hat{\pi}}) \leq F(\boldsymbol{\pi}, \mathbf{d}^{\pi})$. In fact, we will establish the stronger claim that for every j, the cost corresponding to job $\hat{\pi}_j$ under $(\hat{\boldsymbol{\pi}}, \mathbf{d}^{\hat{\pi}})$ is less than the cost corresponding to job π_j under $(\boldsymbol{\pi}, \mathbf{d}^{\pi})$. That is, we will show that

$$\mathbb{E}[f(C_j^{\hat{\pi}} - d_j^{\hat{\pi}})] \leq \mathbb{E}[f(C_j^{\pi} - d_j^{\pi})] \quad \forall j = 1, 2, \ldots, n. \tag{2.3}$$

We proceed to define a due-date vector $\mathbf{d}^{\hat{\pi}}$ that satisfies (2.3).

We define the following notation. Let

$$\begin{aligned} U_j &= \sum_{k=1}^{j} (\mu_{\hat{\pi}_k} - \mu_{\pi_k}), \quad j = 1, 2, \ldots, n, \quad \text{and} \\ \mathbf{U} &= (U_1, U_2, \ldots, U_n). \end{aligned}$$

Now, define $\mathbf{d}^{\hat{\pi}}$ as follows: $\mathbf{d}^{\hat{\pi}} = \mathbf{d}^{\pi} + \mathbf{U}$. Corresponding to this vector of due-dates, the expected cost of job $\hat{\pi}_j$ is

$$\mathbb{E}\Big[f(C_j^{\hat{\pi}} - d_j^{\hat{\pi}})\Big] = \mathbb{E}\Big[f(C_j^{\hat{\pi}} - U_j - d_j^{\pi})\Big]. \tag{2.4}$$

Thus, to show (2.3), it only remains to show that

$$\mathbb{E}\Big[f(C_j^{\hat{\pi}} - U_j - d_j^{\pi})\Big] \leq \mathbb{E}\Big[f(C_j^{\pi} - d_j^{\pi})\Big]. \tag{2.5}$$

Since f is convex, it is sufficient to prove that $C_j^{\hat{\pi}} - U_j - d_j^{\pi} \leq_{cx} C_j^{\pi} - d_j^{\pi}$. That is,

$$\sum_{k=1}^{j} p_{\hat{\pi}_k} - U_j - d_j^{\pi} \leq_{cx} \sum_{k=1}^{j} p_{\pi_k} - d_j^{\pi}. \tag{2.6}$$

Let $\mathcal{I}_{\hat{\pi}} = \{1, 2, \ldots, j\}$, $\mathcal{I}_{\pi} = \{\pi_1, \pi_2, \ldots, \pi_j\}$ and $\mathcal{K} = \mathcal{I}_{\hat{\pi}} \cap \mathcal{I}_{\pi}$. Then,

$$\sum_{k=1}^{j} p_{\hat{\pi}_k} - U_j - d_j^{\pi} = \sum_{k \in \mathcal{I}_{\hat{\pi}} \setminus \mathcal{K}} (p_k - \mu_k) + \sum_{k \in \mathcal{K}} (p_k - \mu_k) + \sum_{k \in \mathcal{I}_{\pi}} \mu_k - d_j^{\pi} \tag{2.7}$$

$$\text{and } \sum_{k=1}^{j} p_{\pi_k} - d_j^{\pi} = \sum_{k \in \mathcal{I}_{\pi} \setminus \mathcal{K}} (p_k - \mu_k) + \sum_{k \in \mathcal{K}} (p_k - \mu_k) + \sum_{k \in \mathcal{I}_{\pi}} \mu_k - d_j^{\pi}. \tag{2.8}$$

Since $\mathcal{I}_{\hat{\pi}} = \{1, 2, \ldots, j\}$ is the set of the j "smallest jobs" in the dilation order, it is easy to see that $\sum_{k \in \mathcal{I}_{\hat{\pi}} \setminus \mathcal{K}} (p_k - \mu_k) \leq_{cx} \sum_{k \in \mathcal{I}_{\pi} \setminus \mathcal{K}} (p_k - \mu_k)$. This observation, along with (2.7), (2.8), and the assumed independence of the processing durations, implies (2.6) as required.

Note : It is possible that the vector $\mathbf{d}^{\hat{\pi}}$ contains some negative due-dates. In that case, it is easy to show that $F(\hat{\pi}, (\mathbf{d}^{\hat{\pi}})^+) \leq F(\hat{\pi}, \mathbf{d}^{\hat{\pi}})$, where for any vector $\mathbf{d} \in \mathbb{R}^n$, $\mathbf{d}^+ \in \mathbb{R}_+^n$ is the vector of component-wise positive parts.

We now consider the optimal due-dates corresponding to the optimal sequence $\hat{\boldsymbol{\pi}}$. The cost corresponding to the j^{th} job is given by $\mathbb{E}[\beta(\sum_{k=1}^{j} p_k - d_j^{\hat{\pi}})^+ + \alpha(d_j^{\hat{\pi}} - \sum_{k=1}^{j} p_k)^+]$. Notice that this expression is identical to the expected cost in a newsvendor problem in which the random demand is $\sum_{k=1}^{j} p_k$, the stocking level is $d_j^{\hat{\pi}}$, the overage cost parameter is α, and the underage cost parameter is β. Thus, the optimal due-dates are given by $d_j^{\hat{\pi}} = \Phi_j^{-1}\left(\dfrac{\beta}{\alpha + \beta}\right)$, where Φ_j is the CDF of $\sum_{k=1}^{j} p_k$. This has been noted in the literature, see, e.g., (Baker, 2014) and (Soroush, 1999). $\square$

A few observations on the result in Theorem 2.4.1 deserve to be mentioned:

- Notice that if $p_1 \leq_{dil} p_2 \leq_{dil} \ldots. \leq_{dil} p_n$, then $\mathrm{Var}(p_1) \leq \mathrm{Var}(p_2) \leq \ldots \leq \mathrm{Var}(p_n)$. Thus, Theorem 2.4.1 guarantees that *an* SVF sequence is optimal for problem SSET when the processing durations of the jobs can be ordered in the dilation order. Furthermore, when the jobs have finite variances, *any* SVF sequence is optimal for problem SSET under the dilation ordering of the processing durations. This is because, for two real-valued random variables X and Y, if $X \leq_{dil} Y$ and $\mathrm{Var}(X) = \mathrm{Var}(Y)$, then $X -$

$E[X] =_{st} Y - E[Y]$; see Theorem 2.2 of (Denuit et al., 2000). Therefore, $X \leq_{dil} Y$ and $Y \leq_{dil} X$. Finally, note that Theorem 1 implies the optimality of the SVF rule for problem SSET if the processing durations are in a convex order. This is because $p_1 \leq_{cx} p_2 \leq_{cx} \ldots \leq_{cx} p_n$ implies that $p_1 \leq_{dil} p_2 \leq_{dil} \ldots \leq_{dil} p_n$ when the means of all the jobs are finite.

- A nice feature of the result in Theorem 2.4.1 is that it does not depend on the values of the cost parameters or the exact expression of the cost. As long as the total cost is a convex function of the deviation from the scheduled due dates, the result provides a rigorous justification for the idea that "more predictable" jobs (or surgeries, in the case of the surgical scheduling application) should be scheduled earlier as opposed to "less predictable" jobs.

- Observe that Theorem 2.4.1 implies the result of (Baker, 2014) that the SVF rule is optimal when the processing times are normally distributed. Let $p_j \sim N(\mu_j, \sigma_j^2)$, $j = 1, 2, \ldots, n$ be independent random variables, and assume $\sigma_1^2 \leq \sigma_2^2 \leq \ldots \leq \sigma_n^2$. Then, it is easy to see that $p_1 - \mu_1 \leq_{cx} p_2 - \mu_2 \leq_{cx} \ldots \leq_{cx} p_n - \mu_n$, or $p_1 \leq_{dil} p_2 \leq_{dil} \ldots \leq_{dil} p_n$, and Theorem 2.4.1 applies.

Theorem 2.4.1 is also applicable to processing durations that follow several other families of distributions; e.g., uniform, gamma, Weibull, lognormal, and beta. For each of these families, if all the processing durations are from the same family and have the same mean, then the durations arranged in the increasing order of their variances are in convex order (and, therefore, dilation order); see (Gupta, 2007), and, (Gupta and Cooper, 2005). Another large class of distributions for the processing durations of the jobs, which are in dilation order, is the popular location-scale family of distributions and can be obtained as follows: Let μ_j and σ_j denote the mean and the standard deviation of the processing duration p_j of job j, $j = 1, 2, \ldots, n$, which is distributed as follows: $p_j \sim \mu_j + z_j\sigma_j$, where z_j's are i.i.d random variables with zero mean and unit standard deviation. In such cases, if $\sigma_1 \leq \sigma_2 \leq \ldots \leq \sigma_n$, then $p_1 \leq_{dil} p_2 \leq_{dil} \ldots \leq_{dil} p_n$.

2.5 Extensions

In this section, we consider two extensions of problem SSET, which we denote as **Problem SSET-E1** and **Problem SSET-E2**. The setting of the extensions is the same as that of problem SSET, but their objective functions are more general. We now define these two problems.

Problem SSET-E1: As in SSET, let $\boldsymbol{\pi} = (\pi_1, \pi_2, \ldots, \pi_n)$ denote an arbitrary sequence in which the jobs are processed. Let $\mathbf{p}^\pi = (p_{\pi_1}, p_{\pi_2}, \ldots, p_{\pi_n})$. Let d_j^π denote the due-date of job π_j in the sequence $\boldsymbol{\pi}$ and let $\mathbf{d}^\pi = (d_1^\pi, d_2^\pi, \ldots, d_n^\pi)$. The realized completion time of job π_j in the sequence $\boldsymbol{\pi}$ is $C_j^\pi = \sum_{i=1}^{j} p_{\pi_i}$. Let the cost corresponding to job π_j be $\theta(C_j^\pi - d_j^\pi)$, where $\theta(\cdot)$ is a non-negative convex function such that $\theta(0) = 0$, i.e., the cost is 0 when there is no earliness or tardiness and the marginal earliness penalty and the marginal tardiness penalty are increasing functions. The objective is the same as that of problem SSET; i.e., minimize the sum of the costs corresponding to all jobs, with the sequence $\boldsymbol{\pi}$ and due-dates $\mathbf{d}^\pi$ as decisions. Problem SSET-E1 is formally defined as follows:

$$\min_{\boldsymbol{\pi}, \mathbf{d}^\pi} \quad F\big(\boldsymbol{\pi}, \mathbf{d}^\pi\big) \quad = \quad \min_{\boldsymbol{\pi}, \mathbf{d}^\pi} \quad \sum_{j=1}^{n} \mathbb{E}[\theta(C_j^\pi - d_j^\pi)]. \tag{SSET-E1}$$

Notice that SSET is a special case of SSET-E1 with $\theta(x) = \alpha(-x)^+ + \beta(x)^+$. The proof of the following remark is identical to that of Theorem 2.4.1.

Remark 1: Theorem 1 holds for problem SSET-E1. ■

Problem SSET-E2: This problem is identical to Problem SSET-E1, except that the cost corresponding to job π_j is given by $\psi(d_j^\pi) + \theta(C_j^\pi - d_j^\pi)$, where $\psi(x)$ is a non-decreasing function of x and $\theta(x)$ is a convex function of x. Here, the term $\psi(d_j^\pi)$ captures the cost associated with giving longer due-dates and the term $\theta(C_j^\pi - d_j^\pi)$ captures the cost of deviating from the due-dates (Cheng, 1991). Formally, problem SSET-E2 is defined as follows:

$$\min_{\boldsymbol{\pi}, \mathbf{d}^\pi} \quad F\big(\boldsymbol{\pi}, \mathbf{d}^\pi\big) \quad = \quad \min_{\boldsymbol{\pi}, \mathbf{d}^\pi} \quad \sum_{j=1}^{n} \Big[\psi(d_j^\pi) + \mathbb{E}[\theta(C_j^\pi - d_j^\pi)]\Big]. \tag{SSET-E2}$$

Problem SSET-E1 is trivially a special case of problem SSET-E2, with $\psi(x) = 0$ for all $x \in \mathbb{R}$. In the following theorem, we show the optimality of the SVF rule for problem SSET-E2, under the additional assumption that $\mu_1 \le \mu_2 \le \ldots \le \mu_n$.

Theorem 2.5.1. *Let p_j and μ_j denote the random processing duration and the mean processing duration of job j, $j = 1, 2, \ldots, n$. Assume that $p_1 \le_{dil} p_2 \le_{dil} \ldots \le_{dil} p_n$ (i.e., $p_1 - \mu_1 \le_{cx} p_2 - \mu_2 \le_{cx} \ldots \le_{cx} p_n - \mu_n$) and that $\mu_1 \le \mu_2 \le \ldots \le \mu_n$. Then, the sequence $(1, 2, \ldots, n)$ is an optimal sequence for Problem SSET-E2.*

Proof. For any $\mathbf{d} = (d_1, d_2, \ldots, d_n)$, let

$$F(\boldsymbol{\pi}, \mathbf{d}) = \sum_{j=1}^{n} \Big[\psi(d_j) + \mathbb{E}[\theta(C_j^\pi - d_j)]\Big]. \tag{2.9}$$

Thus, $F(\boldsymbol{\pi}, \mathbf{d}^\pi)$ is the cost corresponding to $(\boldsymbol{\pi}, \mathbf{d}^\pi)$. Let $\hat{\boldsymbol{\pi}}$ be the special sequence $(1, 2, \ldots, n)$, which is the SVF sequence. We are required to show that

$$\min_{\mathbf{d}} \quad F(\hat{\boldsymbol{\pi}}, \mathbf{d}) \quad \le \quad \min_{\mathbf{d}} \quad F(\boldsymbol{\pi}, \mathbf{d}). \tag{2.10}$$

As in the proof of Theorem 2.4.1, we prove (2.10) by showing the following stronger claim: for any vector $\mathbf{d}^\pi$ of due-dates, there exists a carefully chosen vector $\mathbf{d}^{\hat{\pi}}$ such that for every j, the cost corresponding to job $\hat{\pi}_j$ under $(\hat{\boldsymbol{\pi}}, \mathbf{d}^{\hat{\pi}})$ is less than the cost corresponding to job π_j under $(\boldsymbol{\pi}, \mathbf{d}^\pi)$, i.e.,

$$\psi(d_j^{\hat{\pi}}) + \mathbb{E}[\theta(C^{\hat{\pi}} - d_j^{\hat{\pi}})] \le \psi(d_j^\pi) + \mathbb{E}[\theta(C^\pi - d_j^\pi)]. \tag{2.11}$$

Let $\mathbf{d}^{\hat{\pi}} = \mathbf{d}^\pi + \mathbf{U}$, where $\mathbf{U}$ is as defined in the proof of Theorem 2.4.1. Recall, from the proof of Theorem 2.4.1 that

$$\mathbb{E}\Big[\theta(C_j^{\hat{\pi}} - d_j^{\hat{\pi}})\Big] \le \mathbb{E}\Big[\theta(C_j^\pi - d_j^\pi)\Big]. \tag{2.12}$$

Moreover, the definition of $\hat{\boldsymbol{\pi}}$ and the assumption that $\mu_1 \le \mu_2 \le \ldots \le \mu_n$ implies that $\sum_{i=1}^{j} \mu_{\hat{\pi}_i} \le \sum_{i=1}^{j} \mu_{\pi_i}$. Therefore, $d_j^{\hat{\pi}} \le d_j^{\pi}$, for all $j = 1, 2, \ldots, n$. Since $\psi(\cdot)$ is a non-decreasing function, we have

$$\psi(d_j^{\hat{\pi}}) \le \psi(d_j^{\pi}). \tag{2.13}$$

The claimed inequality in (2.11) follows immediately from (2.12) and (2.13). □

Remark 2 (Optimal Due-Dates): The optimal due-dates $\mathbf{d} = (d_1, d_2, \ldots, d_n)$ can be obtained as follows: For each j, $j = 1, 2, \ldots, n$, d_j is the solution of $\min_{d_j} \left[\psi(d_j) + \mathbb{E}[\theta(C_j^{\hat{\pi}} - d_j)]\right]$, which is a univariate minimization problem. Moreover, if $\psi(\cdot)$ is convex, then this problem is a convex optimization problem. ■

2.6 Effect of Variability on Optimal Cost

In this section, we study the effect of variability of the processing durations of the jobs on the optimal cost of problem SSET-E2. We then illustrate our result using the popular *location-scale family* of distributions and the lognormal distribution for the processing durations of the jobs.

Consider two instances of Problem SSET-E2, $\mathbf{I}$ and $\hat{\mathbf{I}}$, which are defined as follows: Under instance $\mathbf{I}$, the processing duration p_j of the j^{th} job follows a distribution $G_j(\cdot)$, i.e., $p_j \sim G_j(\cdot)$, where the distributions $G_j(\cdot)$ are independent. Similarly, under instance $\hat{\mathbf{I}}$, the processing duration $\hat{p}_j$ of the j^{th} job follows a distribution $\hat{G}_j(\cdot)$, i.e., $\hat{p}_j \sim \hat{G}_j$, where the distributions $\hat{G}_j(\cdot)$ are independent. Let $\mathbb{E}[p_j] = \mu_j$ and $\mathbb{E}[\hat{p}_j] = \hat{\mu}_j$ for all j. Let $F^*(\mathbf{I})$ denote the optimal cost for instance $\mathbf{I}$ and $F^*(\hat{\mathbf{I}})$ denote the optimal cost for instance $\hat{\mathbf{I}}$. The following result compares the optimal cost under the two instances.

Theorem 2.6.1. *For all j, $j = 1, 2, \ldots, n$, assume that $\hat{\mu}_j \le \mu_j$ and $\hat{p}_j \le_{dil} p_j$ (i.e., $\hat{p}_j - \hat{\mu}_j \le_{cx} p_j - \mu_j$). Then $F^*(\hat{\mathbf{I}}) \le F^*(\mathbf{I})$. Moreover, if $\psi(\cdot) = 0$, then the assumption that $\hat{\mu}_j \le \mu_j$ for all j can be dropped.*

Proof. From the definition of Problem SSET-E2, we have

$$F^*(\mathbf{I}) = \min_{\boldsymbol{\pi},\mathbf{d}^\pi} \quad \sum_{j=1}^{n} \Big[\psi(d_j^\pi) + \mathbb{E}[\theta(\sum_{i=1}^{j} p_{\pi_i} - d_j^\pi)]\Big]. \tag{2.14}$$

We prove the following stronger claim: For any sequence $\boldsymbol{\pi}$ and any vector of due-dates $\mathbf{d}^\pi$ for instance $\mathbf{I}$, there exists a carefully chosen vector of due-dates $\hat{\mathbf{d}}^\pi$ for the same sequence $\boldsymbol{\pi}$ for instance $\hat{\mathbf{I}}$ such that for every j, the cost corresponding to job π_j under instance $\hat{\mathbf{I}}$ is less than the cost corresponding to the job π_j under the instance $\mathbf{I}$, i.e.,

$$\psi(\hat{d}_j^\pi) + \mathbb{E}\Big[\theta(\sum_{i=1}^{j} \hat{p}_{\pi_i} - \hat{d}_j^\pi)\Big] \quad \leq \quad \psi(d_j^\pi) + \mathbb{E}\Big[\theta(\sum_{i=1}^{j} p_{\pi_i} - d_j^\pi)\Big]. \tag{2.15}$$

Let $\hat{d}_j^\pi = d_j^\pi + \sum_{i=1}^{j}(\hat{\mu}_{\pi_i} - \mu_{\pi_i})$ for every j, $j = 1, 2, \ldots, n$. Therefore, we have

$$\sum_{i=1}^{j} \hat{p}_{\pi_i} - \hat{d}_j^\pi \quad = \quad \sum_{i=1}^{j}(\hat{p}_{\pi_i} - \hat{\mu}_{\pi_i}) + \sum_{i=1}^{j} \mu_{\pi_i} - d_j^\pi, \quad \text{and} \tag{2.16}$$

$$\sum_{i=1}^{j} p_{\pi_i} - d_j^\pi \quad = \quad \sum_{i=1}^{j}(p_{\pi_i} - \mu_{\pi_i}) + \sum_{i=1}^{j} \mu_{\pi_i} - d_j^\pi. \tag{2.17}$$

Since $\hat{p}_j \leq_{dil} p_j$ for every j, and the processing durations of the jobs are independent, it is easy to see that $\sum_{i=1}^{j}(\hat{p}_{\pi_i} - \hat{\mu}_{\pi_i}) \leq_{cx} \sum_{i=1}^{j}(p_{\pi_i} - \mu_{\pi_i})$. This observation, along with (2.16), (2.17), and the convexity of $\theta(\cdot)$ implies

$$\mathbb{E}\Big[\theta(\sum_{i=1}^{j} \hat{p}_{\pi_i} - \hat{d}_j^\pi)\Big] \quad \leq \quad \mathbb{E}\Big[\theta(\sum_{i=1}^{j} p_{\pi_i} - d_j^\pi)\Big]. \tag{2.18}$$

Moreover, the assumption that $\hat{\mu}_j \leq \mu_j$ for every j, implies that $\sum_{i=1}^{j} \hat{\mu}_{\pi_i} \leq \sum_{i=1}^{j} \mu_{\pi_i}$. Therefore, $\hat{d}_j^\pi \leq d_j^\pi$ for every j. Since $\psi(\cdot)$ is a non-decreasing function, we have

$$\psi(\hat{d}_j^\pi) \quad \leq \quad \psi(d_j^\pi). \tag{2.19}$$

The claimed inequality in (2.15) follows immediately from (2.18) and (2.19).

Note that if $\psi(\cdot) = 0$, we only need to show (2.18), and therefore, the assumption $\hat{\mu}_j \leq \mu_j$, for every j, can be dropped. □

We now discuss the implication of Theorem 2.6.1 to the special case in which all processing durations are drawn from a location-scale family. Let μ_j and σ_j denote the mean and the standard deviation of the processing duration p_j of job j, $j = 1, 2, \ldots, n$, which is distributed as follows: $p_j \sim \mu_j + z_j \sigma_j$, where z_j's are i.i.d random variables with zero mean and unit standard deviation. Let $\boldsymbol{\mu} = (\mu_1, \mu_2, \ldots, \mu_n)$ and $\boldsymbol{\sigma} = (\sigma_1, \sigma_2, \ldots, \sigma_n)$ denote the vector of means and standard deviations. Let $F^*(\boldsymbol{\mu}, \boldsymbol{\sigma})$ denote the optimal cost over all $(\boldsymbol{\pi}, \mathbf{d}^\pi)$, i.e.,

$$F^*(\boldsymbol{\mu}, \boldsymbol{\sigma}) = \min_{\boldsymbol{\pi}, \mathbf{d}^\pi} \sum_{j=1}^{n} \Big[\psi(d_j^\pi) + \mathbb{E}[\theta(\sum_{i=1}^{j}(\mu_{\pi_i} + z_{\pi_i}\sigma_{\pi_i}) - d_j^\pi)]\Big].$$

Corollary 1: *For any vector of mean processing durations $\boldsymbol{\mu}$, the optimal cost, $F^*(\boldsymbol{\mu}, \boldsymbol{\sigma})$ is increasing in $\boldsymbol{\sigma}$.*

Proof. Consider two random vectors $\mathbf{p} = (p_1, p_2, \ldots, p_n)$ and $\hat{\mathbf{p}} = (\hat{p}_1, \hat{p}_2, \ldots, \hat{p}_n)$, where for every j, $j = 1, 2, \ldots, n$, p_j and $\hat{p}_j$ follow the following distribution: $p_j \sim \mu_j + z_j \sigma_j$ and $\hat{p}_j \sim \mu_j + z_j \hat{\sigma}_j$, where the z_j's are i.i.d random variables with zero mean and unit standard deviation. Assume $\hat{\sigma}_j \leq \sigma_j$ for every j (i.e., $\hat{\boldsymbol{\sigma}} \leq \boldsymbol{\sigma}$). We are required to show that

$$F^*(\boldsymbol{\mu}, \hat{\boldsymbol{\sigma}}) \leq F^*(\boldsymbol{\mu}, \boldsymbol{\sigma}).$$

Since $\hat{\sigma}_j \leq \sigma_j$, we have $\hat{\sigma}_j z_j \leq_{cx} \sigma_j z_j$. Thus, $\hat{p}_j \leq_{cx} p_j$ for every j. Since convex ordering implies dilation ordering, the desired result immediately follows from Theorem 2.6.1. $\square$

We note that for surgical applications, lognormal distributions have been widely used to model the duration of surgeries; e.g., (Stepaniak et al., 2010), (May et al., 2000), (Strum et al., 2000). Therefore, we now study the implications of Theorem 2.6.1 for the special case in which all the processing durations are lognormally distributed.

Let the processing duration p_j of job j, $j = 1, 2, \ldots, n$, follow a lognormal distribution: $p_j \sim e^{y_j}$, where y_j, the associated normal random variable, has a mean μ_j and standard deviation σ_j, i.e., $p_j \sim e^{\mu_j + z_j \sigma_j}$, where the z_j's are i.i.d standard normal random variables.

Denote the mean and variance of p_j by m_j and v_j, respectively. Note that $m_j = e^{\mu_j+\sigma_j^2/2}$ and $v_j = (e^{\sigma_j^2} - 1)\ m_j^2$. Let $\boldsymbol{\mu} = (\mu_1, \mu_2, \ldots, \mu_n)$ and $\boldsymbol{\sigma} = (\sigma_1, \sigma_2, \ldots, \sigma_n)$ denote the vector of means and standard deviations of the associated normal random variables. Let $\mathbf{m} = (m_1, m_2, \ldots, m_n)$ and $\mathbf{v} = (v_1, v_2, \ldots, v_n)$ denote the vector of the means and variances of the processing durations. Let $F^*(\boldsymbol{\mu}, \boldsymbol{\sigma})$ denote the optimal cost over all $(\boldsymbol{\pi}, \mathbf{d}^\pi)$, i.e.,

$$F^*(\boldsymbol{\mu}, \boldsymbol{\sigma}) \quad = \quad \min_{\boldsymbol{\pi}, \mathbf{d}^\pi} \quad \sum_{j=1}^{n} \Big[\psi(d_j^\pi) + \mathbb{E}[\theta(\sum_{i=1}^{j} e^{(\mu_{\pi_i}+z_{\pi_i}\sigma_{\pi_i})} - d_j^\pi)]\Big].$$

A result similar to Corollary 1 holds for lognormal distributions too, i.e., *for any vector of mean processing durations* $\mathbf{m}$*, the optimal cost* $F^*(\boldsymbol{\mu}, \boldsymbol{\sigma})$ *is increasing in* $\boldsymbol{\sigma}$. To see this, consider two random vectors $\mathbf{p} = (p_1, p_2, \ldots, p_n)$ and $\hat{\mathbf{p}} = (\hat{p}_1, \hat{p}_2, \ldots, \hat{p}_n)$ where for every j, $j = 1, 2, \ldots, n$, p_j and $\hat{p}_j$ are lognormally distributed with the associated normal random variables, y_j and $\hat{y}_j$, respectively, distributed as follows: $y_j \sim N(\mu_j, \sigma_j^2)$ and $\hat{y}_j \sim N(\hat{\mu}_j, \hat{\sigma}_j^2)$. Assume $\hat{\sigma}_j \leq \sigma_j$ for every j (i.e., $\hat{\boldsymbol{\sigma}} \leq \boldsymbol{\sigma}$). It follows from Table 1.1 of (Müller and Stoyan, 2002) that if for every j, $\hat{\sigma}_j \leq \sigma_j$ and the means of $\hat{p}_j$ and p_j are equal (i.e., $e^{\hat{\mu}_j+\hat{\sigma}_j^2/2} = e^{\mu_j+\sigma_j^2/2}$, or equivalently, $\hat{\mu}_j + \hat{\sigma}_j^2/2 = \mu_j + \sigma_j^2/2$), then $\hat{p}_j \leq_{cx} p_j$. Thus, it follows immediately from Theorem 2.6.1 that $F^*(\hat{\boldsymbol{\mu}}, \hat{\boldsymbol{\sigma}}) \leq F^*(\boldsymbol{\mu}, \boldsymbol{\sigma})$. Also, note that for any vector of mean processing durations $\mathbf{m}$, an increase in $\boldsymbol{\sigma}$ implies an increase in the $\mathbf{v}$, and vice-versa. Thus, *for any vector of mean processing durations, the optimal cost is increasing in the variability of the processing durations.*

2.7 Conclusion

Motivated by applications in surgical scheduling, this paper studies a stochastic, single machine earliness/tardiness problem, with the sequence of processing of the jobs and their due-dates as decisions and the objective of minimizing the sum of earliness and tardiness costs over all jobs. Our main result for this problem is that the SVF rule is optimal under the assumption of dilation ordering of the processing durations. The effect of variability of the processing durations of the jobs on the optimal cost is also discussed.

In practice, three different costs are relevant in the scheduling of surgeries: earliness penalty, idling cost and waiting cost. Our analysis in this paper allows for arbitrary finite values of the the waiting cost and the earliness penalty parameters, but assumes that idling is not allowed (i.e., the idling cost parameter is infinite). Thus, a natural generalization would be to allow each of the three cost parameters to be finite. In this case, the Operating Room (OR) manager has to dynamically decide whether to advance the start time of the next surgery or to idle the OR, should a surgery complete sooner than expected. Another useful generalization is an "online" version of our problem, where patients arrive over time and, hence, the information of all the surgeries is not available at time 0. Future research can also consider these generalizations in the presence of multiple operating rooms that function in parallel and share resources.

CHAPTER 3

AN ECONOMIC ANALYSIS OF AGRICULTURAL SUPPORT PRICES IN DEVELOPING ECONOMIES

3.1 Introduction

Due to its significant contribution to the gross domestic product and its critical role in shaping the livelihood of a majority of the population, the agricultural sector in many developing economies attracts substantial support from the government. Among the various governmental schemes that support agriculture, *Guaranteed Support Prices* (GSPs; also called *Minimum Support Prices*) have been adopted by many developing nations, including Bangladesh, Brazil, China, India, Pakistan, Thailand, and Turkey. A GSP for an agricultural crop is an attractive price at which the government promises to purchase that crop from farmers, regardless of its market price. We first discuss the motivation behind GSPs and then the factors that influence the government's decision to offer such a price for a particular crop.

To support our discussion with data throughout this introduction, we will use India as an example of a developing nation. India first introduced its support-price program in the mid 1960s, during the green revolution, as a *supply-side* incentive: An attractive GSP for a crop protects the farmers from the adverse effects on its market price due to overproduction and naturally incentivizes them to increase input effort. Over the years, GSPs have been widely credited for the consistent increase in productivity as well as the increase in farming inputs; see, e.g., (USDA Foreign Agricultural Service, 2014; Directorate of Economics and Statistics of India, 2014).

Driven by changes in economic and demographic factors, a secondary purpose of a support price has been as a *demand-side* provisioning tool: The increase in farming effort, engineered via a support price, results in high production, of which the government procures a significant amount and distributes it to the economically weak population at a nominal price via a network of ration shops (Government of India, 2010; SMC Investments, 2010; Parikh and Singh, 2007; Planning Commission of India, 2001).

We now discuss the primary factors that influence the value of the support price for a crop. Typically, support prices are determined on an annual basis. In India, the Commission for Agricultural Costs and Prices recommends the values of the support prices for various crops to the government. The recommendation for a specific crop primarily depends on its (i) demand and supply, (ii) cost of production and the total farmland cultivating the crop, and (iii) yield uncertainty; see, e.g., (Ministry of Agriculture, Government of India, 2018; Government of India, 2010; Kang, 2012).

- *Demand-Supply Gap*: Consider a crop that experiences a significantly higher market demand than its supply. The insufficient supply naturally drives the market price high, resulting in the poor finding it difficult to afford the crop at an elevated price. Since the below-poverty-line (BPL) population in developing nations is significant, the government may intervene by offering a support price for this crop to increase its supply and, thereby, its consumption by the population.

  To illustrate the impact of the imbalance between supply and demand on the decision to offer a support price, consider the production of pulses (lentils) in India. When the GSP program was first introduced, pulses were not included. However, with a majority of the vegetarian population relying on pulses for their protein requirement, the per-capita consumption of pulses progressively grew from 0.15 kg/month in 1973-74 to 0.41 kg/month in 2009-10, resulting in a corresponding increase in their market prices. India has eventually introduced GSPs on most of the primary pulses over the past decade or so (Sood, 2015; Srivastava et al., 2010; Reddy, 2012).

- *Crop-Yield Uncertainty*: Yield uncertainty increases fluctuation in supply (i.e., food-grain production), resulting in unstable market prices and, hence, unstable revenue for farmers. Thus, the government again has an incentive to offer a support price and avoid socially undesirable outcomes by safeguarding farmers from adverse price fluctuations (Gomez-Limon et al., 2002).

Yield uncertainty could be a major or a minor factor. If the production of a crop occurs primarily in regions that have a poor irrigation infrastructure – limited resources of water and its inefficient distribution – then farmers depend on rainfall to a large extent. In this case, crop yield could be significantly affected in the event of less-than-normal rainfall; see, e.g., (Timmer et al., 1983; Kochar, 1999; Menon, 2009; Dawande et al., 2013). In recent years, however, the improvement in irrigation infrastructure and water management techniques coupled with the increasing use of genetically-modified seeds, high-quality fertilizers, pesticides, and high-tech farm equipment, have helped reduce yield uncertainty in increasingly larger parts of developing countries (USDA Foreign Agricultural Service, 2014; EXIM Bank of India, 2012).

- *Production Cost and Size of Farmland*: An increase in the cost of farming inputs directly increases the cost of production. One reason for the increase in input costs is the progressive reduction in the average size of a farm. Small farmers are unable to go for mechanization of farming due to physical limitations and, therefore, have to depend on manual labor, which is becoming increasingly scarce and expensive (Kumaraswamy, 2012). As production cost increases, it is natural to expect a decrease in the total input effort, resulting in a reduction in the total production and, ultimately, the food-grains available for consumption. Therefore, to elevate the level of input effort by assuring farmers of "good" income that can compensate for the increase in production cost, the government may offer an attractive support price.

  The reduction in the average size of a farm can be illustrated in the case of India. The percentage of population choosing farming as the main source of livelihood has steadily decreased from 76% in 1960 to 54% by 2001 (Directorate of Economics and Statistics of India, 2014), while the total farmland size has stayed nearly the same during this period (World Bank, 2015). However, due to a high rate of population growth, the absolute number of farmers engaged in farming has, in fact, been increasing (Directorate of Economics and Statistics of India, 2014). Another reason for the reduction in individual land holdings is the repeated divisions of family farmland among siblings (Kumaraswamy, 2012). As a result, the average size of a farm has been steadily falling during this period (NITI Aayog, 2015; New Agriculturist, 1999).

It should be clear that the value of the GSP for a crop should consider the combined influence of these factors. Our goal in this paper is to offer analytically-supported insights on several fundamental aspects of the GSP scheme, including its impact on social welfare. Clearly, this analysis should anticipate the operational decisions of the farmers, i.e., their production decisions and their selling decisions to the government and in the open market, in response to a potential support price. Accordingly, we characterize the equilibrium decisions of the farmers enroute to assessing the impact on social welfare. This analysis then allows us to understand how the effectiveness of the GSP scheme, as a supply-side incentive and as a demand-side provisioning tool, is affected by the budget and the characteristics of the population (e.g., the "poorness" of the BPL population), and by yield uncertainty.

Our analysis captures several important real-world characteristics:

- A majority of farmers in developing countries are small-holders, with each owning or cultivating a small piece of land (on average, less that 2.0 hectares). Accordingly, farmers are assumed to be *price-takers*, i.e., the actions of an individual farmer do not influence the market price. Further, higher the number of farmers, lower is the size of an individual farm and, consequently, the efficiency of production. Accordingly, our analysis uses a production cost that is an increasing function of the total number of farmers.

- In the presence of a support-price program for a crop, a farmer has access to two different outlets: (i) selling the crop to the government at the GSP or (ii) selling in the open market at the market price. In the open market, we consider two different consumer segments: (i) the Above-Poverty-Line or APL consumers and (ii) the Below-Poverty-Line or BPL (or simply "poor") consumers. Our analysis captures both the extent of the "poorness" of the BPL consumers and the relative size of this segment.

- The support-price program for a crop – buying the crop from farmers at the GSP and providing it to the BPL population at a nominal price via a country-wide public distribution system of ration shops – is financed by a limited budget, which the government determines annually based on the country's economical, demographical, and political environment. In India, for instance, the combined budget for the support-price programs of a total of 26 crops was $16 billion in 2016; this is then partitioned into budgets for individual crops (The Wall Street Journal, 2016). Accordingly, our analysis assumes a finite budget for the support-price program of a crop.

Besides the GSP scheme, other schemes that directly transfer monetary subsidies, i.e., cash benefits, to the individual beneficiaries (through their bank accounts) have also received increasing patronage in recent years. For instance, in India, a program to pilot the Direct Benefit Transfer (DBT) scheme was launched in January, 2013, in an effort to increase transparency, eliminate pilferage of funds, and streamline existing processes of government delivery across welfare schemes. By directly transferring a subsidy to the bank accounts of the intended beneficiaries, the government is able to eliminate losses that arise from intermediaries or middlemen. Therefore, the government is better able to realize its objectives, namely *"...simpler and faster flow of information/funds and to ensure accurate targeting of the beneficiaries, de-duplication and reduction of fraud ..."*; see (Direct Benefit Transfer, 2018). Moreover, the DBT scheme would alleviate costly storage and distribution costs that are associated with the GSP scheme.

Despite its potential benefits, several operational factors are critical to the success of the DBT scheme. In particular, the identification and digitization of the database of beneficiaries, the opening of their bank accounts, the enrollment of unique identification (also known as Aadhaar in India) of the beneficiaries and their seeding with the beneficiary database and their bank accounts, and the last-mile delivery of the benefits via banking correspondents (as an alternative to brick and mortar banks) are key to ensuring its success. In a recent study by (Muralidharan et al., 2017) on the pilot implementation of the DBT scheme in certain states, the authors find that most beneficiaries take 1.8 – 3.5 times as much time to withdraw funds from a bank and go to the market to buy grains than it took to get them from the ration shops that distribute food grains under the GSP scheme. Inefficiencies in accurate identification and digitization of the beneficiary database has led to several beneficiaries receiving no subsidy, and in many cases, erratic cash transfers. Besides, the beneficiaries found the cash to be inadequate to buy the same quantity of food grains that they would have received through the ration shops. They conclude that, without assured and timely subsidy payments, consumers are reluctant, and in some cases, unable to buy food grains at the market rate. Producers, on their part, are also concerned about the eventual sale of their produce. Surveys by major media outlets, such as (India Today, 2018) and (The Hindu, 2016), report consistent stories of the shortcomings of the DBT scheme. Besides, other social factors, such as abuse of the subsidy for other activities, including alcohol, have also affected the success of the DBT scheme; see (The Wire, 2018).

In what follows, we review the related literature below in Section 3.1.1, introduce our model in Section 3.2, and analyze the market outcome under (a) the absence of any intervention in Section 3.3, (b) the DBT scheme in Section 3.4 and (c) the GSP scheme in Section 3.5, and conclude in Section 3.6.

3.1.1 Related Literature

While GSPs have been investigated in the agricultural economics literature, a majority of these studies focus on strategic decisions such as the impact of price support on international trade and the number of firms in the industry, and the need for market adjustments due to support prices. Studies that are representative of this line of research include (Fox, 1956; Dantwala, 1967; Spitze, 1978; Sjoquist, 1979; Gulati and Sharma, 1994; Food and Agriculture Organization of United Nations, 2001; Cummings Jr et al., 2006; Josling et al., 2010). In contrast, as discussed above, our aim is to characterize operational decisions of the farmers and the government, and understand the welfare implications of the GSP scheme for the farmers and the consuming population. Besides, we also analyze the DBT scheme and compare the market outcomes under both the GSP and DBT schemes and that under no intervention.

Close to our work, (Kazaz et al., 2016) analyze interventions to improve production of artemisenin (used in the malaria-medicine supply chain) under demand and supply uncertainty, and find that support prices lead to greater production by the farmers. However, their setting consists only of the open-market with exogenous demand. In our setting, a budget-constrained social planner announces a support price, procures grains from farmers, and distributes them among the BPL consumers (one segment of the consumer population). Besides, farmers can also sell in the open-market, which is accessible to both APL and BPL consumers. Further, we model the strategic behavior of the APL and BPL consumers and the farmers explicitly.

Motivated by schemes in developed countries, (Alizamir et al., 2018) consider two subsidy programs: (a) Price Loss Coverage (PLC) and (b) Agricultural Risk Coverage (ARC). Under PLC, the government offers farmers a subsidy if the market price falls below a reference price, while under ARC, the farmers receive a subsidy if their revenue falls below a threshold. They model competition among a finite population of farmers who sell in the open-market and compare the equilibrium market outcome under both these schemes to that under no intervention. Contrary to conventional wisdom, they find that the producer welfare, consumer welfare, and social welfare, can all be higher under PLC than under ARC. Our setting differs from theirs in many aspects. We consider competition among infinitesimally small farmers who can sell to the social planner as well as in the open-market. The social planner distributes the procured quantity among the BPL consumers, while both the APL and the BPL consumers can access the open market.

Relatedly, (Akkaya et al., 2016) study the impact of interventions such as price and cost support under public and private information about government budget on the social welfare. However, in their context, a support price is essentially a price floor that the government announces before the sowing season and pays the farmers the difference between the support price and the open-market price for each unit sold in the open-market. While we consider the interactions among strategic farmers and consumers, in a recent paper, (Hu et al., 2019) study the market outcome under a mix of myopic and strategic farmers. They find that a small fraction of strategic farmers can stabilize fluctuating market prices. (Chintapalli and Tang, 2018) consider farmers' crop-planting decisions when the social planner offers a GSP for two crops. Akin to (Akkaya et al., 2016), a support price in their context is "credit-based", i.e., the government does not procure any quantity from the farmers; rather, it pays the farmers the difference between the support and the market price for each unit sold in the market. They find that farmers do not internalize the externality they impose on other farmers in their planting decisions and therefore, vis-a-vis a setting where the social planner chooses the planting decisions, the decentralized planting decisions of the farmers leads to a loss of producer, consumer, and social welfare. It is worth mentioning that both (Hu et al., 2019) and (Chintapalli and Tang, 2018) model the strategic behavior of producers and consumers but do not consider the effect of yield uncertainty.

The GSP scheme is related to a *price-floor*, a concept well-studied in microeconomics; see (Varian, 1992). In Section 3.5, we comment on the impact of the GSP scheme and the price floor on the decisions of the different stakeholders as well as on their welfare.

Our work caters to the growing interest in the Operations Management (OM) community on agricultural operations; some recent examples include (Huh and Lall, 2013) (crop rotation); (Dawande et al., 2013) (distribution of surface water between farmers); (Chen and Tang, 2015; Parker et al., 2016; Tang et al., 2015) (provision of valuable information to farmers); (An et al., 2015) (farmer aggregation); (Federgruen et al., 2019) (contract farming); (Levi et al., 2018) (adulteration in farming supply chains).

A support price is an incentive aimed at improving production. The design of incentives that enable firms to elicit favorable decisions from supply-chain partners is a well-established research area in OM; see, e.g., the survey by (Cachon, 2003). There is also a growing interest in the OM community in analyzing incentives aimed at improving social welfare and understanding micro-level decisions that consumers make in response to such incentives; some recent examples include (Avci et al., 2014; Cohen et al., 2015) (subsidy programs for adoption of electric vehicles and their environmental impact); (Mu et al., 2015) (programs to reduce adulteration and improve quality in milk supply chains); (Lobel and Perakis, 2011) (subsidy programs for adoption of solar panels); (Atasu et al., 2009) (programs to promote product recycling); (Raz and Ovchinnikov, 2015) (government intervention mechanisms for public interest goods).

3.2 The Model

Consider a social planner (the government), a homogenous farming population of size[1] n that produces a crop, and a consuming population consisting of two segments: (a) the Above-

[1]Throughout this paper, by size, we refer to the mass of a segment.

Poverty-Line, or APL consumers of size M, and (b) the Below-Poverty-Line, or BPL (poor) consumers of size kM. The sequence of events is as follows:

Stage 1: Ahead of the sowing season, the social planner, fuelled by a budget B for the GSP scheme, announces the per-unit GSP p_g for the crop.

Stage 2: Each farmer decides his input effort based on the announced GSP, the cost of production, and the distribution of yield uncertainty. Let q_e denote the input effort of a representative farmer. His production cost is modeled as follows: For a fixed q_e, the farmer incurs a production cost of αq_e^2, where α is the production-cost parameter of the farming population. Several papers in the OM literature have modeled farming cost as a quadratic function of the farmer's effort; see, e.g., (Alizamir et al., 2018).

Stage 3: The production yield uncertainty is realized: Let γ denote the realized yield. For each farmer, the output corresponding to an input effort of q_e is γq_e.

Stage 4: Each farmer decides the quantities to be sold to the social planner and in the open-market, where the per-unit market price p_m is simultaneously realized. Let q_g (resp., q_m) denote the quantity sold by the farmer to the social planner (resp., in the open-market). The social planner distributes some or all[2] of the procured quantity among the BPL consumers at no cost.[3] The BPL consumers have the option of reselling

[2]For simplicity, we assume here that the entire quantity purchased by the social planner is made available for consumption by the BPL population. In reality, the social planner may distribute a fraction of the procured quantity among the BPL consumers and use the rest for other purposes, e.g., to maintain a buffer stock of foodgrains (for instance, in India, there are explicit stocking norms such as the minimum buffer stock which are mandated by the Cabinet Committee on Economic Affairs (CCEA) on a quarterly basis. This is done for operational reasons, such as meeting monthly distributional requirements, or for food security reasons, in events of unexpected shortfall in procurement; see (Food Corporation of India, 2017)). Our analysis can be easily modified in the absence of this assumption, without changing the nature of the results and insights. We discuss this assumption further at the end of Section 3.2.3.

[3]In practice, a nominal unit price is charged from the BPL consumers; we ignore this purely for simplicity of exposition. We further discuss this assumption in Section 3.2.3.

some (or all) of the quantity that they receive from the social planner back into the open market.

The objective of each farmer is to maximize his expected profit while the objective of the social planner is to maximize the expected social welfare. Our main notation is defined in Table A.1. The demand model for the APL and the BPL consumers is presented in Section 3.2.1 while the objective of the farmers (the supply model) is presented in Section 3.2.2. Finally, the social planner's objective is presented in Section 3.2.3.

3.2.1 The Demand Model

Recall the two homogenous consumer segments – APL (size M) and BPL (size kM). The maximum consumption of any consumer in either segment is normalized to 1. Thus, the aggregate consumption by the APL consumers (resp., the BPL consumers) is at most M (resp., kM). We present the consumer utility models for APL and BPL consumers below.

APL Consumers:

Our consumption utility model for the APL consumers works as follows. Suppose w_{APL} ($>> 1$) represents the wealth of the APL consumers. At a consumption level of ξ, the additional consumption utility derived by a consumer from the incremental consumption of an infinitesimal quantity $d\xi$ is $(1-\xi)d\xi$. Thus, the marginal consumption utility for a consumer monotonically decreases from 1 (at the minimum consumption level of 0) to 0 (at the maximum consumption level of 1). Consequently, when a quantity $q \in [0,1]$ is consumed, the net utility derived by an APL consumer is $u_{APL}(q) = \int_0^q (1-\xi)d\xi + (w_{APL} - p_m q)$, where $\int_0^q (1-\xi)d\xi$ is the utility derived from consumption and p_m is the market price. The utility-maximizing quantity consumed by an APL consumer from the open-market at a market price $p_m \in [0,1]$ is

$$q^*_{APL} \;=\; \arg\max_{q \geq 0}\; u_{APL}(q) = \arg\max_{q \geq 0}\; \Big[\int_0^q (1-\xi)d\xi + (w_{APL} - p_m q)\Big].$$

Thus, we have

$$q^*_{APL} \quad = \quad (1 - p_m) \tag{3.1}$$

That is, each APL consumer consumes q^*_{APL} from the open market.

BPL Consumers:

The BPL consumers also have access to the open market. The consumption utility model for the BPL consumers is identical to that of the APL consumers', with two differences: (a) these consumers are assumed to be budget-constrained; i.e., they have lower wealth relative to the APL consumers: Let b (< 1) denote the wealth of a BPL consumer; and (b) the social planner supplements these consumers with additional quantity for consumption: Let q_S denote the quantity that a BPL consumer receives from the social planner.

We allow for the possibility that the BPL consumers may resell a fraction of the quantity they receive from the social planner back in the open market. Suppose the social planner provides each BPL consumer with a quantity q_S. Then, one of the following outcomes hold: either (a) the market price is smaller than the marginal consumption utility at q_S, i.e., $p_m \leq 1 - q_S$, or (b) the market price is strictly larger than the marginal consumption utility at q_S, i.e., $p_m > 1 - q_S$. If the market price is smaller than the marginal consumption utility at q_S (i.e., (a) holds), then the BPL consumers do not find it profitable to sell any quantity they receive from the social planner back into the open-market. Rather, they may purchase some quantity from the open-market, depending on their budget b and the prevailing market-price p_m. On the other hand, if the market-price is strictly higher than the marginal consumption utility at q_S (i.e., (b) holds), then the BPL consumers do not purchase from the open-market (i.e., the quantity they purchase from the open-market is 0). Further, they will find it profitable to sell a fraction of the quantity they receive from the social planner back into the open-market. Combining these two cases, it is straightforward that a BPL consumer either sells in the open-market, or purchases from the open-market, but not both. Let q denote the quantity a BPL consumer consumes from the open market: If the BPL consumer sells in the open-market, then $q \leq 0$; if he purchases from the open market, then $q \geq 0$. The utility-maximizing quantity consumed by a BPL consumer from the open-market at a market price $p_m \in [0, 1]$ is

$$q^*_{BPL} \;=\; \arg \max_{q \in [-q_S, b/p_m]} u_{BPL}(q) = \arg \max_{q \in [-q_S, b/p_m]} \left[\int_0^{q+q_S} (1-\xi) d\xi + (b - p_m q) \right].$$

Thus, we have

$$q^*_{BPL} \;=\; \min \left\{ (1 - q_S - p_m), \frac{b}{p_m} \right\}. \tag{3.2}$$

Observe that the quantity $(1 - q_S - p_m)$ can be negative, if the market price is high (in particular, if it exceeds the marginal utility from consumption at q_S). Since all BPL consumers are homogeneous, we assume that the social planner supplements each consumer with the same quantity q_S and they consume the same quantity q^*_{BPL} from the open market.

Combining (3.1) and (3.2), the aggregate consumer demand at a market price $p_m \in [0, 1]$ can be written as

$$D(p_m) = M q^*_{APL} + kM q^*_{BPL} = M(1 - p_m) + kM \min \left\{ (1 - q_S - p_m), \frac{b}{p_m} \right\}. \tag{3.3}$$

Alternately, given a certain quantity available to be sold in the open-market, the market price p_m is determined by the above equation. We restrict attention to small values of b to reflect the "poorness" of the BPL consumers – we will make this precise in Section 3.3.

3.2.2 The Supply Model: Decision Problems of the Farmers

We now formulate the decision problems of the farmers. Recall that farmers are assumed to be price-takers, i.e., the actions of an individual farmer do not affect the market outcome in any realistic way. We explicitly model the interactions among strategic farmers. Specifically, farmers face competition from other farmers in their selling decisions in the open-market and to the social planner. Moreover, they anticipate the consequences of their selling decisions to the social planner: First, they anticipate a downward shift in the BPL consumers' demand curves in the open market (that is proportional to the quantity sold by the farmers to the social planner). Second, they anticipate that the BPL consumers may resell some of the quantity they receive from the social planner back in the open-market. Let $\hat{p}_m(\gamma)$ denote the belief that a farmer has about the market price (as a function of the realized yield). The homogeneity of farmers allows us to make two assumptions: (a) All farmers hold the same belief $\hat{p}_m(\gamma)$ about the market price, and (b) their production and selling decisions are identical, i.e., all farmers exert an effort q_e, and sell q_m (resp., q_g) in the open-market (resp., to the social planner).[4] Therefore, we assume that farmers hold rational beliefs, i.e., the actions taken by the farmers given their beliefs lead to an outcome that is consistent with their beliefs. This is a standard assumption in the literature, consistent with the theory of rational expectations. In summary, we assume that all farmers hold the same (rational) belief about the market price ($\hat{p}_m(\gamma)$), exert the same effort (q_e), and their selling decisions (q_m, q_g) are identical.

Consider a representative farmer: Let π_f denote the (expected) profit of the farmer, who derives revenue from two sources: (i) by selling to the social planner and (ii) by selling in the open market. For any GSP $p_g \geq 0$ chosen by the social planner, the farmer's belief $\hat{p}_m(\gamma)$ about the market price, the effort q_e chosen by the farmer in the second stage, and the yield γ chosen by nature in the third-stage, the fourth-stage optimization problem is as follows: The farmer decides q_m and q_g, the quantity of produce sold in the open market and

[4]An alternative is to consider asymmetric pure strategies, where each farmer either sells to the social planner or in the open market. In such a case, while the equilibrium effort of all the farmers is identical, we solve for the proportion of farmers that sell in the open market and to the social planner. It can be easily seen that this does not alter the quantity procured by the social planner and made available in the open market. Consequently, the market price, and therefore, the market outcome, are identical.

to the social planner, respectively. That is,

$$\left.\begin{array}{l}\underset{(q_m,q_g)}{\text{Max}}\ \pi_f^{\gamma}(q_m,q_g|p_g,\hat{p}_m(\gamma),q_e) = [\hat{p}_m(\gamma)q_m + p_g q_g] - \alpha q_e^2 \\ \text{subject to:} \\ \qquad\qquad q_m + q_g \le \gamma q_e, \\ \qquad\qquad p_g q_g \ \le B/n, \\ \qquad\qquad (q_m,q_g) \ge 0. \end{array}\right\} \textbf{Problem } \mathrm{P}_f^2$$

Denote the optimal values of q_m (resp., q_g) obtained from P_f^2 by q_m^* (resp., q_g^*). Indeed, q_m^* and q_g^* depend on the realized yield γ; for notational simplicity, we avoid stating this dependence explicitly. The second constraint, $p_g q_g \le B/n$, in the set of constraints above pertains to the maximum quantity that the social planner procures from an individual farmer – we assume that the social planner allocates his budget equally among the farmers (i.e., procures at most B/np_g from each farmer). We then use q_m^* and q_g^* to write the second-stage optimization problem as follows:

$$\left.\underset{q_e\ge 0}{\text{Max}}\ \pi_f(q_e|p_g,\hat{p}_m(\gamma)) = \mathbb{E}_\gamma[\pi_f^{\gamma}(q_m^*,q_g^*|p_g,\hat{p}_m(\gamma),q_e)].\ \right\} \textbf{Problem } \mathrm{P}_f^1$$

3.2.3 Objective of the Social Planner

The objective function Π_S of the social planner is the sum total of the surplus derived by each segment of the population, and consists of the following components: (i) the "consumer surplus" derived by the APL and the BPL consumers from consumption, $Mu_{APL}+kMu_{BPL}$, (ii) the "producer surplus" derived by the farmers, $n\pi_f$ and (iii) unused budget of the social planner. Thus, the social planner's problem is as follows:[5]

$$\max_{p_g\ge 0}\ \Pi_S \ = \ \mathbb{E}_\gamma\left[Mu_{APL} + kMu_{BPL} + n\pi_f + (B - np_g q_g)\right].$$

[5]For ease of notation, we suppress the dependencies of the surplus on the equilibrium decisions that arise as a result of the social planner's decision.

To understand the role of a scheme like the GSP, we first study the market outcome in the absence of any intervention (i.e., the "free-market" outcome). We then demonstrate the reasons that motivate the need for a market intervention.

Remark: Recall, from the sequence of events, that we make two assumptions about the social planner: (a) the social planner distributes *all* of the procured quantity among the BPL consumers, and (b) the social planner distributes the procured quantity at *zero cost*. Both these assumptions are not restrictive, in the sense that our conclusions qualitatively stay the same under a more-general setting where (a$'$) the social planner commits to distributing a fraction, say $x_S \leq 1$, of the procured quantity among the BPL consumers, and (b$'$) charging a nominal positive price, say $p_S \geq 0$, to the BPL consumers. We briefly outline how the analysis can be extended to the general setting. Let q'_{BPL} denote a BPL consumer's demand in the open market. Since p_S is sufficiently small, a BPL consumer's demand in the open-market is $q'_{BPL} = \min\left\{1 - q_S - p_m, \frac{b - p_S q_S}{p_m}\right\}$ such that the total quantity that the social planner distributes among the BPL consumers is $kMq_S \leq x_S n q_g$. Observe that the generalizations (a$'$) and (b$'$) above act in opposite directions in how they affect a BPL consumer's demand. All else equal, relative to the case where $x_S = 1$ and $p_S = 0$, a value of $x_S < 1$ results in the social planner distributing a lower quantity among the BPL consumers. Hence, the first term in the BPL consumer's demand in the above expression is higher, i.e., BPL consumers have a greater need for foodgrains in the open-market. On the other hand, a value of $p_S > 0$ results in lower wealth among the BPL consumers for purchase in the open-market. Hence, the second term in the BPL consumer's demand in the above expression is lower. Consequently, the equilibrium market outcome depends on the exact values of x_S and p_S and also whether the poorness of the BPL consumers is a dominant factor. $\blacksquare$

3.3 Analysis in the Absence of an Intervention

In the absence of any market intervention (henceforth "No Intervention", or NI), the farmers sell their entire produce in the open market. Each farmer decides his (expected) profit-maximizing effort q_e, given his belief about the market price $\hat{p}_m(\gamma)$, by solving the following problem:

$$\max_{q_e \geq 0} \pi_f = \mathbb{E}[\gamma q_e \hat{p}_m(\gamma)] - \alpha q_e^2,$$

where the term inside the expectation is the farmer's belief about his revenue when the realized yield is γ. The farmer's profit maximizing effort q_e^*, given his belief $\hat{p}_m(\gamma)$, is

$$q_e^* = \frac{\mathbb{E}[\gamma \hat{p}_m(\gamma)]}{2\alpha}. \tag{3.4}$$

The numerator in (3.4) is the farmer's belief about the expected market price. Farmers exert greater effort if they believe that the market price is higher. When the realized yield is γ, the total quantity available to be sold in the open market is $nq_e^*\gamma = \frac{n\,\mathbb{E}[\gamma\hat{p}_m(\gamma)]\gamma}{2\alpha}$. Notice that the largest value of $nq_e^*\gamma$ is $\frac{n}{2\alpha}$. Recall that the maximum consumption by any consumer is capped at 1. To avoid settings that result in excess supply, we assume that $\frac{n}{2\alpha} < M(1+k)$, i.e., the maximum production from the farmers does not exceed the maximum consumption by the consumers. This is a reasonable assumption, since many developing countries have a large consumer population.

In the absence of any scheme, the BPL consumers do not receive any support from the social planner, i.e., $q_S = 0$. When the realized yield is γ, the market price $p_m(\gamma)$ is obtained from the following:

$$D(p_m(\gamma)) = nq_e^*\gamma \tag{3.5}$$

$$\hat{p}_m(\gamma) = p_m(\gamma) \tag{3.6}$$

(3.5) states that, at the prevailing market-price, the total demand from the consuming population is equal to the total production from the farmers (i.e., the market clears). (3.6) states that farmers' belief is consistent with the outcome (i.e., farmers hold rational beliefs). Using these two conditions, we can obtain the equilibrium of the game, i.e., the equilibrium effort of the farmers and the equilibrium market price (consistent with the farmers' belief) for any realized yield γ.

Two factors play a key role in determining the equilibrium effort of the farmers – yield uncertainty, as a supply-side impediment, and the "poorness" of the BPL consumers, as a demand-side impediment. In what follows, we isolate the role of each of these factors by solving the equilibrium of this game in two special cases. We then explain the significance of the two results below (namely, Lemmas 3.3.1 and 3.3.2) that motivate the need for an intervention.

3.3.1 Effect of Yield Uncertainty

To isolate the effect of yield uncertainty on the equilibrium outcome, suppose that the wealth b of a BPL consumer is sufficiently high – in this case, the BPL consumers are effectively not budget-constrained. A sufficient condition under which this occurs is $b \geq \frac{1}{4}$; under this condition, we have $\frac{b}{p_m} \geq (1 - p_m)$, since $p_m \in [0, 1]$. Therefore, from (3.2), we have $q^*_{BPL} = 1 - p_m$. Let $\mu = \mathbb{E}[\gamma]$ and $\sigma^2 = \text{Var}(\gamma)$. The following result states the equilibrium outcome under this special case.

Lemma 3.3.1. *In the absence of any intervention, if $b \geq \frac{1}{4}$, the equilibrium effort of a farmer is*

$$q^*_e = \frac{\mu}{2\alpha}\Big(\frac{M(1+k)}{M(1+k) + \frac{n}{2\alpha}(\mu^2 + \sigma^2)}\Big) \tag{3.7}$$

and the equilibrium market price when the realized yield is γ is

$$p_m(\gamma) = 1 - \gamma\Big(\frac{\frac{n}{2\alpha}\mu}{M(1+k) + \frac{n}{2\alpha}(\mu^2 + \sigma^2)}\Big). \tag{3.8}$$

When the BPL consumers have sufficient wealth, the only impediment to high production by the farmers is the uncertainty in yield. All else equal, the equilibrium effort of a farmer q^*_e

increases in mean μ and decreases in the variance σ^2. Consequently, as yield becomes more variable, the equilibrium production effort by the farmers decreases – this argues for an incentive to improve production under yield uncertainty.

3.3.2 Effect of "Poorness" of the BPL Consumers

To better understand the effect of limited wealth of the BPL consumers and in the rest of the paper, we consider a two-point distribution for the yield. Suppose that

$$\gamma = \begin{cases} 1, & \text{w.p. } \theta; \\ 0, & \text{w.p. } (1-\theta). \end{cases}$$

We define a threshold level of the budget, denoted by $b^*(\theta)$, as follows:

$$b^*(\theta) = \frac{M(1+k)\frac{n\theta}{2\alpha}}{\left(M(1+k)+\frac{n\theta}{2\alpha}\right)^2}. \tag{3.9}$$

Observe that $b^*(\theta)$ is a strictly increasing function of θ, $b^*(0) = 0$ and $b^*(1) < \frac{1}{4}$, since $\frac{n}{2\alpha} < M(1+k)$. Intuitively, for a fixed $\theta \in [0,1]$, $b^*(\theta)$ denotes the maximum value of b for a BPL consumer to be deemed as "budget-constrained". In other words, if $b \geq b^*(\theta)$, then the BPL consumers are effectively not budget-constrained; i.e., at the equilibrium consumption q^*_{BPL} of a BPL consumer, his marginal consumption utility equals the market price p_m.

From (3.4), we have that $q_e^* = \frac{\theta \hat{p}_m(1)}{2\alpha}$. The following result states the equilibrium outcome under this setting.

Lemma 3.3.2. *Consider a fixed value of $\theta \in [0,1]$. In the absence of any intervention, the equilibrium effort and the market price are as follows:*

1. *If $b < b^*(\theta)$, then*

$$q_e^{*NI} = \frac{\theta}{2\alpha}\left(\frac{M + \sqrt{M^2 + 4kMb(M+\frac{n\theta}{2\alpha})}}{2(M+\frac{n\theta}{2\alpha})}\right),$$

and the equilibrium market price is

$$p_m(1)^{NI} = \frac{M + \sqrt{M^2 + 4kMb(M + \frac{n\theta}{2\alpha})}}{2(M + \frac{n\theta}{2\alpha})} \text{ and } p_m(0)^{NI} = 1.$$

2. *If $b \geq b^*(\theta)$, then*

$$q_e^{*NI} = \frac{\theta}{2\alpha}\Big(\frac{M(1+k)}{M(1+k) + \frac{n\theta}{2\alpha}}\Big),$$

and the equilibrium market price is

$$p_m(1)^{NI} = \frac{M(1+k)}{M(1+k) + \frac{n\theta}{2\alpha}} \text{ and } p_m(0)^{NI} = 1.$$

Lemma 3.3.2 is interesting for several reasons:

- First, it helps us identify the role of the BPL consumers' poverty as a demand-side impediment to production. Suppose that $b < b^*(\theta)$: In this case, the BPL consumers are "budget-contrained". Precisely, the marginal utility from consumption (for a BPL consumer) is strictly higher than the market price, i.e., $1 - \frac{b}{p_m(1)} > p_m(1)$. Stated differently, if the BPL consumers had more wealth, they would *choose* to purchase more from the open market at the prevailing market price. Farmers rationally anticipate the low buying power of the BPL consumers and therefore their equilibrium production effort is lower. At low values of b, the production by the farmers and the consumption by the BPL consumers is low, which justifies the need for an intervention by the social planner, both as a supply-side and a demand-side stimulant. As b increases, the buying power of the BPL consumers also increases. Consequently, the equilibrium production effort by the farmers also increases, i.e., q_e^* is increasing in b. Beyond $b^*(\theta)$, yield uncertainty plays a dominant role. As a result, beyond $b^*(\theta)$, the equilibrium effort q_e^* is a constant.

- Second, the result helps us compare the roles of the BPL consumers' poverty vis-à-vis yield uncertainty as an impediment to farmers' production. Consider a fixed $b \in [0, b^*(1)]$ and let $\theta^* = b^{*-1}(b)$: The result states that BPL consumers are "budget-constrained" only when $\theta > \theta^*$. A consequence of this result is that if a high yield realization is less likely to occur (i.e., $\theta < \theta^*$), an increase in the wealth of the BPL consumers has no effect on the equilibrium outcome.

- Third, the result identifies the extent to which the BPL consumers' poverty imposes an externality on the APL consumers' consumption. Consider a fixed value of θ and suppose that $b < b^*(\theta)$: A decrease in b leads to a decrease in the equilibrium effort q_e^* and a decrease in the market price $p_m(1)$. Consequently, the quantity consumed by an APL consumer, $1-p_m(1)$, increases. In other words, as BPL consumers become poorer, the total production effort by the farmers decreases but the equilibrium consumption of the APL consumers increases.

To summarize, Lemmas 3.3.1 and 3.3.2 justify the need for an intervention in light of the yield uncertainty and the limited wealth of the poor consumers. In what follows, we analyze the Direct Benefit Transfer mechanism (DBT) as a benchmark intervention and then proceed with the analysis of the Guaranteed Support Price (GSP) scheme.

3.4 Analysis of the Direct Benefit Transfer Scheme

Consider the social planner fueled by a budget B. Let $\beta = \frac{B}{kM}$. Under the Direct Benefit Transfer (henceforth, DBT) scheme, the social planner augments each BPL consumer's wealth using his budget – therefore, the wealth of each BPL consumer becomes $b + \beta$.[6]

[6] An alternate cash transfer scheme is one where the social planner distributes his budget among the farmers, instead of the BPL consumers, as considered in this section. However, it should be obvious to the reader that, relative to NI, such a scheme does not alter incentives of any player. Therefore, the market

As is the case in developing countries, we assume that the budget for the DBT scheme is limited – consequently, β is small relative to b. For given b and β, let $\theta^* = b^{*-1}(b)$ and $\theta^{**} = b^{*-1}\left(\frac{k}{(1+k)}\beta\right)$. Thus, $\theta^{**} < \theta^*$. Recall the definition of $b^*(\theta)$ from (3.9). For a fixed θ, one of the following occurs:

(a) $b < b + \beta \leq b^*(\theta)$,

(b) $b < b^*(\theta) \leq b + \beta$, and

(c) $b^*(\theta) \leq b < b + \beta$.

Since β is small relative to b, we ignore case (b) and focus on cases (a) and (c). In case (a), we have $\theta > \theta^*$. In contrast, in case (c), we have $\theta < \theta^*$. Recall, from Lemma 3.3.2, that under NI, the effect of an increase in the wealth of the BPL consumers on the market outcome depends on the relative comparison between b and $b^*(\theta)$. Therefore, we analyze cases (a) and (c) separately.

3.4.1 Case (a): $b + \beta < b^*(\theta)$

Recall that under NI, this setting corresponds to the case where the "poorness" of the BPL consumers plays a role in determining the equilibrium effort of the farmers (case 1 in Lemma 3.3.2). Further, an increase in the BPL consumers' wealth results in an increase in the equilibrium production effort by the farmers (i.e., q_e^* is increasing in b). Therefore, it is straightforward to see that the DBT scheme – through which the social planner provides additional wealth to the BPL consumers – results in an increase in the production effort of the farmers.

outcome under such a scheme is identical to NI, except that the utility of each farmer increases by an amount equal to the wealth he receives from the social planner, i.e., B/n (under NI, the budget was left unused by the social planner). Consequently, the social planner's surplus is also identical to that under NI. Further, in Section 3.5, we show that this alternate scheme can be theoretically implemented by the GSP scheme as a special case.

Lemma 3.4.1 below compares the equilibrium outcomes and the social surplus under DBT and NI. Under NI, the social planner's budget ($= kM\beta$) is left unused. Under DBT, the BPL consumers strictly prefer to purchase more from the open-market using the additional wealth β (instead of keeping all or part of it unused). Therefore, the surplus of BPL consumers under the DBT scheme exceeds the sum of their surplus under NI and the additional wealth β, i.e., ${u_{BPL}}^{DBT} > {u_{BPL}}^{NI} + \beta$. Relative to NI, the equilibrium effort of the farmers and the market-price are higher under DBT. Although their production costs are higher, the expected profit of the farmers increases due to larger revenues. However, the higher market-price results in the APL consumers being worse-off. Nevertheless, the total increase in the surplus of the BPL consumers and the farmers offsets the decrease in the surplus of the APL consumers and the unused budget; thus, the surplus of the social planner under the DBT scheme is strictly higher than that under NI.

Lemma 3.4.1. *If $b + \beta < b^*(\theta)$, then the farmers' equilibrium effort under the DBT scheme is*

$$q_e^{*DBT} = \frac{\theta}{2\alpha}\left(\frac{M + \sqrt{M^2 + 4kM(b+\beta)(M + \frac{n\theta}{2\alpha})}}{2(M + \frac{n\theta}{2\alpha})}\right),$$

and the equilibrium market price is

$$p_m(1)^{DBT} = \frac{M + \sqrt{M^2 + 4kM(b+\beta)(M + \frac{n\theta}{2\alpha})}}{2(M + \frac{n\theta}{2\alpha})} \text{ and } p_m(0)^{DBT} = 1.$$

Thus, relative to NI, the equilibrium effort of the farmers and the market price under the high-yield realization are higher under the DBT scheme, i.e., $q_e^{*DBT} > q_e^{*NI}$ and $p_m(1)^{DBT} > p_m(1)^{NI}$. Further, the surplus of the social planner is strictly higher under the DBT scheme, i.e., ${\Pi_{SP}}^{DBT} > {\Pi_{SP}}^{NI}$.

3.4.2 Case (c): $b > b^*(\theta)$

Under NI, this setting corresponds to the case where yield uncertainty is dominant in the determination of the equilibrium effort of the farmers and the BPL consumers are effectively not budget-constrained (case 2 in Lemma 3.3.2). Recall that, in this case, an increase in the BPL consumers' wealth has no effect on the market outcome. The additional wealth of the BPL consumers does not alter the incentives of the farmers to increase their production. Therefore, relative to NI, the DBT scheme does not alter the market outcome, i.e., there is no improvement in the production effort of the farmers or the social planner's surplus. The following result states this finding; the proof follows from Lemma 3.3.2.

Lemma 3.4.2. *If $b > b^*(\theta)$, then the farmers' equilibrium effort under the DBT scheme and the market price under the high-yield realization are identical to the respective outcomes under NI. That is,*

$$q_e^{*DBT} = q_e^{*NI} = \frac{\theta}{2\alpha}\Big(\frac{M(1+k)}{M(1+k)+\frac{n\theta}{2\alpha}}\Big) \textit{ and}$$

$$p_m(1)^{DBT} = p_m(1)^{NI} = \frac{M(1+k)}{M(1+k)+\frac{n\theta}{2\alpha}} \textit{ and } p_m(0)^{DBT} = 1.$$

Consequently, the social planner's surplus under the DBT scheme is identical to that under NI; i.e., $\Pi_{SP}{}^{DBT} = \Pi_{SP}{}^{NI}$.

In summary, relative to NI, the DBT scheme leads to a strict improvement in the production effort of the farmers and the social planner's surplus if the poorness of the BPL consumers and yield uncertainty both play a role in determining the equilibrium effort of the farmers and the market price. However, if the poorness of the BPL consumers does not play a role (i.e., only yield uncertainty is a factor), then the DBT scheme is ineffective in improving the market outcome. Next, we study how the GSP scheme affects the market outcome in both these cases. We then contrast the outcome under the GSP scheme with that under the DBT scheme and under NI.

3.5 Analysis of the Guaranteed Support Price Scheme

In this section, we analyze the role and scope of the GSP scheme. We will show the following results about the usefulness of the GSP scheme in the two cases considered in Section 3.4:

1. If the BPL consumers' budget poses a stronger impediment than yield uncertainty to high production by the farmers (i.e., if $b + \beta < b^*(\theta)$), then the equilibrium under the GSP scheme is identical to that under the DBT scheme. Precisely, the equilibrium choice of the support price is $p_g = p_m(1)^{DBT}$ and the market outcome (i.e., the equilibrium effort of the farmers and the social planner's surplus) at the equilibrium support price is identical to that under the DBT scheme. Recall that, in this case, the farmers' effort and social planner's surplus under the DBT scheme are strictly higher than those under NI. Thus, the equilibrium effort of the farmers and the social planner's surplus under the GSP scheme are both strictly higher than those under NI. Further, an increase or a decrease in the support price (i.e., an off-equilibrium support price) leads to a decrease in the effort of the farmers and the social planner's surplus.

2. If, instead, yield uncertainty plays a dominant role, (i.e., if $b > b^*(\theta)$), then there are a continuum of equilibria in which the social planner's surplus is identical, and is equal to that under the DBT scheme (which, in turn, is equal to that under NI). These equilibria, however, differ in the improvement in surplus that they create for the farmers and the BPL consumers. Recall that the DBT scheme allots the entire budget to the BPL consumers, while it is left unused under NI. The GSP scheme allows the social planner to divide his budget so that a proportion of the budget is allotted for the improvement in surplus of the BPL consumers and the remainder is used for the farmers; this split is determined by the choice of the support price. Therefore, at one extreme, the social planner can choose the support price to implement an outcome where the entire budget is allotted to the farmers. At the other extreme, the social planner can choose the support price such that the entire budget (if fully exhausted, or the maximum

budget spent, as applicable[7]) is allotted to the BPL consumers. Moreover, the social planner can also implement any outcome that lies in between these two extremes.

Below, we analyze the market outcome under the GSP scheme for an arbitrary announced value of the support price p_g (i.e., both on- and off-equilibrium values of p_g). We use these results to identify the equilibrium value of p_g and compare the market outcomes under the GSP and DBT schemes. Later, in Section 4.5, we will use the results developed here to analyze the equilibrium outcome for an alternate specification of the social planner's objective consisting of a weighted combination of the BPL consumers' surplus and the farmers' surplus.

Recall the sequence of events under the GSP scheme: The social planner announces a support price p_g ahead of the sowing season. Using this price, the farmers form rational beliefs about the market price – let $\hat{p}_m(1)$ (resp., $\hat{p}_m(0)$) denote the belief about the market price under the high-yield (resp., low-yield) realization. Nature chooses the realized yield γ, and the farmers make their selling decisions (q_g, q_m). The realized open-market price is consistent with the farmers' beliefs.

It is straightforward that $\hat{p}_m(0) = 1$ and $\hat{p}_m(1) \leq \hat{p}_m(0)$. For any choice p_g of the social planner, we have one of the following cases:

(i) $p_g < \hat{p}_m(1)$: The farmers do not sell to the social planner and sell their entire output in the open-market.

(ii) $p_g > \hat{p}_m(1)$: The farmers sell the maximum possible quantity to the social planner, and then sell any remaining quantity in the open market.

(iii) $p_g = \hat{p}_m(1)$: The farmers are indifferent between selling to the social planner and in the open-market.

[7] Under very low values of the yield-uncertainty parameter θ (specifically, $\theta < \theta^{**}$), the farmers' production effort is low enough so that the social planner can procure the entire output of the farmers without exhausting his budget.

In the result below, we solve for the farmer's effort and the selling decisions given his belief about the market price and the announced support price.

Lemma 3.5.1. *For an announced p_g and beliefs $(\hat{p}_m(0), \hat{p}_m(1))$ about the market price, the farmer's equilibrium effort (solution to Problem P_f^1) and his selling decisions (solution to Problem P_f^2) are as follows:*

(i) If $p_g < \hat{p}_m(1)$, then $q_e^* = \frac{\theta}{2\alpha}\hat{p}_m(1)$, $q_g = 0$, and $q_m = q_e$.

(ii) If $p_g > \hat{p}_m(1)$, then

$$q_e^* = \begin{cases} \frac{\theta}{2\alpha}p_g, & \frac{\theta}{2\alpha}\hat{p}_m(1) \leq \frac{\theta}{2\alpha}p_g \leq \frac{B}{np_g}; \\ \frac{B}{np_g}, & \frac{\theta}{2\alpha}\hat{p}_m(1) \leq \frac{B}{np_g} \leq \frac{\theta}{2\alpha}p_g; \\ \frac{\theta}{2\alpha}\hat{p}_m(1), & \frac{B}{np_g} \leq \frac{\theta}{2\alpha}\hat{p}_m(1) \leq \frac{\theta}{2\alpha}p_g. \end{cases}$$

$q_g = \min\left\{q_e, \frac{B}{np_g}\right\}$, *and* $q_m = q_e - q_g = \max\{0, q_e - \frac{B}{np_g}\}$.

(iii) If $p_g = \hat{p}_m(1)$, then $q_e^* = \frac{\theta}{2\alpha}p_g$ and any choice of (q_g, q_m) such that $q_g, q_m \geq 0$ and $q_g + q_m = q_e$ is an equilibrium.

If p_g is strictly lower (resp., higher) than the farmers' belief about the market price, then they sell the maximum quantity in the open market (resp., to the social planner). Since farmers are assumed to be homogeneous, we focus on symmetric strategies in case (*iii*), i.e., all farmers follow the same strategy.[8] Below, we solve and obtain the equilibrium market outcome (the equilibrium effort of the farmers, the market price, and the social welfare) for any choice of the support price announced by the social planner. Then, we find the equilibrium support price that maximizes the social planner's surplus. The analysis of the market outcome for both on- and off-equilibrium support prices is useful because Section 4.5 considers an alternate objective function of the social planner (a weighted combination of the BPL consumers' surplus and the farmers' surplus).

[8] As remarked in Section 3.2.2, an alternative is to consider asymmetric pure strategies, where we solve for the proportion of farmers who sell in the open market and to the social planner – such a consideration leads to an identical market outcome.

3.5.1 Case 1: $b + \beta < b^*(\theta)$

Recall that $b + \beta < b^*(\theta)$ (or equivalently, $\theta > \theta^*$) corresponds to the setting where the poverty of the BPL consumers plays a dominant role in determining the production effort of the farmers. Consequently, the DBT scheme helps in improving the production from the farmers and the social planner's surplus (Lemma 3.4.1). In the results below (Lemmas 3.5.2 and 3.5.3), we obtain the market outcome and the social planner's surplus under any value of the support price p_g announced by the social planner. Subsequently, we use these results to determine the equilibrium value of p_g.

Lemma 3.5.2. (Farmers' production effort and selling decisions) *If $b + \beta < b^*(\theta)$, then for any given value of the support price p_g, the equilibrium effort of the farmers, and the market price (consistent with the farmers' belief) are as follows:*

1. *If $p_g < p_m(1)^{NI}$, then we have $q_e^* = \frac{\theta}{2\alpha}\hat{p}_m(1)$, $q_g^* = 0$, $q_m^* = q_e^*$, and $\hat{p}_m(1) = p_m(1)^{NI}$.*

2. *If $p_g \in [p_m(1)^{NI}, p_m(1)^{DBT}]$, then we have $q_e^* = \frac{\theta}{2\alpha}\hat{p}_m(1)$, $q_g^* = \frac{\theta}{2\alpha}\hat{p}_m(1) - \frac{M}{n}(1 - p_g) - \frac{kMb}{np_g}$, $q_m^* = q_e^* - q_g^*$, and $\hat{p}_m(1) = p_g$.*

3. *If $p_g > p_m(1)^{DBT}$, then $q_e^* = \frac{\theta}{2\alpha}\hat{p}_m(1)$, $q_g^* = \frac{B}{np_g}$, $q_m^* = q_e^* - q_g^*$, and*

$$\hat{p}_m(1) = \frac{\left(M+kM\frac{\beta}{p_g}\right)+\sqrt{\left(M+kM\frac{\beta}{p_g}\right)^2+4kMb\left(M+\frac{n\theta}{2\alpha}\right)}}{2\left(M+\frac{n\theta}{2\alpha}\right)}.$$

Lemma 3.5.3. (Social planner's surplus) *If $b + \beta < b^*(\theta)$, then for any given value of the support price p_g, the social planner's surplus ${\Pi_S}^{GSP}(p_g)$ under the GSP scheme is as follows:*

1. *If $p_g < p_m(1)^{NI}$, then ${\Pi_S}^{GSP}(p_g) = {\Pi_S}^{NI}$.*

2. *If $p_g \in [p_m(1)^{NI}, p_m(1)^{DBT}]$, then ${\Pi_S}^{GSP}(p_g)$ increases in p_g, from ${\Pi_S}^{NI}$ at $p_g = p_m(1)^{NI}$ to ${\Pi_S}^{DBT}$ at $p_g = p_m(1)^{DBT}$.*

3. *If $p_g > p_m(1)^{DBT}$, then ${\Pi_S}^{GSP}(p_g)$ decreases in p_g, from ${\Pi_S}^{DBT}$ at $p_g = p_m(1)^{DBT}$ and approaches ${\Pi_S}^{NI}$ as $p_g \to \infty$.*

Some comments are in order: If the support price p_g is below the market price under NI, $p_m(1)^{NI}$, then the farmers anticipate a higher market price, and therefore do not sell to the social planner. Consequently, the entire budget of the social planner is left unused. The market outcome, and therefore the social planner's surplus, under such low values of p_g, is identical that under NI.

If $p_g \in [p_m(1)^{NI}, p_m(1)^{DBT}]$, then, relative to NI, the social planner distorts the market outcome under GSP. The production effort by the farmers increases (linearly) in p_g. However, the entire budget of the social planner is not exhausted. This is because farmers divide the total produce into the quantities they sell to the social planner and in the open market, (q_g, q_m) such that the market price $p_m(1)^{GSP}$ that results from their selling decisions is equal to p_g. Any deviation from this split, e.g., by increasing (resp., decreasing) q_g, results in a higher (resp., lower) market price, and therefore results in strictly lower profits for the farmers. Although the production effort of the farmers increases in p_g, the quantity they sell in the open market decreases. Therefore, the market price is increasing in p_g. If $p_g = p_m(1)^{NI}$, the market outcome is identical to that under NI. Thus, the social planner's surplus ${\Pi_S}^{GSP}(p_g) = {\Pi_S}^{NI}$. In this case, the entire budget of the social planner is unused. If $p_g = p_m(1)^{DBT}$, the market outcome is identical to that under DBT. Thus, the social planner's surplus ${\Pi_S}^{GSP}(p_g) = {\Pi_S}^{DBT}$. In this case, the entire budget of the social planner is exhausted. An intermediate value of p_g leads to an interesting comparison. Consider any $p_g \in [p_m(1)^{NI}, p_m(1)^{DBT}]$ and define δ as follows:

$$\delta(p_g) = \frac{-b}{\beta} + \frac{1}{kM\beta}\left[\frac{n\theta}{2\alpha}p_g^2 - M(1-p_g)p_g\right].$$

It is straightforward to verify that $\delta(p_g)$ is increasing in p_g, $\delta(p_m(1)^{NI}) = 0$, and $\delta(p_m(1)^{DBT}) = 1$. The market outcome under the GSP scheme for such a value of p_g is identical to the market outcome under the DBT scheme where the social planner operates with a budget $\delta(p_g)B$.

This is because, at such a value of p_g, the social planner uses an amount $\delta(p_g)B$ in procuring from the farmers, while $(1-\delta(p_g))B$ is left unused. Consequently, the social planner's surplus under the GSP scheme is increasing in p_g because the social planner's surplus from operating the DBT scheme with a budget $\delta(p_g)B$ is increasing in $\delta(p_g)$.

Finally, as p_g increases beyond $p_m(1)^{DBT}$, the social planner's budget is fully exhausted. Hence, the quantity procured by the social planner, B/p_g, decreases in p_g. Consequently, the production effort of the farmers also decreases in p_g. The quantity sold in the open market increases and hence, the market price decreases in p_g. In the limit (as $p_g \to \infty$), the social planner procures very little from the farmers, and hence the outcome is identical to the outcome under an alternate distribution scheme (see footnote 6) where the social planner allocates the entire budget to the farmers. The market outcome under such a high value of p_g is identical to that under NI, except that the social planner's budget (which is unused under NI) is distributed among the farmers.

Based on the discussion above, the following result states the equilibrium value of p_g.

Lemma 3.5.4. (Equilibrium Support Price) *If $b+\beta < b^*(\theta)$, then the equilibrium $p_g = p_m(1)^{DBT}$.*

3.5.2 Case 2: $b > b^*(\theta)$

Recall from Section 3.4 that, in this case, the DBT scheme is ineffective in changing the outcome under NI; specifically, the production effort of the farmers, the market price, and the social planner's surplus under the DBT scheme are all identical to those under NI. Further recall that $\theta^{**} = b^{*-1}(\frac{k}{1+k}\beta)$ and $p_m(1)^{NI} = \frac{M(1+k)}{M(1+k)+\frac{n\theta}{2\alpha}}$. In what follows, we analyze the outcome under the GSP scheme. We divide the analysis into two distinct subcases: $\theta \geq \theta^{**}$ and $\theta < \theta^{**}$. The former signifies an environment where yield uncertainty is moderate (intermediate values of θ), while the latter pertains to an environment where yield uncertainty is acute (low values of θ).

Case 2.A: $b > b^*(\theta)$ **and** $\theta \geq \theta^{**}$**:**

In Lemma 3.5.5 below, we identify the equilibrium market outcome under any announced value of the support price p_g. Using this result, we calculate the social planner's surplus that helps us in obtaining the equilibrium value of the support price (Lemma 3.5.6).

Lemma 3.5.5. (Farmers' production effort and selling decisions) *If* $b > b^*(\theta)$ *and* $\theta \geq \theta^{**}$*, then for any* p_g*, the equilibrium effort of the farmers is* $q_e^* = \frac{\theta}{2\alpha}\hat{p}_m(1)$*, and the market price (consistent with farmers' belief) is* $\hat{p}_m(1) = p_m(1)^{NI}$*.*

The proof considers three (mutually exclusive and exhaustive) cases: (a) $p_g < p_m(1)^{NI}$, (b) $p_g = p_m(1)^{NI}$, and (c) $p_g > p_m(1)^{NI}$. Interestingly, the market outcome (i.e., the farmers' effort, the market price, and the consumption by the consumers) under each case is identical to the market outcome under NI. We explain this finding below. If p_g is below the market price under NI, $p_m(1)^{NI}$, then the intuition is identical to that in case 1 of Lemma 3.5.2 – farmers anticipate a higher market price and therefore do not sell to the social planner. The entire budget of the social planner is unused and, hence, the outcome is identical to that under NI.

If $p_g \geq p_m(1)^{NI}$, then the farmers anticipate a decrease in the BPL consumers' demand from the open-market, since the social planner now supplements the consumption needs of this segment via the GSP scheme. In other words, the farmers anticipate the downward shift in the BPL consumers' demand caused by the additional quantity they receive from the social planner. This downward shift is proportional to the quantity that the farmers sell to the social planner. Therefore, relative to NI, the equilibrium effort of the farmers does not change. Consequently, the market outcome is also identical to that under NI. Since the consumption by the population is identical to that under NI, the consumption utilities of the APL and BPL segments are also identical to those under NI. However, the BPL consumers purchase a smaller quantity in the open-market. Thus, relative to NI, their equilibrium wealth under the GSP scheme is higher. Consider a fixed $p_g > p_m(1)^{NI}$. Each farmer's effort is $q_e^* = \frac{\theta}{2\alpha} p_m(1)^{NI}$. Under the high-yield realization, the quantity the farmers sell to the social planner is $q_g^* = \frac{B}{np_g}$. The quantity that each BPL consumer receives from the social planner is $\frac{\beta}{p_g}$, while the quantity he purchases from the open-market is $\big(1 - \frac{\beta}{p_g} - p_m(1)^{NI}\big)$. Therefore, his equilibrium wealth is $\big[b - p_m(1)^{NI}(1 - \frac{\beta}{p_g} - p_m(1)^{NI})\big]$, which can be rewritten as $\big[b - p_m(1)^{NI}(1 - p_m(1)^{NI})\big] + (\frac{p_m(1)^{NI}}{p_g})\beta$. Recall that $\big[b - p_m(1)^{NI}(1 - p_m(1)^{NI})\big]$ is the equilibrium wealth of a BPL consumer under NI. A BPL consumer is, therefore, "richer" under the GSP scheme (relative to NI) by an amount $(\frac{p_m(1)^{NI}}{p_g})\beta$. Each farmer's profit is also higher under the GSP scheme (relative to NI) – this is because the cost of production is identical (since the effort is identical) but they sell a fraction of their produce to the social planner at the support price (which is higher than the market price). Each farmer's utility, therefore, increases by an amount $\frac{B}{n}(1 - \frac{p_m(1)^{NI}}{p_g})$. Relative to NI, the GSP scheme does not improve the social planner's surplus. This is because the social planner's unused budget under NI is used to improve the surplus, by the same amount, of the farmers and the BPL consumers under the GSP scheme.

Indeed, the social planner can use the GSP scheme as a mechanism to divide his budget in any desired proportion to improve the surplus of the BPL consumers and the farmers; the exact split is achieved by setting an appropriate value of the support price. Precisely, let $\zeta \in [0,1]$ define this split, i.e., ζB (resp., $(1-\zeta)B$) denotes the improvement, relative to NI, in the utility of the BPL consumers (resp., farmers) that the social planner intends to achieve. Our analysis shows that the support price that achieves this intended improvement is $p_g = (\frac{1}{\zeta})p_m(1)^{NI}$. If $\zeta = 1$, i.e., if the social planner intends to allot the entire budget to improve the utility of the BPL consumers, then $p_g = p_m(1)^{NI}$. On the other extreme, if $\zeta = 0$, i.e., if the social planner only seeks to improve the utility of the farmers, then $p_g \to \infty$. An intermediate value of ζ leads to a corresponding intermediate value of p_g, which is strictly higher than the market price. The following result formally summarizes this discussion.

Lemma 3.5.6. (Equilibrium Support Price) *If $b > b^*(\theta)$ and $\theta \geq \theta^{**}$, then there are a continuum of equilibria in which for any support price p_g, the social planner's surplus is identical and equal to that under NI. While under NI, the social planner's budget is unspent, under GSP, the social planner can divide his budget in any proportion $\zeta \in [0, 1]$ to achieve corresponding improvements in the surplus of the BPL consumers and the surplus of the farmers, by choosing $p_g = (\frac{1}{\zeta})p_m(1)^{NI}$.*

Case 2.B: $b > b^*(\theta)$ and $\theta < \theta^{**}$:

Similar to the analysis in Section 3.5.2, we derive the equilibrium market outcome for any announced GSP p_g and the social planner's surplus. The key difference between this section and Section 3.5.2 is the extent of yield uncertainty: Under extremely low values of θ, the production of the farmers is low enough so that the social planner can procure the entire output of the farmers. Consequently, there is a threshold in the value of ζ (the proportion of budget that is allotted to the improvement of the BPL consumer's surplus, relative to NI; see discussion above) that can be achieved.

Lemma 3.5.7. (Farmers' production effort and selling decisions) *If $b > b^*(\theta)$ and $\theta < \theta^{**}$, then for any support price p_g, the equilibrium effort of the farmers and the market price (consistent with farmers' belief) are as follows:*

1. *If $p_g < p_m(1)^{NI}$, then $q_e^* = \frac{\theta}{2\alpha}\hat{p}_m(1)$, $q_g^* = 0$, $q_m^* = q_e^*$, and $\hat{p}_m(1) = p_m(1)^{NI}$.*

2. *If $p_g = p_m(1)^{NI}$, then $q_e^* = \frac{\theta}{2\alpha}p_g$, $q_g^* \in [0, \frac{\theta}{2\alpha}p_g]$, $q_m^* = q_e^* - q_g^*$ and $\hat{p}_m(1) = p_m(1)^{NI}$.*

3. *If $p_g \in \left(p_m(1)^{NI}, \sqrt{\frac{B}{\left(\frac{n\theta}{2\alpha}\right)}}\right]$, then $q_e^* = \frac{\theta}{2\alpha}p_g$, $q_g^* = \frac{\theta}{2\alpha}p_g$, $q_m^* = 0$ and $\hat{p}_m(1) = 1 - \frac{\left(\frac{n\theta}{2\alpha}\right)p_g}{M(1+k)}$.*

4. *If $p_g \in \left[\sqrt{\frac{B}{\left(\frac{n\theta}{2\alpha}\right)}}, \frac{B}{n(\frac{\theta}{2\alpha})p_m(1)^{NI}}\right]$, then $q_e^* = \frac{B}{np_g}$, $q_g^* = \frac{B}{np_g}$, $q_m^* = 0$ and $\hat{p}_m(1) = 1 - \frac{B}{M(1+k)p_g}$.*

5. *If* $p_g > \frac{B}{n(\frac{\theta}{2\alpha})p_m(1)^{NI}}$, *then* $q_e^* = \frac{\theta}{2\alpha}\hat{p}_m(1)$, $q_g^* = \frac{B}{np_g}$, $q_m^* = q_e^* - q_g^*$ *and* $\hat{p}_m(1) = p_m(1)^{NI}$.

Lemma 3.5.8. (Social planner's surplus) *If* $b > b^*(\theta)$ *and* $\theta < \theta^{**}$, *then for any support price* p_g, *the social planner's surplus under the GSP scheme,* $\Pi_S^{GSP}(p_g)$, *is as follows:*

1. *If* $p_g \leq p_m(1)^{NI}$, *then* $\Pi_S^{GSP}(p_g) = \Pi_S^{NI}$.

2. *If* $p_g \in \left(p_m(1)^{NI}, \sqrt{\frac{B}{\left(\frac{n\theta}{2\alpha}\right)}}\right]$, *then* $\Pi_S^{GSP}(p_g)$ *decreases in* p_g. *Consequently,* $\Pi_S^{GSP}(p_g) < \Pi_S^{NI}$.

3. *If* $p_g \in \left[\sqrt{\frac{B}{\left(\frac{n\theta}{2\alpha}\right)}}, \frac{B}{n(\frac{\theta}{2\alpha})p_m(1)^{NI}}\right]$, *then* $\Pi_S^{GSP}(p_g)$ *increases in* p_g. *Further,* $\Pi_S^{GSP}(p_g) < \Pi_S^{NI}$ *if* $p_g < \frac{B}{n(\frac{\theta}{2\alpha})p_m(1)^{NI}}$ *and* $\Pi_S^{GSP}(p_g) = \Pi_S^{NI}$ *at* $p_g = \frac{B}{n(\frac{\theta}{2\alpha})p_m(1)^{NI}}$.

4. *If* $p_g > \frac{B}{n(\frac{\theta}{2\alpha})p_m(1)^{NI}}$, *then* $\Pi_S^{GSP}(p_g) = \Pi_S^{NI}$.

We explain these results below. If the support price p_g is below the market price under NI, namely $p_m(1)^{NI}$, then the intuition is identical to that in case 1 of Lemma 3.5.4 – farmers anticipate a higher market price and don't sell to the social planner. Consequently, the market outcome and the social planner's surplus is identical to that under NI.

If $p_g = p_m(1)^{NI}$, the production decisions and the market price are identical to that under NI. Farmers anticipate a downward shift in the BPL consumers' demand curve, with the social planner supplementing their consumption. This downward shift is proportional to the quantity the farmers sell to the social planner. Further, the BPL consumers may sell some of the quantity in the open-market if the market price exceeds their marginal utility from consumption. Consequently, the equilibrium market price remains at $p_m(1)^{NI}$. Since the support price and the market price are identical, farmers are indifferent between selling to the social planner and in the open market. Let $\lambda \in [0, 1]$ denote the proportion of the total production sold to the social planner. If the farmers sell their entire production to the social planner (i.e., $\lambda = 1$), then the proportion of budget spent is $\bar{\zeta} = \frac{n\frac{\theta}{2\alpha}(p_m(1)^{NI})^2}{B}$; since

$\theta < \theta^{**}$, we have that $\bar{\zeta} < 1$. For an intermediate value of $\lambda \in [0,1]$, the budget spent is $\lambda\bar{\zeta}B$. Relative to NI, the surplus of the farmers remains identical, but the surplus of the BPL consumers increases by $\lambda\bar{\zeta}B$ (which depends on the actions of the farmers, viz., the proportion λ). This increase in their surplus is equal to the amount spent by the social planner.

If $p_g \in \left(p_m(1)^{NI}, \sqrt{\frac{B}{\left(\frac{n\theta}{2\alpha}\right)}}\right]$, then, relative to NI, the social planner distorts the market outcome. The total production effort of the farmers increases in p_g. However, the social planner's surplus decreases in p_g: the increase in the surplus of the APL and BPL consumers and the farmers does not offset the decrease in the social planner's unused budget. In other words, the social planner spends more budget than the resulting increase in surplus of the consumers and the farmers. At precisely $p_g = \sqrt{\frac{B}{\left(\frac{n\theta}{2\alpha}\right)}}$, the social planner's budget is fully exhausted. If $p_g \in \left[\sqrt{\frac{B}{\left(\frac{n\theta}{2\alpha}\right)}}, \frac{B}{n(\frac{\theta}{2\alpha})p_m(1)^{NI}}\right]$, then the social planner's budget remains fully exhausted. The quantity sold by the farmers $(\frac{B}{np_g})$ to the social planner decreases in p_g, and hence their total production also decreases in p_g. Further, the social planner's surplus increases in p_g since the farmers' surplus and the BPL consumers' surplus both increase in p_g.

If p_g is beyond $\frac{B}{n(\frac{\theta}{2\alpha})p_m(1)^{NI}}$, then the market outcome (the production decisions of the farmers and the market price) is identical to that under NI. Therefore, the social planner's surplus is a constant and is identical to that under NI. Since the market price is identical, the APL consumers' surplus is identical to that under NI. However, the surplus of the BPL consumers and the farmers depends on the value of p_g. For any $\zeta \in [0, \bar{\zeta}]$, let ζB (resp., $(1-\zeta)B$) denote the improvement in surplus of the BPL consumers (resp., farmers) that the social planner intends to achieve (relative to NI). The support price that achieves this split is given by $(\frac{1}{\zeta})p_m(1)^{NI}$. In the limit (as $p_g \to \infty$), the entire budget is allotted to the farmers (i.e., $\zeta = 0$). We conclude the discussion above by stating this result formally.

Lemma 3.5.9. (Equilibrium Support Price) *If $b > b^*(\theta)$ and $\theta < \theta^{**}$, then the equilibrium value of the support price is $p_g \leq p_m(1)^{NI}$ or $p_g \geq \frac{B}{n(\frac{\theta}{2\alpha})p_m(1)^{NI}}$, and the social planner's surplus is identical and equal to that under NI. While under NI the social planner's budget is unspent, under GSP, the social planner can divide his budget in any proportion $\zeta \in [0, \bar{\zeta}]$ to achieve corresponding improvements in the surplus of the BPL consumers and the surplus of the farmers, by choosing $p_g = (\frac{1}{\zeta})p_m(1)^{NI}$.*

3.5.3 An Alternate Objective of the Social Planner

In our analysis thus far, we have focused on a social planner whose objective consists of four components: the APL and BPL consumers' surplus, the farmers' surplus, and the unused budget. Recall from Section 3.1 that the GSP scheme is primarily intended to benefit two groups: the BPL consumers and the farmers. Therefore, we consider an alternate objective function of the social planner where he weighs the surplus of each segment differently. Specifically, we discuss a special case where the social planner weighs the surplus of the farmers by a factor $\omega_F \in [0, 1]$, the surplus of the BPL consumers by $(1 - \omega_F)$, and the other components by 0.[9] Then, for any $\omega_F \in [0, 1]$, the social planner's problem is:

$$\max_{p_g \geq 0} \Pi_S(\omega_F) \;\; = \;\; \mathbb{E}_\gamma \left[\omega_F(n\pi_f) + (1 - \omega_F)(kMu_{BPL}) \right].$$

Special Case: $\omega_F = 0$

This case corresponds to a setting where the social planner is concerned only with the BPL consumers' welfare. Recall from the proof of Lemma 3.5.3 that if $b + \beta < b^*(\theta)$, then the surplus of the BPL consumers increases in p_g if $[p_m(1)^{NI}, p_m(1)^{DBT}]$ and decreases in p_g

[9] This choice of the social planner's objective allows us to study the effect of larger weights assigned to the surplus of the intended beneficiaries of the scheme on the equilibrium choice of the support price in a straightforward manner. Other choices of a weighted objective function, where the social planner assigns small, non-zero weights to the APL consumers' surplus and the unused budget involve more computation but yield qualitatively similar insights.

if $[p_m(1)^{DBT}, \infty)$. Therefore, if $\omega_F = 0$ and $b + \beta < b^*(\theta)$, then the equilibrium support price is $p_m(1)^{DBT}$. If $b > b^*(\theta)$, then recall from the discussion preceding Lemma 3.5.6 and Lemma 3.5.9 that, for any desirable fraction $\zeta \in [0, 1]$, the GSP scheme enables the social planner to dedicate a ζ fraction of his budget B for improving the surplus of the BPL consumers and the remaining $(1 - \zeta)$ fraction for improving the surplus of the farmers, by setting $p_g = (\frac{1}{\zeta})p_m(1)^{NI}$. Since $\omega_F = 0$ here, the social planner's budget is solely employed for improving the surplus of the BPL consumers; thus, $\zeta = 1$ and the equilibrium support price is $p_g = p_m(1)^{NI}$.

Special Case: $\omega_F = 1$

This case corresponds to a setting where the social planner is concerned with the farmers' welfare. From the proof of Lemma 3.5.3 (resp., Lemma 3.5.5 and Lemma 3.5.8), we have that if $b + \beta < b^*(\theta)$ (resp., $b > b^*(\theta)$), then the surplus of the farmers is increasing in p_g throughout. Therefore, if $\omega_F = 1$ then the equilibrium support price is $p_g = \infty$. In summary, if $\omega_F = 1$, then the social planner implements an outcome wherein he distributes the entire budget among the farmers.

Intermediate value of $\omega_F \in (0, 1)$

Recall from the proof of Lemma 3.5.3 that if $b + \beta < b^*(\theta)$, then the surplus of the BPL consumers and the farmers is increasing in p_g if $p_g \in [p_m(1)^{NI}, p_m(1)^{DBT}]$. Beyond $p_m(1)^{DBT}$ (i.e., $p_g > p_m(1)^{DBT}$), the surplus of the BPL consumers is decreasing in p_g while that of the farmers is increasing in p_g. Therefore, if $b + \beta < b^*(\theta)$ and $\omega_F \in (0, 1)$, we have that the equilibrium support price $p_g \in [p_m(1)^{DBT}, \infty)$, depending on the value of ω_F. In Figure 3.1, we illustrate the weighted objective function of the social planner as a function of ω_F and p_g. At the left extreme (i.e., $\omega_F = 0$), the social planner's objective decreases in p_g, whereas at the right extreme (i.e., $\omega_F = 1$), the social planner's surplus increases in p_g (the numerical

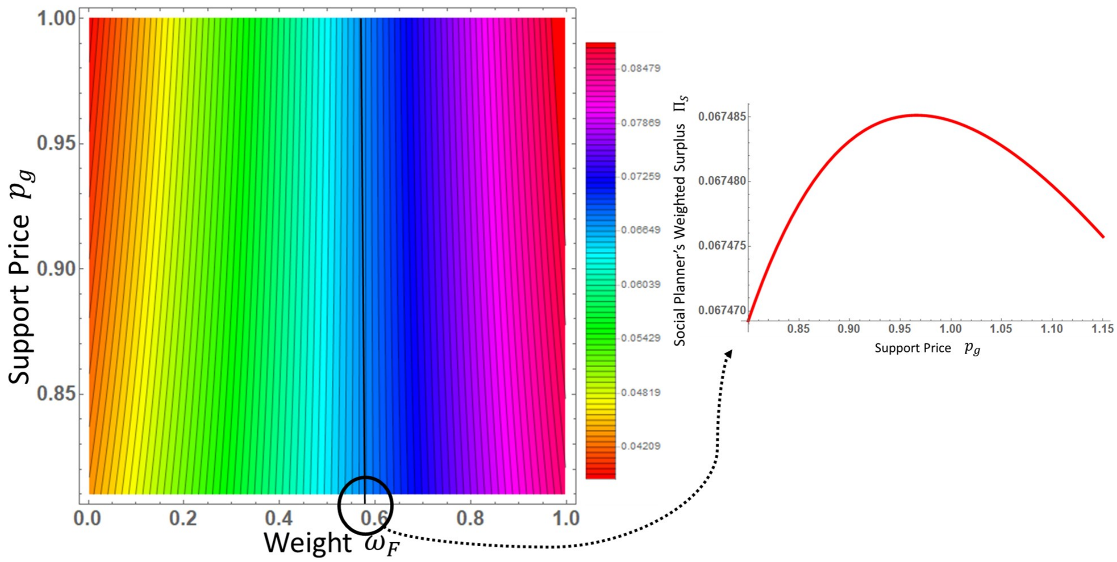

Figure 3.1. Left: Weighted objective of the social planner as a function of the weight ω_F and the support price p_g. Right: Weighted objective of the social planner at $\omega_F = 0.575$ as a function of the support price p_g.

values of the various parameters are: $B = 0.05$, $b = 0.03$, $k = 5$, $M = 1$, $n = 1$, $\alpha = 0.5$, $\theta = 0.5$).

If $b > b^*(\theta)$ and $\theta > \theta^{**}$ (i.e., under medium values of yield uncertainty), then recall from the discussion in Section 3.5.2 that the social planner can choose to divide the budget B among the BPL consumers and the farmers in any proportion $\zeta \in [0, 1]$ by an appropriate choice of $p_g \geq p_m(1)^{NI}$. That is, for any $\zeta \in [0, 1]$, if $p_g = (\frac{1}{\zeta})p_m(1)^{NI}$, then, relative to NI, the BPL consumers' surplus is larger by ζB and the farmers' surplus is larger by $(1 - \zeta)B$. Therefore, if $\omega_F < \frac{1}{2}$, then the social planner chooses $p_g = p_m(1)^{NI}$, and if $\omega_F > \frac{1}{2}$, then the social planner chooses $p_g = \infty$. Formally, corresponding to a support price $p_g \geq p_m(1)^{NI}$, the social planner's weighted objective can be written as $\omega_F\big(n{\pi_F}^{NI} + (1 - \zeta)B\big) + (1 - \omega_F)\big(kM{u_{BPL}}^{NI} + \zeta B\big)$, which can be rewritten as $\omega_F n {\pi_F}^{NI} + (1 - \omega_F)kM{u_{BPL}}^{NI} + \omega_F B + 2B\zeta(\frac{1}{2} - \omega_F)$, where $\zeta = \frac{p_m(1)^{NI}}{p_g}$. Thus, choosing a support price p_g is equivalent to choosing the split ζ. For a fixed ω_F, each of the first three terms is a constant and, hence, the social planner's problem under the weighted objective function is equivalent to maximizing $\zeta(\frac{1}{2} - \omega_F)$. The analysis in the case $b > b^*(\theta)$ and $\theta < \theta^{**}$ (i.e., under low values of yield uncertainty) is identical, since the social planner can divide the "spent" budget ($= \bar{\zeta}B$; see the discussion in Section 3.5.2) between the BPL consumers and the farmers by an appropriate choice of the support price.

The following remark differentiates the action of the GSP scheme from that of a price floor.

Remark (Connection to Price Floors): A price floor for a good is an intervention through which the government imposes a lower bound on its market price. For it to be effective, a price floor should clearly be higher than the free-market equilibrium price. From standard microeconomic theory we know that, under such a price floor, the equilibrium production is higher while the demand is lower, relative to the case where the price floor is absent. Consequently, compared to the outcome in a free market, the consumer surplus is lower, the producer surplus is higher, and the total social welfare is lower; the loss in social welfare from a price floor is referred to as the "dead-weight" loss; see, e.g., (Varian, 1992).

Unlike in a price floor, the government does not regulate the market price directly under the GSP scheme. Indeed, there are two outlets for the producers to sell under a GSP scheme: the open market and the government. The government offers a fixed support price, while the open market price is determined by supply and demand. On their part, the producers (farmers) can sell partial amounts in both these outlets in equilibrium. The other important change in the action of a GSP scheme comes from the structure of the demand side: The consuming population consists of two segments, namely the BPL and APL consumers. The amount procured by the government under the scheme is used to increase the consumption of only one of these two segments (the BPL consumers). As far as the operational decisions are concerned, the presence of two consumer segments and two selling markets for the producers — of which one (the open market) is accessible to both consumer segments and the other (the government) is accessible only to one segment (the BPL consumers) — makes the equilibrium analysis of the producers' selling decisions and the social planner's choice of the support price quite challenging. ■

3.6 Conclusions

Broadly, our goal in this paper is twofold: (a) To understand the role of Guaranteed Support Prices (GSPs) on the operational decisions of its main stakeholders, viz., the farmers, the consuming population and the social planner (government), and (b) To understand the impact of the GSP scheme on the welfare of each stakeholder and compare them with two benchmarks: (a) the absence of an intervention and (b) the Direct Benefit Transfer (DBT) scheme.

Our analysis offers analytically-supported clarity on the impact of the GSP scheme. We expound on two key economic forces – the poorness of the BPL consumers (a demand-side friction) and yield unertainty (a supply-side friction) – that act as impediments to high production by the farmers, and consumption by the poor consumers. If the poorness of the BPL consumers is extreme, then the GSP scheme helps in improving the production of the farmers and the consumption by the BPL consumers. In this case, although the surplus of the APL consumers is lower under the GSP scheme (relative to its absence), the surplus of the farmers, the BPL consumers, and the social planner are all higher. Further, the equilibrium market outcome and the social planner's surplus are identical to that under the DBT scheme. If yield uncertainty is a dominant criterion in determining the equilibrium effort of the farmers, then the surplus of the social planner under the equilibrium GSP is identical to that under the DBT scheme, which in turn is identical to that under no intervention. Nevertheless, under the GSP scheme, the social planner can divide his budget in any desired proportion to improve the surplus of the farmers and the BPL consumers by using an appropriate support price. Finally, we also discuss an alternate objective of the social planner consisting of a weighted combination of the surplus of the two primary intended beneficiaries of the GSP scheme, namely the BPL consumers and the farmers, and demonstrate how different weights affect the equilibrium support price.

Our analysis focused on the GSP scheme for a single crop. In practice, many developing countries offer support prices for multiple crops; see, e.g., (Planning Commission of India, 2001). On the one hand, the government aims for crop-wise targets to "balance" their production quantities based on their relative demands from the consuming population; on the other hand, farmers – constrained by their respective geographical locations – have natural preferences over the crops but may get influenced in their choice via attractive support prices. This makes the analysis of farming effort and governmental decisions under multiple support prices a fairly complex problem. In particular, understanding the influence of support prices on the crop-mix pattern is an important and challenging problem for future research. A recent contribution along this direction is (Chintapalli and Tang, 2018), which considers the case of two crops.

A broader direction for future research is that of a comparison across different classes of subsidies. The World Trade Organization (WTO) classifies agricultural subsidies into *amber-box*, *blue-box*, and *green-box* subsidies (World Trade Organization, 2018; The Guardian, 2013): Amber-box subsidies involve support measures that can significantly distort production; the GSP scheme is one such subsidy. Green-box subsidies do not distort production at all, while blue-box subsidies only cause a moderate amount of distortion. The WTO regulates governmental spending by developing countries on these classes of subsidies – the current ceiling is 10% of the total value of agricultural production for amber-box subsidies and is 8% for blue-box subsidies. There is currently no cap on green-box subsidies, which include policies for environmental and regional protection. Thus, a comparative analysis of the tradeoffs between these classes of schemes can provide meaningful input to policymakers.

CHAPTER 4

YOUR UBER IS ARRIVING: MANAGING ON-DEMAND WORKERS THROUGH SURGE PRICING, FORECAST COMMUNICATION, AND WORKER INCENTIVES[1]

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[1]Portions of this chapter are reprinted with permission: Harish Guda, Upender Subramanian. "Your Uber Is Arriving: Managing On-Demand Workers Through Surge Pricing, Forecast Communication, and Worker Incentives". *Management Science*, **65**(5), 1995 – 2014.

"Was smack in the middle of the zone [with a high surge price] and yet no pings [customer requests]... If it's surging, I would expect to see fewer cars available and as a driver, I would expect instant pings" - Uber driver on online forum (Uber People, 2015).

"When an area is surging [has a surge price], it just kills demand, and a large fraction of the drivers leave the area and drive elsewhere... [Surge pricing] is not incentivizing drivers the way [Uber] hoped it would." - Christo Wilson, co-author of Uber pricing study Chen et al. (2015), as quoted in *The San Francisco Chronicle* (Said, 2015).

4.1 Introduction

The on-demand economy can be defined as "the economic activity created by digital market places that fulfill consumer demand with immediate access to and convenient provisioning of goods and services" (OnDemandEconomy.org, 2016). The on-demand economy has witnessed explosive growth in recent years due, in large part, to the emergence of peer-to-peer online platforms that match consumers in real time with independent workers who are available nearby and can serve consumers at short notice (Economist, 2015; Fowler, 2015). Leading examples of such platforms include Uber and Lyft for cabs; GrubHub, Instacart and Postmates for delivery of food, groceries or other items; TaskRabbit and Handy for household services; and, Glamsquad and Zeel for health and beauty services. One study estimated the annual U.S. consumer spending on on-demand services to be \$5.6 billion for cabs, \$4.6 billion for delivery, and \$8.6 billion for all other services (Colby and Bell, 2016). Given their growing economic significance, it is important to understand the business strategies of on-demand platforms and how these platforms can be managed effectively. In this paper, we examine a pricing strategy adopted by many on-demand platforms.

A fundamental challenge for on-demand platforms is to ensure that workers are available at the right time and locations to serve consumers at short notice. On-demand marketplaces are often characterized by fluctuating demand across market locations. The platforms obtain revenue if demand is met; they typically receive a commission on worker revenues generated from serving consumers. For example, Uber and Lyft receive a commission of 20-25% of their drivers' revenues. However, consumers can be served at short notice only if enough workers are already available nearby, since it can take workers considerable time to move from farther locations. Therefore, the platform must ensure that workers are available when and where they are needed.

This challenge is compounded by the fact that on-demand platforms do not directly control the workers. As Uber's Director of Marketplace Optimization Data Sciences recently noted:

> *"Drivers are independent agents. So they are our customers as much as riders are. We cannot tell the drivers where to move. We can suggest where we think they might want to move"* (Phillips, 2017).

To provide on-demand service at attractive price points, many on-demand platforms rely on what has come to be known as the "gig economy" - a variable workforce of freelancers who work at their convenience and can be hired on-demand for a single project or task (Nunberg, 2016; Torpey and Hogan, 2016). These independent workers are attracted to on-demand platforms by the prospect of earning extra income in their free time and are, hence, willing to work for considerably lower compensation than a full-time worker. For example, workers for on-demand cabs and delivery service can typically be "anyone with a car" - college students, people from other professions, or retirees - who are willing to drive or deliver in between their other daily activities or work. Chen et al. (2017) estimate the value of this flexibility for Uber drivers; they find that, all else equal, even requiring drivers to work eight hours at a stretch would reduce the labor supplied by two-thirds, while regular full-time commitment is practically infeasible. Thus, the platform obtains workers willing to serve consumers at relatively low prices. At the same time, these independent workers can flexibly choose when to work and where to work. Consequently, on-demand platforms cannot plan their supply of workers in advance or directly control worker availability at different market locations.

To operate effectively under such market conditions, on-demand platforms often adopt a two-pronged approach. First, they invest considerable resources to forecast supply and demand patterns ahead of time, and share these forecasts with the workers. For example, Uber uses advanced algorithms to forecast the need for additional drivers at each market location based on historical patterns, holidays, weather, current local events and traffic conditions (Rosenblat and Stark, 2015; Phillips, 2017). Uber shares these forecasts with drivers through a mobile application, and encourages drivers to move to locations where a shortage of drivers is expected.[2]

Second, many on-demand platforms employ a form of dynamic pricing known as *surge pricing*. Under surge pricing, there is a set *regular price* for the entire market region. However, depending on the prevailing supply and demand conditions, the platform can increase the current price at a given market location to be higher than the regular price; this higher price is referred to as a *surge price*. To implement surge pricing, the market is split into several smaller regions or "zones" and the platform periodically updates the current price in each zone. Consumers learn the current price in their zone at the time that they connect to the platform to request service. Surge pricing also affects workers' revenues as they are paid more if they serve consumers in a zone with a surge price. Workers typically have access to "surge heat maps" that display the surge prices (and market forecasts) in their zone and in adjacent zones (Chen et al., 2015; Rosenblat and Stark, 2015). Platforms employing surge pricing include Uber, Lyft, Postmates, Instacart and Handy. Furthermore, surge pricing can be fairly prevalent. Cohen et al. (2016) find the percentage of time that surge pricing was used on Uber's platform in four major U.S. cities to be as follows: 14% in New York City, 17% in Los Angeles, 25% in San Francisco and 28% in Chicago; Chen et al. (2015) find it was used 57% of time in downtown San Francisco, and 14% of time in midtown Manhattan.

[2] The mobile application displays a map highlighting market locations that are likely to need additional workers. Locations are marked yellow, orange or red to indicate the likelihood and extent of the need. Uber instructs workers: "Use this info to your advantage by heading toward surging areas to receive nearby ride requests" Uber Help (2016).

The conventional rationale for these platform strategies is quite straightforward. Sharing market forecasts with workers should encourage workers to be available where they are needed the most. Further, researchers and industry experts agree that surge pricing is useful to balance supply and demand in market zones where there is a shortage of workers relative to demand (Gurley, 2014; Chen and Sheldon, 2015; Hall et al., 2015). Surge pricing is expected to work in two ways. First, by pricing out consumers who have lower willingness to pay, a surge price efficiently allocates or "rations" the limited supply of workers to consumers who value them the most. Second, because a surge price increases workers' compensation in that zone, it should attract more workers to move to that zone from adjacent zones, thus, reducing the extent of shortage in supply. For example, Hall et al. (2015) analyze a substantial surge pricing event that occurred in a particular market zone facing a shortage of drivers on New Year's eve on Uber's platform. They find evidence to suggest that the surge price resulted in rides being allocated to consumers who valued them the most, and attracted more drivers to that zone. Thus surge pricing and forecast information sharing are both expected to improve the functioning of the on-demand marketplace by directing workers to zones requiring additional workers. Moreover, surge pricing is expected to be used in zones where the demand for workers exceeds their supply.

Yet, the above conventional wisdom does not seem to fully capture what is observed in practice. For example, one might expect that if surge pricing is used in zones where demand for workers exceeds their supply, then workers in that zone should not have to wait for relatively long to receive customer requests. Yet, as illustrated by the opening comment, many Uber drivers note that they regularly do not receive customer requests even if they are in a zone where surge pricing is in effect. An empirical study by Chen et al. (2015) adds weight to such reports. They collected data about pricing, demand and supply from Uber's platform in New York City and San Francisco over seven days. Contrary to conventional wisdom that a surge price is used to balance demand with available supply, they find that in certain market zones a surge price on average caused drivers to become idle, in effect creating an imbalance between supply and demand. Further, instead of attracting new drivers, they find that in these zones surge pricing resulted in fewer drivers moving to the zone and more drivers leaving the zone. Consequently, as illustrated by the opening comment, the authors conclude that surge pricing is not working in a manner consistent with conventional wisdom.

Other researchers question whether sharing market forecasts with workers has been effective (Lee et al., 2015; Rosenblat and Stark, 2015). Based on driver reports and surveys, they find that a significant fraction of drivers routinely ignore the forecasts provided by the platform; for example, by staying in their current zone and not driving to market zones that are forecasted to have a shortage of drivers. Some drivers also express a lack of trust regarding whether the forecasts represent true market conditions, or are being used to manipulate drivers to move to different zones (Rosenblat and Stark, 2015). Thus, there may be more to these platform strategies than might initially meet the eye, and calls for more careful investigation.

Our objective in this paper is to conduct a model-based examination of the platform's strategic incentives for surge pricing. We analyze a platform facing uncertain supply and demand conditions in two adjacent market zones over two successive time periods. Workers can move between zones incurring some cost and time. The platform expects to have a surge in demand in one zone (surge zone), and requires workers to move from the adjacent zone (non-surge zone) to meet demand. Workers are independent agents and not directly controlled by the platform. We analyze the strategic interaction between the platform and workers, and the optimal platform pricing policy.

By explicitly accounting for how workers strategically move between zones to maximize their expected earnings, we show how and why, and contrary to conventional wisdom, surge pricing can be profitable even in a zone where the supply of workers exceeds demand. Because of the strategic interaction amongst workers in their decisions to move between zones, too few workers in the non-surge zone may move to the surge zone. Interestingly, surge pricing in the non-surge zone can induce more workers to leave by lowering that zone's revenue potential. In particular, even though it is optimal to charge the regular price in the non-surge zone when this zone is viewed in isolation, distorting the price through surge pricing can increase platform profit across zones by improving worker availability in the surge zone.

We extend our analysis to explore further issues. We show that the platform may have an incentive to misreport market forecasts by exaggerating the need for workers to move. Therefore, the platform may use a surge price to credibly communicate (signal) market conditions to workers. Importantly, a surge price in the zone that the workers should leave is a less costly means for credible communication than a surge price in the zone that workers should move to. We further show when and why surge pricing may be used to retain workers from leaving a zone. We also analyze the implications of offering workers a bonus to move to the surge zone. In what follows, we discuss related literature in Section 4.2, introduce the model in Section 4.3, analyze the strategic role of surge pricing in Section 4.4, extend our analysis to explore further issues in Section 4.5, and conclude in Section 4.6.

4.2 Literature Review

There is growing ongoing research on on-demand platforms. Initial work has focused on surge pricing in a single market zone and has not examined how it influences workers to move between zones. Cachon et al. (2017) find that surge pricing with a revenue-sharing arrangement with workers not only outperforms static pricing but may also capture much of the platform profit that is possible if worker wages are set independently. Castillo et al. (2017) show that, without surge pricing, too few workers may be available close to any customer, causing workers to be unproductively traveling longer durations within a zone to pickup customers. Chen and Sheldon (2015) empirically examine how surge pricing affects the duration for which a driver works on Uber's platform, conditional on having decided to start working. They find that a driver is less likely to stop working if a surge price is in effect at the time the driver finishes a trip. They do not analyze how surge pricing influences drivers to move between zones.

Buchholz (2016) estimates a dynamic structural model of strategic behavior amongst New York City cab drivers in how they compete for customers by driving to different city locations. Similar to his work, we also consider that workers strategically anticipate how other workers will move and account for the explicit and opportunity costs of moving between locations. We further examine platform strategies to manage how workers move between locations. Bimpikis et al. (2017) study an on-demand platform's optimal pricing and worker compensation policy across a network of locations and find that a fixed commission contract is not optimal if demand is not "balanced" across locations. They assume that workers can move costlessly and instantaneously between locations. Hence, the tradeoffs we analyze do not arise in their setting. Ozkan and Ward (2017) study the spatial matching of an arriving customer request with available drivers, and propose a matching policy that significantly outperforms the more commonly used "closest driver" matching policy. Lam and Liu (2017) find that the welfare gains of on-demand ride-hailing from better matching are significantly higher in locations and times that have been underserved by traditional taxis and public transport. Wu et al. (2016) find significant productivity gains for taxi drivers amongst early adopters of mobile-hailing technology, but the gains are lower as more drivers adopt due to competition.

Another line of research examines on-demand platform pricing in a single zone if worker wages are not a fixed proportion of the price. Gurvich et al. (2015) find that worker flexibility to set their own schedules reduces worker participation and increases price levels. With delay-sensitive consumers, Bai et al. (2016) show that worker wages and price-to-wage ratio are both higher during periods of high demand. Taylor (2016) shows that congestion-driven service delays lead to lower prices and higher wages if the platform knows the valuations of consumers and workers; but this may not be the case if the valuations are uncertain. Hu and Zhou (2017) show that the optimal price has a U-shaped relation with the wage depending on market conditions; nevertheless, a fixed commission contract can be optimal or near-optimal. Our work also differs from research on product-sharing platforms where users rent assets from owners, need not be served at short notice, and there is no surge pricing (e.g., Benjaafar et al. 2015; Einav et al. 2015; Fraiberger and Sundararajan 2015; Abhishek et al. 2016; Jiang and Tian 2016; Zervas et al. 2016; Tian and Jiang 2017); see Narasimhan et al. (2017) for a recent review.

Researchers have studied the competitive implications of a firm using location-based pricing to price discriminate amongst consumers based on their location relative to the firm and its competitors (e.g., Corts 1998; Chen et al. 2001; Shaffer and Zhang 2002). Recently, Chen et al. (2016) examine the competitive implications of location-based pricing in the context of mobile geo-targeting. We study location-based pricing for better matching of supply and demand under uncertain market conditions. Our work differs from research on "peak-load pricing" (charging higher prices during periods of high demand) that has by and large focused on situations where a single firm makes capacity and pricing decisions, and does not consider interactions across market regions; see Crew et al. (1995) for a review. Our work also differs from the literature on dynamic pricing under limited inventory, wherein the objective is to manage the sales of a fixed amount of inventory occurring over a period of time; see Elmaghraby and Keskinocak (2003) for a review.

Lastly, we add to the literature on credible sharing of information. Lee et al. (1997) show that a downstream firm can exaggerate demand information to ensure sufficient supply from an upstream firm. Cachon and Lariviere (2001) and Özer and Wei (2006) study supply chain contracts that facilitate credible information sharing. Stock and Balachander (2005) show that a firm may price a product too low and create product scarcity in order to credibly signal higher quality to consumers. We study credible information sharing through surge pricing in a multi-period multi-location setting.

4.3 Model

Our model consists of three sets of players: an on-demand platform, independent workers, and consumers. We analyze the interactions in two adjacent market zones A and B over two successive periods 1 and 2. In each zone, consumers request on-demand service through the platform, and only workers in the same zone as consumers can serve them at short notice. The platform sets prices for on-demand service, and matches consumers requesting service with available workers. A market zone can experience a demand surge (increase in demand), and the workers available in that zone may not be sufficient to satisfy the demand. Workers can move to the zone from an adjacent zone by incurring some cost and time. Workers obtain revenue from providing service, and the platform receives a commission from the revenue generated. Our main interest is to understand when, why and in which market zones the platform will use a surge price, and the implications.

We start by describing consumers. In each zone in each period, there is a continuum of consumers who require one unit of on-demand service at short notice and cannot wait. In zone $i \in \{A, B\}$ in period $j \in \{1, 2\}$, let $D_{ij}(p_{ij})$ denote the consumer demand given the price p_{ij} set by the platform. We model the variation in consumer demand over the two periods as follows. Demand is initially the same in both zones (in period 1), and subsequently (in period 2) surges in one of the two zones. We refer to the zone with the demand surge as the *surge zone*. Either zone can be the surge zone with equal probability. Let $D_R(p)$ be the demand in either zone at price p in period 1; we refer to $D_R(p)$ as the *regular demand*. In period 2, let $D_S(p)$ be the demand in surge zone, where $D_S(p) > D_R(p)$ if $D_S(p) > 0$; we refer to $D_S(p)$ as the *surge demand*. Demand in the non-surge zone remains $D_R(p)$. We assume that $D_R(p)$ and $D_S(p)$ are decreasing, concave and differentiable in p.

Independent workers work at their convenience. Hence, the number of independent workers willing to work in each zone on a given day is uncertain, which we model as follows. There is a continuum of registered workers in each zone. On a given day, a random subset of registered workers are available to serve consumers. Available workers join the platform in their registered "home" zones, which could be, for example, the zone in which they live or get off from other work. Let $N_i \in \{n_H, n_L\}$ denote the number (mass) of workers who join the platform in zone i at the start of period 1, where $n_H > n_L$. We assume that either $(N_A, N_B) = (n_H, n_L)$ or $(N_A, N_B) = (n_L, n_H)$, both being equally likely. In other words, there are either more workers to begin with in zone A than in zone B, or vice-versa. We say the "initial supply in zone i is high" if $N_i = n_H$. The uncertainty in the initial supply of workers is independent of the uncertainty in consumer demand.

While a worker starts in her home zone, she can move to the adjacent zone at a cost $c \geq 0$. Moving between zones requires one period, and the worker is not available to serve consumers in either zone while moving. Workers who stay in their current zone are available to serve consumers in that zone. Let $\tilde{N}_{ij}$ denote the number of workers available to serve consumers in zone i in period j. For analytical convenience, we assume that all service requests take one period to complete, and originate and end in the same market zone. Hence, a worker who serves a consumer is busy for one period and remains in the same market zone at the end of that period.

We should clarify that our two period - two zone setup is a simplification to facilitate analysis. The two periods are meant to represent relatively short durations of time, and not the entire duration of a worker's shift. The two zones are meant to be a discrete approximation of a more continuous spatially dispersed market; they represent locations that require time for workers to move between (e.g., uptown and downtown). Later, in Section 4.6.2, we discuss the implications of considering the interactions over multiple periods and zones.

Each period, the platform matches consumers requesting service in each zone with workers available to serve consumers in the same zone. If more workers are available than consumers requesting service, then demand is rationed and not all workers receive work requests; the platform randomly chooses from the available workers to match with consumers. If there are fewer workers available than consumers requesting service, then supply is rationed and not all consumers obtain service; the platform randomly chooses the consumers who are matched with available workers. The revenue generated from serving a consumer in zone i in period j is p_{ij}. The worker who serves the consumer receives a portion $\lambda \in (0, 1)$ of the revenue; we refer to this portion as the worker's revenue. The platform receives the remaining portion, which we refer to as platform profit. Let $R_{ij} = p_{ij} \min \left\{ D_{ij} \left(p_{ij} \right), \tilde{N}_{ij} \right\}$ denote the market revenue, and $\pi_{ij} = (1 - \lambda) R_{ij}$ denote the corresponding platform profit in zone i in period j. Thus, market revenue and platform profit are constrained by the available supply of workers if $D_{ij} \left(p_{ij} \right) > \tilde{N}_{ij}$.

The platform can set the price p_{ij} to be equal to or higher than the *regular price* p_0. A price $p_{ij} > p_0$ represents a *surge price.* Absent supply constraints, the platform would set prices to maximize the *unconstrained revenue*, given by $pD_R(p)$ under regular demand and $pD_S(p)$ under surge demand. Let $p_R = \arg\max_{p \geq p_0} pD_R(p)$ and $p_S = \arg\max_{p \geq p_0} pD_S(p)$, respectively, denote the unconstrained revenue-maximizing prices under regular and surge demands. Consistent with conventional wisdom, we wish to examine situations where, absent supply constraints, the platform may use a surge price only under surge demand. Therefore, we assume that $p_R = p_0$.[3] We let $p_S \geq p_R$ such that surge demand can be less elastic than regular demand; thus, consumer willingness to pay can be higher under surge demand (e.g., on a rainy day, late night ride home from a pub).

In zone i in period j, let a_{ij} denote the number of workers needed to meet the unconstrained revenue-maximizing demand. We refer to a_{ij} simply as the "number of workers needed to meet demand". Let $a_R = D_R(p_R)$ and $a_S = D_S(p_S)$, respectively, denote the number of workers needed to meet demand under regular and surge demands. We say there

[3]If $p_R > p_0$, then a surge price would be charged even under regular demand, even absent supply constraints.

is a "shortage of workers" in a zone if the number of available workers is not sufficient to meet the unconstrained-revenue maximizing demand level. Thus, there are enough workers to meet demand if $\tilde{N}_{ij} \geq a_{ij}$; otherwise, there is a shortage of workers to the extent of $a_{ij} - \tilde{N}_{ij}$.

We assume that the initial supply in the surge zone is not sufficient to meet the surge demand. To keep the analysis straightforward, we focus on situations where the overall supply of workers across zones is sufficient to meet the overall demand across zones, such that demand can be met in both zones in period 2 if workers move accordingly. Specifically, we assume that: (i) $n_H + n_L > a_S + a_R$, such that there is no "global" shortage of workers, and (ii) $a_S > n_H$, such that there is a "local" shortage of workers in the surge zone if workers do not move from their initial zone.

The sequence of events is as follows. At the start of period 1, Nature decides the identity of the surge zone and the initial number of workers in each zone. Then, in each period j the following events occur. Consumers requiring on-demand service in that period join the platform. Next, the platform sets the price in each zone. Then, workers decide whether to stay in their current zone or to move to the adjacent zone. Consumers decide whether to request service. Lastly, the platform matches consumers requesting service with workers in the same zone who are available to serve consumers. At the end of the period, the workers who were matched with consumers complete their service, the workers who decided to move reach the adjacent zone, and consumers who are not served exit the platform. All players are rational, and maximize their respective expected utility or profit (without discounting) over two periods. Table B.1 summarizes the model notation (see Appendix B).

It is useful to consider the information that is known to the platform and workers. The model setting and parameters are common knowledge. The platform knows the initial number of workers that join in each zone. Further, the platform can forecast future demand. Hence, it knows the identity of the surge zone. Workers too may be knowledgeable about supply and demand conditions, for example, based on their prior experience. Moreover, the platform may share its information with workers. To begin with, in Section 4.4, we examine the *symmetric information* scenario, in which both the platform and workers know the initial supply in each market zone as well as the identity of the surge zone. This scenario allows us to examine the strategic role of surge pricing separate from concerns about truthful communication of market forecasts. This scenario also approximates situations where workers are sufficiently knowledgeable about market conditions based on their past experience, or the platform shares its information truthfully, for example, due to reputational concerns. Later, in Section 4.5.1, we analyze the *asymmetric information* scenario, in which only the platform knows market conditions, and shares this information with workers at the start of period 1. However, the shared information is not verifiable and the platform can misreport the information.

4.4 Managing Independent Workers through Surge Pricing

We start by examining the platform's period 2 pricing strategy, and show that it follows the conventional logic for surge pricing. Next, we show that if the platform has full control over the movement of workers between zones, then the platform will not use surge pricing in period 1. Then, we examine the platform's period 1 pricing strategy given that the platform cannot directly workers.

4.4.1 Surge Pricing in Period 2

In period 2, the platform sets the price in each zone to maximize its profit in that zone given the number of available workers in that zone. Let $p_{i2}^e = \arg\max_{p_{i2} \geq p_0} \pi_{i2}$ denote the platform's optimal price in zone i. If there are enough workers in zone i ($\tilde{N}_{i2} \geq a_{i2}$), then the platform sets the unconstrained revenue-maximizing price $p_{i2}^e = D_{i2}^{-1}(a_{i2})$. If there is a shortage of workers, then, since π_{ij} is concave and decreasing in p_{i2} for $p_{i2} \geq D_{i2}^{-1}(a_{i2})$, the platform will set a market-clearing surge price $p_{i2}^e = D_{i2}^{-1}\left(\tilde{N}_{i2}\right) > p_R$. Thus, in general, $p_{i2}^e = D_{i2}^{-1}\left(\min\left\{a_{i2}, \tilde{N}_{i2}\right\}\right)$.

The platform's pricing strategy in period 2 highlights the conventional logic for surge pricing. Namely, surge pricing is used in the surge zone to manage the shortage of workers in a zone. The platform sets a market-clearing surge price that rations the available workers by allocating them to the $\tilde{N}_{i2}$ consumers willing to pay the most for service. If $p_S > p_R$, then surge pricing will be used in the surge zone even if there is no shortage of workers, in order to extract more consumer surplus under surge demand; but, the surge price is higher if there is a shortage.

Moreover, period 2 prices and profit depend on the number of workers that move to zone A in period 1. We say "enough workers move" if workers move such that there is no shortage in either zone and period ($\tilde{N}_{ij} \geq a_{ij}$). In this case, $p_{A2}^e = p_S$ and $p_{B2}^e = p_R$. We say "not enough workers move" if there is a shortage in zone A in period 2. In this case, $p_{A2}^e > p_S$ and $p_{B2}^e = p_R$; period 2 platform profit is lower than if enough workers move since demand is not fully met in zone A.

Before we proceed, we should note that the platform will not need workers to move from other zones to meet surge demand and surge pricing would be transitory if the supply of workers in the surge zone is not fixed but perfectly elastic; that is to say, if sufficient number of new workers will join the platform in response to a "small", transient surge price. However, as discussed in Section 1, surge pricing is fairly prevalent. Moreover, it is expected to have only a mild effect on the number of workers joining the platform and mainly influence drivers already on the road as, for example, noted by the head of economic research at Uber.[4] And, as a result, platforms have invested significantly in systems to provide market information

[4]He notes: "[T]he main way that we incentivize drivers to move to the places that riders need them and to stay out a little longer if they can afford to is through surge pricing" (Vedantam, 2016); "A more mild impact comes at anticipated periods of high demand, such as New Year's Eve, when the surge brings drivers onto the road" (Said, 2015). Also, Diakopolous (2015) and Chen et al. (2015) find that a surge price did not attract much new drivers to start driving on the Uber platform.

in real time to workers and encourage them to move between zones accordingly (Phillips, 2017). We model this situation by assuming that there is a shortage of workers in the surge zone even after all workers available to work join the platform in period 1.

4.4.2 Surge Pricing in Period 1

If Platform Controls Movement of Workers

The platform decides whether and which workers will move in period 1, and then sets prices. Thus, the platform can essentially choose $\tilde{N}_{ij}$, and set prices taking $\tilde{N}_{ij}$ as given. Clearly, the platform can maximize its profit by having $a_S - N_A$ workers move from zone B to zone A in period 1. In this case, the available workers in each zone and period are: $\tilde{N}_{A1} = N_A$, $\tilde{N}_{A2} = a_S$ and $\tilde{N}_{B1} = \tilde{N}_{B2} = n_H + n_L - a_S$. Since $N_A > a_R$ and $n_H + n_L - a_S > a_R$, enough workers are available in all zones and periods. Therefore, the platform sets the unconstrained revenue-maximizing prices $p_{A1} = p_{A2} = p_{B2} = p_R$ and $p_{A2} = p_S$, and obtains a profit $\Pi_P = \Pi_P^b = (1 - \lambda)(3p_R a_R + p_S a_S)$. Thus, if the platform has full control over movement of workers, then it can move them in advance of the demand surge to ensure that they are available at the right time and locations, such that it can serve the unconstrained revenue-maximizing level of demand in all zones and time periods. Consequently, this outcome also represents the platform's first-best outcome.

Will Enough Workers Move on their Own?

Having established how the platform prefers that workers should move, we now consider how independent workers will move to maximize their respective expected profit. Let r_{ij} denote the expected revenue of a worker who serves zone i in period j, taking into account the probability that the worker receives a customer request if $\tilde{N}_{ij} > D_{ij}(p_{ij})$. We have $r_{ij} = \lambda \frac{R_{ij}}{\tilde{N}_{ij}}$, such that $r_{ij} = \lambda p_{ij}$ if $\tilde{N}_{ij} \leq D_{ij}(p_{ij})$ and $r_{ij} = \lambda p_{ij} \frac{D_{ij}(p_{ij})}{\tilde{N}_{ij}}$ otherwise. Workers' movement decisions will depend on the platform's pricing strategy. The platform's pricing strategy, in turn, will depend on workers' movement decisions. Therefore, we analyze the equilibrium interactions between the platform and workers. We solve for the subgame-perfect equilibrium. In equilibrium, workers correctly anticipate the behavior of all other workers in period 1 and the platform's pricing strategy in period 2. Let $\mu_i \in [0,1]$ denote the proportion of workers initially in zone i who decide to move.[5] Therefore, $\tilde{N}_{i1} = N_i\,(1-\mu_i)$ and $\tilde{N}_{i2} = N_i\,(1-\mu_i) + N_{i'}\,(1-\mu_{i'})$, where i' denotes the adjacent zone.

We start by asking whether simply informing workers about market conditions is sufficient to ensure that enough workers move to zone A such that the platform attains the benchmark profit Π_P^b. In other words, can supply and demand be balanced solely based on the individually rational behavior of workers, without further intervention by the platform. Note that the platform can attain Π_P^b iff enough workers move and the platform charges the regular price in period 1. Therefore, we examine whether enough workers move if $p_{i1} = p_R$ in period 1.

We find that not enough workers may move for two reasons. First, workers anticipate that more workers moving to zone A will drive down the surge price in zone A. If the surge price in zone A will "collapse" to an unprofitable level with enough workers moving to zone A, then not enough workers will move. Specifically, the surge price if there is a shortage of workers in zone A, $p^e_{A2} = D_S^{-1}\left(\tilde{N}_{A2}\right)$, is decreasing in the number of workers that move to zone A. The expected profit of a worker initially in zone B moving to zone A is $r_{A2} - c$, where $r_{A2} = \lambda p^e_{A2}$. Let $\bar{p}_S = D_S^{-1}\left(N_A\right)$ denote the surge price in zone A in period 2 if no worker moves. Clearly, no worker will move if $\lambda\bar{p}_S < c$. Further, not enough workers may move even if $\lambda\bar{p}_S > c$. Note that $p^e_{A2} = p_S$ if enough workers move. Therefore, if $\lambda p_S \leq c$, then workers rationally anticipate that the price in zone A if enough workers move will be too low for it to be profitable for enough workers to move.

[5] Thus, we solve for an asymmetric pure strategy equilibrium in worker decisions to move. Alternatively, one could examine a symmetric mixed strategy equilibrium in which all workers in zone i move with probability μ_i. In our setting with atomistic workers, the mixed strategy equilibrium will yield the same analytical results as the pure strategy equilibrium because of the law of large numbers.

Second, not enough workers may move because workers face an opportunity cost of moving to zone A. A worker moving to zone A gives up the opportunity of serving consumers in zone B. Her expected profit from staying and serving consumers in zone B, given by $r_{B1} + r_{B2}$, is strictly positive. Moreover, this opportunity cost is increasing in the number of workers that move to zone A since there are then fewer workers serving zone B. Specifically, $r_{Bj} = \lambda p_R \frac{a_R}{\tilde{N}_{Bj}}$ is higher if $\tilde{N}_{Bj}$ is lower. For enough workers to move, the expected profit of moving to zone A must not only be positive, but also outweigh the opportunity cost of moving. Proposition 4.4.1 describes our findings.[6]

Proposition 4.4.1. *Even if workers are informed about market conditions, without surge pricing in period* 1*, not enough workers move to the surge zone iff the following condition (C1) holds:* $\lambda p_S \leq c$ *or* $n_H + n_L < a_S + 2a_R \frac{\lambda p_R}{\lambda p_S - c}$.

Many on-demand platforms have come to rely on the gig economy for a low-cost workforce. Proposition 4.4.1 establishes a downside of doing so. Merely informing independent workers about market conditions may not be sufficient to ensure they will be available at the right time and locations. Not enough workers may move to the surge zone because of the strategic interaction in workers' moving decisions. Workers anticipate that the surge price in the surge zone will "collapse" as more workers move, which discourages them from moving. Moreover, workers face an opportunity cost of leaving their current zone that is increasing in the number of workers that leave. It is interesting to note that the platform does not face an opportunity cost from workers moving to zone A: even though a worker is unavailable to serve consumers while moving, there are enough workers in zone B to take the place of a worker who moves. Consequently, the platform strictly prefers that enough workers move at the right time to the right locations even if the workers do not.

[6]The proofs for results in this section are provided in the Appendix B.

At this juncture, we should mention some mechanisms through which the platform may be able to allocate workers efficiently. First, similar to a traditional cab company, the platform can employ a full-time workforce that is paid a salary and is under the direct control of the platform. A full-time workforce can, however, be considerably more costly than independent workers from the gig economy; full-time workers expect higher wages since they are not just working in their free time (Chen et al., 2017), and have to be provided additional benefits required for full-time workers (e.g., health insurance). Alternatively, the platform could require that independent workers who participate on the platform should obey its instructions to move. However, as noted in Section 4.1, platform managers do not consider this form of control to be practical in the gig economy.[7] Therefore, we assume that such mechanisms are not available and examine the implications for surge pricing.

Strategic Surge Pricing to Make More Workers to Move

We now investigate whether and how surge pricing in period 1 can be used to influence more workers to move to the surge zone, if enough workers will not move on their own. We refer to this form of surge pricing as *strategic surge pricing,* since it represents a distortion relative to the pricing that maximizes platform profit in period 1. We solve for the subgame equilibrium in period 1 given the platform's period 1 prices $p_{i1} \geq p_R$. Let $\bar{\mu} = \frac{a_S - N_A}{N_B}$, denote the proportion of workers in zone B that must move to meet demand in zone A. Throughout the analysis, we make the dependence of the subgame equilibrium outcomes on period 1 prices explicit only where necessary.

If $\mu_A > 0$ and $\mu_B > 0$, then workers' incentive to move between zones will be influenced by period 1 prices in both zones. We show, however, that $\mu_A = 0$, i.e., workers in the

[7] We conjecture that this may be because requiring workers to move when it is not profitable for them to do so makes it less attractive for workers to participate on the platform. Also, because the platform does not know the constraints facing different workers (e.g., preferred working locations, time available on a given day, rest or break requirements), the platform may not be able to determine which workers are able to move and which workers are not.

surge zone remain in the surge zone. Intuitively, workers will not move from a zone with a shortage of workers to a zone with excess supply. Hence, $\tilde{N}_{A1} = N_A$, $\tilde{N}_{A2} = N_A + \mu_B N_B$ and $\tilde{N}_{B1} = \tilde{N}_{B2} = N_B(1 - \mu_B)$. Consequently, the incentive for workers in zone B to move to zone A can be influenced only by the period 1 price in zone B and not in zone A.

Specifically, consider workers' incentive to move from zone B to zone A. In the relevant range of analysis, $\mu_B < 1$. If $\mu_B = 0$ in equilibrium, then we require $r_{B1} + r_{B2} \geq r_{A2} - c$ such that workers in zone B (weakly) prefer to stay in zone B. If $\mu_B \in (0, 1)$, then we require $r_{B1} + r_{B2} = r_{A2} - c$ such that neither the workers that move nor the workers that stay have an incentive to deviate from their respective strategies. Therefore, the equilibrium worker movement condition is

$$r_{B1} + r_{B2} \geq r_{A2} - c, \tag{4.1}$$

where equality holds if $\mu_B > 0$. Condition (4.1) determines μ_B given the period 1 prices. This condition is directly affected only by p_{B1}. Hence, the equilibrium μ_B is not influenced by p_{A1}.[8]

We find that a surge price $p_{B1} > p_R$ can make more workers move to zone A. If there are enough workers to meet demand in zone B, then the regular price also maximizes expected revenue of serving zone B in period 1. Specifically, $r_{B1} = \lambda \frac{p_{B1} D_R(p_{B1})}{N_B(1-\mu_B)}$. All else equal, r_{B1} is strictly decreasing in p_{B1} since the regular price maximizes the unconstrained revenue under regular demand. Therefore, a surge price lowers a worker's expected revenue. Intuitively, a surge price primarily throttles demand in zone B and, thus, lowers the worker's opportunity cost of moving to zone A. If $\mu_B > 0$ for $p_{B1} = p_R$, then condition (4.1) holds as an equality and μ_B is strictly increasing in p_{B1}. If condition (4.1) holds as an inequality for $p_{B1} = p_R$, then either a sufficiently high surge price is needed to induce more workers to move, or a

[8]Recall that $r_{ij} = \lambda \frac{R_{ij}}{\tilde{N}_{ij}}$, and $R_{B1} = p_{B1} \min\left\{D_R(p_{B1}), \tilde{N}_{B1}\right\}$, $R_{A2} = \min\left\{a_S p_S, \tilde{N}_{A2} D_S^{-1}\left(\tilde{N}_{A2}\right)\right\}$ and $R_{B2} = \min\left\{a_R p_R, \tilde{N}_{B2} D_R^{-1}\left(\tilde{N}_{B2}\right)\right\}$.

strategic surge price cannot make more workers move. We find that a strategic surge price can influence more workers to move if the surge price under surge demand when no workers move ($\bar{p}_S$) is sufficiently high and the extent of oversupply in the non-surge zone ($\frac{N_i}{a_R}$) is sufficiently high. Proposition 4.4.2 summarizes our findings.

Proposition 4.4.2. *In period* 1*, only surge pricing in the non-surge zone can make more workers move to the surge zone, and does so iff* $\lambda \bar{p}_S - c > \lambda \frac{a_R}{N_i} p_R$*, where* i *denotes the non-surge zone. The number of workers that move is increasing in the surge price.*

Proposition 4.4.2 establishes that the platform can use a strategic surge price in the non-surge zone (and not in the surge zone) to make more workers move to the surge zone. The regular price maximizes worker revenue in the non-surge zone. Distorting the price through surge pricing lowers workers' opportunity cost of moving. By throttling demand and exacerbating the oversupply, the strategic surge price lowers a worker's probability of obtaining a customer request and increases the number of idle workers in the non-surge zone, thereby forcing workers to leave to serve other zones.

A strategic surge price differs from a surge price used to ration supply or extract consumer surplus in some important ways. The latter forms of surge pricing are used in the surge zone during a surge in demand, and maximize platform profit and worker revenue in that zone. Consequently, both these types of surge price make it more attractive for workers to move in from an adjacent zone. In contrast, a strategic surge price is used in advance of the surge demand, in a non-surge zone that has an excess supply of workers. It increases the number of idle workers and lowers platform profit and worker revenue in that zone, and induces workers to leave to serve other zones.

Can Strategic Surge Pricing be Profitable?

Strategic surge pricing increases platform profit in zone A in period 2 at the expense of platform profit in zone B in period 1. The platform's tradeoff depends on how effective strategic surge pricing is in inducing workers to move for a given loss of profit in zone B. Consider the workers' equilibrium movement condition (4.1). Since $\mu_B > 0$ if workers move and $r_{Bj} = \lambda \frac{R_{Bj}}{N_B(1-\mu_B)}$ if not enough workers move, condition (4.1) can be rewritten as

$$R_{B1} + R_{B2} = N_B \left(1 - \mu_B\right) \left(p^e_{A2} - \frac{c}{\lambda}\right). \tag{4.2}$$

Equation (4.2) determines how a given decrease in R_{B1} (because of a strategic surge price) results in an increase in μ_B.[9] Noting that $R_{A1} = D_R\left(p_{A1}\right) p_{A1}$, $R_{A2} = \left(N_A + \mu_B N_B\right) p^e_{A2}$, and substituting for $R_{B1} + R_{B2}$ from equation (4.2), the platform's equilibrium profit can be written as:

$$\Pi_P = (1 - \lambda) \left[D_R\left(p_{A1}\right) p_{A1} + \left(n_H + n_L\right) p^e_{A2} - N_B \left(1 - \mu_B\right) \frac{c}{\lambda}\right], \tag{4.3}$$

Equation (4.3) represents the platform's tradeoff between R_{B1} and R_{A2} accounting for how workers move in response to a strategic surge price. The effect of the strategic surge price is represented implicitly through its effect on μ_B. Hence, it is sufficient to consider how μ_B affects platform profit.

We find that strategic surge pricing can be profitable only if $c > 0$. If $c = 0$, then platform profit in equation (4.3) is always decreasing in μ_B, since p^e_{A2} is decreasing in μ_B. Essentially, the decrease in R_{B1} needed for a given increase in μ_B (as determined by Equation (4.2)) more than offsets the corresponding increase in R_{A2}. If $c > 0$, then platform profit can be increasing in μ_B because strategic surge pricing is more effective in making workers move. This can be seen from equation (4.2): holding p^e_{A2} constant, a given change in R_{B1} leads to a larger change in μ_B if c is higher. This represents the direct effect of c on the effectiveness of the surge price, and corresponds to the last term within the square brackets in equation (4.3). Theorem 4.4.1 below provides a sufficient condition for strategic surge pricing to be

[9] Since $p^e_{A2} = D_S^{-1}\left(N_A + \mu_B N_B\right)$ is decreasing in μ_B, the RHS is decreasing in μ_B. Therefore, a decrease in R_{B1} leads to an increase in the equilibrium μ_B.

profitable. Let μ_0 denote the proportion of workers that move to zone B without strategic surge pricing in period 1, and let $p_S^0 = D_S^{-1}(N_A + \mu_0 N_B)$ denote the corresponding surge price in zone A in period 2.

Theorem 4.4.1. *If not enough workers move to the surge zone without surge pricing in period* 1*, then a surge price in period* 1 *in the non-surge zone is profitable if:* $\lambda \bar{p}_S - c > 2\lambda \frac{a_R}{N_i} p_R$ *and* $-cD_S'(p_S^0) > \lambda(n_H + n_L)$*, where* i *denotes the non-surge zone.*

Intuitively, strategic surge pricing is more effective in making workers move if $\mu_0 > 0$, such that more workers move for any strategic surge price $p_{B1} > p_R$, and if the surge price in zone A in period 2 (p_{A2}^e) does not fall too fast as more workers move to zone A.[10] We have $\mu_0 > 0$ if the surge price under surge demand when no workers move ($\bar{p}_S$) is sufficiently high and the extent of oversupply in the non-surge zone ($\frac{N_i}{a_R}$) is sufficiently high. Further, since $\frac{dp_{A2}^e}{d\mu_B} = \frac{N_B}{D_S'(p_{A2}^e)}$, p_{A2}^e is sufficiently insensitive to workers moving to zone A if surge demand is sufficiently price-sensitive. We note that it may not be optimal for the platform to induce enough workers to move to the surge zone. In particular, if $p_S > p_R$, then the platform's incremental gain in zone A from having an additional worker move goes to zero if enough workers move, whereas the incremental loss in zone B is strictly negative. Thus, the platform may still use surge pricing in zone A in period 2. We next show that there always exists a surge demand such that the conditions in Theorem 4.4.1 hold.

Proposition 4.4.3. *Given any regular demand, initial supply of workers, workers' cost to move and revenue-sharing rate, there exists surge demand such that the total demand across zones does not exceed the total number of available workers and strategic surge pricing is profitable for the platform.*

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[10] If $\mu_0 = 0$, then we require additional functional form assumptions to determine whether surge pricing is profitable; equation (4.3) holds only for p_{B1} sufficiently high such that $\mu_B > 0$, and the first order condition is only necessary and not sufficient for strategic surge pricing to be profitable.

It is straightforward to show that strategic surge pricing is profitable if surge demand is of sufficiently larger scale than the regular demand, i.e., $D_S(p) = kD_R(p)$, where $k > 1$ is sufficiently large. Such a surge demand may, however, exceed the available number of workers across both zones (after regular demand is met), i.e., $a_S = ka_R > n_H + n_L - a_R$. In Proposition 4.4.3, we show that there are always more "reasonable" surge demands that do not exceed the available number of workers, for which strategic surge pricing is profitable. Three characteristics of the surge demand matter. First, the number of workers needed to meet surge demand (a_S) must be sufficiently high, such that there is a shortage of workers in the surge zone without surge pricing in period 1. Second, the period 2 surge price in the surge zone if no worker moves ($\bar{p}_S$) must be sufficiently high, such that it will be attractive for workers to move and the platform can use a strategic surge price to make them move. Lastly, the inverse demand should be sufficiently insensitive such that the period 2 surge price in the surge zone does not fall too fast as more workers move. All three characteristics can be thought of as different aspects of how "strong" the surge demand is relative to regular demand. Thus, strategic surge pricing is profitable if surge demand is sufficiently strong on these dimensions.

4.5 Further Considerations in Managing Independent Workers

4.5.1 Sharing Market Forecasts with Workers

On-demand platforms such as Uber and Lyft continually provide market forecasts to workers about locations needing additional workers and the extent of the need. While workers who are new to the platform are especially likely to rely on these forecasts, even experienced workers may rely on these forecasts to learn about actual conditions on a particular day. If the platform is known to share information truthfully, for example, due to reputational concerns, then the outcome will be the same as under symmetric information. However, some workers have raised the concern whether the forecasts shared by the platform represent true market conditions, or are being used to manipulate them to move to different zones (Rosenblat and Stark, 2015). Therefore, we examine the platform's incentive to share forecasts truthfully in period 1, and whether a surge price in period 1 can serve as a credible "signal" for workers to move. Of particular interest is in which zone - the surge or non-surge zone - such a surge price will be used. Thus, we shed light on an additional reason why, contrary to conventional wisdom, a surge price may be useful in a zone with excess supply of workers.

To develop our insights in a straightforward manner, we proceed as follows. First, we establish how workers' beliefs about market conditions influences their decisions to move. Next, in order to highlight the signaling role of strategic surge pricing, we focus on situations where the platform would not resort to strategic surge pricing under symmetric information, and ask whether truthful communication is feasible without strategic surge pricing under asymmetric information. Then, we examine whether a period 1 surge price can facilitate credible communication. We refer to this form of surge pricing also as strategic surge pricing, since it too represents a distortion from the price that maximizes platform revenue in period 1.

At the start of period 1, the platform is informed about the identity of the surge zone and the initial supply in each zone. Together, these determine which zone will face a shortage of workers in period 2, and the extent of the shortage. The platform communicates this information to workers at the start of period 1. Note that the platform does not have an incentive to misinform workers about which market zone is the surge zone. For example, if surge demand will occur in zone A, then the platform benefits if workers in zone A know that they must stay in that zone and workers in zone B know that they must move. Hence, this information will be shared truthfully. Therefore, it is sufficient to analyze the implications of asymmetric information about the initial supply condition. Without loss of generality, we fix the identity of the surge zone to be zone A. We denote the platform's private information about the initial supply through the platform's type: the platform is type H if the initial supply in zone A is low ($= n_L$), and type L if the initial supply in zone A is high ($= n_H$). Thus, the type H platform faces a larger shortage of workers in zone A given the initial supply. Let $s \in \{H, L\}$ denote the platform's communication to workers about its type. We use the notation $x|_t$ to denote variable x for a type $t \in \{H, L\}$ platform, and simply use x to denote the variable for either platform type. We solve for a perfect Bayesian equilibrium.

At the start of period 1, workers have a prior belief that the platform is type H with probability $\frac{1}{2}$. Workers update their belief based on the platform's communication s and the period 1 price $\boldsymbol{p_1} = (p_{A1}, p_{B1})$. Let $\theta \in [0, 1]$ denote their posterior belief that the platform is type H. We show the dependence of outcomes on θ and $\boldsymbol{p_1}$ explicitly only where necessary. Given θ and $\boldsymbol{p_1}$, it can be shown as before that only workers in zone B may move in period 1. Let μ_B denote the proportion of workers in zone B who move to zone A, and $\tilde{N}_{ij}(\mu_B)$ denote the resulting number of workers available to serve zone i in period j. Therefore, $\tilde{N}_{A1}(\mu_B) = N_A$, $\tilde{N}_{A2}(\mu_B) = N_A + N_B\mu_B$ and $\tilde{N}_{Bj}(\mu_B) = N_B(1 - \mu_B)$; where μ_B depends on θ and $\boldsymbol{p_1}$, and $(N_A, N_B) = (n_L, n_H)$ if the platform is type H and $(N_A, N_B) = (n_H, n_L)$ otherwise.[11] For a type t platform, let $R_{ij}(\mu_B)|_t = p_{ij} \min \left\{ D_{ij}(p_{ij}), \tilde{N}_{ij}(\mu_B) \right\} \Big|_t$ denote the market revenue in zone i in period j, and let $r_{ij}(\mu_B)|_t = \lambda \left. \frac{R_{ij}(\mu_B)}{\tilde{N}_{ij}(\mu_B)} \right|_t$ denote the expected revenue of a worker serving zone i in period j.

In period 2, the platform sets prices to maximize its profit in each zone given the available workers. Therefore, $p^e_{i2}(\mu_B) = D^{-1}_{i2}\left(\min \left\{ a_{i2}, \tilde{N}_{i2}(\mu_B) \right\} \right)$. At the start of period 1, given their posterior belief θ and $\boldsymbol{p_1}$, workers decide whether to move, anticipating the behavior of all other workers and the platform's pricing strategy in period 2. Given a worker's posterior

[11] To determine the behavior of workers in a consistent and straightforward manner under all possible beliefs, including incorrect beliefs, we assume the following: given θ and $\boldsymbol{p_1}$, in zone i, the proportion of workers that move in any randomly chosen subset of workers in that zone is the same and, hence, equal to μ_i. That is to say the behavior of workers in a zone is uniform and does not differ systematically across different groups of workers.

belief θ, let $\hat{r}_{ij}(\theta) = \theta\, r_{ij}(\mu_B)|_H + (1-\theta)\, r_{ij}(\mu_B)|_L$ denote her expectation of the expected revenue of serving zone i in period j. The equilibrium worker movement condition is

$$\hat{r}_{B1}(\theta) + \hat{r}_{B2}(\theta) \geq \hat{r}_{A2}(\theta) - c, \tag{4.4}$$

where equality holds if $\mu_B > 0$. The following lemma describes how workers' moving behavior is affected by their belief about the platform type.[12]

Lemma 4.5.1. *Given period* 1 *prices and workers' posterior belief* θ*, there is a unique equilibrium proportion* $\mu_B(\theta)$ *of workers that move;* $\mu_B(\theta)$ *is strictly increasing in* θ *if* $\mu_B(\theta) \in (0,1)$*.*

Lemma 4.5.1 shows that more workers will move if their belief that the platform is type H is higher (except in the boundary cases where either no worker moves or all workers move regardless of their belief). Intuitively, compared to a type L platform, workers face greater competition for service requests in zone B and lesser competition in zone A on a type H platform. This is because, to begin with, there is greater undersupply of workers in zone A and greater oversupply in zone B on a type H platform. Therefore, it is more attractive for workers in zone B to move if they believe that there is higher probability that the platform is type H.

Can the Platform Communicate Truthfully without Strategic Surge Pricing?

As mentioned before, we focus on situations where neither platform type will resort to strategic surge pricing under symmetric information.[13] We analyze a separating equilibrium

[12] The proofs for results in this section are provided in Appendix C.

[13] From Theorem 4.4.1, a sufficient condition for neither platform type to use strategic surge pricing under symmetric information is: $-cD_S'\left(p_S^0|_t\right) \leq \lambda(n_H + n_L)$ for $t \in \{H, L\}$. It can be shown that if the sufficient condition holds for a type H platform then it also holds for a type L platform. Specifically, $p_S^0|_H \geq p_S^0|_L$ since $\mu_0|_H \geq \mu_0|_L$ from Lemma 4.5.1. Then, $-D_S'\left(p_S^0|_H\right) \geq -D_S'\left(p_S^0|_L\right)$ since $D_S(\cdot)$ is concave.

in which the platform truthfully communicates its type $s|_t = t$ and charges the regular price $\boldsymbol{p_1} = (p_R, p_R)$, and the workers trust the message by updating their belief to $\theta = 0$ if $s = L$ and $\theta = 1$ if $s = H$. For truthful communication to be feasible, the platform must not have an incentive to misreport its type given that workers trust the message.

We find that the type L platform can benefit from misreporting its type in order to induce more workers to move to zone A. Influencing workers in this manner can be profitable for the type L platform only if it faces a shortage of workers in zone A (under symmetric information), i.e., $\mu_B(\theta = 0) < \bar{\mu}|_L$, which is the case iff condition (C1) in Proposition 4.4.1 holds. The type L platform can influence workers through its communication only if workers respond to their belief about the platform type. From Lemma 4.5.1, this occurs if $\mu_B(\theta = 1) > 0$ and $\mu_B(\theta = 0) < 1$ such that $\mu_B(\theta = 1) > \mu_B(\theta = 0)$. At the same time, not too many workers should move if they believe the platform is type H; otherwise, the type L platform can face a significant shortage of workers in zone B and the ensuing loss of revenue in zone B offsets the gain from having more workers move to zone A. A sufficient condition, therefore, is that the type L platform does not face a shortage in zone B if it misreports its type, i.e., $\tilde{N}_{Bj}(\mu_B(\theta = 1))\Big|_L = n_L(1 - \mu_B(\theta = 1)) \geq a$. In Proposition 4.5.1, conditions (i) and (ii) respectively, ensure that there is a shortage of workers under symmetric information and some workers move if they believe the platform is type H. As shown in the proof of Proposition 4.4.3, there always exists surge demand such that these conditions hold. Condition (iii) ensures that misreporting its type will not cause a shortage in zone B for the type L platform. This condition is satisfied if n_L is sufficiently close to n_H. Thus, if the platform types are sufficiently similar in their initial supply conditions, then there exists surge demand such that the platform cannot communicate forecasts truthfully.

Proposition 4.5.1. *Under asymmetric information, the platform cannot truthfully communicate market forecasts with workers without strategic surge pricing if the following conditions all hold: (i) condition (C1) from Proposition 4.4.1; (ii)* $\bar{p}_S|_H - \frac{c}{\lambda} > 2p_R\frac{a_R}{n_H}$*; (iii)* $1 - \frac{a_R}{n_L} > \frac{a_S - n_L}{n_H}$.

Proposition 4.5.1 suggests why there can be a lack of trust amongst workers in the forecasts shared by the platform. Because workers are more reluctant to move than what is optimal for the platform if the true market conditions were known to them, the platform has an incentive to exaggerate market conditions to induce more workers to move. We next examine whether and how a strategic surge price can facilitate truthful communication.

Can Strategic Surge Pricing Facilitate Credible Communication?

We analyze a separating equilibrium in which the platform truthfully communicates $s|_t = t$, and the type H platform signals its type by using a surge price in one or both market zones in period 1, i.e., $p_{i1}|_L = p_R$ and $p_{i1}|_H \geq p_R$, where $p_{i1}|_H > p_R$ for some i. At the start of period 1, workers update their belief to $\theta = 1$ if $s = H$ and $\boldsymbol{p_1} = \boldsymbol{p_1}|_H$, and to $\theta = 0$ otherwise. We solve for the least-cost separating equilibrium (e.g., Moorthy and Srinivasan 1995; Desai 2001); the least-cost separating equilibrium essentially minimizes the surge price that is used to signal the platform's type and is the most profitable separating equilibrium for the type H platform.

Since a strategic surge price reduces platform revenue in period 1, it is a "costly" signal. For such a signal to facilitate credible communication, we require that the signal is sufficiently costly such that the type L platform will not mimic the type H platform. The platform's revenue in period 1 is strictly decreasing in p_{i1}. Therefore, higher the surge price, the more costly the signal. We will focus on situations where the signal can be made sufficiently costly by using a strategic surge price in only one of the zones, either zone A or zone B. Doing so allows us to examine whether a strategic surge price will be used in zone B even when it is not necessary to do so. We show that a strategic surge price in either one of the zones can be a sufficiently costly signal if either the surge in demand is not too high or the two platform types are sufficiently similar in their initial supply conditions; in either case, the incremental profit from mimicking the type H platform is limited and can be offset by the profit loss from the strategic surge price.

For credible communication, we also require that the type H platform benefits more from using a strategic surge price to signal its true type than the type L platform benefits from using it to misrepresent its type.[14] We find that this is indeed the case. In a separating equilibrium, a larger proportion of workers in zone B move to zone A if they infer the platform's type to be type H than type L (from the strategic surge price); specifically, more workers move if they believe that the platform is type H (as shown in Lemma 4.5.1), and more workers also move if a strategic surge price is used in zone B (for the same reasons as in Proposition 4.4.2 under symmetric information). Let $\Delta\mu$ denote the increase in proportion of workers that move in response to the strategic surge price. We find that the type H platform benefits more than the type L platform from this increase $\Delta\mu$ for two reasons. First, the type H platform faces a greater shortage of workers in zone A. Hence, the marginal benefit of having an additional worker move to zone A is higher for the type H platform. Second, the actual number of workers that move in response to the strategic surge price, given by $n_t\Delta\mu$, is higher for a type H platform since $n_H > n_L$.

We further find that the separating equilibrium in which the type H platform uses a strategic surge price only in zone B is the unique least-cost separating equilibrium. A surge price in zone A shrinks demand in zone A (making the signal costly), but does not otherwise influence workers' incentives to move to zone A (as in Proposition 4.4.2). As a result, as a signaling instrument, a surge price in zone A is equally costly for both platform types. In contrast, a surge price in zone B is relatively less costly for the type H platform. A surge price in zone B not only shrinks demand in zone B, but also increases the proportion of workers that move to zone A. This increase in proportion of workers moving to zone A is more valuable for the type H platform for the same two reasons as discussed above; namely, the type H platform profits more from having an additional worker move, and more workers move if the platform is type H. Consequently, a surge price in zone B is a more efficient

[14] This condition is usually referred to as the single-crossing property in signaling games.

signaling instrument. Intuitively, a surge price in zone A is akin to signaling by simply "burning money" and results in the same profit loss for both platform types. Whereas, a surge price in zone B results in a lower profit loss if there is truly a greater need for workers to move, making it more efficient than burning money. Proposition 4.5.2 summarizes our results.

Proposition 4.5.2. *Under the conditions of Proposition 4.5.1, if* $a_R p_R + n_H\, \bar{p}_S|_L > a_S p_S$ *or* n_L *is sufficiently close to* n_H*, then the platform can credibly communicate market forecasts by using a strategic surge price in either one of the surge or non-surge zones. In the unique least-cost separating equilibrium, the strategic surge price is used only in the non-surge zone.*

Thus, a strategic surge price can also be used to facilitate forecast communication by credibly signaling greater need for workers to move. Importantly, a strategic surge price in the market zone that workers should leave minimizes the platform's cost to credibly signal market conditions. This use of surge pricing is also consistent with the market observations discussed before. Specifically, workers leave the market zone in which a surge price is used, and the platform does not serve all consumers in this zone despite there being sufficient number of idle workers available to serve them in equilibrium. These results shed further light on the rationale for the counterintuitive use of surge pricing in a market zone with excess supply of workers.

4.5.2 Offering Workers Bonuses to Move

While forcing workers to move may not be practical in the gig economy, an alternative could be to offer a "bonus" for workers who will move and let workers decide whether to accept the offer. For instance, recently, Uber has started a driver incentive program known as *Boost*, wherein drivers are offered a higher wage if they will be available to serve consumers at certain zones and specified times (Uber Boost, 2018).[15] While there can be some practical constraints to offering bonuses in real time, we now examine the implications if such bonuses are feasible. At the start of period 1, the platform announces the bonus (if any) and the period 1 prices. Workers who accept the offer and move are paid the bonus; we refer to these workers as "bonused" workers. We allow for the platform to offer a limited number of bonuses that are assigned to workers who accept the offer on a first-come first-served basis (or randomly). Let b_i denote the bonus offered to workers in zone i to move to the adjacent zone, and m_i denote the number of bonuses offered. We focus on situations where not enough workers move without a bonus or a strategic surge price, i.e., when condition (C1) in Proposition 4.4.1 holds. We first show that to induce more workers to move, the platform must offer the bonus to all workers that move, and not just a subset of them. We then compare whether strategic surge pricing or offering bonuses is more profitable.

Using Bonuses to Make More Workers Move

Given the period 1 prices and bonus offers, let M_B denote the number of workers that move in the subgame equilibrium. Suppose that the bonus is offered to a subset of workers that move, i.e., $m_B < M_B$. All else equal, bonused workers have a higher incentive to move than non-bonused workers; the expected profit from moving for bonused workers is $r_{A2} + b_B - c$, and their expected profit from staying is $r_{B1} + r_{B2}$. Therefore, in the subgame equilibrium, if some workers move, then at least some bonused workers must move. Moreover, if some non-bonused workers move, then all bonused workers must move. Since $M_B > m_B$, at least some non-bonused workers must move, and the equilibrium worker movement condition for bonused workers is

$$r_{B1} + r_{B2} \leq r_{A2} + b_B - c. \tag{4.5}$$

[15]In the case of Uber's Boost, the bonus is in the form of a higher revenue-sharing rate for customer rides originating in the specified zones and times. We consider an upfront lump-sum bonus, which will be equivalent to offering a higher revenue-sharing rate in a specific zone in period 2.

The equilibrium worker movement condition for non-bonused workers is as before in condition (4.1), and holds as an equality since some non-bonused workers move. Note that r_{ij} is only influenced by the total number of available workers $\tilde{N}_{ij}$. Consequently, the equilibrium worker movement condition (4.1) for non-bonused workers does not depend on the mix of bonused and non-bonused workers that move, but only on the total number of workers that move. We find, therefore, that if non-bonused workers will move in equilibrium, then the total number of workers that move must remain the same as in the case where a bonus is not offered; otherwise condition (4.1) cannot hold as an equality. Essentially, the non-bonused workers rationally anticipate the behavior of bonused workers, and fewer of them move than if bonuses were not offered. For example, suppose a number $\hat{\mu}N_B > 0$ of workers will move from zone B if bonuses were not offered (given any period 1 prices). If bonuses are offered to less than $\hat{\mu}N_B$ workers, then the total number of workers that move remains $\hat{\mu}N_B$: all bonused workers will move and, anticipating which, a smaller proportion than $\hat{\mu}$ of non-bonused workers will move such that the total number that move remains $\hat{\mu}N_B$. The following proposition describes this result.

Proposition 4.5.3. *To induce more workers to move from the non-surge zone to the surge zone using a bonus, the platform must offer a bonus to all workers that move.*

Proposition 4.5.3 establishes an important dimension to offering bonuses to move. Because workers are strategic, offering bonuses to some workers discourages other workers from moving. Thus, more workers cannot be induced to move by simply bonusing the additional workers that move.

Bonuses vs. Strategic Surge Pricing

We first compare whether strategic surge pricing or offering bonuses is less costly for the platform to induce a given number of workers to move. Recall that μ_0 is the proportion of workers in zone B that move without strategic surge pricing and bonuses. Let $\mu > \mu_0$ be the proportion of workers that are made to move. We show that strategic surge pricing is less costly if μ is sufficiently high.

Proposition 4.5.4. *To induce a proportion $\mu > \mu_0$ of workers in the non-surge zone to move to the surge zone, strategic surge pricing is more profitable than bonusing workers iff $\mu > 1 - \lambda$.*

Proposition 4.5.4 shows that even if the platform is able to offer a bonus for workers to move, it should not do so if the number of workers that will move is sufficiently high, i.e., if $\mu > 1 - \lambda$. Instead, it should use strategic surge pricing. In this case, a bonus may still be offered if the maximum possible surge price (such that $D_R(p_{B1}) = 0$) is not sufficient to induce a proportion μ of workers to move. Nevertheless, strategic surge pricing is profitable for the platform and will be used to the maximum extent possible. Intuitively, because all workers that move must be bonused, the bonus is less attractive if more workers move. Moreover, offering a bonus is less attractive if the revenue-sharing rate λ is higher. If λ is higher, then the bonus needed to compensate workers for their opportunity cost of moving is higher. Also, the same bonus is relatively more costly for the platform, because the platform's share of the incremental revenue is lower. Consequently, the threshold μ above which strategic surge pricing is more profitable, is decreasing in λ.

Proposition 4.4.3 showed that in the case where bonuses are not feasible, there always exists a surge demand such that strategic surge pricing will be used in equilibrium. We now extend this result to the case where bonuses are feasible. In Corollary 4.5.1, we show that if $\lambda > \frac{a_R}{N_i}$, then there exists surge demand such that strategic surge pricing is profitable if bonuses are not feasible and $\mu > 1 - \lambda$; hence, the platform will use strategic surge pricing even if bonuses are feasible.

Corollary 4.5.1. *If bonuses are feasible and* $\lambda > \frac{a_R}{N_i}$ *(where* i *denotes the non-surge zone), then there exists surge demand such that the platform will use strategic surge pricing in equilibrium, and the total demand across market zones does not exceed the total number of available workers.*

To match supply and demand, on-demand service platforms have three instruments available - sharing forecasts with workers, strategic surge pricing, and bonuses. Our results taken together shed light on the appropriateness of each instrument. All else equal, when the magnitude of demand surge (or the extent of undersupply in the surge zone) is low, we find that the platform should only share forecasts with workers and not intervene any further to influence their moving decisions. When the magnitude of demand surge is higher, it behooves the platform to intervene to make more workers move. The intervention can be in the form of strategic surge pricing or bonuses, depending on the number of workers that need to move. If the demand surge is substantially high, then the platform would require a large number of workers to move. In such a case, strategic surge pricing is more profitable. Otherwise, if fewer workers need to move, then bonuses are more profitable.

4.5.3 Workers Flocking the Surge Zone

Some industry observers have noted that surge pricing in one location may attract workers from adjacent zones leading to a shortage of workers and surge pricing at those adjacent locations (e.g., Diakopolous, 2015). We now examine whether workers may in fact rationally "flock" the surge zone and the implications for surge pricing.

Note that if condition (C1) in Proposition 4.4.1 does not hold, then more than enough workers move to zone A and there is excess supply in zone A, i.e., $\tilde{N}_{A2} = N_A + \mu_B N_B > a_S$ iff $\lambda p_S > c$ and $n_H + n_L > a_S + 2a_R \frac{\lambda p_R}{\lambda p_S - c}$. Then, the platform charges a price p_S in the surge zone in period 2. We find that if p_S is sufficiently high, then "too many workers" may in fact move to the surge zone, resulting in a shortage of workers in the non-surge zone in both periods. i.e., $\tilde{N}_{Bj} = N_B (1 - \mu_B) < a_R$. In this case, the platform will use a surge price in the non-surge zone in both periods. Intuitively, workers flock the surge zone because it supports a sufficiently higher price than does the regular zone. In equilibrium, workers move to the surge zone till the excess supply there offsets the benefit of the higher surge price in relation to the surge price in the non-surge zone. We show that the surge price in the non-surge zone is a "market-clearing price" that is less than p_S.

Proposition 4.5.5. *Too many workers move to the surge zone iff the following condition (C2) holds:* $\lambda p_S > \left(\frac{n_H+n_L-a_R}{a_S}\right)(2\lambda p_R + c)$. *In this case, the platform uses a market-clearing surge price in the non-surge zone in both periods that is less than the surge price in the surge zone in period* 2.

Proposition 4.5.5 provides further insights regarding the role of surge pricing in non-surge zones to influence independent workers. Workers rationally flock the surge zone if the surge demand is sufficiently inelastic compared to regular demand, which results in surge prices in the surge and non-surge zones in period 2. Nevertheless, the surge price in the surge zone is higher, such that workers are attracted to move to the surge zone. In period 2, the platform charges the market clearing surge price $p_{B2} = D_R^{-1} (N_B (1 - \mu_B)) > p_R$ in zone B. The platform charges the same market clearing surge price also in period 1 in zone B. In period 1, this surge price plays an additional role. It reduces the number of workers moving to the surge zone by raising their opportunity cost of moving. Intuitively, since there is a shortage of workers in zone B, raising the price above the regular price increases a worker's expected revenue of serving that zone, i.e., if $\tilde{N}_{B1} < D_R (p_{B1})$, then $r_{B1} = \lambda p_{B1}$, which is increasing in p_{B1}. Charging a surge price higher than the equilibrium market clearing price, however, causes r_{B1} to decrease since it causes workers to idle, and induces more workers to leave. Therefore, the platform charges the market clearing price also in period 1.

Thus, workers flocking the surge zone provides an additional reason for the platform to charge a surge price in the non-surge zone ahead of the demand surge. Such a surge price not only rations supply, but also enables the platform to minimize the extent of flocking. Similar to the case of a strategic surge price, the market-clearing surge price is used in a non-surge zone and workers leave the zone with a surge price. However, unlike in the case of a strategic surge price, the market-clearing surge price balances supply and demand, does not cause workers to idle, and minimizes the number of workers that leave.

4.6 Concluding Remarks

A fundamental challenge for on-demand platforms that rely on independent workers from the gig-economy is to ensure that these workers are available at the right times and right market locations. In this paper, we examine the role of surge pricing, forecast communication and worker incentives in addressing this challenge. In particular, the conventional perspective on surge pricing has by and large viewed workers at different locations in isolation. We extend the conventional thinking on surge pricing by explicitly considering how workers strategically move between market locations to maximize their expected earnings. Some of our findings are as follows.

First, simply sharing market information with independent workers may not be sufficient to obtain the optimal distribution of workers. Not enough workers may move to the zone requiring additional workers because of the strategic interaction in workers' moving decisions. In particular, workers anticipate that the price in this zone will "collapse" as more workers move, which discourages them from moving. Moreover, workers face an opportunity cost to leave their current zone, as they must give up the opportunity to serve consumers in this zone, and this cost is increasing in the number of workers that leave. Second, distorting the price in the zone that workers must leave through surge pricing lowers workers' opportunity cost of moving and, thereby, increases the number of workers that move. In particular, the surge price lowers a worker's expected revenue from serving the current zone by deliberately throttling demand and lowering a worker's probability of obtaining a customer request in that zone. Third, even though distorting the price in this manner lowers the platform profit in that zone when this zone is viewed in isolation, it can improve platform profit across zones if the demand in the zone requiring additional workers is sufficiently "strong".

Fourth, in sharing market forecasts with workers, information about which market zone the workers should move to can be shared credibly. However, the platform can have an incentive to exaggerate the need for workers to move, making the forecast information non-credible. Fifth, in such instances, a surge price accompanying the forecasts can facilitate truthful communication by serving as a credible signal that workers must move; importantly, it is more profitable for the platform to use a surge price in the market zone that the workers should leave. Sixth, in offering a bonus to encourage more workers to move, the bonus cannot be offered simply to the additional workers that move; fewer non-bonused workers will move because they strategically anticipate that the bonused workers will move.

Seventh, our results suggest when platforms should adopt different strategies to manage workers. If the demand surge (or the extent of under supply) is low, then the platform should only inform workers where to move. If the demand surge is high and the proportion of workers needed to move from an adjacent zone is not high, then the platform should incentivize workers with bonuses to move. If the demand surge is high and the proportion of workers that need to move from an adjacent zone is also high, then the platform should use strategic surge pricing in the adjacent zone ahead of the demand surge. Lastly, workers may also rationally "flock" a zone requiring additional workers, creating a shortage in the zone that they leave. In this case, a surge price can be used in the zone that workers leave to minimize the number of workers that leave.

4.6.1 Model Predictions and Market Observations

We briefly discuss how key model predictions might manifest in practice. Our analysis pertains to marketplaces with independent workers who move between market locations to serve consumers. Our model predicts that such marketplaces will be characterized by too many workers staying in locations that already have sufficient supply of workers, instead of moving to locations facing a shortage of workers. In other words, there is spatial mismatch of supply and demand that persists even if workers are informed about market conditions. Our model further predicts that more workers will move from a zone with excess supply if a surge price is used in that zone; all else, equal, a higher surge price should cause more workers to leave that zone. Thus, contrary to conventional wisdom, one should observe that a surge price systematically causes workers to leave a zone, and the total supply of workers to decline. Our model also predicts that an on-demand platform will use a surge price not only in zones where there is a shortage of workers, but also in zones where there is an excess supply of workers; we note that a surge price in a zone with excess supply of workers will be characterized by the number of idle workers in that zone increasing following the surge price.

We next discuss how some of the market observations and empirical findings may be consistent with our analysis and predictions. Using data from the New York City taxicab market, Buchholz (2016) estimates a structural model of how drivers strategically compete with other drivers by driving to different city locations. He finds that there is significant spatial mismatch in supply and demand because drivers face substantial direct and opportunity costs to move between locations. Based on driver interviews and comments on online discussion forums, Lee et al. (2015) and Rosenblat and Stark (2015) find that a significant fraction of drivers on on-demand cab service platforms do not drive towards zones indicated by the platform as requiring more drivers; in particular, drivers anticipate how the moving decisions of other drivers might drive down the surge price in the indicated zone, and prefer to stay in their current zone. Also, many experienced drivers advice new drivers not to respond to the market information provided by the platform.[16] In their study of Uber surge pricing in the San Francisco and New York City areas, Chen et al. (2015) find that in certain market locations a surge price is systematically linked to lower demand levels, more drivers becoming idle, fewer drivers moving to that location, and more drivers leaving that location. Their market observations, while contrary to conventional wisdom, are consistent with what our model predicts regarding the platform's strategic use of surge pricing in a zone with excess supply.

4.6.2 Directions for Further Research

Further work is necessary to test our model predictions and distinguish them from other alternative explanations. For example, we identify different reasons why workers may leave a zone with a surge price; namely, either the surge price exacerbates the oversupply in that zone, or because it is used to prevent too many workers from leaving that zone. In the former case, the number of idle workers in that zone should increase following the surge price; in both cases, the total number of workers in that zone should decline. Another possibility is that a surge price attracted too many new workers to the platform in that zone, causing existing workers to leave that zone; in which case, the total number of workers in that zone should increase. Empirical research is needed to distinguish between these different scenarios.

Since surge pricing is expected to mainly influence workers already on the platform, we focused on a setting where all workers who are available to work have joined the platform in period 1. We expect qualitatively similar results if supply is elastic such that some available workers join only in response to a surge price and there is still a shortage in the surge zone. In particular, strategic surge pricing would be profitable for a sufficiently "strong" surge in demand. If, on the other hand, sufficient workers join, then strategic surge pricing will not be

[16]Example driver comment on online forum uberpeople.net: "Uber urges drivers heading to price surging areas. When you arrived there, just like other drivers did, the price went back to normal because all of sudden, the supply was more than the demand. My suggestion is, ignore the heat map."

necessary. For analytical ease, we assumed only two states for the initial number of workers in each zone, causing the initial supply to be negatively correlated across zones. Allowing for more supply states and independent initial supply will not affect our insights under symmetric information. Under asymmetric information, as before, the platform will not always have an incentive to truthfully share information since a larger fraction of workers will move under some supply states than for others. Consequently, a surge price will be necessary to credibly communicate a greater need for workers to move. Moreover, separation through a surge price in the non-surge zone can be more profitable for the same reasons as in our current model - either the separating type has more workers moving for a given increase in the proportion of workers that move, or has a larger need for workers to move. However, the analysis of a separating equilibrium with several supply states is more involved.

We assumed both periods to be of equal duration. It is straightforward to extend our analysis and results to a setting where period 2 is $k > 1$ times the duration of period 1; by weighting period 2 revenues by k times relative to period 1 revenues.[17] We describe in Appendix C how the specific conditions for our results change. We assumed that it takes finite time for workers to move between zones. We expect that allowing for instantaneous movement between zones would not alter our results qualitatively. In particular, not enough workers may move to meet the demand surge because of the direct costs to move and, consequently, strategic surge pricing can still be profitable. A more general model would consider multiple periods and market locations. We expect our key insights to extend to this setting since workers face qualitatively similar tradeoffs in moving between zones. Also, under asymmetric information, the platform will not have an incentive to mislead workers about which zone(s) to leave and to move to. We further expect that, all else equal, a surge price in the market zone(s) that workers should leave will be a more efficient means

[17]Under the assumption that demand remains the same throughout period 2, it can be shown formally that if workers will move to zone B at all then they should do so in period 1 and not in the middle of period 2.

to credibly signal the need for workers to leave those zone(s). Nevertheless, future work is necessary to analyze this situation formally. Lastly, we assumed the platform's commission rate and regular price to be exogenous. In practice, these are likely to be decided by various factors such as workers' reservation wage and platform competition. Future research could study the resulting implications.

APPENDIX A

PROOFS OF RESULTS IN CHAPTER 3

Table A.1. The main notation used in our analysis.

Notation	Description
Parameters	
n	The size of the farming population ($n > 0$).
α	The production-cost parameter ($\alpha > 0$).
M	The size of the APL consumer segment ($M > 0$).
k	The size of the BPL consumer segment relative to the APL consumer segment ($k > 0$).
B	The budget of the social planner ($B > 0$).
b	The wealth of a BPL consumer ($b \geq 0$).
Decision Variables	
p_g	The guaranteed support price (GSP).
q_e	The effort exerted by a farmer.
q_m	The quantity sold by a farmer in the market for a realized value of the yield uncertainty γ.
q_g	The quantity sold by a farmer to the social planner for a realized value of the yield uncertainty γ.
q_{APL}	The quantity consumed by an APL consumer from the open-market.
q_{BPL}	The total quantity consumed by a BPL consumer.
q_S	The quantity provided to a BPL consumer by the social planner (government).

Proof of Lemma 3.3.1: If $b \geq \frac{1}{4}$, then for any $p_m \in [0, 1]$, we have that $0 \leq 1 - p_m \leq \frac{b}{p_m}$. Since $q_S = 0$, from (3.1) and (3.2), we have that $q^*_{BPL} = 1 - p_m$. Therefore, the total consumer demand in the open-market at a market price $p_m \in [0, 1]$ is

$$D(p_m) = M(1 + k)(1 - p_m).$$

We assume that all farmers have the same yield realization γ and hold (identical) beliefs $\hat{p}_m(\gamma)$ about the market-price for any yield realization γ. From (3.4), their effort (given their beliefs about the market price) is given by

$$q_e^* = \frac{\mathbb{E}[\hat{p}_m(\gamma)\gamma]}{2\alpha}. \tag{A.1}$$

Suppose the realized yield is γ: The total quantity available in the open-market is given by $n\frac{\mathbb{E}[\hat{p}_m(\gamma)\gamma]\gamma}{2\alpha}$. Therefore, the market price (denoted by $p_m(\gamma)$) is obtained from (3.5) as follows:

$$M(1+k)(1-p_m(\gamma)) = \frac{n}{2\alpha}\,\mathbb{E}[\hat{p}_m(\gamma)\gamma]\gamma.$$
$$\Rightarrow \quad p_m(\gamma) = 1 - \left(\frac{n}{2\alpha M(1+k)}\,\mathbb{E}[\hat{p}_m(\gamma)\gamma]\right)\gamma.$$

From (3.6), we have that $p_m(\gamma) = \hat{p}_m(\gamma)$. We substitute (3.6) in the above equation. Observe, from the equation above, that $p_m(\gamma)$ is linear in γ, with the intercept term equal to 1. Using this observation, we solve for $p_m(\gamma)$ and obtain the following:

$$p_m(\gamma) = 1 - \Big(\frac{\frac{n}{2\alpha}\mu}{M(1+k) + \frac{n}{2\alpha}(\mu^2+\sigma^2)}\Big)\gamma. \tag{A.2}$$

Substituting (A.2) in (A.1), we have

$$q_e^* = \frac{\mu}{2\alpha}\Big(\frac{M(1+k)}{M(1+k) + \frac{n}{2\alpha}(\mu^2+\sigma^2)}\Big).$$

■

Proof of Lemma 3.3.2: Consider a fixed value of $\theta \in [0,1]$. It is straightforward to see that $\hat{p}_m(0) = p_m(0) = 1$. We solve for the equilibrium value of $p_m(1)$ below. Using (3.4), we have that $q_e^* = \frac{\theta}{2\alpha}\hat{p}_m(1)$. Recall, from (3.9), that $b^*(\theta) = \frac{M(1+k)\frac{n\theta}{2\alpha}}{\left(M(1+k)+\frac{n\theta}{2\alpha}\right)^2}$. In the absence of an intervention, $q_S = 0$. Substituting (3.3) and (3.6) in (3.5), we have

$$M(1-p_m(1)) + kM\min\left\{1-p_m(1), \frac{b}{p_m(1)}\right\} = n\frac{\theta}{2\alpha}p_m(1). \tag{A.3}$$

Observe that the LHS of (A.3) is strictly decreasing in $p_m(1)$, while the RHS is strictly increasing in $p_m(1)$. At $p_m(1) = 1$, the LHS is strictly smaller than the RHS, while at $p_m(1) = 0$, the LHS is strictly larger than the RHS. Consequently, (A.3) has a unique solution for $p_m(1)$.

When $b > b^*(\theta)$,

$$p_m(1) = \frac{M(1+k)}{M(1+k) + \frac{n\theta}{2\alpha}}$$

solves (A.3) above. Further, at this value of $p_m(1)$, we have that $\frac{b}{p_m(1)} > 1 - p_m(1)$.

When $b < b^*(\theta)$,

$$p_m(1) = \frac{M + \sqrt{M^2 + 4kMb(M + \frac{n\theta}{2\alpha})}}{2(M + \frac{n\theta}{2\alpha})}$$

solves (A.3) above. Further, at this value of $p_m(1)$, we have that $\frac{b}{p_m(1)} < 1 - p_m(1)$.

Using the above solutions and the observation that (A.3) has a unique solution for $p_m(1)$, we have the required result (depending on whether $b > b^*(\theta)$ or $b < b^*(\theta)$). Substituting the solution to $p_m(1)$ obtained above in (3.4), we get the equilibrium value of the farmer's effort.

■

Proof of Lemma 3.4.1: In this case, we have that $b+\beta < b^*(\theta)$. Recall, from Lemma 3.3.2, that under NI, if $b < b^*(\theta)$, then the equilibrium effort of a farmer is

$$q_e^* = \frac{\theta}{2\alpha}\left(\frac{M + \sqrt{M^2 + 4kMb(M + \frac{n\theta}{2\alpha})}}{2(M + \frac{n\theta}{2\alpha})}\right),$$

and the equilibrium market price under high yield realization is

$$p_m(1) = \frac{M + \sqrt{M^2 + 4kMb(M + \frac{n\theta}{2\alpha})}}{2(M + \frac{n\theta}{2\alpha})}.$$

Both these quantities are increasing in the wealth b of a BPL consumer. Consequently, under the DBT scheme, if $b+\beta < b^*(\theta)$, the production effort and the market-price are both higher. The equilibrium quantities are obtained by substituting $b \to (b+\beta)$ in the above expressions. Therefore, we have the first part of the result.

Under NI, the (expected) social planner's surplus under NI can be written as

$$\begin{aligned}
{\Pi_{SP}}^{NI} &= \underbrace{M\Big[\theta\Big(\int_0^{1-p_m(1)}(1-\xi)d\xi + (w_{APL} - p_m(1)(1-p_m(1)))\Big) + (1-\theta)w_{APL}\Big]}_{\text{APL Consumers' Surplus}} \\
&\quad \underbrace{+kM\Big[\theta\Big(\int_0^{\frac{b}{p_m(1)}}(1-\xi)d\xi + \Big(b - p_m(1)\Big(\frac{b}{p_m(1)}\Big)\Big)\Big) + (1-\theta)(b)\Big]}_{\text{BPL Consumers' Surplus}} \\
&\quad \underbrace{+n\Big[\theta\Big(p_m(1)\frac{\theta}{2\alpha}p_m(1) - \alpha(\frac{\theta}{2\alpha}p_m(1))^2\Big) + (1-\theta)\Big(-\alpha\Big(\frac{\theta}{2\alpha}p_m(1)\Big)^2\Big)\Big]}_{\text{Farmers' Surplus}} \\
&\quad + \underbrace{kM\beta}_{\text{Unused Budget}}, \qquad\qquad (A.4) \\
\text{where } p_m(1) &= p_m(1)^{NI} = \frac{M + \sqrt{M^2 + 4kMb(M + \frac{n\theta}{2\alpha})}}{2(M + \frac{n\theta}{2\alpha})}.
\end{aligned}$$

In (A.4), the first term represents the APL consumers' expected surplus, the second term represents the BPL consumers' expected surplus, the third term is the farmers' expected profit, and the fourth term is the unused budget of the social planner (which is the entire budget B under NI). We rewrite (A.4) as follows:

$$\begin{aligned}
{\Pi_{SP}}^{NI} &= \underbrace{Mw_{APL} + kMb + kM\beta}_{\text{Wealth Across All Segments}} + \\
&\quad M\Big[\theta\Big(\int_0^{1-p_m(1)}(1-\xi-p_m(1))d\xi\Big)\Big] + \\
&\quad kM\Big[\theta\Big(\int_0^{\frac{b}{p_m(1)}}(1-\xi-p_m(1))d\xi\Big)\Big] + \\
&\quad n\Big[-\alpha\Big(\frac{\theta}{2\alpha}p_m(1)\Big)^2 + \theta\Big(p_m(1)(\frac{\theta}{2\alpha}p_m(1))\Big)\Big], \qquad\qquad (A.5)
\end{aligned}$$

where the first term represents the total wealth available across both segments of the consuming population along with the budget of the social planner. The second and third terms are the consumption utilities from each segment of the consuming population.

Under the DBT scheme, the social planner's surplus can be written as

$$\begin{aligned}
{\Pi_{SP}}^{DBT} &= Mw_{APL} + kM(b+\beta) + \\
& M\Big[\theta\Big(\int_0^{1-p_m(1)}(1-\xi-p_m(1))d\xi\Big)\Big] + \\
& kM\Big[\theta\Big(\int_0^{\frac{b+\beta}{p_m(1)}}(1-\xi-p_m(1))d\xi\Big)\Big] + \\
& n\Big[-\alpha\Big(\frac{\theta}{2\alpha}p_m(1)\Big)^2 + \theta\Big(p_m(1)\big(\frac{\theta}{2\alpha}p_m(1)\big)\Big)\Big], \qquad \text{(A.6)} \\
\text{where } p_m(1) &= p_m(1)^{DBT} = \frac{M+\sqrt{M^2+4kM(b+\beta)(M+\frac{n\theta}{2\alpha})}}{2(M+\frac{n\theta}{2\alpha})}.
\end{aligned}$$

We show that the difference in the social planner's surplus, i.e., the difference in the RHS of (A.6) and (A.5) is strictly positive. Observe that the wealth available across all segments of the population ($Mw_{APL} + kMb + kM\beta$) comprises of internal monetary transfer among the agents. Consequently, it is sufficient to show that the derivative of the sum of the remaining three terms in (A.5) w.r.t. b is *strictly* positive. Let

$$\begin{aligned}
\check{\Pi}_{SP}^{NI} &= M\Big[\theta\Big(\int_0^{1-p_m(1)}(1-\xi-p_m(1))d\xi\Big)\Big] + \\
& kM\Big[\theta\Big(\int_0^{\frac{b}{p_m(1)}}(1-\xi-p_m(1))d\xi\Big)\Big] + \\
& n\Big[-\alpha\Big(\frac{\theta}{2\alpha}p_m(1)\Big)^2 + \theta\Big(p_m(1)\big(\frac{\theta}{2\alpha}p_m(1)\big)\Big)\Big], \qquad \text{(A.7)} \\
\text{where } p_m(1) &= p_m(1)^{NI} = \frac{M+\sqrt{M^2+4kMb(M+\frac{n\theta}{2\alpha})}}{2(M+\frac{n\theta}{2\alpha})}; \\
\text{i.e., } \Pi_{SP}^{NI} &= \check{\Pi}_{SP}^{NI} + (Mw_{APL} + kMb + kM\beta).
\end{aligned}$$

We will show that $\frac{d\check{\Pi}_{SP}^{NI}}{db} > 0$. We can write $\frac{d\check{\Pi}_{SP}^{NI}}{db}$ as

$$\frac{d\check{\Pi}_{SP}^{NI}}{db} = \frac{\partial\check{\Pi}_{SP}^{NI}}{\partial b} + \frac{\partial\check{\Pi}_{SP}^{NI}}{\partial p_m(1)}\frac{\partial p_m(1)}{\partial b} \qquad \text{(A.8)}$$

The first term represents the direct effect of an increase in b. The second term represents the indirect effect of the increase in b, because an increase in b leads to an increase in $p_m(1)$.

From Lemma 3.3.2, it is straightforward that $\frac{\partial p_m(1)}{\partial b} > 0$. We have that

$$\begin{aligned}
\frac{\partial \check{\Pi}_{SP}^{NI}}{\partial b} &= \frac{kM\theta}{p_m(1)}[1 - p_m(1) - \frac{b}{p_m(1)}] > 0, \\
\frac{\partial \check{\Pi}_{SP}^{NI}}{\partial p_m(1)} &= -\frac{\theta k M b}{p_m(1)^2}[1 - p_m(1) - \frac{b}{p_m(1)}] < 0, \text{ and} \\
\frac{dp_m(1)}{db} &= \frac{4\alpha^2 kM^2 + 2\alpha\theta kMn}{2(2\alpha M + \theta n)\sqrt{4\alpha^2 bkM^2 + 2\alpha b\theta kMn + \alpha^2 M^2}} = \frac{k}{2\sqrt{kb(1 + \frac{n\theta}{2M\alpha}) + \frac{1}{4}}}, \\
\text{where } p_m(1) &= \frac{M + \sqrt{M^2 + 4kMb(M + \frac{n\theta}{2\alpha})}}{2(M + \frac{n\theta}{2\alpha})}.
\end{aligned}$$

The direct effect (of an increase in b) is positive, while the indirect effect is negative. Substituting these expressions back in (A.8) and using straightforward algebraic manipulations, we have the required result. ■

Proof of Lemma 3.5.1: Consider an announced p_g and farmers' belief $(\hat{p}_m(1), \hat{p}_m(0))$, where $\hat{p}_m(1) \le \hat{p}_m(0)$ and $\hat{p}_m(0) = 1$. Recall the farmer's decisions and objective from Section 3.2.2. One of the following holds: either $p_g < \hat{p}_m(1)$, or $p_g = \hat{p}_m(1)$, or $p_g > \hat{p}_m(1)$. For each of these cases, we solve the optimal selling decisions (q_g, q_m) for a fixed q_e when the realized yield is high (i.e., solution to **Problem** $\mathbf{P}_f^2$). Then, we substitute this solution back into **Problem** $\mathbf{P}_f^1$ to find the equilibrium q_e that maximizes the farmer's expected profit.

(*i*) Suppose $p_g < \hat{p}_m(1)$: Then, for any given q_e, it is straightforward that $q_g = 0$ and $q_m = q_e$ (i.e., the solution to **Problem** $\mathbf{P}_f^2$). Substituting these solutions in **Problem** $\mathbf{P}_f^1$, we have $\pi_f = -\alpha q_e^2 + \mathbb{E}[\hat{p}_m(\gamma)\gamma q_e]$, which gives us $q_e^* = \frac{\mathbb{E}[\hat{p}_m(\gamma)\gamma]}{2\alpha} = \frac{\theta}{2\alpha}\hat{p}_m(1)$.

(*ii*) Suppose $p_g > \hat{p}_m(1)$: Then, for any given q_e, it is straightforward that $q_g = \min\{\frac{B}{np_g}, q_e\}$, and $q_m = q_e - q_g$ (i.e., the solution to **Problem** $\mathbf{P}_f^2$). Substituting these back in **Problem** $\mathbf{P}_f^1$, we have $\pi_f = -\alpha q_e^2 + \mathbb{E}[p_g \min\{q_e, \frac{B}{np_g}\} + \hat{p}_m(1)(q_e - \frac{B}{np_g})^+]$, which gives us

$$q_e^* = \begin{cases} \frac{\theta}{2\alpha}p_g, & \frac{\theta}{2\alpha}\hat{p}_m(1) \le \frac{\theta}{2\alpha}p_g \le \frac{B}{np_g}; \\ \frac{B}{np_g}, & \frac{\theta}{2\alpha}\hat{p}_m(1) \le \frac{B}{np_g} \le \frac{\theta}{2\alpha}p_g; \\ \frac{\theta}{2\alpha}\hat{p}_m(1), & \frac{B}{np_g} \le \frac{\theta}{2\alpha}\hat{p}_m(1) \le \frac{\theta}{2\alpha}p_g \end{cases}$$

(*iii*) Suppose $p_g = \hat{p}_m(1)$: Then, for any given q_e, it is straightforward that any $(q_g, q_m) \geq 0$ s.t. $q_g + q_m = q_e$ is optimal (the solution to **Problem** $\mathbf{P}_f^2$). Substituting these solutions in **Problem** $\mathbf{P}_f^1$, we have $\pi_f = -\alpha q_e^2 + \mathbb{E}[p_g \gamma q_e]$, which gives us $q_e^* = \frac{p_g \mathbb{E}[\gamma]}{2\alpha} = \frac{\theta}{2\alpha} p_g$. ■

Proof of Lemma 3.5.2: In this case, we have $b + \beta < b^*(\theta)$. Consider case 1, where the announced support price $p_g < p_m(1)^{NI}$. We show the result under this case in two steps. First, we show that the only rational belief of the farmers is $p_g < \hat{p}_m(1)$ and $\hat{p}_m(1) = p_m(1)^{NI}$. Second, using Lemma 3.5.1, we identify the equilibrium outcome.

We show the first part (i.e., the only consistent beliefs) by contradiction. Suppose not: suppose $p_g = \hat{p}_m(1)$. Then, from Lemma 3.5.1, we have $q_e^* = \frac{\theta}{2\alpha} p_g$. Using (3.5), $p_g = \hat{p}_m(1)$, and decisions q_g and $q_m = q_e - q_g$, we have

$$M(1 - p_g) + kM \min\left\{1 - p_g - \frac{n}{kM} q_g, \frac{b}{p_g}\right\} = n\left(\frac{\theta}{2\alpha} p_g - q_g\right).$$

We rewrite this as

$$M(1 - p_g) + kM \min\left\{1 - p_g, \frac{b}{p_g} + \frac{n}{kM} q_g\right\} = n\left(\frac{\theta}{2\alpha} p_g\right).$$

All else equal, the LHS of the above expression is decreasing in p_g and increasing in q_g. Since $p_g < p_m(1)^{NI}$, the LHS is strictly higher than $M(1 - p_m(1)^{NI}) + kM \min\{1 - p_m(1)^{NI}, \frac{b}{p_m(1)^{NI}}\}$. Since the RHS is strictly increasing in p_g, we have that the RHS is strictly lower than $n\frac{\theta}{2\alpha} p_m(1)^{NI}$. However, we have $M(1 - p_m(1)^{NI}) + kM \min\{1 - p_m(1)^{NI}, \frac{b}{p_m(1)^{NI}}\} = n\frac{\theta}{2\alpha} p_m(1)^{NI}$ (since $b + \beta < b^*(\theta)$) leading to a contradiction. Therefore, we eliminate the possible belief $p_g = \hat{p}_m(1)$. Using a similar argument, we eliminate the possible belief $p_g > \hat{p}_m(1)$. Therefore, the only rational beliefs are $p_g < \hat{p}_m(1)$.

We now find the equilibrium outcome. Using the expressions in case (*i*) of Lemma 3.5.1, we have that $q_e^* = \frac{\theta}{2\alpha} \hat{p}_m(1)$, $q_g = 0$ and $q_m = q_e$. Substituting these in (3.5), we obtain $\hat{p}_m(1) = p_m(1)^{NI}$.

The proof for case 2, where $p_g \in [p_m(1)^{NI}, p_m(1)^{DBT}]$ and case 3, where $p_g > p_m(1)^{DBT}$ follow identical steps as above, where we first identify the unique beliefs (on the market price) by eliminating impossible beliefs and then obtain the equilibrium outcome in the respective cases. In case 2, where $p_g \in [p_m(1)^{NI}, p_m(1)^{DBT}]$, we show that the only rational belief of the farmers is $p_g = \hat{p}_m(1)$ (by eliminating the beliefs $p_g < \hat{p}_m(1)$ and $p_g > \hat{p}_m(1)$). Then, from Lemma 3.5.1, we have that $q_e = \frac{\theta}{2\alpha}p_g$, and any $(q_g, q_m) \geq 0$ such that $q_g + q_m = q_e$ is optimal. We focus on symmetric strategies (i.e., all farmers adopt the same strategy). Substituting these back in (3.5) and using algebraic manipulations, we have $M(1-p_g) + kM\min\{1 - p_g, \frac{b}{p_g} + \frac{n}{kM}q_g\} = n\frac{\theta}{2\alpha}p_g$. Since $b + \beta < b^*(\theta)$, we have that for $p_g \in [p_m(1)^{NI}, p_m(1)^{DBT}]$, $\frac{b}{p_g} + \frac{n}{kM}q_g \leq 1 - p_g$. Therefore, we have $M(1-p_g) + kM(\frac{b}{p_g} + \frac{n}{kM}q_g) = n\frac{\theta}{2\alpha}p_g$. Therefore, $q_g = \frac{\theta}{2\alpha}p_g - \frac{M}{n}(1-p_g) - \frac{kM}{n}\frac{b}{p_g}$.

In case 3, where $p_g > p_m(1)^{DBT}$, we show that the only rational belief is $p_g > \hat{p}_m(1)$ (by eliminating the beliefs $p_g \leq \hat{p}_m(1)$). Further, since $b + \beta < b^*(\theta)$, for any $p_g > p_m(1)^{DBT}$, we have that $\frac{B}{np_g} < \frac{\theta}{2\alpha}\hat{p}_m(1) < \frac{\theta}{2\alpha}p_g$. Therefore, from Lemma 3.5.1, $q_e^* = \frac{\theta}{2\alpha}\hat{p}_m(1)$, $q_g = \frac{B}{np_g}$ and $q_m = q_e - q_g$. Substituting these back in (3.5) and using some algebraic manipulations, we have that $M(1 - \hat{p}_m(1)) + kM\min\{1 - \hat{p}_m(1), \frac{b}{\hat{p}_m(1)} + \frac{\beta}{p_g}\} = n(\frac{\theta}{2\alpha}\hat{p}_m(1))$. Since $b + \beta < b^*(\theta)$, we have $M(1 - \hat{p}_m(1)) + kM(\frac{b}{\hat{p}_m(1)} + \frac{\beta}{p_g}) = n(\frac{\theta}{2\alpha}\hat{p}_m(1))$. Therefore, we have that $\hat{p}_m(1) = \frac{\left(M + kM\frac{\beta}{p_g}\right) + \sqrt{\left(M + kM\frac{\beta}{p_g}\right)^2 + 4kMb\left(M + \frac{n\theta}{2\alpha}\right)}}{2\left(M + \frac{n\theta}{2\alpha}\right)}$. ■

Proof of Lemma 3.5.3: The analysis in case 1, where $p_g < p_m(1)^{NI}$ is straightforward: Using case 1 of Lemma 3.5.2, we have that the market outcome in this case is identical to the outcome under NI. Therefore, the social planner's surplus is also identical to that under NI.

In case 2, where $p_g \in [p_m(1)^{NI}, p_m(1)^{DBT}]$, the social planner's surplus, using Lemma 3.5.2, can be written as follows:

$$\Pi_S = Mw_{APL} + kMb + kM\beta - n\alpha q_e^{*2} + \theta\Big(M\int_0^{1-p_g}(1-\xi)d\xi + kM\int_0^{\frac{1}{kM}(\frac{n\theta}{2\alpha}p_g - M(1-p_g))}(1-\xi)d\xi\Big),$$

where $q_e^* = \frac{\theta}{2\alpha} p_g$. Then,

$$\frac{d\Pi_S}{dp_g} = \Big(\frac{\theta(M + \frac{n\theta}{2\alpha})}{kM}\Big)\Big(M(1+k) + \frac{n\theta}{2\alpha}\Big)\Big(\frac{M(1+k)}{(M(1+k) + \frac{n\theta}{2\alpha})} - p_g\Big).$$

The first two terms in the above expression are positive. The third term is the difference between the market price $p_m(1)^{NI}$ when $b > b^*(\theta)$ (i.e., the BPL consumers are effectively not budget constrained) and p_g and is decreasing in p_g. At $p_g = p_m(1)^{DBT}$, this difference is positive, since $b + \beta < b^*(\theta)$ and $p_m(1)^{NI}$ is increasing in b if $b < b^*(\theta)$. Therefore, at any value of $p_g \in [p_m(1)^{NI}, p_m(1)^{DBT}]$, this difference is positive.

In case 3 where $p_g > p_m(1)^{DBT}$, the social planner's surplus is given by:

$$\Pi_S = Mw_{APL} + kMb + kM\beta - n\alpha q_e^{*2} + \theta\Big(M\int_0^{1-p_m(1)}(1-\xi)d\xi + kM\int_0^{\frac{\beta}{p_g}+\frac{b}{p_m(1)}}(1-\xi)d\xi\Big),$$

where

$$p_m(1) = \frac{\big(M + kM\frac{\beta}{p_g}\big) + \sqrt{\big(M + kM\frac{\beta}{p_g}\big)^2 + 4kMb\big(M + \frac{n\theta}{2\alpha}\big)}}{2\big(M + \frac{n\theta}{2\alpha}\big)}$$

is decreasing in p_g. Then,

$$\frac{d\Pi_S}{dp_g} = \Big(\theta kM\Big)\Big(\frac{d}{dp_g}\Big(\frac{\beta}{p_g} + \frac{b}{p_m(1)}\Big)\Big)\Big(1 - p_m(1) - \Big(\frac{\beta}{p_g} + \frac{b}{p_m(1)}\Big)\Big).$$

The second term is the derivative of the quantity consumed by a BPL consumer with p_g. Since the quantity consumed by a BPL consumer decreases in p_g, this term is negative. The third term is positive, since $b + \beta < b^*(\theta)$. ■

Proof of Lemma 3.5.5: In this case, we have $b > b^*(\theta)$ and $\theta \geq \theta^{**}$. The proof consists of three (mutually exclusive and exhaustive) cases, depending on the value of p_g: (a) $p_g < p_m(1)^{NI}$, (b) $p_g = p_m(1)^{NI}$, and (c) $p_g > p_m(1)^{NI}$. The approach is identical to the proof of Lemma 3.5.2. That is, for each case, we first show the only consistent beliefs on $\hat{p}_m(1)$ by eliminating impossible beliefs. Then, we find the equilibrium market outcome using Lemma 3.5.1.

Consider the case (a) where $p_g < p_m(1)^{NI}$ (resp., case (b) where $p_g = p_m(1)^{NI}$, and case (c) where $p_g > p_m(1)^{NI}$): Using an approach identical to the proof of Lemma 3.5.2, we can show that the only rational beliefs are $p_g < \hat{p}_m(1)$ (resp., $p_g = \hat{p}_m(1)$ and $p_g > \hat{p}_m(1)$) and that $\hat{p}_m(1) = p_m(1)^{NI}$. Further, using Lemma 3.5.1, we can show in each of the three cases that the market price $\hat{p}_m(1)$ is unique and is equal to $p_m(1)^{NI}$. A detailed proof is, therefore, avoided for brevity. ■

Proof of Lemma 3.5.7: In this case, we have $b > b^*(\theta)$ and $\theta < \theta^{**}$. The proof consists of five cases, but the approach is identical to the proofs of Lemma 3.5.2 and Lemma 3.5.5. That is, for each case, we first show the only consistent beliefs on $\hat{p}_m(1)$ by eliminating impossible beliefs. Then, we find the equilibrium market outcome using Lemma 3.5.1.

Consider case 1, where $p_g < p_m(1)^{NI}$: Using an approach identical to the proof of Lemma 3.5.2, we can show that the only rational beliefs are $p_g < \hat{p}_m(1)$. Then, using Lemma 3.5.1, we have that $q_e^* = \frac{\theta}{2\alpha}\hat{p}_m(1)$, $q_g = 0$ and $q_m = q_e^*$. Substituting these back in (3.5) and using some algebraic manipulations, we have that $M(1-\hat{p}_m(1)) + kM\min\{1-\hat{p}_m(1), \frac{b}{\hat{p}_m(1)}\} = n(\frac{\theta}{2\alpha})\hat{p}_m(1)$. Since the LHS is strictly decreasing in $\hat{p}_m(1)$, the RHS is strictly increasing in $\hat{p}_m(1)$ and $b > b^*(\theta)$, we have that the unique solution to this equation is $\hat{p}_m(1) = p_m(1)^{NI}$.

Consider case 2, where $p_g = p_m(1)^{NI}$: Using an approach identical to the proof of Lemma 3.5.2, we can show that the only rational beliefs are $p_g = \hat{p}_m(1)$. Therefore, $\hat{p}_m(1) = p_m(1)^{NI}$. Then, using Lemma 3.5.1, we have that $q_e^* = \frac{\theta}{2\alpha}p_g$, any $q_g \in [0, \frac{\theta}{2\alpha}p_g]$ and $q_m = q_e^* - q_g^*$.

Consider case 3, where $p_g \in \left(p_m(1)^{NI}, \sqrt{\frac{B}{\left(\frac{n\theta}{2\alpha}\right)}}\right)$ (resp., case 4 where $p_g \in \left[\sqrt{\frac{B}{\left(\frac{n\theta}{2\alpha}\right)}}, \frac{B}{n(\frac{\theta}{2\alpha})p_m(1)^{NI}}\right]$ and case 5 where $p_g > \frac{B}{n(\frac{\theta}{2\alpha})p_m(1)^{NI}}$). Using an approach identical to the proof of Lemma 3.5.2, we can show that the only rational beliefs are $p_g > \hat{p}_m(1)$ and $\frac{\theta}{2\alpha}\hat{p}_m(1) < \frac{\theta}{2\alpha}p_g < \frac{B}{np_g}$ (resp., $\frac{\theta}{2\alpha}\hat{p}_m(1) \leq \frac{B}{np_g} \leq \frac{\theta}{2\alpha}p_g$ and $\frac{B}{np_g} < \frac{\theta}{2\alpha}\hat{p}_m(1) < \frac{\theta}{2\alpha}p_g$). Then, using Lemma 3.5.1, we can show in each of the three cases that the market price $\hat{p}_m(1)$ is unique using a similar monotonicity argument used in case 1. Therefore, we have that $\hat{p}_m(1) = 1 - \frac{\left(\frac{n\theta}{2\alpha}\right)p_g}{M(1+k)}$ (resp., $\hat{p}_m(1) = 1 - \frac{B}{M(1+k)p_g}$ and $\hat{p}_m(1) = p_m(1)^{NI}$). A detailed proof is, therefore, avoided for brevity. ■

Proof of Lemma 3.5.8: The analysis in case 1, where $p_g \leq p_m(1)^{NI}$, is straightforward. Since the market outcome is identical to NI, the social planner's surplus is also identical to NI.

In case 2, where $p_g \in \left(p_m(1)^{NI}, \sqrt{\frac{B}{\left(\frac{n\theta}{2\alpha}\right)}}\right)$, the social planner's surplus is:

$$\begin{aligned}\Pi_S &= Mw_{APL} + kMb + kM\beta - n\alpha {q_e^*}^2 + \\ &\quad \theta\Big(M\int_0^{1-p_m(1)}(1-\xi)d\xi + kM\int_0^{\left(1-\frac{n}{kM}\frac{\theta}{2\alpha}p_g - p_m(1)\right)+\left(\frac{n}{kM}\frac{\theta}{2\alpha}p_g\right)}(1-\xi)d\xi\Big),\end{aligned}$$

where $q_e^* = \frac{\theta}{2\alpha}p_g$ and $p_m(1) = 1 - \frac{\frac{n\theta}{2\alpha}p_g}{M(1+k)}$. Then,

$$\frac{d\Pi_S}{dp_g} = -\theta\Big(\frac{\left(\frac{n\theta}{2\alpha}\right)}{\frac{M(1+k)}{M(1+k)+\frac{n\theta}{2\alpha}}}\Big)\big(p_g - \frac{M(1+k)}{M(1+k)+\frac{n\theta}{2\alpha}}\big).$$

The third term is positive, since $p_g > p_m(1)^{NI}$. Therefore, $\frac{d\Pi_S}{dp_g} < 0$. Therefore, the social planner's surplus is strictly decreasing in p_g.

In case 3, where $p_g \in \left[\sqrt{\frac{B}{\left(\frac{n\theta}{2\alpha}\right)}}, \frac{B}{n(\frac{\theta}{2\alpha})p_m(1)^{NI}}\right]$, the social planner's surplus is:

$$\Pi_S = B + Mw_{APL} + kMb - n\alpha {q_e^*}^2 + \theta\Big(M\int_0^{1-p_m(1)}(1-\xi)d\xi + kM\int_0^{\left(1-\frac{B}{kMp_g}-p_m(1)\right)+\left(\frac{B}{kMp_g}\right)}(1-t)dt\Big),$$

where $q_e^* = \frac{B}{np_g}$ and $p_m(1) = 1 - \frac{B}{M(1+k)p_g}$. Then,

$$\frac{d\Pi_S}{dp_g} = \frac{B\theta}{{p_g}^3}\left[\frac{B}{n\frac{\theta}{2\alpha}\frac{M(1+k)}{M(1+k)+\frac{n\theta}{2\alpha}}} - p_g\right].$$

The RHS is decreasing in p_g. Further, at $p_g = \frac{B}{n(\frac{\theta}{2\alpha})p_m(1)^{NI}}$, we have that $\frac{d\Pi_S}{dp_g} = 0$. Therefore, $\frac{d\Pi_S}{dp_g} \geq 0$ in $p_g \in \left[\sqrt{\frac{B}{\left(\frac{n\theta}{2\alpha}\right)}}, \frac{B}{n(\frac{\theta}{2\alpha})p_m(1)^{NI}}\right]$, i.e., Π_S is increasing in p_g if $p_g \in \left[\sqrt{\frac{B}{\left(\frac{n\theta}{2\alpha}\right)}}, \frac{B}{n(\frac{\theta}{2\alpha})p_m(1)^{NI}}\right]$.

Finally, in case 4, we have that the market outcome does not change with p_g and the total production effort of the farmers is identical to that under NI. Consequently, $\frac{d\Pi_S}{dp_g} = 0$ and $\Pi_S(p_g) = {\Pi_S}^{NI}$. ■

APPENDIX B

MODEL NOTATION AND PROOFS OF MAIN RESULTS IN CHAPTER 4

B.1 Model Notation

Table B.1. Model Notation

i	: Market zone, $i \in \{A, B\}$	j	: Market period, $j \in \{1, 2\}$
p_0	: Regular price for on-demand service	p_{ij}	: Price in zone i in period j, $p_{ij} \geq p_0$.
$D_R(\cdot)$	: Regular demand	$D_S(\cdot)$	: Surge demand
p_R	: Unconstrained revenue maximizing price under $D_R(\cdot)$	p_S	: Unconstrained revenue maximizing price under $D_S(\cdot)$
a_R	: Number of workers needed to meet regular demand	a_S	: Number of workers needed to meet surge demand
λ	: Workers' share of revenue	c	: Workers' cost to move to adjacent zone
r_{ij}	: Expected revenue of worker serving zone i, period j	π_{ij}	: Platform profit in zone i, period j
μ_i	: Proportion of zone i workers who move	μ_0	: μ_i in non-surge zone without strategic surge pricing
p_S^0	: period 2 price in surge zone without strategic surge pricing	$\bar{p}_S$	: period 2 price in surge zone if no workers move
t	: Platform's type, $t \in \{H, L\}$	$x\|_t$	: Variable x on a type t platform
θ	: Workers' posterior belief that platform is type H	$\hat{r}_{ij}$	: Expected r_{ij} given workers' belief θ
b_i	: Bonus offered to workers in zone i	m_i	: Number of bonuses offered in zone i
N_i	: Initial number of workers in zone i; $(N_A, N_B) = (n_H, n_L)$ or (n_L, n_H), where $n_H > n_L$	$\tilde{N}_{ij}$	: Number of workers available to serve consumers in zone i, period j
$\bar{\mu}$	: minimum μ_i in non-surge zone such that demand can be met in surge zone $(= \frac{a_S - N_A}{N_B})$		

B.2 Proofs for Main Results (in Section 4.4)

We develop two intermediate results under symmetric information that are useful for our analysis.

Lemma B.2.1. *In the subgame equilibrium given period* 1 *prices, workers in the surge zone stay in the surge zone.*

Proof. Suppose, towards a contradiction, $\mu_A > 0$. In period 1, a worker will move from zone i to zone $i^{'}$ iff $r_{i1} + r_{i2} \leq r_{i^{'}2} - c$. Therefore, if it is attractive for workers to move from zone i to zone $i^{'}$, then it is not attractive for workers to move from zone $i^{'}$ to zone i. Since $\mu_A > 0$ (by assumption), $\mu_B = 0$. Then $\tilde{N}_{A2} < a_S$ and $\tilde{N}_{B2} > a_R$. Consequently, the optimal period 2 prices are such that $p^e_{A2} > p_S$ and $p^e_{B2} = p_R$. Therefore, $r_{A2} > \lambda p_S$ and $r_{B2} < \lambda p_R$. But, then $r_{A1} + r_{A2} > r_{B2} - c$, which is a contradiction. Therefore, $\mu_A = 0$. $\square$

Lemma B.2.2. *If condition* $(C1)$ *holds, then the platform sets period* 1 *prices such that* $\mu_B \leq \bar{\mu}$ *in equilibrium.*

Proof. Suppose towards a contradiction, in equilibrium, the platform sets $p_{B1} = p^{'}_{B1}$ such that $\mu_B > \bar{\mu}$. Let μ_0 denote the value of μ_B when $p_{i1} = p_R$. Since (C1) holds, we have $\mu_0 < \bar{\mu}$. Since $\mu_0 < \bar{\mu}$ and $\mu_B\left(p^{'}_{B1}\right) > \bar{\mu}$, from Proposition 4.4.2, there exists $p^{''}_{B1} \in \left(p_R, p^{'}_{B1}\right)$ such that $\mu_B = \bar{\mu}$. The platform profit is given by

$$\Pi_P = (1 - \lambda)\left(R_{B1} + R_{B2} + R_{A1} + R_{A2}\right) \tag{B.1}$$

Comparing the platform profit in the two cases (where the platform sets $p_{B1} = p^{'}_{B1}$ and $p_{B1} = p^{''}_{B1}$), the platform profit in zone A is identical in both the cases while the platform profit in zone B is strictly lower for $p_{B1} = p^{'}_{B1}$, which is the desired contradiction. $\square$

Proof for Proposition 4.4.1: We obtain conditions such that $\mu_B < \bar{\mu}\left(= \frac{a_S - N_A}{N_B}\right)$ for a subgame equilibrium in which $p_{i1} = p_R$. From Lemma B.2.1 above, we know that $\mu_A = 0$.

Thus, $\tilde{N}_{A2} = N_A + \mu_B N_B$ and $\tilde{N}_{Bj} = N_B(1 - \mu_B)$. Consider workers in zone B. Since $\tilde{N}_{Bj}$ is decreasing in μ_B, r_{Bj} is strictly increasing in μ_B. This is because having fewer workers in zone B reduces worker competition and increases worker compensation. Similarly, $\tilde{N}_{A2}$ is increasing in μ_B and hence r_{A2} is strictly decreasing in μ_B. Therefore, if $r_{B1} + r_{B2} > r_{A2} - c$ for $\mu_B = \bar{\mu}$, then $r_{B1} + r_{B2} > r_{A2} - c$ for any $\mu_B > \bar{\mu}$, and $\mu_B < \bar{\mu}$ in equilibrium. We have $r_{Bj} = \lambda p_R \frac{a_R}{n_H + n_L - a_S}$, and $r_{A2} = \lambda p_S$ for $\mu_B = \bar{\mu}$. It follows that $r_{B1} + r_{B2} > r_{A2} - c$ iff $\lambda p_S < c$ or $n_H + n_L < a_S + a_R \frac{2\lambda p_R}{\lambda p_S - c}$. Conversely, if $r_{B1} + r_{B2} \leq r_{A2} - c$ for $\mu_B = \bar{\mu}$, then $r_{B1} + r_{B2} < r_{A2} - c$ for any $\mu_B < \bar{\mu}$. Hence, $\mu_B \geq \bar{\mu}$ in equilibrium.

Proof for Proposition 4.4.2: From Lemma B.2.1 above, $\mu_A = 0$. We first show that there is a unique μ_B given p_{i1}. The expected revenue of workers serving zone B $(= r_{B1} + r_{B2})$ is strictly increasing in μ_B, whereas the expected revenue of workers serving zone A $(= r_{A2})$ is strictly decreasing in μ_B.[1] This is because having fewer workers in zone B reduces worker competition in zone B and increases worker competition in zone A. Therefore, the LHS of condition (4.1) is strictly increasing in μ_B and the RHS is strictly decreasing in μ_B. This leads to the following observation regarding condition (4.1). Given period 1 prices: (i) condition (4.1) can hold as an equality for at most one $\mu_B \in [0, 1]$, (ii) if it holds as an inequality for $\mu_B = 0$ then $\mu_B = 0$ is the unique equilibrium, (iii) if it does not hold for any $\mu_B < 1$ then $\mu_B = 1$ is the unique equilibrium. Therefore an equilibrium exists and is unique. Further, condition (4.1), is influenced only by p_{B1} and not by p_{A1}. Therefore, μ_B is not influenced by p_{A1}.

Next, we show that μ_B is increasing in p_{B1} iff $\lambda \overline{p}_S - c > \lambda \left(\frac{a_R}{N_B}\right) p_R$. We divide the proof into the following three cases. First, suppose $\mu_B > 0$ for $p_{B1} = p_R$. This is the case if $r_{B1} + r_{B2} < r_{A2} - c$ for $\mu_B = 0$ and $p_{B1} = p_R$; substituting for μ_B and p_{B1}, we require

[1] For any p_{B1}, $r_{B1} = \lambda p_{B1} \min\left\{\frac{D_R(p_{B1})}{N_B(1-\mu_B)}, 1\right\}$, $r_{B2} = \lambda D_R^{-1}\left(N_B\left(1 - \mu_B\right)\right)$ if $N_B\left(1 - \mu_B\right) < a_R$ and $r_{B2} = \lambda \frac{a_R p_R}{N_B(1-\mu_B)}$ otherwise and $r_{A2} = \lambda D_S^{-1}\left(N_A + N_B \mu_B\right)$ if $N_A + N_B \mu_B < a_S$ and $r_{A2} = \lambda p_S \frac{a_S}{N_A + N_B \mu_B}$ otherwise.

$2\lambda\left(\frac{a_R}{N_B}\right)p_R < \lambda\overline{p}_S - c$. In this case, r_{B1} and hence the LHS of condition (4.1) is strictly decreasing in p_{B1}. Also, as noted before, the LHS of condition (4.1) is strictly increasing in μ_B and the RHS is strictly decreasing in μ_B. Since condition (4.1) holds as an equality, it follows that μ_B is strictly increasing in p_{B1} if $\mu_B < 1$. Next, suppose $\mu_B = 0$ even for $p_{B1} > p_R$ such that $r_{B1} = 0$. This is the case if for $r_{B1} = 0$ and $\mu_B = 0$, condition (4.1) holds; substituting for μ_B and r_{B1}, we require $\lambda\left(\frac{a_R}{N_B}\right)p_R \geq \lambda\overline{p}_S - c$. In this case, for any $p_{B1} \geq p_R$ such that $r_{B1} \geq 0$, condition (4.1) holds as an inequality and hence $\mu_B = 0$ for any value of $p_{B1} \geq p_R$. Finally, suppose $\mu_B = 0$ at $p_{B1} = p_R$ but condition (4.1) holds as an equality for some $p_{B1} > p_R$. It must be that $r_{B1} + r_{B2} \leq r_{A2} - c$ for $\mu_B = 0$ and $r_{B1} = 0$; substituting for μ_B and r_{B1}, we require $\lambda\left(\frac{a_R}{N_B}\right)p_R < \lambda\overline{p}_S - c \leq 2\lambda\left(\frac{a_R}{N_B}\right)p_R$. In this case, there is a sufficiently high p_{B1} such that $\mu_B > 0$ thereafter, and $\mu_B = 0$ otherwise. Using the same arguments as above, μ_B is strictly increasing in p_{B1} for $\mu_B \in (0, 1)$.

Proof for Theorem 4.4.1: If condition $(C1)$ holds, we know from Proposition 4.4.1 that $\mu_0 < \bar{\mu}$. If $\mu_0 < \bar{\mu}$, then the platform may have an incentive to induce more workers to move. Moreover, from Lemma B.2.2 above, the platform will not induce a proportion more than $\bar{\mu}$ because inducing more workers to move requires distorting p_{B1} and the distortion is increasing in the number of workers that must move. Therefore, over the relevant range of analysis, $\tilde{N}_{A2} \leq a_S$ and $\tilde{N}_{B2} > a_R$. Hence the platform's optimal period 2 prices are $p^e_{A2} = D_S^{-1}(N_A + \mu_B N_B)$ and $p^e_{B2} = p_R$, where $D_S^{-1}(N_A + \mu_B N_B)$ is the highest price p such that $D_S(p) = N_A + \mu_B N_B$. As shown in the proof of Proposition 4.4.2, $\mu_0 > 0$ if $\lambda\overline{p}_S - c > \lambda p_R\left(\frac{2a_R}{N_i}\right)$. Therefore, condition (4.1) holds as an equality over the relevant range of analysis and can be written as shown in (4.2). Substituting for $R_{B1} + R_{B2}$ from condition (4.2), the platform profit Π_P is given by (4.3). Since $\mu_0 > 0$ and μ_B is increasing in p_{B1}, the necessary and sufficient condition for strategic surge pricing is that $\frac{d\Pi_P}{d\mu_B} > 0$ at $\mu_B = \mu_0$, which yields $-cD_S'\left(p_S^0\right) > \lambda\left(n_H + n_L\right)$.[2]

Proof for Proposition 4.4.3: We construct $D_S(p)$ from $D_R(p)$, progressively, using three successive transformations that ensure that the sufficient conditions in Theorem 4.4.1 are met. To conserve on notation, we refer to the resulting demand function after each successive step as $D_S(p)$, keeping in mind that the desired $D_S(p)$ is the one obtained after the last step. The first transformation rescales $D_R(p)$ by multiplying it with a constant larger than 1. We show that if the multiplier is large enough then not enough workers move to zone A. The second transformation shifts $D_S(p)$ to the right, if necessary, to ensure that some workers move when $p_{i1} = p_R$, i.e., $\lambda\bar{p}_S - c > 2\lambda\frac{a_R}{N_i}p_R$. The third transformation increases the demand slope, if necessary, to ensure that the inverse demand is sufficiently insensitive, such that the first-order condition in Theorem 4.4.1 is met.

Step 1 (Scaling): Let $D_S(p) = kD_R(p)$, where $k > 1$. Then $p_S = p_R$ and $a_S = ka_R$. From Proposition 1, not enough workers move iff

$$\left(n_H + n_L - ka_R\right)\left(\lambda p_R - c\right) < 2\lambda a_R p_R. \tag{B.2}$$

Note that we require $a_S = ka_R \leq n_H + n_L - a_R$ such that there is no global shortage of workers. We have that for $k = \frac{n_H+n_L-a_R}{a_R}$ such that $a_S = n_H + n_L - a_R$, the above condition holds since $\lambda p_R - c < 2\lambda p_R$. Hence, there exists $k \leq \frac{n_H+n_L-a_R}{a_R}$ such that $\mu_0 < \bar{\mu}$.

Step 2 (Right-Shift): Let

$$D_S(p) = \begin{cases} k^* D_R(p_R) & p_R \leq p \leq p_R + \delta; \\ k^* D_R(p-\delta) & \text{otherwise,} \end{cases}$$

[2]From (4.3), we have $\frac{d\Pi_P}{d\mu_B} = (1-\lambda)\left[\frac{c}{\lambda}N_B + (n_H+n_L)\frac{d}{d\mu_B}\left(D_S^{-1}\left(N_A + N_B\mu_B\right)\right)\right]$. However, $\frac{d}{d\mu_B}\left(D_S^{-1}\left(N_A + N_B\mu_B\right)\right) = \frac{N_B}{D_S'\left(D_S^{-1}\left(N_A+N_B\mu_B\right)\right)}$. Therefore, at $\mu_B = \mu_0$, $\frac{d\Pi_P}{d\mu_B} > 0 \Leftrightarrow \frac{c}{\lambda} + \frac{(n_H+n_L)}{D_S'\left(p_S^0\right)} > 0$, where $p_S^0 = D_S^{-1}\left(N_A + N_B\mu_0\right)$.

where $k^* = \frac{n_H + n_L - a_R}{a_R}$ and $\delta \geq 0$. Then, $p_S = p_R + \delta$ and $a_S = k^* a_R$, and the platform's optimal period 2 price is $p^e_{A2} = D_S^{-1}(N_A + \mu N_B) = D_R^{-1}\left(\frac{N_A + \mu N_B}{k^*}\right) + \delta$. The equilibrium worker movement condition for $p_{i1} = p_R$ is

$$\lambda \frac{2a_R p_R}{N_B(1-\mu_0)} = \lambda D_R^{-1}\left(\frac{N_A + \mu_0 N_B}{k^*}\right) + \lambda\delta - c. \tag{B.3}$$

The RHS above is increasing in δ, whereas the RHS is decreasing in μ_0 and LHS is increasing in μ_0. Therefore, the equilibrium μ_0 is strictly increasing in δ. Now, either $\mu_0 > 0$ for $\delta = 0$, in which case we take $\delta = 0$, or there exists $\delta > 0$ such that (B.3) holds as an equality at $\mu_0 = 0$. Let δ^* be such that μ_0 is greater than or just equal to 0 for $\delta = \delta^*$. Let μ_0^* denote the corresponding μ_0, and $p_0^* = D_R^{-1}\left(\frac{N_A + \mu_0^* N_B}{k^*}\right) + \delta^*$ denote the corresponding surge price in zone A in period 2.

Step 3 (Slope-Shift): Consider,

$$D_S(p) = \begin{cases} k^* D_R(p_R) & p_R \leq p \leq \frac{p_R + (t-1)p_0^* + \delta^*}{t}; \\ k^* D_R(p_0^* - \delta^* + t(p - p_0^*)) & \frac{p_R + (t-1)p_0^* + \delta^*}{t} < p \leq p_0^*; \\ k^* D_R(p - \delta^*) & \text{otherwise,} \end{cases}$$

where $t \geq 1$ is a slope shifting parameter. This transformation increases the slope of $D_S(p)$ to the left of $p = p_0^*$ without affecting μ_0, such that the inverse surge demand can be made sufficiently insensitive at this point. In particular, if $p_{i1} = p_R$ then $\mu_0 = \mu_0^*$ and $p_{A2} = p_0^*$ for any $t \geq 1$. Note that $-D_S'(p_0^*) = -k^* t D_R'(p_0^* - \delta^*)$, which is increasing in t. Hence, t can be sufficiently large if necessary such that the first-order condition in Theorem 4.4.1 is met.

APPENDIX C

PROOFS OF ADDITIONAL RESULTS IN CHAPTER 4

Proofs for Additional Results in Section 4.5

Proof for Lemma 4.5.1: To establish uniqueness, we show that $\hat{r}_{B1}(\theta)+\hat{r}_{B2}(\theta)-\hat{r}_{A2}(\theta)$ is strictly increasing in μ_B. It follows from our analysis under symmetric information that the expected revenue of workers serving zone B given that the platform is type t $(= r_{B1}(\mu_B)|_t + r_{B2}(\mu_B)|_t)$ is strictly increasing in μ_B, because having fewer workers in zone B reduces worker competition in zone B. We have

$$\hat{r}_{B1}(\theta)+\hat{r}_{B2}(\theta)=\theta\left(r_{B1}(\mu_B)|_H + r_{B2}(\mu_B)|_H\right)+(1-\theta)\left(r_{B1}(\mu_B)|_L + r_{B2}(\mu_B)|_L\right)$$

Thus, $\hat{r}_{B1}(\theta)+\hat{r}_{B2}(\theta)$ is strictly increasing in μ_B. Similarly, the expected revenue of workers serving zone A in period 2 given that the platform is type t $(r_{A2}(\mu_B)|_t)$ is strictly decreasing in μ_B, because having more workers in zone A increases worker competition in zone A. Hence $\hat{r}_{A2}(\theta)$ is strictly decreasing in μ_B. Thus, $\hat{r}_{B1}(\theta)+\hat{r}_{B2}(\theta)-\hat{r}_{A2}(\theta)$ is strictly increasing in μ_B. If the worker movement condition in (4.4) holds as an inequality at $\mu_B = 0$, then it would hold as an inequality for any $\mu_B \in [0,1]$. Therefore, the equilibrium $\mu_B = 0$. If $\hat{r}_{B1}(\theta)+\hat{r}_{B2}(\theta) < \hat{r}_{A2}(\theta)-c$ at $\mu_B = 1$, then $\hat{r}_{B1}(\theta)+\hat{r}_{B2}(\theta) < \hat{r}_{A2}(\theta)-c$ for any value of $\mu_B \in [0,1]$, and hence the equilibrium $\mu_B = 1$. Otherwise, (4.4) holds as an equality for a unique value of $\mu_B \in (0,1)$.

We next show that $\hat{r}_{B1}(\theta)+\hat{r}_{B2}(\theta)-\hat{r}_{A2}(\theta)$ is also strictly decreasing in θ. Then, since condition (4.4) must hold as an equality for $\mu_B(\theta) \in (0,1)$, it follows that $\mu_B(\theta)$ must be decreasing in θ. Note that $\tilde{N}_{Bj}(\mu_B)\Big|_H > \tilde{N}_{Bj}(\mu_B)\Big|_L$ since $n_H > n_L$. Therefore, there is more worker competition in zone B in the case of a type H platform, and we have: (i) $r_{B1}(\mu_B)|_H \leq r_{B1}(\mu_B)|_L$, where the inequality is strict iff $\tilde{N}_{B1}(\mu_B)\Big|_H > D_R(p_{B1})$, and (ii) $r_{B2}(\mu_B)|_H < r_{B2}(\mu_B)|_L$. Hence, from (4.4), $\hat{r}_{B1}(\theta)+\hat{r}_{B2}(\theta)$ is strictly decreasing in θ.

Similarly, there is less worker competition in zone A in the case if a type H platform, i.e., $\left.\tilde{N}_{A2}(\mu_B)\right|_H < \left.\tilde{N}_{A2}(\mu_B)\right|_L$, and hence $\left.r_{A2}(\mu_B)\right|_H > \left.r_{A2}(\mu_B)\right|_L$. Therefore, $\hat{r}_{A2}(\theta)$ is strictly increasing in θ.

Proof for Proposition 4.5.1: For the platform not to share market information truthfully, we require that condition $(C1)$ holds. We also require that $\mu_B(\theta = 1) > 0$ such that $\mu_B(\theta = 1) > \mu_B(\theta = 0)$ and more workers move if they believe that the platform is type H (than type L). Therefore, the equilibrium worker condition must not hold for $\mu_B = 0$ for $\theta = 1$, which yields

$$\begin{aligned}
&\left.r_{B1}(\mu_B = 0)\right|_H + \left.r_{B2}(\mu_B = 0)\right|_H < \left.r_{A2}(\mu_B = 0)\right|_H - c, \\
&\qquad \implies \lambda \frac{2a_R p_R}{n_H} < \lambda \left.\bar{p}_S\right|_H - c \implies \left.\bar{p}_S\right|_H - \frac{c}{\lambda} > 2p_R \frac{a_R}{n_H}. \quad \text{(C.1)}
\end{aligned}$$

Lastly, a sufficient condition for the type L platform to strictly benefit from misreporting its type is that there are enough workers to serve consumers in zone B even if it misreports its type: $\left.\tilde{N}_{B1}\right|_L = n_L(1 - \mu_B(\theta = 1)) > a_R$ which yields $\mu_B(\theta = 1) < 1 - \frac{a_R}{n_L}$. Since $\mu_B(\theta = 1) < \left.\bar{\mu}\right|_H$, a sufficient condition for this to hold is $\left.\bar{\mu}\right|_H < 1 - \frac{a_R}{n_L}$, which yields condition (iii).

Proof for Proposition 4.5.2: Given that truthful communication is not feasible without an accompanying surge price, we obtain the least-cost separating equilibrium by construction. Let $\boldsymbol{p}_j = (p_{Aj}, p_{Bj})$ denote the vector of prices in zone A and zone B in period j. Consider a separating equilibrium in which $\left.s\right|_t = t$, $\left.\boldsymbol{p}_1\right|_L = (p_R, p_R)$ and $\left.\boldsymbol{p}_1\right|_H = (p_R, \hat{p}_R)$ where $\hat{p}_R \geq p_R$ and $\boldsymbol{p}_2 = (p^e_{A2}, p^e_{B2})$. Workers update their belief to $\theta = 1$ if $s = H$ and $\boldsymbol{p}_1 = \left.\boldsymbol{p}_1\right|_H$, and to $\theta = 0$ otherwise. Therefore, $\hat{r}_{ij} = \left.r_{ij}\right|_t$ in equilibrium on a type t platform. Let $\left.\mu^*_B\right|_L = \mu_B(\left.\boldsymbol{p}_1\right|_L, \theta = 0)$ and $\left.\mu^*_B\right|_H = \mu_B(\left.\boldsymbol{p}_1\right|_H, \theta = 1)$ denote the equilibrium proportion of workers that move if the platform is type L and type H, respectively. Let $\left.\Pi^*_P\right|_L = \left.\Pi_P(\left.\boldsymbol{p}_1\right|_L, \theta = 0)\right|_L$ and $\left.\Pi^*_P\right|_H = \left.\Pi_P(\left.\boldsymbol{p}_1\right|_H, \theta = 1)\right|_H$ denote the corresponding equilibrium platform profits for the type L and type H platform respectively. We say $\boldsymbol{p}'_1 < \left.\boldsymbol{p}_1\right|_H$ if $\boldsymbol{p}'_1 = (p_R, p')$ and

$p_R \leq p' < \hat{p}_R$. Let $\Pi'_P\big|_L = \Pi_P\left(\boldsymbol{p}_1|_H, \theta = 1\right)\big|_L$ denote the type L platform profit if it mimics the type H platform. Let $\Pi'_P\big|_H = \Pi_P\left(\boldsymbol{p}'_1, \theta = 0\right)\big|_H$ denote the type H platform profit if it deviates to $\boldsymbol{p}'_1 < \boldsymbol{p}_1|_H$. We use $\mu_B\left(\boldsymbol{p}_1, \theta\right)$ to make the dependence of $\mu_B\left(\theta\right)$ on $\boldsymbol{p}_1$ explicit. We break the analysis into several lemmas to improve clarity.

Lemma C.0.1. *$\mu_B\left(\boldsymbol{p}_1, \theta\right)$ is unaffected by p_{A1}, and is strictly increasing in p_{B1} if $\mu_B\left(\boldsymbol{p}_1, \theta\right) > 0$.*

Proof. We characterize $\mu_B\left(\boldsymbol{p}_1, \theta\right)$ by considering how $\boldsymbol{p}_1$ affects the terms in equilibrium condition (4.4) for a given value of $\mu_B\left(\boldsymbol{p}_1, \theta\right)$. As before, $\mu_A\left(\boldsymbol{p}_1, \theta\right) = 0$, i.e., workers in the surge zone remain in the surge zone. Since p_{A1} does not affect any of the terms in (4.4), $\mu_B\left(\boldsymbol{p}_1, \theta\right)$ is not affected by p_{A1}. $\hat{r}_{B1}\left(\theta\right)$ is strictly decreasing in p_{B1}, while the other terms are unaffected. Specifically, for a given μ_B, $r_{B1}\left(\mu_B\right)\big|_t$ is strictly decreasing in p_{B1}. Since equilibrium condition (4.4) holds as an equality for $\mu_B\left(\boldsymbol{p}_1, \theta\right) > 0$ and $\hat{r}_{B1}\left(\theta\right) + \hat{r}_{B2}\left(\theta\right) - \hat{r}_{A2}\left(\theta\right)$ is strictly increasing in μ_B (see proof of Lemma 4.5.1), it follows that $\mu_B\left(\boldsymbol{p}_1, \theta\right)$ is strictly increasing in p_{B1} if $\mu_B\left(\boldsymbol{p}_1, \theta\right) > 0$. □

Lemma C.0.2. *$\mu^*_B|_L < \bar{\mu}|_L$, $\mu^*_B|_H > \mu^*_B|_L$.*

Proof. Since the type L platform will misreport its type if $\hat{p}_R = p_R$, we require: $\mu^*_B|_L < \bar{\mu}|_L$ such that not enough workers move if the platform is type L; and $\mu_B\left(\boldsymbol{p}_1|_L, \theta = 1\right) > \mu^*_B|_L \geq 0$ such that more workers move from zone B if they believe the platform is type H. Also, $\mu^*_B|_H > \mu_B\left(\boldsymbol{p}_1|_L, \theta = 1\right)$ since $\mu_B\left(\boldsymbol{p}_1|_L, \theta = 1\right) > 0$ and $\hat{p}_R > p_R$ (from Lemma C.0.1). □

Let $\bar{p}_R$ denote the price at which regular demand goes to zero, i.e., $D_R\left(\bar{p}_R\right) = 0$. Therefore, $r_{B1} = 0$ if $p_{B1} = \bar{p}_R$. For separation to be feasible using only a surge price in zone B, we require that $\Pi^*_P|_L > \Pi'_P\big|_L$ for $\hat{p}_R = \bar{p}_R$. Since truthful communication is not feasible without surge pricing, we have $\Pi^*_P|_L < \Pi'_P\big|_L$ for $\hat{p}_R = p_R$. Hence, if $\Pi^*_P|_L > \Pi'_P\big|_L$ for $\hat{p}_R = \bar{p}_R$, then it follows by continuity that there exists some $\hat{p}_R \in \left(p_R, \bar{p}_R\right)$ such that $\Pi^*_P|_L = \Pi'_P\big|_L$ such that the type L platform will not mimic the type H platform. In the following result, we obtain two sufficient conditions for $\Pi^*_P|_L < \Pi'_P\big|_L$ if $\hat{p}_R = \bar{p}_R$.

Lemma C.0.3. *$\Pi'_P\big|_L < \Pi^*_P|_L$ if $\hat{p}_R$ is sufficiently close to $\bar{p}_R$ and either $a_R p_R + n_H\ \bar{p}_S|_L > a_S p_S$ or n_L is close to n_H.*

Proof. The type L platform's equilibrium profit is

$$\Pi^*_P|_L = (1-\lambda)\left[3a_R p_R + (n_H + n_L\ \mu^*_B|_L)\, D_S^{-1}\,(n_H + n_L\ \mu^*_B|_L)\right]$$

Specifically, the platform realizes profit of $\pi_{ij} = (1-\lambda)\, a_R p_R$ except in zone A in period 2, and $\pi_{A2} = (1-\lambda)\,(n_H + n_L\ \mu^*_B|_L)\, D_S^{-1}\,(n_H + n_L\ \mu^*_B|_L)$ since $\mu^*_B|_L < \bar{\mu}|_L$. Since the RHS is strictly increasing in $\mu^*_B|_L$, we obtain a lower bound for $\Pi^*_P|_L$ by setting $\mu^*_B|_L = 0$

$$\Pi^*_P|_L \geq (1-\lambda)\left[3a_R p_R + n_H\ \bar{p}_S|_L\right] \tag{C.2}$$

Suppose $\boldsymbol{p}_1|_H = \overline{\boldsymbol{p}}_1 = (p_R, \overline{p}_R)$. Consider the type L platform's profit from deviating to $\overline{\boldsymbol{p}}_1$. Its profit in zone B in period 1 is zero since no consumers will request service. To obtain an upper bound on its profit in period 2, note that at best the platforms attains the maximum revenue in both zones, resulting in a profit of $(1-\lambda)\,(a_S p_S + a_R p_R)$. Also, its profit in period 1 in zone A, is $(1-\lambda)\, a_R p_R$. Therefore, its deviation profit

$$\Pi'_P\Big|_L \leq (1-\lambda)\left[2a_R p_R + a_S p_S\right] \tag{C.3}$$

From (C.2) and (C.3), the profit gain from the deviation is bounded by

$$\Pi^*_P|_L - \Pi'_P\Big|_L \geq (1-\lambda)\left[a_R p_R + n_H\ \bar{p}_S|_L - a_S p_S\right] \tag{C.4}$$

Therefore, if $a_R p_R + n_H\ \bar{p}_S|_L > a_S p_S$, then the RHS is strictly positive. By continuity, $\Pi^*_P|_L > \Pi'_P\big|_L$ for $\hat{p}_R$ sufficiently close to $\overline{p}_R$.

We next show that $\Pi'_P\big|_L < \Pi^*_P|_L$ if n_L is close to n_H and $\hat{p}_R$ is sufficiently close to $\bar{p}_R$. Since by construction, strategic surge pricing is unprofitable under symmetric information, we have $\Pi_P\left(\boldsymbol{p}_1|_H, \theta = 1\right)|_H$ is strictly decreasing in $\hat{p}_R$. Therefore, $\Pi^*_P|_H < \Pi_P\left(\boldsymbol{p}_1|_L, \theta = 1\right)|_H$ for $\hat{p}_R = \bar{p}_R$. Letting $n_L \to n_H$, we have $\Pi^*_P|_H \to \Pi'_P\big|_L$ and

$$\Pi_P\left(\boldsymbol{p}_1|_L, \theta = 1\right)|_H \to \Pi^*_P|_L.$$

Therefore, $\Pi'_P\big|_L < \Pi^*_P|_L$ for $\hat{p}_R = \bar{p}_R$, which is the desired result. $\square$

Thus, there exists $\hat{p}_R \in (p_R, \bar{p}_R)$ such that $\Pi^*_P|_L = \Pi'_P\big|_L$. Let $\tilde{p}_R = \min\left\{\hat{p}_R : \Pi^*_P|_L = \Pi'_P\big|_L\right\}$ be the minimum such $\hat{p}_R$ and $\tilde{\boldsymbol{p}}_1 = (p_R, \tilde{p}_R)$. Consider a separating equilibrium in which $\boldsymbol{p}_1|_H = \tilde{\boldsymbol{p}}_1$ is feasible. By construction the type L platform will not mimic the type H platform. We now show that the type H platform will not deviate to a lower price. Hence, the separating equilibrium is feasible. Lemma C.0.4 establishes bounds for the proportion of workers that move for off-equilibrium prices. Lemma C.0.5 then shows the desired result.

Lemma C.0.4. *For $\boldsymbol{p}'_1 \leq \tilde{\boldsymbol{p}}_1$, $\mu_B\left(\boldsymbol{p}'_1, \theta = 0\right) < \bar{\mu}|_L$ and $\mu_B\left(\boldsymbol{p}'_1, \theta = 1\right) < \bar{\mu}|_H$.*

Proof. Recall that by assumption, neither platform type have an incentive to use surge price under full information. Therefore, $\Pi^*_P|_L > \Pi_P\left(\boldsymbol{p}'_1, \theta = 0\right)\big|_L$ for any $\boldsymbol{p}'_1 > \boldsymbol{p}_1|_L$ (where $\boldsymbol{p}'_1 = \left(p_R, p'\right)$ such that $p' > p_R$). Since $\Pi^*_P|_L = \Pi'_P\big|_L$ when $\boldsymbol{p}_1|_H = \tilde{\boldsymbol{p}}_1$, we have $\Pi'_P\big|_L \geq \Pi_P\left(\boldsymbol{p}'_1, \theta = 0\right)\big|_L$ for any $\boldsymbol{p}'_1 < \tilde{\boldsymbol{p}}_1$ where equality holds iff $\boldsymbol{p}'_1 = \boldsymbol{p}_1|_L$. But this is possible iff there is a shortage of workers in zone A for $\boldsymbol{p}'_1 \leq \tilde{\boldsymbol{p}}_1$ and $\theta = 0$, i.e., $\mu_B\left(\boldsymbol{p}'_1, \theta = 0\right) < \bar{\mu}|_L$. Then, equilibrium worker movement condition (4.4) must not hold for $\theta = 0$, $\boldsymbol{p}_1 = \boldsymbol{p}'_1$ and $\mu_B = \bar{\mu}|_L$, which yields

$$\lambda\left[\frac{D_R\left(p'\right)p' + a_R p_R}{n_H + n_L - a_S}\right] > \lambda p_S - c.$$

Note that $\tilde{N}_{A2}\left(\mu_B = \bar{\mu}|_H\right)\Big|_H = a_S = \tilde{N}_{A2}\left(\mu_B = \bar{\mu}|_L\right)\Big|_L$. Therefore, given $\boldsymbol{p}'_1$, the terms in the equilibrium condition (4.4) are identical for $\theta = 0$, $\mu_B = \bar{\mu}|_L$ and for $\theta = 1$, $\mu_B = \bar{\mu}|_H$. Consequently, the equilibrium condition (4.4) will also not hold for $\theta = 1$, $\boldsymbol{p}_1 = p'_1$ and $\mu_B = \bar{\mu}|_H$. Hence, $\mu_B\left(\boldsymbol{p}'_1, \theta = 1\right) < \bar{\mu}|_H$. $\square$

Lemma C.0.5. *For* $\boldsymbol{p}_1|_H = \tilde{\boldsymbol{p}}_1$ *and* $\boldsymbol{p}_1^{'} < \tilde{\boldsymbol{p}}_1$*,* $\Pi_P^*|_H - \Pi_P^{'}\big|_H > 0$.

Proof. We show that $\Pi_P^*|_H - \Pi_P^{'}\big|_H > \Pi_P^{'}\big|_L - \Pi_P\left(\boldsymbol{p}_1^{'}, \theta = 0\right)\big|_L$. Since $\Pi_P^{'}\big|_L = \Pi_P^*|_L > \Pi_P\left(\boldsymbol{p}_1^{'}, \theta = 0\right)\big|_L$ (see proof of Lemma C.0.4), we obtain the desired result. Consider a change in period 1 price from $\boldsymbol{p}_1^{'}$ to $\tilde{\boldsymbol{p}}_1$. For such a change in price, let $\Delta\mu = \mu_B^*|_H - \mu_B\left(\boldsymbol{p}_1^{'}, \theta = 0\right) > 0$ denote the increase in proportion of workers in zone B that move. Let $\Delta\, \pi_{ij}|_t = \pi_{ij}\left(\tilde{\boldsymbol{p}}_1, \theta = 1\right)|_t - \pi_{ij}\left(\boldsymbol{p}_1^{'}, \theta = 0\right)\big|_t$ denote the corresponding difference in profit in zone i in period j for a type t platform. Note that $\Pi_P^*|_H - \Pi_P^{'}\big|_H = \sum \Delta\, \pi_{ij}|_H$ and $\Pi_P^*|_L - \Pi_P^{'}\big|_L = \sum \Delta\, \pi_{ij}|_L$. We will show that $\sum \Delta\, \pi_{ij}|_H \geq \sum \Delta\, \pi_{ij}|_L$ for all i, j and the inequality is strict for zone A in period 2.

Consider zone A in period 1. The platform serves a_R consumers at both the prices (i.e., at $\tilde{\boldsymbol{p}}_1$ and at $\boldsymbol{p}_1^{'}$) for either platform type. Therefore, $\Delta\, \pi_{A1}|_L = \Delta\, \pi_{A1}|_H = 0$. In period 2, if the platform sets period 1 price $\boldsymbol{p}_1^{'}$, then $\tilde{N}_{A2} < a_S$ (from Lemma C.0.4). The type H platform has fewer available workers in zone A than the type L platform. Specifically, $\tilde{N}_{A2} = (n_H + n_L) - n_t\left(1 - \mu_B\left(\boldsymbol{p}_1^{'}, \theta = 0\right)\right)$ and $\tilde{N}_{A2}\Big|_H < \tilde{N}_{A2}\Big|_L$. Now, platform profit $\pi_{A2} = (1 - \lambda)\, \tilde{N}_{A2} D_S^{-1}\left(\tilde{N}_{A2}\right)$ is strictly increasing and strictly concave in $\tilde{N}_{A2}$. Therefore, since $\tilde{N}_{A2}\Big|_H < \tilde{N}_{A2}\Big|_L$, the incremental profit from having a given number of additional workers in zone A is higher for a type H platform. Moreover, if the platform shifts to period 1 price $\tilde{\boldsymbol{p}}_1$, then the number of additional workers that move to zone A is $n_t\Delta\mu$, which is also higher for a type H platform. Therefore, $\Delta\, \pi_{A2}|_L < \Delta\, \pi_{A2}|_H$.

Next, consider zone B. In period 1, either platform type serves a_R consumers if the price is $\boldsymbol{p}_1^{'}$ since $\mu_B\left(\boldsymbol{p}_1^{'}, \theta = 0\right) < \bar{\mu}|_L$ and the platform profit is the same for both types $\left(=(1 - \lambda)\, p^{'} D_R\left(p^{'}\right)\right)$. At the price $\tilde{\boldsymbol{p}}_1$, all consumers requesting service will be served if the platform is type H since $\mu_B^*|_H < \bar{\mu}|_H$. Since $\bar{\mu}|_H < 1 - \frac{a_R}{n_L}$ (from Proposition 4.5.1), at the price $\tilde{\boldsymbol{p}}_1$, we have $\tilde{N}_{B1}\Big|_L > a_R$; the platform profit is the same for both types if a_R consumers are served. Therefore, $\Delta\, \pi_{B1}|_H = \Delta\, \pi_{B1}|_L$. In period 2, we have $\pi_{B2} = (1 - \lambda)\, a_R p_R$ if $\tilde{N}_{B2} \geq a_R$. For either period 1 price, $\tilde{N}_{B2}\Big|_H \geq a_R$ since $\mu_B\left(\boldsymbol{p}_1^{'}, \theta = 0\right) < \mu_B^*|_H < \bar{\mu}|_H$.

Therefore, $\Delta\,\pi_{B2}|_H = 0$. For the type L platform, $\tilde{N}_{B2}\Big|_L > a_R$ if the period 1 price is $\boldsymbol{p}_1^{'}$ since $\mu_B\left(\boldsymbol{p}_1^{'}, \theta = 0\right) < \bar{\mu}|_L$. Since $\bar{\mu}|_H < 1 - \frac{a_R}{n_L}$ (from Proposition 4.5.1), $\tilde{N}_{B2}\Big|_L \geq a_R$ for period 1 price $\tilde{\boldsymbol{p}}_1$, and hence $\Delta\,\pi_{B2}|_L = 0$. Therefore, $\Delta\,\pi_{B2}|_H = \Delta\,\pi_{B2}|_L$. $\square$

We now establish that the candidate separating equilibrium in which $\boldsymbol{p}_1|_H = \tilde{\boldsymbol{p}}_1$ is the unique least-cost separating equilibrium if the conditions in Propositions 4.5.1 and 4.5.2 hold. If these conditions hold, then the type H platform profit conditional on separation is decreasing in the surge price. Also, separation is not feasible for $\boldsymbol{p}_1|_H < \tilde{\boldsymbol{p}}_1$ since the type L platform will mimic. Hence, there is no separating equilibrium in which $p_{A1}|_H = p_R$ that yields higher profit for the type H platform than the candidate equilibrium. Suppose, there is an alternate least cost equilibrium in which $p_{A1}|_H > p_R$. Let $\boldsymbol{p}_1|_H = \boldsymbol{p}_1^A = \left(p_A^A, p_B^A\right)$ in this separating equilibrium, where $p_A^A > p_R$ and $p_B^A \geq p_R$. Since by assumption, neither platform type have an incentive to use surge price under full information, $\Pi_P\left(\boldsymbol{p}_1^A, \theta = 1\right)\big|_H$ is strictly decreasing in p_i^A. Therefore, for the alternate separating equilibrium to be least-cost, we require that $p_B^A < \tilde{p}_R$; otherwise, $\Pi_P\left(\boldsymbol{p}_1^A, \theta = 1\right)\big|_H > \Pi_P\left(\tilde{\boldsymbol{p}}_1, \theta = 1\right)|_H$, which is a contradiction. Let $\boldsymbol{p}_1^{'} = \left(p_{R,}p_B^A\right)$. Therefore, $\boldsymbol{p}_1^{'} < \tilde{\boldsymbol{p}}_1$. Also, $\mu_B\left(\boldsymbol{p}_1^{'}, \theta\right) = \mu_B\left(\boldsymbol{p}_1^A, \theta\right)$ from Lemma C.0.1. Note that the platform profit in zone A in period 1 is determined only by p_{A1} ($\pi_{A1} = (1-\lambda)\,p_{A1} D_R\left(p_{A1}\right)$) and the platform profit in the remaining zones and periods is determined by p_{B1} and μ_B. Therefore, $\Pi_P\left(\boldsymbol{p}_1^A, \theta\right)\big|_t$ and $\Pi_P\left(\boldsymbol{p}_1^{'}, \theta\right)\big|_t$ differ only in the profit in zone A in period 1, and this difference is independent of the platform type t.

$$\Pi_P\left(\boldsymbol{p}_1^A, \theta = 1\right)\big|_H - \Pi_P\left(\boldsymbol{p}_1^{'}, \theta = 1\right)\bigg|_H = \Pi_P\left(\boldsymbol{p}_1^A, \theta = 1\right)\big|_L - \Pi_P\left(\boldsymbol{p}_1^{'}, \theta = 1\right)\bigg|_L. \tag{C.5}$$

For separation to be feasible in the alternate equilibrium, we require $\Pi_P\left(\boldsymbol{p}_1^A, \theta = 1\right)\big|_L \leq \Pi_P^*|_L$. By construction, $\Pi_P\left(\tilde{\boldsymbol{p}}_1, \theta = 1\right)|_L = \Pi_P^*|_L$. Hence, from (C.5),

$$\Pi_P\left(\boldsymbol{p}_1^A, \theta = 1\right)\big|_H - \Pi_P\left(\boldsymbol{p}_1^{'}, \theta = 1\right)\bigg|_H \leq \Pi_P\left(\tilde{\boldsymbol{p}}_1, \theta = 1\right)|_L - \Pi_P\left(\boldsymbol{p}_1^{'}, \theta = 1\right)\bigg|_L. \tag{C.6}$$

We will show that

$$\Pi_P\left(\tilde{\boldsymbol{p}}_1,\theta=1\right)\big|_H-\Pi_P\left(\boldsymbol{p}_1^{'},\theta=1\right)\Big|_H>\Pi_P\left(\tilde{\boldsymbol{p}}_1,\theta=1\right)\big|_L-\Pi_P\left(\boldsymbol{p}_1^{'},\theta=1\right)\Big|_L \tag{C.7}$$

and therefore $\Pi_P\left(\tilde{\boldsymbol{p}}_1,\theta=1\right)\big|_H>\Pi_P\left(\boldsymbol{p}_1^{A},\theta=1\right)\big|_H$ from condition (C.6), which is the desired contradiction. Rearranging the above inequality, we need to show

$$\Pi_P\left(\tilde{\boldsymbol{p}}_1,\theta=1\right)\big|_H-\Pi_P\left(\tilde{\boldsymbol{p}}_1,\theta=1\right)\big|_L>\Pi_P\left(\boldsymbol{p}_1^{'},\theta=1\right)\Big|_H-\Pi_P\left(\boldsymbol{p}_1^{'},\theta=1\right)\Big|_L. \tag{C.8}$$

The following lemma shows that $\Pi_P\left(\boldsymbol{p}_1^{'},\theta=1\right)\big|_H-\Pi_P\left(\boldsymbol{p}_1^{'},\theta=1\right)\big|_L$ is strictly increasing in $\hat{p}_B$, which is the desired result.

Lemma C.0.6. *For $\boldsymbol{p}_1^{'}=(p_R,\hat{p}_B)\leq\tilde{\boldsymbol{p}}_1$, $\Pi_P\left(\boldsymbol{p}_1^{'},\theta=1\right)\big|_H-\Pi_P\left(\boldsymbol{p}_1^{'},\theta=1\right)\big|_L$ is strictly increasing in $\hat{p}_B$.*

Proof. Let $\mu=\mu_B\left(\boldsymbol{p}_1^{'},\theta=1\right)$. From Lemma C.0.4, we have $\mu<\bar{\mu}|_H$. Hence, $\tilde{N}_{A2}\Big|_H<a_S$, $\tilde{N}_{Bj}\Big|_H>a_R$ and

$$\Pi_P\left(\boldsymbol{p}_1^{'},\theta=1\right)\Big|_H=(1-\lambda)\left[2a_Rp_R+\hat{p}_BD_R\left(\hat{p}_B\right)+\left(n_L+n_H\mu\right)D_S^{-1}\left(n_L+n_H\mu\right)\right]$$

where the terms correspond to $\pi_{A1}|_H+\pi_{B2}|_H$, $\pi_{B1}|_H$ and $\pi_{A2}|_H$, respectively. Since $\bar{\mu}|_H<1-\frac{a_R}{n_L}$ (from Proposition 4.5.1), the type L platform will not face a shortage in zone B, i.e., $\tilde{N}_{Bj}\Big|_L>a_R$. However, either $\mu>\bar{\mu}|_L$ or $\mu\leq\bar{\mu}|_L$. If $\mu>\bar{\mu}|_L$. then $\tilde{N}_{A2}\Big|_L>a_S$ and

$$\Pi_P\left(\boldsymbol{p}_1^{'},\theta=1\right)\Big|_L=(1-\lambda)\left[2a_Rp_R+a_Sp_S+\hat{p}_BD_R\left(\hat{p}_B\right)\right]$$

where the terms correspond to $\pi_{A1}|_L+\pi_{B2}|_L$, $\pi_{A2}|_L$ and $\pi_{B1}|_L$, respectively. Therefore,

$$\Pi_P\left(\boldsymbol{p}_1^{'},\theta=1\right)\Big|_H-\Pi_P\left(\boldsymbol{p}_1^{'},\theta=1\right)\Big|_L=(1-\lambda)\left[\left(n_L+n_H\mu\right)D_S^{-1}\left(n_L+n_H\mu\right)-a_Sp_S\right].$$

The first term is the revenue in zone A for the type H platform, which is strictly increasing in μ since $\mu<\bar{\mu}|_H$. Since μ is strictly increasing in $\hat{p}_B$ (Lemma C.0.1), the above expression is strictly increasing in $\hat{p}_B$, which is the desired result. If $\mu\leq\bar{\mu}|_L$, then $\tilde{N}_{A2}\Big|_L\leq a_S$ and

$$\Pi_P\left(\boldsymbol{p}_1^{'},\theta=1\right)\Big|_L=(1-\lambda)\left[2a_Rp_R+\left(n_H+n_L\mu\right)D_S^{-1}\left(n_H+n_L\mu\right)+\hat{p}_BD_R\left(\hat{p}_B\right)\right]$$

where the terms correspond to $\pi_{A1}|_L + \pi_{B2}|_L$, $\pi_{A2}|_L$ and $\pi_{B1}|_L$, respectively. Therefore,

$$\Pi_P\left(\boldsymbol{p}_1^{'}, \theta = 1\right)\Big|_H - \Pi_P\left(\boldsymbol{p}_1^{'}, \theta = 1\right)\Big|_L =$$
$$(1-\lambda)\left[(n_L + n_H\mu)\, D_S^{-1}(n_L + n_H\mu) - (n_H + n_L\mu)\, D_S^{-1}(n_H + n_L\mu)\right].$$

The first term ($= (1-\lambda)\,\tilde{N}_{A2}(\mu)\Big|_H D_S^{-1}\left(\tilde{N}_{A2}(\mu)\Big|_H\right)$) is the profit in zone A for the type H platform, while the second term ($= (1-\lambda)\,\tilde{N}_{A2}(\mu)\Big|_L D_S^{-1}\left(\tilde{N}_{A2}(\mu)\Big|_L\right)$) is the profit in zone A for the type L platform. Since $D_S(p)$ is concave and decreasing in p, the profit for either platform type in zone A, $(1-\lambda)\,\tilde{N}_{A2} D_S^{-1}\left(\tilde{N}_{A2}\right)$, is concave and increasing in $\tilde{N}_{A2}$ and therefore in μ if $\tilde{N}_{A2} < a_S$. For a given change in μ, the gain in the profit for the type H platform is larger than the type L platform because the type H platform faces a higher shortage (since $\tilde{N}_{A2}(\mu)\Big|_H < \tilde{N}_{A2}(\mu)\Big|_L$), and more workers move for a given change in μ (since $n_H > n_L$). Therefore, this difference is increasing in μ and hence in $\hat{p}_B$ (from Lemma C.0.1). □

Proof for Proposition 4.5.3: Given period 1 prices, let $\hat{\mu}$ denote the proportion of workers that move if the platform did not offer a bonus. Condition (4.1) must hold in this subgame equilibrium, either as an equality or as an inequality. Now, suppose more workers move with bonus b_B being offered to m_B workers, such that the total number of workers that move is $M_B > N_B\hat{\mu}$. We have $\tilde{N}_{Bj} = N_B - M_B$ and $\tilde{N}_{A2} = N_A + M_B$. Note that $r_{B1} + r_{B2}$ is strictly increasing in M_B because the expected worker revenue is higher if there are fewer workers available. Similarly, r_{A2} is strictly decreasing in M_B. Therefore, condition (4.1) must hold as an inequality for any $M_B > N_B\hat{\mu}$, and none of the non-bonused workers move. Hence, either only bonused workers move and the bonus is being offered to all the workers that move, or there is no equilibrium where $M_B > N_B\hat{\mu}$ for this bonus level. In the former case, condition (4.5) holds as an equality, and defines the bonus that must be offered for M_B workers to move. In the latter case, $M_B = N_B\hat{\mu}$ in equilibrium. If $\hat{\mu} = 0$, then no workers move and the bonus is not sufficient to make any worker move. If $\hat{\mu} > 0$ and $m_B < M_B$, then condition (4.1) holds as an equality; all the m_B bonused workers move and $M_B - m_B$ non-bonused workers move. If $\hat{\mu} > 0$ and $m_B \geq M_B$, then M_B bonused workers move.

Proof for Proposition 4.5.4: We solve for the optimal period 1 price p_{B1} and bonus b_B such that $\mu_B = \mu > \mu_0$ in the subgame equilibrium. The platform profit

$$\Pi_P = (1-\lambda)\left(R_{B1} + R_{B2} + R_{A1} + R_{A2}\right) - \mu N_B b_B. \tag{C.9}$$

In any subgame equilibrium in which $\mu_B = \mu$, workers in zone B must be indifferent between moving and not moving to zone A. The equilibrium worker movement condition then is

$$\lambda \frac{R_{B1} + R_{B2}}{N_B (1-\mu)} = \lambda p_{A2} + b_B - c. \tag{C.10}$$

Notice that for a given μ, an increase in p_{B1} leads to a decrease in b_B. Substituting for b_B from (C.10) in (C.9),

$$\Pi_P = \left(R_{B1} + R_{B2}\right)\left(\frac{1-\lambda-\mu}{1-\mu}\right) + (1-\lambda)\left(R_{A1} + R_{A2}\right) + \mu N_B \left(\lambda p_{A2} - c\right). \tag{C.11}$$

Note that R_{B1} is strictly decreasing in p_{B1}, and R_{A2} is not affected by p_{B1} for a given μ. Hence, Π_P is strictly decreasing in p_{B1} iff $\mu < 1-\lambda$ and p_{B1} is such that $r_{B1} > 0$. Similarly, Π_P is strictly increasing in p_{B1} iff $\mu > 1-\lambda$ and p_{B1} is such that $r_{B1} > 0$. Otherwise, Π_P is not affected by p_{B1}.

Proof of Corollary 4.5.1: We proceed as in the proof of Proposition (4.4.3), except that we ensure that $\mu_0 > 1-\lambda$ after the second transformation step.

Step 1: Let $D_S(p) = kD_R(p)$, where $k > 1$. Then $p_S = p_R$ and $a_S = ka_R$. As before, there exists $k \leq \frac{n_H + n_L - a_R}{a_R}$ such that $\mu_0 < \bar{\mu}$.

Step 2: Let

$$D_S(p) = \begin{cases} k^* D_R(p_R) & p_R \leq p \leq p_R + \delta; \\ k^* D_R(p-\delta) & \text{otherwise,} \end{cases}$$

where $k^* = \frac{n_H+n_L-a_R}{a_R}$ and $\delta > 0$. Then, $p_S = p_R + \delta$ and $a_S = k^* a_R$, and the platform's optimal period 2 price is $p_{A2} = D_S^{-1}(N_A + \mu N_B) = D_R^{-1}\left(\frac{N_A+\mu N_B}{k^*}\right) + \delta$. As before, the equilibrium μ_0 is strictly increasing in δ. We require $\mu_0 > 1 - \lambda$. Since $\mu_0 < \bar{\mu} = \frac{a_S - N_A}{N_B} = \frac{N_B - a_R}{N_B}$, this is feasible iff $\bar{\mu} > 1 - \lambda$. Therefore, $\lambda > \frac{a_R}{N_B}$. Now, either $\mu_0 \geq 1-\lambda$ for $\delta = 0$, in which case we take $\delta = 0$, or there exists $\delta > 0$ such that $\mu_0 = 1-\lambda$. Let δ^* be such that $\mu_0 \geq 1-\lambda$ for $\delta = \delta^*$. Let μ_0^* denote the corresponding μ_0, and $p_0^* = D_R^{-1}\left(\frac{N_A+\mu_0^* N_B}{k^*}\right) + \delta^*$ denote the corresponding surge price in zone A in period 2.

Step 3: Consider,

$$D_S(p) = \begin{cases} k^* D_R(p_R) & p_R \leq p \leq \frac{p_R + (t-1)p_0^* + \delta^*}{t}; \\ k^* D_R(p_0^* - \delta^* + t(p - p_0^*)) & \frac{p_R + (t-1)p_0^* + \delta^*}{t} < p \leq p_0^*; \\ k^* D_R(p - \delta^*) & \text{otherwise,} \end{cases}$$

where $t \geq 1$ is a slope shifting parameter. As before, this transformation increases the slope of $D_S(p)$ to the left of $p = p_0^*$ without affecting μ_0, such that the inverse surge demand can be made sufficiently insensitive at this point. As before, t can be sufficiently large if necessary such that the first-order condition in Theorem 4.4.1 is met.

Proof for Proposition 4.5.5: Let $\bar{\bar{\mu}} = 1 - \frac{a_R}{N_B}$, i.e., $\bar{\bar{\mu}}$ denote the proportion of workers that move such that $\tilde{N}_{Bj} = a_R$. We first obtain conditions conditions such that $\mu_B > \bar{\bar{\mu}}$ for a subgame equilibrium in which $p_{i1} = p_R$. From Lemma B.2.1, we have $\mu_A = 0$. Consider workers in zone B. Using arguments similar to Proposition B.2.1, if $r_{B1} + r_{B2} < r_{A2} - c$ for $\mu_B = \bar{\bar{\mu}}$, then $r_{B1} + r_{B2} < r_{A2} - c$ for any $\mu_B < \bar{\bar{\mu}}$. Hence, the equilibrium $\mu_B > \bar{\bar{\mu}}$. If $\mu_B = \bar{\bar{\mu}}$, then $r_{Bj} = \lambda p_R$, and $r_{A2} = \lambda p_S \frac{a_S}{n_H+n_L-a_R}$. It follows that $r_{B1} + r_{B2} < r_{A2} - c$ iff $2\lambda p_R < \lambda p_S \frac{a_S}{n_H+n_L-a_R} - c$, i.e., $\lambda p_S > \left(\frac{n_H+n_L-a_R}{a_S}\right)(2\lambda p_R + c)$. We now develop some intermediate results.

Lemma C.0.7. *If condition $(C2)$ holds, then $\mu_B > \bar{\bar{\mu}}$ in the subgame for any $p_{i1} \geq p_R$.*

Proof. Towards a contradiction, suppose $\mu_B \leq \bar{\bar{\mu}}$ for some $p_{i1} \geq p_R$. Then, $\tilde{N}_{Bj} \geq a_R$. Therefore, $r_{B1} \leq \lambda p_{B1} \frac{D_R(p_{B1})}{a_R}$, $r_{B2} \leq \lambda p_R$ and $r_{A2} \geq \lambda p_S \frac{a_S}{N_A+N_B-a_R}$. Since $p_{B1} \geq p_R$, we have $r_{B1} \leq \lambda p_R$. Therefore, $r_{B1} + r_{B2} \leq 2\lambda p_R$ and $r_{A2} - c \geq \lambda p_S \frac{a_S}{N_A+N_B-a_R} - c$. Since $(C2)$ holds, we have $r_{B1} + r_{B2} < r_{A2} - c$, which is the desired contradiction. □

Lemma C.0.8. *If condition $(C2)$ holds, then the number of workers moving out of the non-surge zone is decreasing in the surge price in that zone up to the market-clearing price and increases thereafter, and is not influenced by the price in the surge zone in that period, where the market clearing price $p_{B1} = p_R^c$ is the unique solution to*

$$2\lambda p_{B1} = \lambda p_S \frac{a_S}{n_H + n_L - D_R(p_{B1})} - c. \tag{C.12}$$

Proof. Using arguments identical to those in Lemma B.2.1 and Proposition 4.4.2, it can be shown that μ_B is unique given p_{i1} and does not depend on p_{A1}. Let p_R^c denote the market clearing price as defined above. We first show that p_R^c is unique. The LHS of equation (C.12) is increasing in p_{B1} and the RHS is decreasing in p_{B1}. Since condition $(C2)$ holds, there exists a unique $p_{B1} > p_R$ that solves equation (C.12). We next establish how p_{B1} affects μ_B. From Lemma C.0.7, we have $\tilde{N}_{A2} > a_S$ and $\tilde{N}_{Bj} < a_R$ for any $p_{B1} \geq p_R$. Therefore, $r_{B2} = \lambda D_R^{-1}(N_B(1-\mu_B))$ and $r_{A2} = \lambda p_S \frac{a_S}{N_A+N_B\mu_B}$. We divide the analysis into three cases.

1. If $p_{B1} > p_R^c$, then we first show that $D_R(p_{B1}) < N_B(1-\mu_B)$ in the subgame equilibrium. Towards a contradiction, suppose $D_R(p_{B1}) \geq N_B(1-\mu_B)$. Then, $r_{B1} = \lambda p_{B1}$, $r_{B2} = \lambda D_R^{-1}(N_B(1-\mu_B)) \geq \lambda p_{B1}$ and $r_{A2} \leq \lambda p_S \frac{a_S}{N_A+N_B-D_R(p_{B1})}$. Since $p_{B1} > p_R^c$, we have $r_{B1} + r_{B2} > 2\lambda p_R^c$. Since $D_R(p_{B1}) < D_R(p_R^c)$, we have $r_{A2} - c < 2\lambda p_R^c$ (using (C.12)). Therefore, $r_{B1} + r_{B2} > r_{A2} - c$ for $\mu_B \geq \bar{\bar{\mu}}$, which is the desired contradiction. Therefore, $D_R(p_{B1}) < N_B(1-\mu_B)$. The worker movement condition can be expressed

as $\lambda\left(R_{B1}+R_{B2}\right)=N_B\left(1-\mu_B\right)\left(r_{A2}-c\right)$, where the LHS is decreasing in p_{B1} and increasing in μ_B, while the RHS is decreasing in μ_B. Therefore, μ_B is increasing in p_{B1}.

2. If $p_{B1}<p_R^c$, then we first show that $D_R\left(p_{B1}\right)>N_B\left(1-\mu_B\right)$ in the subgame equilibrium. Towards a contradiction, suppose $D_R\left(p_{B1}\right)\leq N_B\left(1-\mu_B\right)$. Then $r_{B1}\leq\lambda p_{B1}$, $r_{B2}=\lambda D_R^{-1}\left(N_B\left(1-\mu_B\right)\right)\leq\lambda p_{B1}$, and $r_{A2}\geq\lambda p_S\frac{a_S}{N_A+N_B-D_R(p_{B1})}$. Since $p_{B1}<p_R^c$, we have $r_{B1}+r_{B2}<2\lambda p_R^c$. Since $D_R\left(p_{B1}\right)>D_R\left(p_R^c\right)$, we have $r_{A2}-c>2\lambda p_R^c$ (using (C.12)). However, the worker movement condition is given by $r_{B1}+r_{B2}=r_{A2}-c$ for $\mu_B\geq\bar{\bar{\mu}}$, which is the desired contradiction. Therefore, $D_R\left(p_{B1}\right)>N_B\left(1-\mu_B\right)$ and $r_{B1}=\lambda p_{B1}$. Thus, r_{B1} is increasing in p_{B1}, r_{B2} is increasing μ_B, and $r_{A2}-c$ is decreasing in μ_B. Since $r_{B1}+r_{B2}=r_{A2}-c$ in equilibrium, it follows that μ_B is decreasing in p_{B1}.

3. If $p_{B1}=p_R^c$, then it follows from above that $D_R\left(p_{B1}\right)=N_B\left(1-\mu_B\right)$. Therefore, p_R^c is the unique market clearing price where the supply equals the demand in zone B.

□

We now obtain the platform's optimal pricing strategy. From Lemma C.0.7, we have $\tilde{N}_{A2}>a_S$ and $\tilde{N}_{Bj}<a_R$. Therefore, the platform always attains the maximum unconstrained revenue in zone A, i.e., $\pi_{A1}=(1-\lambda)\,p_Ra_R$ and $\pi_{A2}=(1-\lambda)\,p_Sa_S$. Thus, we focus on platform profit in zone B. If $p_{B1}\leq p_R^c$, $\tilde{N}_{Bj}$ is increasing in p_{B1} (from Lemma C.0.8). The platform profit in zone B is $\pi_{B1}+\pi_{B2}$, where $\pi_{B1}=(1-\lambda)\,p_{B1}\tilde{N}_{Bj}$ and $\pi_{B2}=(1-\lambda)\,D_R^{-1}\left(\tilde{N}_{Bj}\right)\tilde{N}_{Bj}$, both of which are increasing in p_{B1}. Therefore, Π_P is increasing in p_{B1}. If $p_{B1}\geq p_R^c$, $\tilde{N}_{Bj}$ is decreasing in p_{B1} (from Lemma C.0.8). The platform profit in zone B is $\pi_{B1}=(1-\lambda)\,p_{B1}D_R\left(p_{B1}\right)$ and $\pi_{B2}=(1-\lambda)\,D_R^{-1}\left(\tilde{N}_{Bj}\right)\tilde{N}_{Bj}$, both of which are decreasing in p_{B1}. Therefore, Π_P is decreasing in p_{B1}. Thus, the equilibrium price in zone B is $p_{B1}=p_R^c$, which is the unique market-clearing price.

Results for Period 2 Revenues Weighted by $k > 1$

Our analysis and results are qualitatively unchanged. The specific conditions for some results change as shown below.

Table C.1. How Weighting Period 2 Revenues by $k > 1$ Affects Results

Result	**Condition in Original Model**	**Condition in Model with Weighted Revenue**
Proposition 4.4.1	$\lambda p_S \leq c$, or $n_H + n_L < a_S + 2a_R \frac{\lambda p_R}{\lambda p_S - c}$.	$\lambda p_S \leq \frac{c}{k}$, or $n_H + n_L < a_S + \left(\frac{k+1}{k}\right) a_R \frac{\lambda p_R}{\lambda p_S - \frac{c}{k}}$.
Proposition 4.4.2	$\lambda \bar{p}_S - c > \lambda \frac{a_R}{N_B} p_R$	$\lambda \bar{p}_S - \frac{c}{k} > \lambda \frac{a_R}{N_B} p_R$
Theorem 4.4.1	$\lambda \bar{p}_S - c > 2\lambda \frac{a_R}{N_B} p_R$, and $-cD_S^{'}\left(p_S^0\right) > \lambda \left(n_H + n_L\right)$.	$\lambda \bar{p}_S - \frac{c}{k} > \left(\frac{k+1}{k}\right) \lambda \frac{a_R}{N_B} p_R$, and $-\frac{c}{k} D_S^{'}\left(p_S^0\right) > \lambda \left(n_H + n_L\right)$.
Proposition 4.5.1	(i) Condition in Proposition 4.4.1; (ii) $\bar{p}_S\vert_H - \frac{c}{\lambda} > 2p_R \frac{a_R}{n_H}$; (iii) $1 - \frac{a_R}{n_L} > \frac{a_S - n_L}{n_H}$	(i) Modified condition above in Proposition 4.4.1; (ii) $\bar{p}_S\vert_H - \frac{c}{k\lambda} > \left(\frac{k+1}{k}\right) p_R \frac{a_R}{n_H}$; (iii) No Change
Proposition 4.5.2	$a_R p_R + n_H \, \bar{p}_S\vert_L > a_S p_S$, or n_L is sufficiently close to n_H.	$\frac{1}{k} a_R p_R + n_H \, \bar{p}_S\vert_L > a_S p_S$, or n_L is sufficiently close to n_H.
Proposition 4.5.5	$\lambda p_S > \left(\frac{n_H + n_L - a_R}{a_S}\right) \left(2\lambda p_R + c\right)$	$\lambda p_S > \left(\frac{n_H + n_L - a_R}{a_S}\right) \left(\left(\frac{k+1}{k}\right) \lambda p_R + \frac{c}{k}\right)$

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Harish Guda was born in Nellore, India. After completing high school, he joined the Indian Institute of Technology, Madras, in Fall 2009 to pursue an undergraduate degree in Mechanical Engineering. Subsequently, he pursued his doctoral studies in Management Science at The University of Texas at Dallas (UTD). His work has been accepted for publication in the journals *Production and Operations Management* and *Management Science*. His work has also been recognized by the POMS College of Supply Chain Management in their Student Paper Competitions in 2018 and 2019.

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