# SUPPLY CHAIN WITH DISRUPTION RISKS, STRATEGIC PLAYERS, AND DISOBEYING PLAYERS 

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Thank you to my academic adviser
who guided me in this process and the committee who kept me on track.

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by<br>XI SHAN, BA<br>\section*{DISSERTATION}<br>Presented to the Faculty of The University of Texas at Dallas in Partial Fulfillment of the Requirements for the Degree of

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# SUPPLY CHAIN WITH DISRUPTION RISKS, STRATEGIC PLAYERS, AND DISOBEYING PLAYERS 

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We consider three problems in supply chain management. First one is a single period problem where a retailer sources from a supplier, whose reliability is private information and whose efforts to improve reliability is unobservable (hidden action). Second one is a problem of a retailer who orders from competing strategic suppliers subject to independent or correlated disruptions, and responds by setting the retail price upon delivery, called responsive pricing. The suppliers set their wholesale prices in a Nash game. Finally, we develop a model where firms decide to disobey some regulating rules by considering economic, moral, as well as behavioral factors in their decisions.

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## CHAPTER 1

## INTRODUCTION

We consider three problems in supply chain management in this dissertation. In the first problem, two research questions are focused in which there is one retailer and one supplier based on this problem. The first question considers how to obtain better information about the supplier to assess the suppliers reliability and hedge against the supply uncertainty. The second question considers how the right amount of incentive can be implemented to make the supplier exert a certain level of effort to improve his reliability. This chapter considers a single-period problem in which a retailer sources from a supplier whose reliability is private information and efforts to improve reliability are unobservable (hidden actions). At the beginning of the game, the retailer offers the supplier a contract menu with a transfer payment, order quantity, and unit penalty rate for nondelivery for each reliability, which is modeled as the type of the supplier, and the supplier chooses a contract that most benefits him. As the supplier chooses a contract, he also decides the optimal level of effort to invest in reliability improvement.

In the second problem, we investigate a supply chain consisting of a responsive-pricing retailer sourcing from multiple suppliers having different production costs and reliabilities. The reliability of a supplier is modeled by the probability with which he defaults on the retailer's order. Moreover, the suppliers' default events may or may not be independent. The retailer's procurement problem is formulated as a Stackelberg-Nash game in which the suppliers are the Stackelberg leaders by setting their wholesale prices simultaneously in a Nash game, and the retailer is the Stackelberg follower when determining her respective order quantities from the suppliers ex-ante (i.e, before the default events) and retail price ex-post.

In the third chapter, we introduce a new decision variable termed the disobedience level of a given rule by each firm engaged in a Cournot competition along with parameters of moral
standard and punishment. Thus the firms choose not only their production quantities but also their levels of disobedience in a Nash equilibrium setting. We solve the game to obtain these decisions and then study the amount of punishment to reduce the disobedience levels and their effect on the firms profits. We identify the situations of Creative delinquency and Destructive Selfishness, respectively, as those with firms disobediences causing an increase in surplus and those with a firms disobedience benefiting the self and hurting the other in way that decreases surplus. After that, we consider the behavioral factor of unfairness the firms perceive over the rule as well as the disobedience action of a firm induced by rivals disobedience behavior. Finally, we consider obtaining an optimal rule that can be imposed by a regulatory authority.

## CHAPTER 2

## RELIABILITY IMPROVEMENT UNDER INFORMATION ASYMMETRY

### 2.1 Introduction

As manufacturing companies outsource more parts to concentrate on their own core competitiveness and firms become less vertically integrated than they were in the past, they increasingly expect their suppliers to deliver products on time. However, it is impossible for suppliers to meet this need when they experience supply chain disruptions due to natural disasters, cyber-attacks, or delivery uncertainties. Thus, there is a growing recognition of the importance of activities a buyer can undertake to improve the reliability of suppliers, which is one of the main considerations in supplier development (Krause et al. 2007).

To improve suppliers performance in terms of higher order fulfillment reliability, both Honda and Toyota invest significant resources in their suppliers, as is common in the automotive industry (Liker and Choi 2004). When a suppliers financial state is related to its ability to fulfill orders, a buyer can improve the suppliers reliability by using subsidy funds, which works for the suppliers liability reduction or asset investment (Babich 2010).

However, information asymmetry becomes the main obstacle between a retailer and an outsourced supplier, when the retailer intends to improve the reliability of the supplier. A supplier usually refuses to share information about its reliability (Yang et al. 2009), resource allocation and capabilities, and other sensitive and confidential information. For example, Solectron has difficulty capturing suppliers status changes on time because their suppliers information systems are not integrated with theirs.

Little attention has been given to incentive mechanisms designed to mitigate supply disruption risks at the upstream end of a supply chain under information asymmetry. In this chapter, two research questions are focused in which there is one retailer (principal, female) and one supplier (agent, male) based on this problem. The first question considers
how to obtain better information about the supplier to assess the suppliers reliability and hedge against the supply uncertainty. The second question considers how the right amount of incentive can be implemented to make the supplier exert a certain level of effort to improve his reliability.

Yang et al. (2009) studied the optimal procurement contract when the suppliers reliability is private information. This chapter differs from Yang et al.s (2009) work in the following ways. First, this chapter relaxes their assumption that a supplier cannot improve his reliability. I assume that the suppliers reliability improvement effort is a hidden action and cannot be observed by the buyer, and each level of effort is associated with an improvement cost. In this way, it can be deemed that the reliability improving cost is always infinitely high in Yang et al.s (2009) model. Second, in order to focus on the impact of the suppliers asymmetric information and hidden action on the disruption risk mitigation strategy of the buyer, this chapter does not consider a back-up option.

Several studies have examined firms decisions regarding efforts to mitigate supply disruption risks. Wang et al. (2010) explored a model in which a buyer can exert effort to improve supplier reliability or source from multiple suppliers and compared the two mitigation strategies under both random capacity and random yield. Hu et al. (2013) studied buyers incentive mechanisms to motivate a suppliers investment in capacity restoration and compares this approach with the traditional approach of diversifying part of the order to an expensive but reliable supplier. Babich (2010) examined manufacturers joint capacity reservation and financial subsidy decisions facing a risky supplier that could file for bankruptcy. Differing from the studies above, this chapter considers the supplier a game player whose reliability is private information and improvement efforts are hidden actions.

This chapter is also related to agency models in economics. A advanced treatment of the topic is provided in Dewatripont (2005). This basic model resembles models used to study the regulation of monopolists with unknown costs (Baron and Myerson 1982).

In this chapter I (who is the single author of this chapter) consider a single-period problem in which a retailer sources from a supplier whose reliability is private information and efforts to improve reliability are unobservable (hidden actions). At the beginning of the game, the retailer offers the supplier a contract menu with a transfer payment, order quantity, and unit penalty rate for nondelivery for each reliability, which is modeled as the type of the supplier, and the supplier chooses a contract that most benefits him. As the supplier chooses a contract, he also decides the optimal level of effort to invest in reliability improvement.

### 2.2 Model and Analysis

Consider a supply chain with a (female) risk-neutral retailer who sources products from a (male) risk-neutral supplier and sells them to customers in a single selling season. Let $\theta_{i}$ denote the reliability of the supplier which can be high or low, where $i=H, L$. Without loss of generality, I assume $\theta_{H}>\theta_{L}$. Let $\alpha$ denote the fraction of low-type suppliers among all suppliers. The supplier is unreliable because his production is subject to a random disruption. If a supplier has a reliability $\theta \in(0,1)$, his random yield of production is subject to a Bernoulli random variable $\rho(\theta)$ with success probability $\theta$ as follows:

$$
\rho(\theta)= \begin{cases}1, & \text { with probability } \theta \\ 0, & \text { with probability } 1-\theta\end{cases}
$$

Let $c$ denote the unit production cost of the supplier. Note that I normalize the unit production cost of the high-type and low-type supplier to be the same. Let demand be deterministic and denoted as $D=1$. Let $r$ denote the unit retail price of the product. The contract offered by the retailer to the supplier is given by $\left(X_{i}, q_{i}, p_{i}\right)$, where $X_{i}$ is the transfer payment to supplier type- $\theta_{i}, q_{i}$ is the order quantity from supplier type- $\theta_{i}$ and $p_{i}$ is the penalty rate if the supplier fails to deliver.

In the following, I first consider the problem under the integrated system, in which there is only one player in this channel. Then I consider the first-best and second-best solutions
when the supplier and retailer belong to different business groups. After that, I consider the problem when reliability improvement is allowed.

### 2.3 Integrated System

In the integrated system, there is only one player in this channel. When the reliability for this system is $\theta_{i}$, the system maximizes his integrated channel profit by choosing the size of regular production $z_{i}$. Since the supplier has disruption risks, the selling amount is a random variable $\rho\left(\theta_{i}\right) z_{i}$. The system's problem is as follows.

$$
\pi_{I}=\max _{z_{i} \geq 0} E\left\{r \cdot \min \left\{D, \rho\left(\theta_{i}\right) z_{i}\right\}-c z_{i}\right\}
$$

Since $\min \left\{D, z_{i}\right\}=\min \left\{1, z_{i}\right\} \leq 1$, it is never optimal to choose $z_{i}>1$. Thus, the constraint $z_{i} \leq 1$ is present, as well as $E\left\{r \cdot \min \left\{D, \rho\left(\theta_{i}\right) z_{i}\right\}-c z_{i}\right\}=E\left\{r \cdot \rho\left(\theta_{i}\right) z_{i}-c z_{i}\right\}=$ $\left(\theta_{i} r-c\right) z_{i}$. Thus, the following theorem is applicable.

Theorem 1. When the reliability of the system is $\theta_{i}, i=H, L, z_{i}^{*}=1$ if $\theta_{i} \geq \frac{c}{r}$, and $z_{i}^{*}=0$ otherwise.

The system chooses not to produce if the systems reliability is too low, which means that doing business is unprofitable. Next, this chapter discusses whether this contract can also coordinate the system when the supply chain members belong to different groups.

### 2.4 First-best

In this section, it is assumed that the suppliers reliability is public information to make a comparison with the second-best case. For a given contract ( $X_{i}, q_{i}, p_{i}$ ), the supplier's problem is:

$$
\pi_{S}=\max _{z_{i} \geq 0}\left\{X_{i}-c z_{i}-E\left[p_{i}\left(q_{i}-\rho\left(\theta_{i}\right) z_{i}\right)^{+}\right]\right\}
$$

where the revenue comes from the fixed payment offered by the retailer and the total cost is composed of the production cost and the possible total penalty; $\left(q_{i}-\rho\left(\theta_{i}\right) z_{i}\right)$ is a random variable and the following should apply:

$$
\begin{gathered}
X_{i}-c z_{i}-E\left[p_{i}\left(q_{i}-\rho\left(\theta_{i}\right) z_{i}\right)^{+}\right] \\
=X_{i}-c z_{i}-\left(1-\theta_{i}\right) p_{i} q_{i}-\theta_{i} p_{i}\left(q_{i}-z_{i}\right)^{+} .
\end{gathered}
$$

Since $z_{i} \leq q_{i}$, the following should apply:

$$
\begin{gathered}
X_{i}-c z_{i}-\left(1-\theta_{i}\right) p_{i} q_{i}-\theta_{i} p_{i}\left(q_{i}-z_{i}\right) \\
=X_{i}-c z_{i}-p_{i} q_{i}+\theta_{i} p_{i} z_{i} \\
=X_{i}-p_{i} q_{i}+\left(\theta_{i} p_{i}-c\right) z_{i} .
\end{gathered}
$$

If $\theta_{i} p_{i}-c \geq 0$, then $z_{i}^{*}=q_{i}$. If $\theta_{i} p_{i}-c<0$ then $z_{i}^{*}=0$. This result is summarized as follows.

Lemma 2. When the reliability of the supplier is $\theta_{i}, i=H, L, z_{i}^{*}=q_{i}$ if $\theta_{i} \geq \frac{c}{p_{i}}$, and $z_{i}^{*}=0$ otherwise.

Note that those are only the best responses of a supplier for a given contract ( $X_{i}, q_{i}, p_{i}$ ). Next, the retailers optimal contract by solving the retailers problem is identified. The retailers problem is:

$$
\begin{gathered}
\pi_{R}=\max _{X_{i}, q_{i}, p_{i} \geq 0} E\left\{r \cdot \min \left\{D, \rho\left(\theta_{i}\right) z_{i}\right\}-X_{i}+p_{i} E\left(q_{i}-\rho\left(\theta_{i}\right) z_{i}\right)\right\} \\
\text { s.t. } \pi_{S} \geq 0
\end{gathered}
$$

where the retailers profit is composed of selling revenue from the market and the possible penalty. The selling amount from the market is the minimum demand and quantity delivered. The retailer either receives revenue from the market or receives a penalty from the supplier. The retailers cost is the transfer payment to the supplier. The retailer needs to guarantee that
the supplier receives a non-negative profit. The retailer can always subtract the supplier's profit by choosing $X_{i}=p_{i} q_{i}-\left(\theta_{i} p_{i}-c\right) z_{i}$, where the retailer obtains the channel profit. Thus, this contract helps the retailer achieve channel efficiency.

Since $E\left\{r \cdot \min \left\{D, \rho\left(\theta_{i}\right) z_{i}\right\}-X_{i}+p_{i} E\left(q_{i}-\rho\left(\theta_{i}\right) z_{i}\right)\right\}=r \theta_{i} z_{i}-p_{i} q_{i}+\left(\theta_{i} p_{i}-c\right) z_{i}+$ $\left(1-\theta_{i}\right) p_{i} q_{i}+\theta_{i} p_{i}\left(q_{i}-z_{i}\right)$, there are two cases to be considered. First, if $\theta_{i} \geq \frac{c}{p_{i}}$, then $r \theta_{i} z_{i}-p_{i} q_{i}+\left(\theta_{i} p_{i}-c\right) z_{i}+\left(1-\theta_{i}\right) p_{i} q_{i}+\theta_{i} p_{i}\left(q_{i}-z_{i}\right)=r \theta_{i} q_{i}-c q_{i}$, and thus $q_{i}=1, p_{i}=r$, $X_{i}=c, \pi_{R}=r \theta_{i}-c$. Note that $p_{i}$ may have more than one results and without loss of generality we let $p_{i}=r$. Second, if $\theta_{i}<\frac{c}{p_{i}}$ then $q_{i}=0, p_{i}<\theta_{i} c$ and $X_{i}=0$. The retailer's optimal contract is summarized as follows.

Theorem 3. When the reliability of the supplier is $\theta_{i}, i=H, L,\left(X_{i}^{*}, q_{i}^{*}, p_{i}^{*}\right)=(c, 1, r)$ if $\theta_{i} \geq \frac{c}{r}$, and the retailer leaves the market otherwise.

The theorem above shows that the supplier is excluded from the market if production is not profitable due to low reliability. This will result in the following three cases: the retailer orders from both types of suppliers, the retailer only orders from the high-type supplier, and the retailer does not order from any supplier.

### 2.5 Second-best

In this section, the suppliers initial reliability is private information. In this way, the retailer is not able to directly distinguish between the two types of suppliers, and thus must use a contract menu to screen the suppliers types. This problem is classified as an adverse selection problem. The game sequence is defined as follows: (1) Nature reveals the type to the supplier; (2) The retailer offers a contract menu in which $\left(X_{i}, q_{i}, p_{i}\right)$ is designed for a type- $\theta_{i}$ supplier; (3) The type- $\theta_{i}$ supplier chooses a contract and decides his optimal production size.

Let $\pi_{S}\left(\theta_{i}, \theta_{j}\right)$ denote the optimal profit of type- $\theta_{i}$ supplier when he chooses a contract designed for type- $\theta_{j}$. First, the supplier chooses the optimal effort level and production size
for a given contract $\left(X_{j}, q_{j}, p_{j}\right)$. The supplier's problem is as follows:

$$
\max _{z_{i} \geq 0}\left\{X_{j}-c z_{i}-E\left[p_{j}\left(q_{j}-\rho\left(\theta_{i}\right) z_{i}\right)^{+}\right]\right\}
$$

Therefore, all possible combinations for $i, j=H, L$ are as follows.
(1) $\pi_{S}\left(\theta_{L}, \theta_{L}\right)=X_{L}-p_{L} q_{L}+\left(\theta_{L} p_{L}-c\right) z_{i}$. If $\theta_{L} p_{L}-c \geq 0, z_{i}=q_{L}$, the optimal profit is $X_{L}-p_{L} q_{L}+\left(\theta_{L} p_{L}-c\right) q_{L}$; if $\theta_{L} p_{L}-c<0, z_{i}=0$, the optimal profit is $X_{L}-p_{L} q_{L}$.
(2) $\pi_{S}\left(\theta_{H}, \theta_{L}\right)=X_{L}-p_{L} q_{L}+\left(\theta_{H} p_{L}-c\right) z_{i}$. If $\theta_{H} p_{L}-c \geq 0, z_{i}=q_{L}$, the optimal profit is $X_{L}-p_{L} q_{L}+\left(\theta_{H} p_{L}-c\right) q_{L}$; if $\theta_{H} p_{L}-c \geq 0, z_{i}=q_{L}$, the optimal profit is $X_{L}-p_{L} q_{L}+$ $\left(\theta_{H} p_{L}-c\right) q_{L}$.
(3) $\pi_{S}\left(\theta_{H}, \theta_{H}\right)=X_{H}-p_{H} q_{H}+\left(\theta_{H} p_{H}-c\right) z_{i}$. If $\theta_{H} p_{H}-c \geq 0, z_{i}=q_{H}$, the optimal profit is $X_{H}-p_{H} q_{H}+\left(\theta_{H} p_{H}-c\right) q_{H}$; if $\theta_{H} p_{H}-c<0, z_{i}=0$, the optimal profit is $X_{H}-p_{H} q_{H}$.
(4) $\pi_{S}\left(\theta_{L}, \theta_{H}\right)=X_{H}-p_{H} q_{H}+\left(\theta_{L} p_{H}-c\right) z_{i}$. If $\theta_{L} p_{H}-c \geq 0, z_{i}=q_{H}$, the optimal profit is $X_{H}-p_{H} q_{H}+\left(\theta_{L} p_{H}-c\right) q_{H}$; if $\theta_{L} p_{H}-c<0, z_{i}=0$, the optimal profit is $X_{H}-p_{H} q_{H}$.

The equations above help the retailer consider her constraints. The retailers contract menu must guarantee that each supplier receives a non-negative profit; thus, $\pi_{S}\left(\theta_{H}, \theta_{H}\right) \geq 0$ $(\mathrm{IRH})$ and $\pi_{S}\left(\theta_{L}, \theta_{L}\right) \geq 0(\mathrm{IRL})$. If the suppliers are truthful, then $\pi_{S}\left(\theta_{H}, \theta_{H}\right) \geq \pi_{S}\left(\theta_{H}, \theta_{L}\right)$ (ICH) and $\pi_{S}\left(\theta_{L}, \theta_{L}\right) \geq \pi_{S}\left(\theta_{L}, \theta_{H}\right)$ (ICL). In the final result, IRL and ICH bind, and then IRH and ICL bind automatically. The retailers problem is solved by the following details. It is assumed that if $p_{H} q_{H} \geq p_{L} q_{L}$, after the solution is obtained, this condition is confirmed to hold.

Case (1): $\frac{c}{\theta_{H}}<\frac{c}{\theta_{L}} \leq p_{L} \leq p_{H}$. In this case, $\pi_{S}\left(\theta_{L}, \theta_{L}\right)=X_{L}-p_{L} q_{L}+\left(\theta_{L} p_{L}-c\right) q_{L}$, $z_{L}=q_{L}, \pi_{S}\left(\theta_{H}, \theta_{L}\right)=X_{L}-p_{L} q_{L}+\left(\theta_{H} p_{L}-c\right) q_{L}, \pi_{S}\left(\theta_{H}, \theta_{H}\right)=X_{H}-p_{H} q_{H}+\left(\theta_{H} p_{H}-c\right) q_{H}$, $z_{H}=q_{H}, \pi_{S}\left(\theta_{L}, \theta_{H}\right)=X_{H}-p_{H} q_{H}+\left(\theta_{L} p_{H}-c\right) q_{H}$

The retailer's problem is as follows:

$$
\pi_{R}=\max _{X, q, p \geq 0}\left\{\begin{array}{c}
(1-\alpha) E\left\{r \cdot \min \left\{D, \rho\left(\theta_{H}\right) z_{H}\right\}-X_{H}+p_{H} E\left(q_{H}-\rho\left(\theta_{H}\right) z_{H}\right)\right\} \\
+\alpha E\left\{r \cdot \min \left\{D, \rho\left(\theta_{L}\right) z_{L}\right\}-X_{L}+p_{L} E\left(q_{L}-\rho\left(\theta_{L}\right) z_{L}\right)\right\}
\end{array}\right\}
$$

or

$$
\begin{gathered}
\max _{X, q, p \geq 0}\left\{\begin{array}{c}
(1-\alpha)\left(r \theta_{H} q_{H}-X_{H}+\left(1-\theta_{H}\right) p_{H} q_{H}\right) \\
+\alpha\left(r \theta_{L} q_{L}-X_{L}+\left(1-\theta_{L}\right) p_{L} q_{L}\right)
\end{array}\right\} \\
\text { s.t. } X_{L}-p_{L} q_{L}+\left(\theta_{L} p_{L}-c\right) q_{L} \geq 0 ; \\
X_{H}-p_{H} q_{H}+\left(\theta_{H} p_{H}-c\right) q_{H} \geq 0 ; \\
X_{L}-p_{L} q_{L}+\left(\theta_{L} p_{L}-c\right) q_{L} \geq X_{H}-p_{H} q_{H}+\left(\theta_{L} p_{H}-c\right) q_{H} ; \\
X_{H}-p_{H} q_{H}+\left(\theta_{H} p_{H}-c\right) q_{H} \geq X_{L}-p_{L} q_{L}+\left(\theta_{H} p_{L}-c\right) q_{L} .
\end{gathered}
$$

If IRL and ICH bind, then IRH and ICL bind automatically. Now the retailers problem is as follows:

$$
\begin{gathered}
\max _{X, q, p \geq 0}\left\{\begin{array}{c}
(1-\alpha)\left(r \theta_{H} q_{H}-X_{H}+\left(1-\theta_{H}\right) p_{H} q_{H}\right) \\
+\alpha\left(r \theta_{L} q_{L}-X_{L}+\left(1-\theta_{L}\right) p_{L} q_{L}\right)
\end{array}\right\} \\
\text { s.t. } X_{L}-p_{L} q_{L}+\left(\theta_{L} p_{L}-c\right) q_{L}=0 ; \\
X_{H}-p_{H} q_{H}+\left(\theta_{H} p_{H}-c\right) q_{H}=X_{L}-p_{L} q_{L}+\left(\theta_{H} p_{L}-c\right) q_{L} .
\end{gathered}
$$

or

$$
\begin{gathered}
\max _{X, q, p \geq 0}\left\{\begin{array}{c}
(1-\alpha)\left(r \theta_{H} q_{H}-X_{H}+\left(1-\theta_{H}\right) p_{H} q_{H}\right) \\
+\alpha\left(r \theta_{L} q_{L}-X_{L}+\left(1-\theta_{L}\right) p_{L} q_{L}\right)
\end{array}\right\} \\
\text { s.t. } X_{L}=p_{L} q_{L}-\left(\theta_{L} p_{L}-c\right) q_{L} \\
X_{H}=p_{H} q_{H}-\left(\theta_{H} p_{H}-c\right) q_{H}+\left(\theta_{H}-\theta_{L}\right) p_{L} q_{L} .
\end{gathered}
$$

or

$$
\begin{gathered}
\max _{X, q, p \geq 0}\left\{\begin{array}{c}
(1-\alpha)\left(r \theta_{H} q_{H}-X_{H}+\left(1-\theta_{H}\right) p_{H} q_{H}\right) \\
+\alpha\left(r \theta_{L} q_{L}-X_{L}+\left(1-\theta_{L}\right) p_{L} q_{L}\right)
\end{array}\right\} \\
\text { s.t. } X_{L}=p_{L} q_{L}-\left(\theta_{L} p_{L}-c\right) q_{L} ; \\
X_{H}=p_{H} q_{H}-\left(\theta_{H} p_{H}-c\right) q_{H}+\left(\theta_{H}-\theta_{L}\right) p_{L} q_{L} .
\end{gathered}
$$

or

$$
\max _{X, q, p \geq 0}\left\{\begin{array}{c}
(1-\alpha)\left(r \theta_{H} q_{H}-X_{H}+\left(1-\theta_{H}\right) p_{H} q_{H}\right) \\
+\alpha\left(r \theta_{L} q_{L}-X_{L}+\left(1-\theta_{L}\right) p_{L} q_{L}\right)
\end{array}\right\}
$$

or

$$
\max _{q, p \geq 0}\left\{\begin{array}{c}
(1-\alpha)\left(r \theta_{H} q_{H}-c q_{H}\right)+\alpha\left(r \theta_{L} q_{L}-c q_{L}\right) \\
-(1-\alpha)\left(\theta_{H}-\theta_{L}\right) p_{L} q_{L}
\end{array}\right\}
$$

The result gives $p_{L}^{*}=\frac{c}{\theta_{L}}, q_{H}^{*}=1, p_{H}^{*} \geq \frac{c}{\theta_{L}}, q_{L}^{*}=1$ if $\theta_{L} \geq \frac{c}{r}\left[1+\frac{1-\alpha}{\alpha} \frac{\theta_{H}-\theta_{L}}{\theta_{L}}\right]$ and $q_{L}^{*}=0$ if $\theta_{L}<\frac{c}{r}\left[1+\frac{1-\alpha}{\alpha} \frac{\theta_{H}-\theta_{L}}{\theta_{L}}\right]$. If $\theta_{L} \geq \frac{c}{r}\left[1+\frac{1-\alpha}{\alpha} \frac{\theta_{H}-\theta_{L}}{\theta_{L}}\right], q_{L}^{*}=1$, the retailer's profit is $(1-\alpha)\left(r \theta_{H}-c\right)+\alpha\left(r \theta_{L}-c\right)-(1-\alpha)\left(\theta_{H}-\theta_{L}\right) \frac{c}{\theta_{L}}$; If $\theta_{L}<\frac{c}{r}\left[1+\frac{1-\alpha}{\alpha} \frac{\theta_{H}-\theta_{L}}{\theta_{L}}\right], q_{L}^{*}=0$, the retailer profit's is $(1-\alpha)\left(r \theta_{H}-c\right)$. And $p_{H}^{*} q_{H}^{*} \geq p_{L}^{*} q_{L}^{*}$ is verified.

Case (2): $\frac{c}{\theta_{H}} \leq p_{L}<\frac{c}{\theta_{L}} \leq p_{H} . \pi_{S}\left(\theta_{L}, \theta_{L}\right)=X_{L}-p_{L} q_{L}, z_{L}=0, \pi_{S}\left(\theta_{H}, \theta_{L}\right)=$ $X_{L}-p_{L} q_{L}+\left(\theta_{H} p_{L}-c\right) q_{L}, \pi_{S}\left(\theta_{H}, \theta_{H}\right)=X_{H}-p_{H} q_{H}+\left(\theta_{H} p_{H}-c\right) q_{H}, \pi_{S}\left(\theta_{L}, \theta_{H}\right)=X_{H}-$ $p_{H} q_{H}+\left(\theta_{L} p_{H}-c\right) q_{H}$.

The retailer's problem becomes:

$$
\max _{X, q, p \geq 0}\left\{\begin{array}{c}
(1-\alpha) E\left\{r \cdot \min \left\{D, \rho\left(\theta_{H}\right) z_{H}\right\}-X_{H}+p_{H} E\left(q_{H}-\rho\left(\theta_{H}\right) z_{H}\right)\right\} \\
+\alpha E\left\{r \cdot \min \left\{D, \rho\left(\theta_{L}\right) z_{L}\right\}-X_{L}+p_{L} E\left(q_{L}-\rho\left(\theta_{L}\right) z_{L}\right)\right\}
\end{array}\right\}
$$

or

$$
\begin{gathered}
\max _{X, q, p \geq 0}(1-\alpha)\left(r \theta_{H} q_{H}-X_{H}+\left(1-\theta_{H}\right) p_{H} q_{H}\right)+\alpha\left(-X_{L}+p_{L} q_{L}\right) \\
\text { s.t. } X_{L}-p_{L} q_{L} \geq 0 \\
X_{H}-p_{H} q_{H}+\left(\theta_{H} p_{H}-c\right) q_{H} \geq 0 \\
X_{L}-p_{L} q_{L} \geq X_{H}-p_{H} q_{H}+\left(\theta_{L} p_{H}-c\right) q_{H} \\
X_{H}-p_{H} q_{H}+\left(\theta_{H} p_{H}-c\right) q_{H} \geq X_{L}-p_{L} q_{L}+\left(\theta_{H} p_{L}-c\right) q_{L} .
\end{gathered}
$$

If IRL and ICH bind then $X_{H}-p_{H} q_{H}+\left(\theta_{L} p_{H}-c\right) q_{H} \leq X_{H}-p_{H} q_{H}+\left(\theta_{H} p_{H}-c\right) q_{H}=$ $X_{L}-p_{L} q_{L}$ and thus IRH and ICL bind automatically. Now the retailer's problem becomes

$$
\begin{gathered}
\max _{X, q, p \geq 0}(1-\alpha)\left(r \theta_{H} q_{H}-X_{H}+\left(1-\theta_{H}\right) p_{H} q_{H}\right)+\alpha\left(-X_{L}+p_{L} q_{L}\right) \\
\text { s.t. } X_{L}-p_{L} q_{L}=0 \\
X_{H}-p_{H} q_{H}+\left(\theta_{H} p_{H}-c\right) q_{H}=X_{L}-p_{L} q_{L}+\left(\theta_{H} p_{L}-c\right) q_{L}
\end{gathered}
$$

or

$$
\max _{X, q, p \geq 0}(1-\alpha)\left(r \theta_{H} q_{H}-c q_{H}-\left(\theta_{H} p_{L}-c\right) q_{L}\right)
$$

or

$$
\max _{X, q, p \geq 0}(1-\alpha)\left(r \theta_{H} q_{H}-c q_{H}\right)-(1-\alpha)\left(\theta_{H} p_{L}-c\right) q_{L}
$$

The result gives $p_{L}^{*}=\frac{c}{\theta_{H}}, q_{H}^{*}=1, p_{H}^{*} \geq \frac{c}{\theta_{L}}$ and $q_{L}^{*} \in[0,1], X_{L}^{*}-p_{L}^{*} q_{L}^{*}$. The retailer's optimal profit is $(1-\alpha)\left(r \theta_{H}-c\right)$.

Case (3): $p_{L}<\frac{c}{\theta_{H}}<\frac{c}{\theta_{L}} \leq p_{H} . \pi_{S}\left(\theta_{L}, \theta_{L}\right)=X_{L}-p_{L} q_{L}, \pi_{S}\left(\theta_{H}, \theta_{L}\right)=X_{L}-p_{L} q_{L}$, $\pi_{S}\left(\theta_{H}, \theta_{H}\right)=X_{H}-p_{H} q_{H}+\left(\theta_{H} p_{H}-c\right) q_{H}, z_{H}=q_{H}, \pi_{S}\left(\theta_{L}, \theta_{H}\right)=X_{H}-p_{H} q_{H}+\left(\theta_{L} p_{H}-c\right) q_{H}$.

The retailer's problem is

$$
\begin{gathered}
\max _{X, q, p \geq 0}(1-\alpha)\left(r \theta_{H} q_{H}-X_{H}+\left(1-\theta_{H}\right) p_{H} q_{H}\right)+\alpha\left(-X_{L}+p_{L} q_{L}\right) \\
\text { s.t. } X_{L}-p_{L} q_{L} \geq 0 \\
X_{H}-p_{H} q_{H}+\left(\theta_{H} p_{H}-c\right) q_{H} \geq 0 \\
X_{L}-p_{L} q_{L} \geq X_{H}-p_{H} q_{H}+\left(\theta_{L} p_{H}-c\right) q_{H} \\
X_{H}-p_{H} q_{H}+\left(\theta_{H} p_{H}-c\right) q_{H} \geq X_{L}-p_{L} q_{L}
\end{gathered}
$$

If IRL and ICH bind then IRH and ICL bind automatically. Now the retailer's problem becomes

$$
\begin{gathered}
\max _{X, q, p \geq 0}(1-\alpha)\left(r \theta_{H} q_{H}-X_{H}+\left(1-\theta_{H}\right) p_{H} q_{H}\right)+\alpha\left(-X_{L}+p_{L} q_{L}\right) \\
\text { s.t. } X_{L}-p_{L} q_{L}=0 \\
X_{H}-p_{H} q_{H}+\left(\theta_{H} p_{H}-c\right) q_{H}=X_{L}-p_{L} q_{L} .
\end{gathered}
$$

or

$$
\max _{X, q, p \geq 0}(1-\alpha)\left(r \theta_{H} q_{H}-p_{H} q_{H}+\left(\theta_{H} p_{H}-c\right) q_{H}+\left(1-\theta_{H}\right) p_{H} q_{H}\right)
$$

or

$$
\max _{X, q, p \geq 0}(1-\alpha)\left(r \theta_{H} q_{H}-c q_{H}\right)
$$

The result gives $p_{L}^{*}<\frac{c}{\theta_{H}}, p_{H}^{*} \geq \frac{c}{\theta_{L}}, q_{H}^{*}=1, q_{L}^{*} \in(0,1), X_{L}^{*}-p_{L}^{*} q_{L}^{*}$. The retailer optimal profit is $(1-\alpha)\left(r \theta_{H}-c\right)$

Case (4): $\frac{c}{\theta_{H}} \leq p_{L} \leq p_{H}<\frac{c}{\theta_{L}} . \pi_{S}\left(\theta_{L}, \theta_{L}\right)=X_{L}-p_{L} q_{L}, \pi_{S}\left(\theta_{H}, \theta_{L}\right)=X_{L}-p_{L} q_{L}+$ $\left(\theta_{H} p_{L}-c\right) q_{L}, \pi_{S}\left(\theta_{H}, \theta_{H}\right)=X_{H}-p_{H} q_{H}+\left(\theta_{H} p_{H}-c\right) q_{H}, \pi_{S}\left(\theta_{L}, \theta_{H}\right)=X_{H}-p_{H} q_{H}$.

The retailer's problem is

$$
\begin{gathered}
\max _{X, q, p \geq 0}(1-\alpha)\left(r \theta_{H} q_{H}-X_{H}+\left(1-\theta_{H}\right) p_{H} q_{H}\right)+\alpha\left(-X_{L}+p_{L} q_{L}\right) \\
\text { s.t. } X_{L}-p_{L} q_{L} \geq 0 \\
X_{H}-p_{H} q_{H}+\left(\theta_{H} p_{H}-c\right) q_{H} \geq 0 \\
X_{L}-p_{L} q_{L} \geq X_{H}-p_{H} q_{H} \\
X_{H}-p_{H} q_{H}+\left(\theta_{H} p_{H}-c\right) q_{H} \geq X_{L}-p_{L} q_{L}+\left(\theta_{H} p_{L}-c\right) q_{L}
\end{gathered}
$$

If IRL and ICH bind then IRH and ICL bind automatically. Now the retailer's problem becomes

$$
\max _{X, q, p \geq 0}(1-\alpha)\left(r \theta_{H} q_{H}-c q_{H}-\left(\theta_{H} p_{L}-c\right) q_{L}\right)
$$

The result gives $p_{L}^{*}=\frac{c}{\theta_{H}}, \frac{c}{\theta_{H}} \leq p_{H}^{*}<\frac{c}{\theta_{L}}, q_{H}^{*}=1, q_{L}^{*} \in[0,1], X_{L}^{*}-p_{L}^{*} q_{L}^{*}$. The the retailer optimal profit is $(1-\alpha)\left(r \theta_{H}-c\right)$.

Case (5): $p_{L}<\frac{c}{\theta_{H}} \leq p_{H}<\frac{c}{\theta_{L}} . \pi_{S}\left(\theta_{L}, \theta_{L}\right)=X_{L}-p_{L} q_{L}, \pi_{S}\left(\theta_{H}, \theta_{L}\right)=X_{L}-p_{L} q_{L}$, $\pi_{S}\left(\theta_{H}, \theta_{H}\right)=X_{H}-p_{H} q_{H}+\left(\theta_{H} p_{H}-c\right) q_{H}, \pi_{S}\left(\theta_{L}, \theta_{H}\right)=X_{H}-p_{H} q_{H}$.

The retailer's problem is

$$
\begin{gathered}
\max _{X, q, p \geq 0}(1-\alpha)\left(r \theta_{H} q_{H}-X_{H}+\left(1-\theta_{H}\right) p_{H} q_{H}\right)+\alpha\left(-X_{L}+p_{L} q_{L}\right) \\
\text { s.t. } X_{L}-p_{L} q_{L} \geq 0 \\
X_{H}-p_{H} q_{H}+\left(\theta_{H} p_{H}-c\right) q_{H} \geq 0 \\
X_{L}-p_{L} q_{L} \geq X_{H}-p_{H} q_{H} \\
X_{H}-p_{H} q_{H}+\left(\theta_{H} p_{H}-c\right) q_{H} \geq X_{L}-p_{L} q_{L}
\end{gathered}
$$

If IRL and ICH bind then IRH and ICL bind automatically. Now the retailer's problem becomes

$$
\max _{X, q, p \geq 0}(1-\alpha)\left(r \theta_{H} q_{H}-c q_{H}\right)
$$

The result gives $p_{L}^{*}<\frac{c}{\theta_{H}}, q_{H}^{*}=1, \frac{c}{\theta_{H}} \leq p_{H}^{*}<\frac{c}{\theta_{L}}, q_{L}^{*} \in[0,1], X_{L}^{*}-p_{L}^{*} q_{L}^{*}$. The retailer optimal profit is $(1-\alpha)\left(r \theta_{H}-c\right)$.

Case (6) $p_{L}<p_{H}<\frac{c}{\theta_{H}}<\frac{c}{\theta_{L}}$. Since no production takes place, the retailer's profit is zero.

The result is summarized as follows.
Theorem 4. (1) If $\theta_{L} \geq \frac{c}{r}\left[1+\frac{1-\alpha}{\alpha} \frac{\theta_{H}-\theta_{L}}{\theta_{L}}\right]$, then $p_{L}^{*}=\frac{c}{\theta_{L}}, q_{H}^{*}=1$, $p_{H}^{*}=\frac{c}{\theta_{L}}+\varepsilon$, $q_{L}^{*}=1$, where $\varepsilon>0$.
(2) Otherwise, $p_{L}^{*}=\frac{c}{\theta_{L}}, q_{H}^{*}=1, p_{H}^{*}=\frac{c}{\theta_{L}}+\varepsilon, q_{L}^{*}=0$.

The transfer payment $X_{L}^{*}$ and $X_{H}^{*}$ can be obtained by active IRL and ICH. Each type of supplier earns zero profits under public information. However, under private information and hidden actions, the high-type supplier receives information rent to tell the truth, while the low-type supplier still receives zero profit. This leads to possible channel loss when the retailer decides to no longer use the low-type supplier to prevent non-profitable information rent. Thus, the retailer incurs either information rent or channel loss due to private information and hidden actions, but not both.

### 2.6 Integrated System with Reliability Improvement

In this section, this chapter considers a problem in which the supplier can improve his reliability. That is, if a supplier improves his reliability $\theta$ by $e$ and his reliability becomes $\theta+e$, then I assume the supplier has an improvement $\operatorname{cost} C(\theta, e)=\frac{\beta e^{2}}{2(1-\theta)}$. Note that $\partial C(\theta, e) / \partial e>0$ and $\partial^{2} C(\theta, e) / \partial e^{2}>0$, which implies that the improvement cost is strictly convex in $e$, and it is more and more costly to make one more unit of effort. The effort cost
function also depends on the suppliers initial reliability. If the suppliers initial reliability is lower, then making improvements is less costly. In an extreme case, when $\theta \rightarrow 1$, the cost for improvement becomes extremely high. Since the level of effort exerted to improve a suppliers reliability is usually very high in practice, I assume the effort cost coefficient $\beta>r$. For example, Intel continuously invests billions of dollars in production facilities to seek to minimize their shortage and increase supply as much as possible (Pyrinis 2019). Note that when $\beta \rightarrow+\infty$, improving reliability is too costly to be considered by the supplier, and the problem reduces to the one considered earlier, in which the supplier does not have the option to improve his reliability.

In the integrated system where the channel is owned by only one player, the system's reliability is $\theta_{i}, i=H, L$. The player chooses his optimal effort level $e_{i}$ and production size $z_{i}$. The problem of the player in the integrated system is as follows:

$$
\max _{e_{i}, z_{i} \geq 0} E\left\{r \cdot \min \left\{D, \rho\left(\theta_{i}+e_{i}\right) z_{i}\right\}-c z_{i}-\frac{\beta e_{i}^{2}}{2(1-\theta)}\right\}
$$

where the revenue comes from the minimum of demand and production realization. The total cost is composed of two parts: the production cost and the reliability improvement cost. The profit of the integrated system can also be written as

$$
\begin{aligned}
& E\left\{r \cdot \min \left\{D, \rho\left(\theta_{i}+e_{i}\right) z_{i}\right\}-c z_{i}-\frac{\beta e_{i}^{2}}{2\left(1-\theta_{i}\right)}\right\} \\
& =\left(\theta_{i}+e_{i}\right) r \cdot \min \left\{D, z_{i}\right\}-c z_{i}-\frac{\beta e_{i}^{2}}{2(1-\theta)} \\
& =\left(\theta_{i}+e_{i}\right) r z_{i}-c z_{i}-\frac{\beta e_{i}^{2}}{2\left(1-\theta_{i}\right)}
\end{aligned}
$$

where $z_{i} \leq D$, otherwise the profit is never optimal. Therefore, this chapter presents the following theorem.

Theorem 5. When the reliability of the integrated system is $\theta_{i}, i=H, L$,
(1) The channel profit is $\theta_{i} r-c+\frac{r^{2}}{2 \beta}\left(1-\theta_{i}\right)$, which is positive as $\theta_{i} \geq \frac{c}{r} \frac{1-\frac{r}{2 \beta} \cdot \frac{r}{c}}{1-\frac{r}{c}}$;
(2) If $\theta_{i} \geq \frac{c}{r} \frac{1-\frac{r}{2 \beta} \frac{r}{c}}{1-\frac{r}{2 \beta}}, z_{i}^{*}=1$ and $e_{i}^{*}=\frac{r}{\beta}\left(1-\theta_{i}\right)$; the system stops working otherwise.

Since it is assumed that $\theta_{i} \geq \frac{c}{r}$, it is always the case that $\left(\theta_{i}+e_{i}\right) r-c>0$ and thus $z_{i}^{*}=1$ is the optimal choice of the system. By taking the first-order conditions, $e_{i}^{*}=\frac{r}{\beta}\left(1-\theta_{i}\right)$ which is the optimal effort level. Since $e_{i}^{*}+\theta_{i}<1$, the system will not push the improved reliability to 1 bacause exerting effort is too costly. Note that the system makes less effort as he is more reliable initially. In addition, the optimal effort level increases in the unit retail price, and decreases in the effort cost coefficient.

The channel profit increases as the system is more reliable. When $\theta_{i}=\frac{c}{r} \frac{1-\frac{r}{2 \beta} \cdot \frac{r}{c}}{1-\frac{r}{2 \beta}}$, the channel profit is zero. Note that $\frac{c}{r} \frac{1-\frac{r}{2 \beta} \cdot \frac{r}{c}}{1-\frac{r}{2 \beta}}<\frac{c}{r}$, which implies that when efforts to improve reliability are allowed to be exerted, more systems with low reliability are able to participate in this game.

### 2.7 First-best with Reliability Improvement

Now consider a case in which there is one supplier and one retailer in the system, and the suppliers initial reliability and effort level is public information to the retailer. In this problem, as before, the retailer offers a contract $\left(X_{i}, q_{i}, p_{i}\right)$ to supplier type- $\theta_{i}$. For a given contract $\left(X_{i}, q_{i}, p_{i}\right)$, the supplier chooses the optimal level of effort and production size. In summary, the supplier's problem is:

$$
\max _{e_{i}, z_{i} \geq 0}\left\{X_{i}-c z_{i}-E\left[p_{i}\left(q_{i}-\rho\left(\theta_{i}+e_{i}\right) z_{i}\right)^{+}\right]-\frac{\beta e_{i}^{2}}{2\left(1-\theta_{i}\right)}\right\},
$$

where the revenue comes from the fixed payment $X_{i}$. The total cost is composed of three parts: the production $\operatorname{cost} c z_{i}$, the effort cost and the possible penalty that should be paid to the retailer if the supplier fails to deliver. The objective function of the supplier can also
be written as follows.

$$
\begin{gathered}
X_{i}-c z_{i}-E\left[p_{i}\left(q_{i}-\rho\left(\theta_{i}+e_{i}\right) z_{i}\right)^{+}\right]-\frac{\beta e_{i}^{2}}{2\left(1-\theta_{i}\right)} \\
=X_{i}-c z_{i}-\left(1-\theta_{i}-e_{i}\right) p_{i} q_{i}-\left(\theta_{i}+e_{i}\right) p_{i}\left(q_{i}-z_{i}\right)^{+}-\frac{\beta e_{i}^{2}}{2(1-\theta)} \\
=X_{i}-p_{i} q_{i}+\left[\left(\theta_{i}+e_{i}\right) p_{i}-c\right] z_{i}-\frac{\beta e_{i}^{2}}{2(1-\theta)} .
\end{gathered}
$$

Note that the supplier's profit is never optimal when $z_{i}>q_{i}$, and thus the solution must guarantee that $z_{i} \leq q_{i}$. The first-order conditions are as follows

$$
\begin{aligned}
& \left(\theta_{i}+e_{i}\right) p_{i}-c \\
& p_{i} z_{i}-\frac{\beta e_{i}}{(1-\theta)}
\end{aligned}
$$

Therefore, $z_{i}^{*}=q_{i}$ and $e_{i}^{*}=\frac{p_{i} q_{i}}{\beta}\left(1-\theta_{i}\right)$ if the marginal profit of producing one more unit is positive, that is, $\left(\theta_{i}+e_{i}^{*}\right) p_{i}-c>0$. Otherwise, the supplier finds that production is not optimal and thus chooses $z_{i}^{*}=0$ and $e_{i}^{*}=0$. See that the supplier's chosen effort level increases in the penalty rate given by the retailer. Thus, the retailer can adjust the suppliers optimal effort level by choosing the right penalty rate to achieve her optimal profit.

Note that those are only the best responses of a supplier from a given contract ( $X_{i}, q_{i}, p_{i}$ ). Next it shows the retailer's optimal contract by solving the retailer's problem. The retailer's problem is

$$
\begin{gathered}
\max _{X_{i}, q_{i}, p_{i} \geq 0} E\left\{r \cdot \min \left\{D, \rho\left(\theta_{i}+e_{i}^{*}\right) z_{i}^{*}\right\}-X_{i}+p_{i}\left(q_{i}-\rho\left(\theta_{i}+e_{i}^{*}\right) z_{i}^{*}\right)^{+}\right\} \\
\text {s.t. } X_{i}-c z_{i}^{*}-E\left[p_{i}\left(q_{i}-\rho\left(\theta_{i}+e_{i}^{*}\right) z_{i}^{*}\right)^{+}\right]-\frac{\beta\left(e_{i}^{*}\right)^{2}}{2\left(1-\theta_{i}\right)} \geq 0,
\end{gathered}
$$

where the retailer's profit is composed of two parts: selling revenue from the market and a possible penalty. The selling amount from the market is the minimum of the demand and quantity delivered. The retailer either receives revenue from the market or receives a penalty from the supplier. The retailer's cost is the transfer payment to the supplier. The
retailer needs to guarantee that the supplier receives non-negative profit. Since the retailer can always subtract the supplier's profit by choosing $X_{i}=c z_{i}^{*}+E\left[p_{i}\left(q_{i}-\rho\left(\theta_{i}+e_{i}^{*}\right) z_{i}^{*}\right)^{+}\right]+$ $\frac{\beta\left(e_{i}^{*}\right)^{2}}{2\left(1-\theta_{i}\right)}$, the retailer obtains the channel profit. Thus, this contract helps the retailer achieve channel efficiency. The retailer's optimal contract is summarized as follows

Theorem 6. (1) When the reliability of the supplier is $\theta_{i}, i=H, L$, the retailer's optimal contract is $\left(X_{i}^{*}, q_{i}^{*}, p_{i}^{*}\right)=\left(c+r\left(1-\theta_{i}\right)-\frac{r^{2}}{2 \beta}\left(1-\theta_{i}\right), 1, r\right)$ if $\theta_{i} \geq \frac{c}{r} \frac{1-\frac{r}{2 \beta} \frac{r}{c}}{1-\frac{r}{2 \beta}}$, and the retailer leaves the market otherwise.
(2) The contract coordinates the system.

Note that the retailer charges the supplier $p_{i}^{*}=r$ and thus the supplier chooses $e_{i}^{*}=$ $\frac{r}{\beta}\left(1-\theta_{i}\right)$, which is equal to the optimal effort level under the integrated system. Therefore, the retailer offers the supplier the same trade-off and the supplier exerts an optimal level of effort, then the retailer subtracts all his profit. In this way, the retailer designs the contract and channel efficiency is achieved.

### 2.8 Second-best with Reliability Improvement

In this section, the supplier's initial reliability is private information, and his effort level to improve his reliability is a hidden action. In this way, the retailer is not able to directly distinguish between the two types of suppliers, and must use a contract menu to screen the suppliers' types. This problem is classified as an adverse selection problem with moral hazards.

The game sequence is defined as follows: (1) Nature reveals the type to the supplier; (2) The retailer offers a contract menu where $\left(X_{i}, q_{i}, p_{i}\right)$ is designed for a type- $\theta_{i}$ supplier; (3) The supplier of type- $\theta_{i}$ chooses a contract and decides his optimal production size and effort level.

To focus on the impact of private information on the channel, this section omits trivial cases for $\theta_{i}<\frac{c}{r}$ where production is not profitable and only displays the results under $\theta_{i} \geq \frac{c}{r}$.

When there is private information, it may cost the distortion of the penalty rate and order quantity. The focus of this chapter is how the retailer motivates the supplier to improve his reliability. Therefore, to facilitate a comparison between the reliability improvement case and non-reliability improvement case, this chapter first restricts $q_{i} \in\{0,1\}$ in which the production activity is considered a project and the supplier chooses whether to produce. In the extension part this chapter relax this assumption and show the optimal quantity and penalty level chosen by the supplier.

## The Supplier's Problem

Let $\pi_{S}\left(\theta_{i}, \theta_{j}\right)$ denote the optimal profit of type- $\theta_{i}$ supplier when he chooses a contract designed for type- $\theta_{j}$. First, the supplier chooses the optimal effort level and production size for the given contract $\left(X_{j}, q_{j}, p_{j}\right)$. From the analysis above, the supplier's optimal production size is $z_{i}^{*}=q_{j}$ or $z_{i}^{*}=0$, and the supplier's optimal effort is $e_{i}^{*}=\frac{p_{j} z_{i}^{*}}{\beta}\left(1-\theta_{i}\right)$. If $q_{j}>0$ and the supplier chooses to make positive quantity production, then $e_{i}^{*}=\frac{p_{j} q_{j}}{\beta}\left(1-\theta_{i}\right)$. Therefore, all possible combinations for $i, j=H, L$ are presented as follows.

$$
\begin{gathered}
\pi_{S}\left(\theta_{H}, \theta_{H}\right)=X_{H}-c q_{H}-\left(1-\theta_{H}-\frac{p_{H} q_{H}}{\beta}\left(1-\theta_{H}\right)\right) p_{H} q_{H}-\frac{p_{H}^{2} q_{H}^{2}}{2 \beta}\left(1-\theta_{H}\right) \\
\pi_{S}\left(\theta_{H}, \theta_{L}\right)=X_{L}-c q_{L}-\left(1-\theta_{H}-\frac{p_{L} q_{L}}{\beta}\left(1-\theta_{H}\right)\right) p_{L} q_{L}-\frac{p_{L}^{2} q_{L}^{2}}{2 \beta}\left(1-\theta_{H}\right) \\
\pi_{S}\left(\theta_{L}, \theta_{L}\right)=X_{L}-c q_{L}-\left(1-\theta_{L}-\frac{p_{L} q_{L}}{\beta}\left(1-\theta_{L}\right)\right) p_{L} q_{L}-\frac{p_{L}^{2} q_{L}^{2}}{2 \beta}\left(1-\theta_{L}\right) \\
\pi_{S}\left(\theta_{L}, \theta_{H}\right)=X_{H}-c q_{H}-\left(1-\theta_{L}-\frac{p_{H} q_{H}}{\beta}\left(1-\theta_{L}\right)\right) p_{H} q_{H}-\frac{p_{H}^{2} q_{H}^{2}}{2 \beta}\left(1-\theta_{L}\right) .
\end{gathered}
$$

The aforementioned equations help the retailer consider her constraints. The retailers contract menu must guarantee that each supplier receives non-negative profit, and thus $\pi_{S}\left(\theta_{H}, \theta_{H}\right) \geq 0(\mathrm{IRH})$ and $\pi_{S}\left(\theta_{L}, \theta_{L}\right) \geq 0(\operatorname{IRL})$. If the suppliers are truthful, then $\pi_{S}\left(\theta_{H}, \theta_{H}\right) \geq$ $\pi_{S}\left(\theta_{H}, \theta_{L}\right)(\mathrm{ICH})$ and $\pi_{S}\left(\theta_{L}, \theta_{L}\right) \geq \pi_{S}\left(\theta_{L}, \theta_{H}\right)$ (ICL). In the final result, IRL and ICH bind, and IRH and ICL then bind automatically.

## The Retailer's Problem

The retailers problem is summarized as follows.

$$
\max _{X, q, p \geq 0}\left\{\begin{array}{c}
(1-\alpha) E\left\{\begin{array}{c}
r \cdot \min \left\{D, \rho\left(\theta_{H}+e_{H}^{*}\right) z_{H}^{*}\right\} \\
-X_{H}+p_{H}\left(q_{H}-\rho\left(\theta_{H}+e_{H}^{*}\right) z_{H}^{*}\right)^{+}
\end{array}\right\} \\
+\alpha E\left\{\begin{array}{c}
r \cdot \min \left\{D, \rho\left(\theta_{L}+e_{L}^{*}\right) z_{L}^{*}\right\}-X_{L} \\
+p_{L}\left(q_{L}-\rho\left(\theta_{L}+e_{L}^{*}\right) z_{L}^{*}\right)^{+}
\end{array}\right\} \\
\text {s.t. } \pi_{S}\left(\theta_{H}, \theta_{H}\right) \geq 0 \\
\pi_{S}\left(\theta_{L}, \theta_{L}\right) \geq 0 \\
\pi_{S}\left(\theta_{H}, \theta_{H}\right) \geq \pi_{S}\left(\theta_{H}, \theta_{L}\right) \\
\pi_{S}\left(\theta_{L}, \theta_{L}\right) \geq \pi_{S}\left(\theta_{L}, \theta_{H}\right)
\end{array}\right.
$$

Note that since IRL and ICH bind, and IRH and ICL then bind automatically, the retailers problem becomes

$$
\begin{gathered}
\max _{X, q, p \geq 0}\left\{\begin{array}{c}
(1-\alpha) E\left\{\begin{array}{c}
r \cdot \min \left\{D, \rho\left(\theta_{H}+e_{H}^{*}\right) z_{H}^{*}\right\} \\
-X_{H}+p_{H}\left(q_{H}-\rho\left(\theta_{H}+e_{H}^{*}\right) z_{H}^{*}\right)^{+}
\end{array}\right\} \\
+\alpha E\left\{\begin{array}{c}
r \cdot \min \left\{D, \rho\left(\theta_{L}+e_{L}^{*}\right) z_{L}^{*}\right\}-X_{L} \\
+p_{L}\left(q_{L}-\rho\left(\theta_{L}+e_{L}^{*}\right) z_{L}^{*}\right)^{+}
\end{array}\right\}
\end{array}\right\} \\
\left\{\begin{array}{c}
X_{H}-c q_{H}-\left(1-\theta_{H}-\frac{p_{H} q_{H}}{\beta}\left(1-\theta_{H}\right)\right) p_{H} q_{H} \\
-\frac{p_{H}^{2} q_{H}^{2}}{2 \beta}\left(1-\theta_{H}\right)-\left(\theta_{H}-\theta_{L}\right)\left(p_{L} q_{L}-\frac{p_{L}^{2} q_{L}^{2}}{2 \beta}\right)
\end{array}\right\}=0
\end{gathered}
$$

By further simplification, the retailer's problem becomes

$$
\max _{q, p \geq 0}\left\{\begin{array}{c}
(1-\alpha)\left\{\begin{array}{c}
r\left(\theta_{H}+\frac{p_{H} q_{H}}{\beta}\left(1-\theta_{H}\right)\right) q_{H}-c q_{H} \\
-\frac{p_{H}^{2} q_{H}^{2}}{2 \beta}\left(1-\theta_{H}\right)-\left(\theta_{H}-\theta_{L}\right)\left(p_{L} q_{L}-\frac{p_{L}^{2} q_{L}^{2}}{2 \beta}\right)
\end{array}\right\} \\
+\alpha\left\{r\left(\theta_{L}+\frac{p_{L} q_{L}}{\beta}\left(1-\theta_{L}\right)\right) q_{L}-c q_{L}-\frac{p_{L}^{2} q_{L}^{2}}{2 \beta}\left(1-\theta_{L}\right)\right\}
\end{array}\right\}
$$

The first-order condition gives $p_{H}^{*}=r$ and $q_{H}^{*}=1$. Since $q_{L}^{*} \in\{0,1\}$, there are two cases for the low-type supplier. If $q_{L}^{*}=0$, then it has to be the case $p_{L}^{*} \geq \frac{\alpha\left\{r \theta_{L}-c\right\}}{(1-\alpha)\left(\theta_{H}-\theta_{L}\right)}$ so that the retailer finds it is not profitable to order from the low-type supplier and pays rent to the high-type supplier to buy the truth. In this way, the retailer only orders from the high-type supplier. And then the retailer's optimal profit equals $(1-\alpha)\left\{r \theta_{H}-c+\frac{r^{2}}{2 \beta}\left(1-\theta_{H}\right)\right\}$. If $q_{L}^{*}=1$, then $p_{L}^{*}$ satisfies the following condition

$$
\left.\alpha\left\{\frac{r}{\beta}\left(1-\theta_{L}\right)\right)-\frac{p_{L}^{*}}{\beta}\left(1-\theta_{L}\right)\right\}-(1-\alpha)\left(\theta_{H}-\theta_{L}\right)\left(1-\frac{p_{L}^{*}}{\beta}\right)=0
$$

where it shows that $p_{L}^{*}<r$, which implies that the retailer gives less motivation to the lowtype supplier to avoid a higher information rent, which is defined as $\left(\theta_{H}-\theta_{L}\right)\left(p_{L}^{*}-\frac{\left(p_{L}^{*}\right)^{2}}{2 \beta}\right)$. The optimal channel profit is as follows.

$$
\pi_{C}^{*}=\left\{\begin{array}{c}
(1-\alpha)\left\{r \theta_{H}-c+\frac{r^{2}}{2 \beta}\left(1-\theta_{H}\right)-\left(\theta_{H}-\theta_{L}\right)\left(p_{L}^{*}-\frac{\left(p_{L}^{*}\right)^{2}}{2 \beta}\right)\right\} \\
+\alpha\left\{r\left(\theta_{L}+\frac{p_{L}^{*}}{\beta}\left(1-\theta_{L}\right)\right)-c-\frac{\left(p_{L}^{*}\right)^{2}}{2 \beta}\left(1-\theta_{L}\right)\right\}
\end{array}\right\} .
$$

The result is summarized as follows.

Theorem 7. (1) If $\pi_{C}^{*} \geq(1-\alpha)\left\{r \theta_{H}-c+\frac{r^{2}}{2 \beta}\left(1-\theta_{H}\right)\right\}$, then $p_{H}^{*}=r, q_{H}^{*}=1$, $p_{L}^{*}=$ $\frac{r \alpha-\beta \theta_{H}+\alpha \beta \theta_{H}-r \alpha \theta_{L}+\beta \theta_{L}-\alpha \beta \theta_{L}}{\alpha-\theta_{H}+\alpha \theta_{H}+\theta_{L}-2 \alpha \theta_{L}}<r, q_{L}^{*}=1$.
(2) Otherwise, $p_{L}^{*} \geq \frac{\alpha\left\{r \theta_{L}-c\right\}}{(1-\alpha)\left(\theta_{H}-\theta_{L}\right)}, q_{L}^{*}=0$.

The transfer payment $X_{L}^{*}$ and $X_{H}^{*}$ can be obtained by active IRL and ICH. Each type of supplier earns zero profit under public information. However, under private information and hidden actions, the high-type supplier receives information rent to tell the truth, while the low-type supplier still receives zero profit. This leads to possible channel loss when the retailer decides to stop working with the low-type supplier to prevent non-profitable information rent. Thus, the retailer incurs either information rent or channel loss due to private information and hidden actions, but not both.

If the supplier is a high-type, then making him produce is more profitable than making a low-type supplier produce. Therefore, a high-type supplier is more likely to mimic the lowtype supplier. The retailer pays information rent to the high-type supplier by involving the low-type supplier less in the market. This may lead to channel loss. When that happens, the retailer sets $p_{L} \geq \frac{\alpha\left\{r \theta_{L}-c\right\}}{(1-\alpha)\left(\theta_{H}-\theta_{L}\right)}$ to subtract all of the high-type suppliers' profits and thus prevents ordering from the low-type supplier.

The effect of the penalty is different between allowing reliability improvement and not allowing reliability improvement. Under the case of not allowing reliability improvement, the penalty is set to influence the production activity of the supplier. Under the case of allowing reliability improvement, the penalty is mainly used to motivate the efforts of the supplier. Only under extreme cases in which the low-type supplier is excluded from the market is the penalty set to realize that objective. In this way, the case allowing reliability improvement is shown to only be a special case.

## Extension

Here, the assumption that $q_{i} \in\{0,1\}$ in which the production activity is considered a project and the supplier chooses whether to produce is relaxed. In this case, when the retailer orders from both types of suppliers, the following conditions are satisfied.

$$
\begin{aligned}
& p_{H}^{*}=r, q_{H}^{*}=1, \\
& \left\{\begin{array}{c}
\left.\alpha\left\{r \frac{q_{L}^{*}}{\beta}\left(1-\theta_{L}\right)\right) q_{L}^{*}-\frac{p_{L}^{*}\left(q_{L}^{*}\right)^{2}}{\beta}\left(1-\theta_{L}\right)\right\} \\
-(1-\alpha)\left(\theta_{H}-\theta_{L}\right)\left(q_{L}^{*}-\frac{p_{L}^{*}\left(q_{L}^{*}\right)^{2}}{\beta}\right)
\end{array}\right\}=0, \\
& \left\{\begin{array}{c}
\left.\alpha\left\{r \theta_{L}+2 r \frac{p_{L}^{*} q_{L}^{*}}{\beta}\left(1-\theta_{L}\right)\right)-c-\frac{\left(p_{L}^{*}\right)^{2} q_{L}^{*}}{\beta}\left(1-\theta_{L}\right)\right\} \\
-(1-\alpha)\left(\theta_{H}-\theta_{L}\right)\left(p_{L}^{*}-\frac{p_{L}^{2} q_{L}^{*}}{\beta}\right)
\end{array}\right\}=0 .
\end{aligned}
$$

The transfer payment $X_{L}^{*}$ and $X_{H}^{*}$ can be obtained by active IRL and ICH. Managers are encouraged to use the equations above to find equilibrium.

### 2.9 Conclusion

This chapter considers a single-period problem in which a retailer sources from a supplier whose reliability is private information and efforts to improve reliability are unobservable (hidden actions). At the beginning of the game, the retailer offers the supplier a contract menu with a transfer payment, order quantity, and unit penalty rate for nondelivery for each reliability, which is modeled as the type of supplier, and the supplier chooses a contract that most benefits him. As the supplier chooses a contract, he also decides the optimal level of effort to invest in reliability improvement.

Under an integrated system, when exerting effort to improve reliability is allowed, more systems with low reliability are able to participate in this game. Under the first-best scenario, note that the retailer charges the supplier $p_{i}^{*}=r$ and thus the supplier chooses $e_{i}^{*}=\frac{r}{\beta}\left(1-\theta_{i}\right)$, which is equal to the optimal effort level under the integrated system. Therefore, the retailer offers the supplier the same trade-off and the supplier exerts an optimal level of effort, after which the retailer subtracts all his profit. In this way, the retailer designs the contract and channel efficiency is achieved. Each type of supplier earns zero profit under public information. However, under private information and hidden actions, the high-type supplier receives information rent to tell the truth, while the low-type supplier still receives zero profit. This leads to possible channel loss when the retailer decides to no longer work with the low-type supplier to prevent non-profitable information rent. Thus, the retailer incurs either information rent or channel loss due to private information and hidden actions, but not both.

If the supplier is of the high-type, then making him produce is more profitable than making a low-type supplier produce. Therefore, a high-type supplier is more likely to mimic the low- type supplier. The retailer pays information rent to the high-type supplier by involving less low-type suppliers in the market. This may lead to channel loss. When that
happens, the retailer sets $p_{L} \geq \frac{\alpha\left\{r \theta_{L}-c\right\}}{(1-\alpha)\left(\theta_{H}-\theta_{L}\right)}$ to subtract all of the high-type suppliers' profits and thus prevents ordering from the low-type supplier.

The effect of penalties is different between allowing reliability improvement and not allowing reliability improvement. When reliability improvement is not allowed, the penalty is set to influence the production activity of the supplier. When reliability improvement is allowed, the penalty is mainly used to motivate the efforts of the supplier. Only under extreme cases in which the low-type supplier is excluded from the market is the penalty set to realize that objective. In this way, the case allowing reliability improvement is shown to be a special case.

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## CHAPTER 3

## A RESPONSIVE-PRICING RETAILER SOURCING FROM COMPETING SUPPLIERS FACING DISRUPTIONS ${ }^{1}$

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### 3.1 Introduction

A critical issue in supply chain management is the uncertainty of supplies. Among the main reasons behind supply uncertainty is disruption due to natural disasters, strikes, terrorist or cyber attacks, and delivery uncertainties. The impact of supply disruption on supply chain members' decisions has been studied in a variety of settings in the literature; see, e.g., Tomlin (2006), Yang et al. (2009), Gumus et al. (2012), Yang and Babich (2015) and references therein.

To mitigate the impact of supply uncertainty, supply diversification has been adopted by retailers in practice as an effective strategy (Yano and Lee 1995). Supply diversification, when suppliers are not decision makers and thus wholesale prices are exogenous, is a wellresearched problem. The existing literature has established that cost always takes precedence over reliability when it comes to selecting a supplier, and reliability affects the order quantity requested from a chosen supplier. In other words, cost is the order qualifier and reliability is the order winner (Dada et al. 2007, Federgruen and Yang 2009, Hu and Kostamis 2014).

In addition to the direct benefit of risk pooling, an indirect advantage of using supply diversification is that it induces unreliable suppliers to compete. One example used in the empirical work of Wu and Choi (2005) is that Coach Company's two suppliers in Malaysia compete via their prices to reduce supply chain disruption risk and improve their operational performance. Moreover, it is well known in economics that competition results in lower prices. It is certainly reasonable then for competing suppliers to decide on prices. In this case, is high reliability truly beneficial to them? The answer is yes or no, depending on the problem parameters, as shown in our chapter. Moreover, our chapter provides detailed guidance for managers from the suppliers' perspectives.

It is also worth noting that the retailer's strategy of supply diversification is heavily dependent on the correlation among different unreliable suppliers. As shown in Li et al.
(2013), the aforementioned insight "cost is the order qualifier" may not hold with multiple correlated suppliers.

Supplier disruption would be correlated for a number of reasons such as product and service designs, geographic proximity, common customers, operations input sources, and political and economic reasons (Babich et al. 2007). Another reason for supplier correlation is that tier- 1 suppliers may prefer a more overlapped supply chain and hence may strategically choose to form a "diamond-shaped" supply chain by sharing the same tier-2 suppliers (Ang et al. 2016). A diamond-shaped supply chain is very fragile in response to disruption risks since the retailer's tier-1 suppliers are highly correlated. This problem struck Toyota mercilessly after the March 11, 2011 tsunami/earthquake, leading to an immediate shortage of more than 400 parts (Tabuchi 2011, Greimel 2012, Masui and Nishi 2012).

In the presence of supplier disruption correlation, the combined effect of risk pooling and competition is subtler. Babich et al. (2007) show that with two suppliers, a price-taking retailer prefers more positively correlated suppliers despite the loss of risk pooling, because the induced competition effect dominates the risk-pooling effect. They also show that the suppliers become worse off when the correlation between their default risks increases. But this does not explain why in practice a supplier has the incentive to become more correlated to other suppliers by, for example, strategically forming a diamond-shaped supply chain. We delve into this and provide a possible explanation for this phenomenon.

In addition to supply diversification, another effective strategy of mitigating the impact of supply uncertainty is responsive pricing. That is, after the supply uncertainty is resolved, most retailers do have the ability to adjust the product price to some extent to better match demand with supply. We call this type of retailers responsive-pricing retailers (Tang and Yin 2007, Kazaz 2008, Li et al. 2017). For example, Dell changes its memory card demand by adjusting the price as a tactic to deal with supply disruptions (Tomlin 2006). Furthermore, many suppliers in the semiconductor and electronic component industries set prices according
to the availability of inventory on hand (Li and Zheng 2006, Feng 2010, Feng and Shi 2012). Owing to the prevalence of responsive-pricing retailers and correlated suppliers in practice, it is important to investigate their sourcing and pricing problems with correlated suppliers (Li et al. 2013, Feng et al. 2017). As has been shown in the literature, the retailer's diversification strategy can be dramatically different when endowed with different pricing power (Li et al. 2013). But, how would the suppliers adapt their competing strategies to the retailer's pricing power remains an important question. This chapter sheds light on this question from the suppliers' perspectives.

While there is a considerable literature on both supply diversification and responsive pricing for managing supply uncertainty and there are a few chapters studying a price-taking retailer's supply diversification problem in a game-theoretic setting, there are none devoted to a responsive-pricing retailer's supply diversification problem with multiple competing correlated unreliable suppliers. As has been shown that each of these features is significant on its own account, it is definitely worthwhile to study their combined impact. Moreover, as we shall see that such an analysis offers interesting new insights in addition to filling an important gap in the literature.

We investigate a supply chain consisting of a responsive-pricing retailer (she) sourcing from multiple suppliers having different production costs and reliabilities. The reliability of a supplier (he) is modeled by the probability with which he defaults on the retailer's order. Moreover, the suppliers' default events may or may not be independent. The retailer's procurement problem is formulated as a Stackelberg-Nash game in which the suppliers are the Stackelberg leaders by setting their wholesale prices simultaneously in a Nash game, and the retailer is the Stackelberg follower when determining her respective order quantities from the suppliers ex-ante (i.e, before the default events) and retail price ex-post.

This leads to a number of significant managerial insights. First, counter to the intuition - reliability is an order winner - prescribed by the literature (Dada et al. 2007, Federgruen
and Yang 2009), a supplier may be worse off as his reliability increases. We identify two effects that drive this result: a reliability effect (first-order effect of the reliability) and a competition effect of the reliability (second-order effect of the reliability). How reliability affects a supplier depends on the magnitude of these two effects. Specifically, a supplier becomes worse off from an increased reliability when the competition effect dominates the reliability effect. In absence of the competition effect, that is, when the suppliers' wholesale prices are given, it is straightforward to show that a higher reliability benefits a supplier on account of receiving a larger order from the retailer. However, when there is competition and the suppliers set their wholesale prices as Nash players, an increase in the reliability of a supplier induces his rivals to reduce their wholesale prices. This consequently not only forces this supplier to reduce his wholesale price, but also results in a smaller order for this supplier. This analysis has important implications for an unreliable supplier in cases when it is possible for him to invest in improving his reliability.

Second, different from Babich et al. (2007), we demonstrate that a supplier's profit may increase in the disruption correlation between this supplier and one of his rivals. We identify two effects behind this result: a correlation effect (first-order effect of the correlation) and a competition effect of the correlation (second-order effect of the correlation). The correlation effect may be positive or negative depending on how the retailer's order quantities respond to the correlation change. As the correlation increases, the substitutability in terms of risk between the suppliers increases, leading to a reduced necessity for the retailer to diversify. This naturally results in a lower order quantity from the retailer. From this perspective, the correlation effect is negative. However, when the suppliers' wholesale prices differ a lot, a responsive-pricing retailer may choose to order more from the cheaper supplier because the derived cost benefit outweighs the reduced risk-pooling benefit. In this sense, the correlation effect is positive. When the suppliers set their wholesale prices as Nash players, knowing that a higher correlation increases their substitutability (a surrogate for the competition
intensity), both suppliers will have to reduce their wholesale prices (and thus profit margins) to compete. Therefore, the competition effect serves as a negative force as is usual. When the correlation effect is positive and dominates the competition effect of the correlation, the low-cost supplier can benefit from the correlation increase. This happens when a supplier has a sufficient inherent production cost advantage over the other. Note that with a price-taking retailer, the correlation effect is always negative because the retailer is less flexible in terms of adjusting her order quantities. This is why it was found in Babich et al. (2007) that an increased correlation is always harmful to the suppliers.

Third, in contrast to Li et al. (2013) who find that the retailer's total order quantity always decreases when the correlation between the non-decision-making suppliers increases, we demonstrate that with strategic suppliers the total order quantity may increase as the correlation increases. Two drivers are behind this result. The aforementioned positive correlation effect is the first driver. The second driver is that as the correlation increases, both suppliers lower their wholesale prices due to the intensified competition (increased substitutability). This induces the retailer to order more from the suppliers.

The remainder of this article proceeds as follows. Section 3.2 summarizes the related literature. In Section 3.3, we describe our general model with basic assumptions. In Section 3.4, we analyze the general case of two correlated unreliable suppliers. We investigate the impact of reliability on the retailer and the suppliers in Section 3.5, and the impact of the supplier correlation in Section 3.6. We discuss in Section 3.7 the case when the retailer must choose to order from only one of the two competing suppliers. In Section 3.8 we examine the case of more than two suppliers. Section 3.9 concludes the chapter. Proofs are relegated to an online supplement.

### 3.2 Literature Review

To highlight our contribution to the literature on supply risk management, we shall review two steams of the literature in detail: supplier level competition and delivery-dependent retail pricing. Prior to doing that, let us mention in passing that supply risk management has attracted a great deal of attention from researchers over the years, and the earlier reviews of the literature has been conducted by Yano and Lee (1995) and Tang (2006). Most of this reviewed work and subsequent research such as Anupindi and Akella (1993), Tomlin and Wang (2005), Tomlin (2006, 2009), Dada et al. (2007), Tang and Yin (2007), Federgruen and Yang (2009), Wang et al. (2010), Hu and Kostamis (2014), and Gupta et al. (2014) has focused on a price-taking retailer sourcing from unreliable suppliers who do not compete in setting their wholesale prices.

In the first stream, the most related paper is that of Babich et al. (2007) who study a price-taking retailer sourcing from multiple competing unreliable suppliers. They show that an increase in the suppliers' default risks correlation renders the retailer better off and the suppliers worse off. We incorporate the retailer's ability to set the retail price in response to the actual amount delivered, and show that the model in Babich et al. (2007) is a special case of our model when the price sensitivity approaches zero. This richer model (responsive pricing) enables us to obtain new insights including some that differ from Babich et al. (2007). Specifically, we find that a supplier may not necessarily benefit from a higher reliability or a lower correlation with the other suppliers. Yang et al. (2012) study a retailer's choice between the winner-takes-all strategy (resulting in a competition effect) and the strategy of sourcing from both suppliers (resulting in a diversification effect) when the suppliers have private information about their disruption likelihoods and the retailer is endowed with the power to offer procurement contracts. Unlike their paper that focuses on the buyer's welfare, we examine the reliability, correlation, and competition effects on the suppliers. Demirel et al. (2017) evaluate the costs and benefits of a manufacturer's sourcing policies which include
(i) sole sourcing from one supplier and (ii) sourcing primarily from an unreliable supplier while using a reliable supplier to provide a backup capacity to the manufacturer when the delivery from the unreliable supplier is disrupted. They find that the manufacturer may be harmed by the presence of the backup option in some cases. Different from Demirel et al. (2017), our study investigates the wholesale price competition among multiple unreliable suppliers before random disruptions occur, and we find that the retailer can always benefit from a higher correlation among suppliers' default risks due to competition.

In the second stream, as another important tool in the retailer's arsenal to mitigate disruption risks, alongside supply diversification, the retailer sets the retail price upon deliveries; see Tang and Yin (2007), Li and Zheng (2006), and Feng (2010) for models with one unreliable supplier. Li et al. (2017) consider a firm's sourcing problem from one reliable supplier and one unreliable supplier in a committed-pricing scenario (where the firm makes the pricing decision prior to deliveries) and in a responsive-pricing scenario. They examine the interplay between the supply-diversification strategy and the responsive-pricing strategy in mitigating supply uncertainty. For an examination of dynamic pricing decisions under supply capacity uncertainty with multiple unreliable suppliers, the readers are referred to Feng and Shi (2012) and Feng et al. (2017).

Li et al. (2013), the most related to our chapter in this stream, study a responsive-pricing retailer's multisourcing problem with multiple correlated unreliable suppliers with given wholesale prices. They show that the retailer's total order quantity always decreases in the correlation among the suppliers' disruption risks. Our chapter, on the other hand, considers importantly that the suppliers compete in deciding their wholesale prices, representing a significant conceptual and methodological change from Li et. al (2017). Owing to the supplier level competition, we see that the retailer's total order quantity may increase in the suppliers' disruption risks correlation and the supplier with a significant inherent cost advantage can benefit from high disruption correlation. These results stand in a sharp contrast to those in Li et al. (2013).

### 3.3 Model

Consider a supply chain with a risk-neutral retailer, who orders identical products from multiple, possibly unreliable, risk-neutral suppliers, and sells them to customers in a single selling season. The suppliers have different reliabilities and production costs. They compete among themselves by setting their wholesale prices to the retailer in a Nash game. The suppliers act as the Stackelberg leaders in a sequential game with the retailer as the Stackelberg follower deciding on the order quantities and the retail price. The goal of each supplier and retailer is to maximize his/her respective expected profit.

A supplier can be either reliable or unreliable, where the distinction is viewed from the retailer's perspective. If a supplier can always completely fill the retailer's order, then he is reliable. Otherwise, he is unreliable. We assume that the probability of a default by supplier $i$ is $1-\theta_{i}, i=1,2, \ldots, n$, and when in default, he delivers nothing, irrespective of the order quantity. Further, supplier $i$ 's disruption risk is exogenously determined and is therefore independent of the retailer's order quantity. Thus, supplier $i$ is reliable if $\theta_{i}$ equals to 1 and unreliable if $0<\theta_{i}<1$. We do not consider the trivial case of $\theta_{i}=0$. We label supplier $i$ to be more reliable than supplier $j$ when $\theta_{i}>\theta_{j}, i, j=1,2, \ldots, n$.

The selling season consists of three stages. In the first stage, the suppliers compete by deciding their wholesale prices $w_{i}, i=1,2, \ldots, n$. We denote supplier $i$ 's production cost as $c_{i}$. In the second stage, the retailer chooses the quantity $Q_{i}$ to order from supplier $i$ at the wholesale price of $w_{i}$. Let $S_{i}\left(Q_{i}\right)$, which is either $Q_{i}$ or 0 , denote the delivered quantity from supplier $i$. We assume that $S_{i}\left(Q_{i}\right)$ is Bernoulli distributed so that $S_{i}\left(Q_{i}\right)=Q_{i}$ with probability $\theta_{i}$, and 0 otherwise. The retailer only pays each supplier for the delivered quantity. In the third stage, based on the total delivered quantity $S\left(Q_{1}, Q_{2}, \ldots, Q_{n}\right)=S_{1}\left(Q_{1}\right)+S_{2}\left(Q_{2}\right)+$ $\ldots+S_{n}\left(Q_{n}\right)$, abbreviated as $S$, the retailer decides the unit retail price $p$ for the product. Such a retailer is termed a responsive-pricing retailer (see Li et al. 2013) because she sets the retail price in response to the actual delivered quantity. To better focus on the impact
of supply uncertainty, we assume that demand is deterministic and linear in price, that is, $D(p)=a-b p$, where $a>0, b>0$. To ensure that the retailer can make a positive profit and to avoid trivial cases, we assume $c_{i}<a / b$. Holdback rather than clearance, we presume, can result in possible unsold units. Note that the clearance assumption is used mostly for mathematical tractability, while the holdback assumption is often more general. The retailer may salvage unsold products in a secondary market at a unit price $\gamma$ with $0<\gamma<c_{i}$. The unit cost of lost goodwill is $\delta \geq 0$ owing to any unfulfilled demand.

## Retailer's Problem

The objective of the retailer is to determine the retail price $p$ in the third stage of the game and the order quantities $Q_{1}, Q_{2}, \ldots, Q_{n}$ in the second stage so as to maximize her expected profit $\Pi\left(Q_{1}, Q_{2}, \ldots, Q_{n}\right)$, which is equal to the expected optimal third-stage profit $E\left[\pi\left(Q_{1}, Q_{2}, \ldots, Q_{n}\right)\right]$ minus the expected purchase $\operatorname{cost} C\left(Q_{1}, Q_{2}, \ldots, Q_{n}\right)$ in the second stage. The retailer's problem is:

$$
\begin{align*}
\max _{Q_{1}, Q_{2}, \ldots, Q_{n} \geq 0}\left\{\Pi\left(Q_{1}, Q_{2}, \ldots, Q_{n}\right)=\right. & \left.E\left[\pi\left(Q_{1}, Q_{2}, \ldots, Q_{n}\right)\right]-C\left(Q_{1}, Q_{2}, \ldots, Q_{n}\right)\right\}  \tag{3.1}\\
\text { where } \pi\left(Q_{1}, Q_{2}, \ldots, Q_{n}\right)= & \max _{p \geq 0}\left\{p \cdot \min \left\{D(p), S\left(Q_{1}, Q_{2}, \ldots, Q_{n}\right)\right\}\right. \\
& +\gamma \cdot\left(S\left(Q_{1}, Q_{2}, \ldots, Q_{n}\right)-D(p)\right)^{+}  \tag{3.2}\\
& \left.-\delta \cdot\left(D(p)-S\left(Q_{1}, Q_{2}, \ldots, Q_{n}\right)\right)^{+}\right\} \\
C\left(Q_{1}, Q_{2}, \ldots, Q_{n}\right)= & E\left[\sum_{i=1}^{n} w_{i} S_{i}\left(Q_{i}\right)\right] .
\end{align*}
$$

The retailer's third-stage profit consists of three terms: the first term is her sales revenue at the retail price $p$, the second term is her salvage revenue if there are leftover products, and the third term is the shortage cost if there is unsatisfied demand. It is worth noting that, while Babich et al. (2007) assume the suppliers to be paid based on the retailer's order quantities, it is straightforward to show that our payment scheme would not change the managerial insights (such as Theorem 1) obtained in their paper.

## Suppliers' Problems

In the first stage, the $n$ suppliers decide their wholesale prices simultaneously by playing a Nash game. We solve it by finding the best response wholesale price of each supplier to his rivals' wholesale prices. As a Stackelberg leader, by knowing the retailer's best sourcing decisions $\widetilde{Q}_{i}\left(w_{1}, w_{2}, \ldots, w_{n}\right)$ for $i=1,2, \ldots, n$, supplier $i$ 's objective is to maximize his expected profit $\Pi_{i}\left(w_{i} \mid \mathbf{w}_{-i}\right)$ by choosing his wholesale price $w_{i}$ given his rivals' wholesale price vector $\mathbf{w}_{-i}$. For example, if $i=1$, then $\mathbf{w}_{-i}=\left(w_{2}, \ldots, w_{n}\right)$. Therefore, supplier $i$ 's problem is:

$$
\begin{equation*}
\max _{w_{i}}\left\{\Pi_{i}\left(w_{i} \mid \mathbf{w}_{-i}\right)=\left(w_{i}-c_{i}\right) E\left[S_{i}\left(\widetilde{Q}_{i}\left(w_{i}, \mathbf{w}_{-i}\right)\right)\right]\right\}, i=1,2, \ldots, n . \tag{3.3}
\end{equation*}
$$

We use backward induction to obtain the equilibrium of the Stackelberg-Nash game under consideration. The analysis of the third stage is included in this section. In Sections 3.4 3.6, we provide a complete analysis of the first and second stages for the benchmark setting with two suppliers. We extend our analysis to the general setting with $n$ suppliers in Section 3.8.

## Analysis of Third Stage

In the third stage, the retailer's problem (4.2) can be formulated as:

$$
\max _{p \geq 0}\left\{p \cdot \min \{D(p), S\}+\gamma \cdot(S-D(p))^{+}-\delta \cdot(D(p)-S)^{+}\right\}
$$

where $S$ is the realized total delivered quantity. Referring to Theorem 1 of Li et al. (2013), the optimal retail price for any given $S$ in the third stage is

$$
\widetilde{p}= \begin{cases}\frac{a+\gamma b}{2 b}, & \text { if } S \geq A  \tag{3.4}\\ \frac{a-S}{b}, & \text { otherwise }\end{cases}
$$

where $A$, referred to as the abundant supply, equals $(a-\gamma b) / 2$, and this denotes a threshold above which excess units are salvaged. That is, when the total delivered quantity is abundant, the retailer sets the price at $(a+\gamma b) / 2 b$ and salvages the leftover products for a total profit of $(a-\gamma b)^{2} / 4 b+\gamma S$. Otherwise, the retailer sets the price so as to clear the inventory and obtains $S(a-S) / b$ as her profit.

In either scenario, the price is set such that the demand is less than or equal to the delivered quantity, and therefore no shortage is incurred. This explains why we do not see the presence of the shortage cost parameter $\delta$ in the price formula (4.4). Note that if the demand were assumed to be stochastic, then the shortage cost parameter would influence the optimal price. However, the resulting extra complexity would not qualitatively alter the main managerial insights obtained in this study.

### 3.4 Two Suppliers

In this section, we study the case in which the retailer orders from two correlated unreliable suppliers and obtain the game equilibrium. To model the codependence between the two suppliers' disruption risks, we follow the approach used in Babich et al. (2007) and introduce a parameter $\psi_{11}$ to represent the probability that both suppliers provide the retailer with her order quantities successfully. In particular, with two independent suppliers, $\psi_{11}=\theta_{1} \theta_{2}$. Let $\psi_{10}$ denote the probability that supplier 1 delivers the order and supplier 2 does not, $\psi_{01}$ represent the probability that supplier 2 delivers the order and supplier 1 does not, and $\psi_{00}$ denote the probability that disruption happens to both suppliers. Clearly, $\psi_{10}=\theta_{1}-\psi_{11}$, $\psi_{01}=\theta_{2}-\psi_{11}$, and $\psi_{00}=1-\theta_{1}-\theta_{2}+\psi_{11}$. Moreover, with fixed $\theta_{1}$ and $\theta_{2}$, when the codependence increases, $\psi_{11}$ and $\psi_{00}$ increase while $\psi_{10}$ and $\psi_{01}$ decrease.

A number of approaches can model the joint disruption distribution in order to display the effects of the increasing codependence between the supplier disruption risks. For example, based on these event probabilities, we can calculate the Pearson correlation coefficient $\rho$ that measures the strength and direction of a linear relationship between the two suppliers' disruption probabilities as follows:

$$
\begin{equation*}
\rho=\frac{\psi_{11}-\theta_{1} \theta_{2}}{\sqrt{\theta_{1} \theta_{2}\left(1-\theta_{1}\right)\left(1-\theta_{2}\right)}} \tag{3.5}
\end{equation*}
$$

Note that there is a one-to-one correspondence between the correlation coefficient $\rho$ and probability $\psi_{11}$ when the marginal distributions are fixed, which provides us with the theoretical support for the analysis in Section 3.6. To ease the exposition, it is convenient to use $\psi_{11}$ as a proxy measure of the correlation between the unreliable suppliers.

Note that the feasible range of $\psi_{11}$ is dependent on the values of $\theta_{1}$ and $\theta_{2}$, and $\psi_{11}$ cannot exceed $\theta_{1}$ and $\theta_{2}$. To avoid trivial cases, we assume that $\psi_{11} \in\left(0, \min \left\{\theta_{1}, \theta_{2}\right\}\right)$. In particular, when $\theta_{1}=\theta_{2}=1 / 2$, the feasible range of $\psi_{11}$ is $(0,1 / 2)$, and both perfectly negative and positive correlations are possible.

### 3.4.1 Analysis of Second Stage

The analysis of the third stage is independent of the correlation between the two suppliers. In this subsection, we provide the analysis of the retailer's optimal sourcing strategy in the second stage where the retailer decides the optimal order quantities ( $\left.\widetilde{Q}_{1}, \widetilde{Q}_{2}\right)$ to maximize her expected profit $\Pi\left(Q_{1}, Q_{2}\right)$, given the optimal retail price obtained in equation (4.4). The following lemma summarizes her optimal sourcing strategy with the suppliers.

Lemma 8. For $i=1,2$, the retailer's optimal order quantities from the two correlated unreliable suppliers are

$$
\left\{\begin{array}{l}
\widetilde{Q}_{1}=\frac{a-b w_{1}}{2}, \widetilde{Q}_{2}=0, \quad \text { if } w_{2} \geq C_{E 1}\left(w_{1}\right) ; \\
\widetilde{Q}_{i}=\frac{\theta_{3-i}\left(a \psi_{i 0}-b \theta_{i} w_{i}+b \psi_{11} w_{3-i}\right)}{2\left(\theta_{1} \theta_{2}-\psi_{11}^{2}\right)}, \quad \text { if } \Phi\left(w_{1}, w_{2}\right) \geq 0, w_{i}<C_{E 3-i}\left(w_{3-i}\right) ; \\
\widetilde{Q}_{i}=\frac{a \psi_{i 0}-b \theta_{i} w_{i}+b \gamma \psi_{11}}{2\left(\theta_{i}-\psi_{11}\right)}, \quad \text { if } \Phi\left(w_{1}, w_{2}\right)<0, w_{i}<C_{E 3-i}\left(w_{3-i}\right) ; \\
\widetilde{Q}_{1}=0, \widetilde{Q}_{2}=\frac{a-b w_{2}}{2}, \quad \text { if } w_{1} \geq C_{E 2}\left(w_{2}\right),
\end{array}\right.
$$

where $\Phi\left(w_{1}, w_{2}\right) \equiv b \theta_{2} \psi_{10} w_{2}+b \theta_{1} \psi_{01} w_{1}-a \psi_{10} \psi_{01}-b \gamma\left(\theta_{1} \theta_{2}-\psi_{11}^{2}\right), C_{E i}\left(w_{i}\right)=w_{i}+(a-$ $\left.b w_{i}\right)\left(\theta_{3-i}-\psi_{11}\right) / b \theta_{3-i}$.

To facilitate the analysis, we adopt the notion of (unit) effective purchase cost introduced by Li et al. (2013), which for the retailer when purchasing from supplier $i$ is

$$
C_{E i}\left(w_{i}\right)=w_{i}+\left(a-b w_{i}\right)\left(\theta_{3-i}-\psi_{11}\right) / b \theta_{3-i}, i=1,2 .
$$

The significance of the concept of effective purchase cost is that it allows us to delineate the single and dual sourcing regions. Specifically, if $w_{3-i} \geq C_{E i}\left(w_{i}\right)$, then the retailer sources only from supplier $i, i=1,2$. Otherwise, she dual sources. Note that, under single sourcing, her optimal order quantity does not depend on the chosen supplier's reliability.

Since the retailer's effective purchase cost from a supplier is always no less than that supplier's wholesale price, the supplier with a lower wholesale price will always be chosen by the retailer according to Lemma 8. This is consistent with the insight "cost is the order qualifier" stated in Dada et al. (2007).

With two unreliable suppliers, the retailer's optimal total order quantity may be greater than the abundant supply, which potentially leads to a holdback situation. This occurs when both suppliers' wholesale prices are sufficiently low (i.e., $\Phi\left(w_{1}, w_{2}\right)<0$ ). Under this condition, the order quantity from supplier $i$ does not depend on $w_{3-i}, i=1,2$. However, as we shall demonstrate in the next subsection, in equilibrium, strategic suppliers do not have incentives to reduce wholesale prices to satisfy this condition. Therefore, the equilibrium wholesale prices always lead to the retailer's clearance sale. That is, the equilibrium wholesale prices satisfy $\Phi\left(w_{1}, w_{2}\right) \geq 0$, and consequently, the total order quantity is lower than the abundant supply and the order quantity from each supplier depends on both suppliers' wholesale prices, reliabilities, and the disruption correlation.

The retailer's sourcing strategy with two independent unreliable suppliers can be given by Lemma 8 when $\psi_{11}$ is replaced by $\theta_{1} \theta_{2}$. In the case of two independent unreliable suppliers, $C_{E i}\left(w_{i}\right)$ is decreasing in $\theta_{i}$ for $i=1,2$, and as a result, the retailer's dual sourcing region shrinks when either supplier's reliability increases. Clearly, with two reliable suppliers, the
two effective purchase cost lines and the line $w_{1}=w_{2}$ superpose, and the diversification zone disappears.

A noteworthy result from Lemma 8 is that with one reliable and one unreliable supplier, a responsive-pricing retailer may diversify, while a price-taking retailer will always single source, as shown by Babich et al. (2007). Although the payment scheme is different in their paper, it can be verified that this result does not depend on the payment scheme. The comparison of this preliminary result between a responsive-pricing retailer and a price-taking retailer highlights the fundamental difference between these two retailer types. A responsivepricing retailer has more flexibility when dealing with supply uncertainty, while the choices are more rigid for a price-taking retailer. This fundamental difference enables us to obtain, in the remainder of this chapter, a number of interesting and important results that do not emerge under the assumption of a price-taking retailer.

### 3.4.2 Analysis of First Stage

In anticipation of the retailer's optimal ordering and pricing decisions found in the second and third stages, the two suppliers compete in a Nash game by setting their wholesale prices simultaneously. Considering both suppliers' reaction curves (see Online Supplement) are piecewise functions and depend on the retailer's diversification decision, the Nash equilibrium can lead to single sourcing or dual sourcing depending on the suppliers' cost and reliability attributes. We encapsulate these results in the following theorem:

Theorem 9. There exists a unique Nash equilibrium $\left\{w_{1}^{*}, w_{2}^{*}\right\}$ for the two correlated unreliable suppliers:
(1) If $c_{1}<\bar{C}_{2}$ and $c_{2}<\bar{C}_{1}$, then $\left\{w_{1}^{*}, w_{2}^{*}\right\}=\left\{\left(b \theta_{2}\left(2 c_{1} \theta_{1}+c_{2} \psi_{11}\right)+a\left(2 \theta_{1} \theta_{2}-\psi_{11} \theta_{2}-\right.\right.\right.$ $\left.\left.\left.\psi_{11}^{2}\right)\right) / b\left(4 \theta_{1} \theta_{2}-\psi_{11}^{2}\right),\left(b \theta_{1}\left(2 c_{2} \theta_{2}+c_{1} \psi_{11}\right)+a\left(2 \theta_{1} \theta_{2}-\psi_{11} \theta_{1}-\psi_{11}^{2}\right)\right) / b\left(4 \theta_{1} \theta_{2}-\psi_{11}^{2}\right)\right\} ;$
(2) If $\bar{C}_{2} \leq c_{1}<\hat{C}_{2}$, then $\left\{w_{1}^{*}, w_{2}^{*}\right\}=\left\{c_{1}, C_{E 2}^{-1}\left(c_{1}\right)\right\}$;
(3) If $\bar{C}_{1} \leq c_{2}<\hat{C}_{1}$, then $\left\{w_{1}^{*}, w_{2}^{*}\right\}=\left\{C_{E 1}^{-1}\left(c_{2}\right), c_{2}\right\}$;
(4) If $c_{1} \geq \hat{C}_{2}$, then $\left\{w_{1}^{*}, w_{2}^{*}\right\}=\left\{c_{1}, w_{2}^{M}\right\}$;
(5) If $c_{2} \geq \hat{C}_{1}$, then $\left\{w_{1}^{*}, w_{2}^{*}\right\}=\left\{w_{1}^{M}, c_{2}\right\}$,
where $\bar{C}_{i}=\left(2 a \theta_{1} \theta_{2}-a \psi_{11} \theta_{i}+b c_{i} \theta_{i} \psi_{11}-a \psi_{11}^{2}\right) / b\left(2 \theta_{1} \theta_{2}-\psi_{11}^{2}\right)<\hat{C}_{i}=\psi_{11} w_{i}^{M} / \theta_{3-i}+\left(\theta_{3-i}-\right.$ $\left.\psi_{11}\right) a /\left(\theta_{3-i} b\right), w_{i}^{M}=\left(a+b c_{i}\right) / 2 b, C_{E 3-i}^{-1}\left(c_{i}\right)=\left[b c_{i} \theta_{i}-a\left(\theta_{i}-\psi_{11}\right)\right] / b \psi_{11}$ for $i=1,2$.

For the ease of exposition, for $i=1,2$, we term $\hat{C}_{i}$ as supplier $i$ 's threat threshold, which means that there is a threat for supplier $i$ when $c_{3-i}$ is below $\hat{C}_{i}$. We term $\bar{C}_{i}$ as supplier $i$ 's entry barrier for supplier $3-i$, and supplier $3-i$ receives an order only when $c_{3-i}$ is below the entry barrier of his rival.

Our research continues with the explanation of the results in Theorem 9 with respect to the change in the production cost of supplier 1 when supplier 2 receives an order. Without loss of generality, we assume $c_{1} \geq c_{2}$ in the following analysis. There are three regimes to be examined. We term the case $c_{1}<\bar{C}_{2}$ and $c_{2}<\bar{C}_{1}$ regime $1, \bar{C}_{2} \leq c_{1}<\hat{C}_{2}$ regime 2, and $c_{1} \geq \hat{C}_{2}$ regime 3. This analysis could be interpreted for convenience in exposition by considering supplier 2 as the incumbent, since he will always be chosen, and supplier 1 as a (potential) entrant who is not in the market in regime 3 when $c_{1}$ is very high and could be in the market when $c_{1}$ is lower. The sensitivity analysis that we implement concerning $c_{1}$ can be explained alternatively by considering supplier 1 transitioning from regime 3 to regime 2 , and then to regime 1 as his cost decreases. Regime 3 can be viewed as the monopoly regime in the sense that only supplier 2 receives an order, and he can set his wholesale price to be the monopoly price $w_{2}^{M}$. As $c_{1}$ becomes low enough to be in regime 2 , the wholesale price charged by supplier 2 is lower than the monopoly price, which can be viewed as a defensive action by the incumbent to prevent the entrant from entering the market. Finally, when $c_{1}$ is low enough to be in regime 1, any further defensive action is not in the interest of supplier 2 and supplier 1 receives an order from the retailer, which can be deemed as the entrant finally entering the market. In what follows, we provide a further elaboration of the above description.

In regime 3 , the equilibrium is single sourcing from supplier 2 only and $w_{2}^{*}=w_{2}^{M}$. We call $w_{2}^{M}$ supplier 2's monopoly wholesale price, because this is the price supplier 2 would charge if there were no supplier 1 . Clearly, when $c_{1} \geq \hat{C}_{2}$, the presence of supplier 1 does not threaten supplier 2 at all. The threat threshold $\hat{C}_{i}$ can be viewed as the expected value of two prices $a / b$ and $w_{i}^{M}$ given no disruption occurs for supplier $3-i$. These two prices are the maximum marginal prices the retailer is willing to pay conditional on whether supplier $i$ is disrupted or not, respectively. In other words, if a disruption does not occur for suppler $i$, then the retailer will pay the monopoly price $w_{i}^{M}$ charged by supplier $i$, and if it does, then the retailer is willing to pay up to $a / b$ per unit to make a positive profit. In summary, in order to become a threat to the incumbent, the potential entrant's production cost must be lower than the retailer's expected willingness to pay for an additional unit.

In regime 2, when supplier 1 breaks the threat threshold $\hat{C}_{2}$ and consequently deters supplier 1 from being sourced by the retailer, supplier 2 takes a defensive action by lowering his wholesale price to $C_{E 2}^{-1}\left(c_{1}\right)$, which is the inverse function of $C_{E 2}(\cdot)$. In this way, from Lemma 8, the retailer will single source from supplier 2. Even though supplier 2 can deter supplier 1's entry by doing so, his expected profit is harmed by the threat from supplier 1. As long as $c_{1}$ remains above $\bar{C}_{2}$, single sourcing remains in equilibrium. However, when $c_{1}<\bar{C}_{2}$, it is no longer beneficial for supplier 2 to deter supplier 1's entry. Hence, in regime 2 , because of the threat from supplier 1 , supplier 2 reduces his equilibrium wholesale price as supplier 1's production cost decreases. Once supplier 1's production cost is below the entry barrier $\bar{C}_{2}$, supplier 2 is better off by allowing supplier 1 to enter the market and charging a higher equilibrium wholesale price.

In regime 1, the equilibrium is dual sourcing, which is tantamount to saying that supplier 1's production cost is low enough for him to be in the market. The retailer only sources from supplier 1 under the condition $c_{2} \geq \bar{C}_{1}$, when $c_{2}$ is above the entry barrier of supplier 1. The retailer can then charge supplier 1 the monopoly wholesale price when $c_{2}$ is above supplier 1's threat threshold.

When the two suppliers are independent, i.e., $\psi_{11}=\theta_{1} \theta_{2}$, we have the equilibrium described in the following corollary to Theorem 9 .

Corollary 10. There exists a unique Nash equilibrium $\left\{w_{1}^{*}, w_{2}^{*}\right\}$ with two independent unreliable suppliers:
(1) If $c_{1}<\bar{C}_{2}$ and $c_{2}<\bar{C}_{1}$, then $\left\{w_{1}^{*}, w_{2}^{*}\right\}=\left\{\left[b\left(2 c_{1}+c_{2} \theta_{2}\right)+a\left(2-\theta_{2}-\theta_{1} \theta_{2}\right)\right] / b(4-\right.$ $\left.\left.\theta_{1} \theta_{2}\right),\left[b\left(2 c_{2}+c_{1} \theta_{1}\right)+a\left(2-\theta_{1}-\theta_{1} \theta_{2}\right)\right] / b\left(4-\theta_{1} \theta_{2}\right)\right\} ;$
(2) If $\bar{C}_{2} \leq c_{1}<\hat{C}_{2}$, then $\left\{w_{1}^{*}, w_{2}^{*}\right\}=\left\{c_{1}, C_{E 2}^{-1}\left(c_{1}\right)\right\}$;
(3) If $\bar{C}_{1} \leq c_{2}<\hat{C}_{1}$, then $\left\{w_{1}^{*}, w_{2}^{*}\right\}=\left\{C_{E 1}^{-1}\left(c_{2}\right), c_{2}\right\}$;
(4) If $c_{1} \geq \hat{C}_{2}$, then $\left\{w_{1}^{*}, w_{2}^{*}\right\}=\left\{c_{1}, w_{2}^{M}\right\}$;
(5) If $c_{2} \geq \hat{C}_{1}$, then $\left\{w_{1}^{*}, w_{2}^{*}\right\}=\left\{w_{1}^{M}, c_{2}\right\}$,
where $\bar{C}_{i}=\left(2 a-a \theta_{i}+b c_{i} \theta_{i}-a \theta_{1} \theta_{2}\right) / b\left(2-\theta_{1} \theta_{2}\right)<\hat{C}_{i}=\theta_{i} w_{i}^{M}+\left(1-\theta_{i}\right) a / b, w_{i}^{M}=\left(a+b c_{i}\right) / 2 b$, $C_{E 3-i}^{-1}\left(c_{i}\right)=\left[b c_{i}-a\left(1-\theta_{3-i}\right)\right] / b \theta_{3-i}, i=1,2$.

Corollary 10 is important because it will enable us to perform a sensitivity analysis with respect to the suppliers' reliabilities in Section 3.5. As stated in Lemma 8, if the suppliers' wholesale prices are sufficiently low (i.e., $\Phi\left(w_{1}, w_{2}\right)<0$ ) so that the retailer dual sources, then the retailer's optimal total order quantity is larger than the abundant supply, which could result in a holdback situation. Corollary 11 shows that this holdback situation does not occur in equilibrium, because the suppliers, being strategic, will not reduce their wholesale prices to a level at which the retailer has an incentive to order more than the abundant supply. Thus, in equilibrium, the retailer will always choose a clearance sale in the third stage.

Corollary 11. In equilibrium, the retailer always chooses clearance instead of holdback.
We now proceed to the special case in which supplier 1 is perfectly reliable and supplier 2 is unreliable. In practice, the reliable supplier would likely have a higher production cost to achieve his reliability status. For completeness, however, we also analyze this unusual situation when the reliable supplier has a lower production cost as the following.

Corollary 12. If supplier 1 is reliable and supplier 2 is unreliable and $c_{1}<c_{2}$, then the retailer dual sources as long as the two production costs are not too far apart (i.e., $c_{2}<\bar{C}_{1}$ ).

When the reliable supplier has a lower production cost, it is natural to believe that he will dominate the unreliable supplier by pricing him out of the market. Surprisingly, Corollary 12 shows that it may not be in the reliable supplier's best interest to do so. Note that, to price the unreliable supplier out of the market, the reliable supplier would have to set the wholesale price the same as the unreliable supplier's production cost $c_{2}$. With this, the reliable supplier's received order quantity is $\left(a-b c_{2}\right) / 2$. When the reliable supplier chooses not to price the unreliable supplier out of the market, it is optimal for him to set the wholesale price at $\frac{2 a+2 b c_{1}-2 a \theta_{2}+b c_{2} \theta_{2}}{b\left(4-\theta_{2}\right)}>c_{2}$ when the two suppliers' production costs are relatively close ( $c_{1}<c_{2}<\bar{C}_{1}$ ). Because of the increased wholesale price, the reliable supplier's received order quantity decreases to $\frac{2 a-2 b c_{1}-2 a \theta_{2}+b c_{1} \theta_{2}-b c_{2} \theta_{2}}{2\left(4-\theta_{2}\right)\left(1-\theta_{2}\right)}<\left(a-b c_{2}\right) / 2$. Therefore, although the reliable supplier could exclude the unreliable supplier from the market, the sacrifice on the profit margin in doing so would outweigh the gain in terms of the received order quantity when the two suppliers' production costs are relatively close $\left(c_{1}<c_{2}<\bar{C}_{1}\right)$.

Note that this result does not hold with a price-taking retailer (see Babich et al. (2007)), because the retailer's optimal order quantity is either the same as the exogenous demand or zero. With a responsive-pricing retailer, the capability of adjusting demand through pricing enables the retailer to tailor her order quantities based on the suppliers' wholesale prices. Since the reliable supplier's profit is decided not only by the suppliers' wholesale prices, but also by his received order quantity as a reaction to those wholesale prices, the wholesale prices affect the reliable supplier in a more complicated way than a straightforward argument. The discussion progresses in greater detail throughout the remainder of the chapter.

### 3.5 Impact of Reliability

From Theorem 10, it is clear that the threat thresholds and entry barriers are important boundaries that influence the suppliers' competing strategies as well as the retailer's sourcing structure. The following theorem manifests how the threat thresholds and entry barriers are affected by the suppliers' reliabilities.

Theorem 13. For $i=1,2$, (1) supplier $i$ 's threat threshold $\hat{C}_{i}$ decreases in $\theta_{i}$, and is independent of $\theta_{3-i}$; (2) supplier $i$ 's entry barrier $\bar{C}_{i}$ decreases in $\theta_{1}$ and $\theta_{2}$.

A supplier's production cost and reliability can be viewed as two dimensions of his competitiveness. By definition, the threat threshold and entry barrier are both boundaries in terms of the production cost. From Corollary 10, the equilibrium sourcing structure depends on the comparison between the incumbent's threat threshold (or entry barrier) and the entrant's production cost. Surprisingly, an increase in the entrant's reliability does not make it easier for him to become a threat or enter the market. To be more specific, the incumbent's threat threshold is independent of the entrant's reliability because the comparison between the incumbent's threat threshold and the entrant's production cost does not change the equilibrium sourcing structure (i.e., single sourcing from the incumbent). Thus, the entrant's reliability does not come into play. Furthermore, the incumbent's entry barrier decreases in the entrant's reliability, because in anticipation of a more competitive opponent in the dual sourcing equilibrium, the incumbent is more determined to make it harder for the entrant to enter the market by reducing the entry barrier, i.e., to require the potential entrant to be even more cost efficient to make a profitable entry. This analysis implies that, reducing the production cost is the entrant's only way to enter the market, and improving reliability without lowering cost will make it increasingly difficult to enter the market. As shown next, even after entering the market, an improvement in reliability could hurt a supplier's profit due to the intensified competition.

We now examine the impact of the suppliers' reliabilities on the equilibrium profits of the retailer and the suppliers when the equilibrium is dual-sourcing. We omit the details for the single-sourcing scenario because they are straightforward and the results are intuitively obvious. As shown in equation (3.5), we are aware that $\psi_{11}$ has to change with $\theta_{1}$ and $\theta_{2}$ to keep the Pearson correlation coefficient constant. This change makes it difficult for us to perform a sensitivity analysis when the suppliers' reliabilities are correlated. Owing to this issue, we will only consider the case of two independent suppliers in this section, and discuss correlated suppliers in Section 3.6.

Intuitively, (i) the retailer would be better off if the suppliers' reliabilities were higher; (ii) a supplier would normally benefit from an increase (resp. a decrease) in his (resp. his rival's) reliability. However, the second intuition does not always hold as we see below. Based on the equilibrium result shown in Corollary 10, we have the following theorem:

Theorem 14. When the retailer diversifies in equilibrium, then for $i=1,2$,
(1) her profit increases in each supplier's reliability;
(2) supplier $i$ 's profit decreases in $\theta_{3-i}$, and increases in $\theta_{i}$ if $c_{3-i}-c_{i} \geq-\left(a-b c_{i}\right)$ $\left(8-4 \theta_{3-i}-10 \theta_{1} \theta_{2}-\theta_{i} \theta_{3-i}^{2}+5 \theta_{1}^{2} \theta_{2}^{2}+2 \theta_{i}^{2} \theta_{3-i}^{3}\right) / b \theta_{3-i}\left(4+\theta_{1} \theta_{2}-2 \theta_{1}^{2} \theta_{2}^{2}\right)$ and decreases otherwise;
(3) both suppliers' wholesale prices decrease in their reliabilities;
(4) the retailer's order quantity from supplier $i$ increases in $\theta_{i}$ if $c_{i}-c_{3-i} \leq\left(a-b c_{3-i}\right)(6-$ $\left.5 \theta_{3-i}-4 \theta_{1} \theta_{2}+2 \theta_{i} \theta_{3-i}^{2}+\theta_{1}^{2} \theta_{2}^{2}\right) / b\left(6-4 \theta_{1} \theta_{2}+\theta_{1}^{2} \theta_{2}^{2}\right)$ and decreases otherwise, and increases in $\theta_{3-i}$ if $c_{i}-c_{3-i} \leq\left(a-b c_{3-i}\right)\left(-4+6 \theta_{i}-4 \theta_{i}^{2} \theta_{3-i}+\theta_{1}^{2} \theta_{2}^{2}+\theta_{i}^{3} \theta_{3-i}^{2}\right) / b \theta_{i}\left(6-4 \theta_{1} \theta_{2}+\theta_{1}^{2} \theta_{2}^{2}\right)$ and decreases otherwise.

The first intuition that the retailer would be better off if the suppliers' reliabilities were higher is borne out (see Theorem 14 result (1)). This is because a higher reliability intensifies the competition between the suppliers, leading to lower wholesale prices that benefit the retailer.

The second intuition that a supplier would benefit from an increase (resp. a decreases) in his (resp. his rival's) reliability, however, does not necessarily hold for unreliable suppliers. Then we provide a complete explanation of this counter-intuitive result.

Intuitively, an increase in the reliability of supplier $i$ can increase the probability of successful delivery and may help him win more orders. However, this also leads to his rival setting a lower wholesale price and thus, the supplier may ultimately be hurt by the more severe competition. To isolate the effects caused by an increase in supplier $i$ 's reliability, we introduce the following two definitions.

The optimal expected profit function of supplier $i$ is a function of his reliability, and thus we define $\Pi_{i}^{*}\left(\theta_{i}\right) \equiv \theta_{i}\left(w_{i}^{*}\left(\theta_{1}, \theta_{2}\right)-c_{i}\right) \widetilde{Q}_{i}\left(\theta_{i}, w_{1}^{*}\left(\theta_{1}, \theta_{2}\right), w_{2}^{*}\left(\theta_{1}, \theta_{2}\right)\right)$. Note that $\widetilde{Q}_{i}$ is from Lemma 8. By taking the derivative of $\Pi_{i}^{*}\left(\theta_{i}\right)$ with respect to $\theta_{i}$, we develope the following result:

$$
\frac{d \Pi_{i}^{*}\left(\theta_{i}\right)}{d \theta_{i}}=\frac{\partial \Pi_{i}^{*}\left(\theta_{i}\right)}{\partial \theta_{i}}+\frac{\partial \Pi_{i}^{*}\left(\theta_{i}\right)}{\partial w_{i}^{*}} \cdot \frac{d w_{i}^{*}}{d \theta_{i}}+\frac{\partial \Pi_{i}^{*}\left(\theta_{i}\right)}{\partial w_{3-i}^{*}} \cdot \frac{d w_{3-i}^{*}}{d \theta_{i}} .
$$

From the Envelope theorem, we have $\frac{\partial \Pi_{i}^{*}\left(\theta_{i}\right)}{\partial w_{i}^{*}}=0$, which implies that supplier $i$ always chooses the optimal wholesale price (achieving a zero profit margin). Thus, the derivative above has only two components on the right-hand side: $\frac{\partial \Pi_{i}^{*}\left(\theta_{i}\right)}{\partial \theta_{i}}$ and $\frac{\partial \Pi_{i}^{*}\left(\theta_{i}\right)}{\partial w_{3-i}^{*}} \cdot \frac{d w_{3-i}^{*}}{d \theta_{i}}$. We define $\frac{\partial \Pi_{i}^{*}\left(\theta_{i}\right)}{\partial \theta_{i}}$ as supplier $i$ 's first-order effect of the reliability (we term this the reliability effect), and we have

$$
\frac{\partial \Pi_{i}^{*}\left(\theta_{i}\right)}{\partial \theta_{i}}=\theta_{i}\left(w_{i}^{*}\left(\theta_{1}, \theta_{2}\right)-c_{i}\right)\left(\frac{\widetilde{Q}_{i}}{\theta_{i}}+\frac{\partial \widetilde{Q}_{i}}{\partial \theta_{i}}\right)>0
$$

The reliability effect of supplier $i$ has a positive value since higher reliability can help him win more orders when the suppliers are non-strategic (derived from Lemma 8), namely $\frac{\partial \widetilde{Q}_{i}}{\partial \theta_{i}}>0$. This does not hold in Babich et al. (2007), where the order quantities from the suppliers under dual sourcing do not depend on their reliabilities. Responsive pricing thus enables the retailer to adjust demand in response to the delivered quantity. Consequently, her order quantities depend on the suppliers' reliabilities.

We define $\frac{\partial \Pi_{i}^{*}\left(\theta_{i}\right)}{\partial w_{3-i}^{*}} \cdot \frac{d w_{3-i}^{*}}{d \theta_{i}}$ as supplier $i$ 's second-order effect of the reliability (we term this the competition effect of the reliability), and we have

$$
\frac{\partial \Pi_{i}^{*}\left(\theta_{i}\right)}{\partial w_{3-i}^{*}} \cdot \frac{d w_{3-i}^{*}}{d \theta_{i}}=\theta_{i}\left(w_{i}^{*}\left(\theta_{1}, \theta_{2}\right)-c_{i}\right)\left(\frac{\partial \widetilde{Q}_{i}}{\partial w_{3-i}^{*}} \cdot \frac{d w_{3-i}^{*}}{d \theta_{i}}\right)<0
$$

With strategic suppliers, when one supplier's reliability increases, the resulting more intensified competition forces the other supplier to reduce his wholesale price to stay competitive, namely $\frac{d w_{3-i}^{*}}{d \theta_{i}}<0$. Owing to the retailer's ability to change her retail price responsively, her order quantity from one supplier decreases when the other supplier lowers his wholesale price, namely $\frac{\partial \widetilde{Q}_{i}}{\partial w_{3-i}^{*}}>0$. As a result, the competition effect of the reliability has a negative impact on a supplier's expected profit. From this perspective, our study is significantly different from Li et al. (2013), who only consider non-strategic suppliers and consequently do not analyze the competition effect.

Next, we examine whether this counter-intuitive result continues to hold without responsive pricing (i.e., when the demand no longer responds to the retail price). In our model, the demand becomes less sensitive to the retail price as $b$ decreases. When $b$ goes to zero, the condition under which a supplier gets hurt by increasing his reliability nearly disappears as well (see the condition in Theorem 14 result (2)). We summarize this result in the following corollary.

Corollary 15. As $b \rightarrow 0$, supplier $i$ 's profit tends to always increase in $\theta_{i}, i=1,2$.

As $b$ approaches 0 , our problem tends to reduce to the problem studied by Babich et al. (2007), who find that, with a price-taking retailer, a supplier can always benefit from improving his reliability. Corollary 15 demonstrates that responsive pricing is the reason why a supplier may not want to improve his reliability.

### 3.6 Impact of Correlation

A supplier disruption correlation exists for a large number of reasons as described in Section 1. In this section, we examine the impact of the disruption correlation between the two suppliers on the game equilibrium obtained in Theorem 9, and provide thorough explanations.

Theorem 16. For $i=1,2$, supplier $i$ 's threat threshold $\hat{C}_{i}$ and entry barrier $\bar{C}_{i}$ decrease in $\psi_{11}$.

With the other parameters fixed, a higher disruption correlation implies that the two suppliers are more substitutable in terms of their reliabilities, and therefore the need for the retailer to diversify reduces. The reduced reliability differentiation leads to a more intense wholesale price competition between the two suppliers. As a result, the cost advantage of the incumbent becomes more pronounced, and consequently, it is more difficult for the entrant to threaten the incumbent or to enter the market. As a result, the diversification zone shrinks as the disruption correlation increases. Next, we examine the impact of the disruption correlation on the retailer and the two suppliers, assuming that the entrant has entered the market.

Theorem 17. When the retailer diversifies in equilibrium, then for $i=1,2$,
(1) her profit increases in $\psi_{11}$;
(2) supplier $i$ 's profit increases in $\psi_{11}$ if $c_{i} \leq \tilde{C}_{3-i}$, and decreases otherwise, where $\tilde{C}_{3-i}=$ $c_{3-i}+\left(a-b c_{3-i}\right)\left(-4 \theta_{i}^{2} \theta_{3-i}^{3}+4 \theta_{1}^{2} \theta_{2}^{2} \psi_{11}-\theta_{i} \theta_{3-i}^{2} \psi_{11}^{2}-2 \theta_{1} \theta_{2} \psi_{11}^{3}+2 \theta_{3-i} \psi_{11}^{4}+\psi_{11}^{5}\right) / b \psi_{11}\left(4 \theta_{1}^{2} \theta_{2}^{2}-\right.$ $\left.2 \theta_{1} \theta_{2} \psi_{11}^{2}+\psi_{11}^{4}\right) ;$
(3) both suppliers' wholesale prices decrease in $\psi_{11}$;
(4) her order quantity from supplier $i$ increases in $\psi_{11}$ if $c_{i} \leq \check{C}_{3-i}$, and decreases otherwise, where $\check{C}_{3-i}=c_{3-i}+\left(a-b c_{3-i}\right)\left(-4 \theta_{i}^{2} \theta_{3-i}^{3}+12 \theta_{1}^{2} \theta_{2}^{2} \psi_{11}-5 \theta_{i} \theta_{3-i}^{2} \psi_{11}^{2}-8 \theta_{1} \theta_{2} \psi_{11}^{3}+3 \theta_{3-i} \psi_{11}^{4}+\right.$ $\left.2 \psi_{11}^{5}\right) / 2 b \psi_{11}\left(6 \theta_{1}^{2} \theta_{2}^{2}-4 \theta_{1} \theta_{2} \psi_{11}^{2}+\psi_{11}^{4}\right) ;$
(5) her total order quantity increases in $\psi_{11}$ if
$\left(4 \theta_{1}^{3} \theta_{2}^{2}-12 \theta_{1}^{2} \theta_{2}^{2} \psi_{11}+5 \theta_{1}^{2} \theta_{2} \psi_{11}^{2}+8 \theta_{1} \theta_{2} \psi_{11}^{3}-3 \theta_{1} \psi_{11}^{4}-2 \psi_{11}^{5}\right)\left(a-b c_{1}\right)$
$+\left(4 \theta_{1}^{2} \theta_{2}^{3}-12 \theta_{1}^{2} \theta_{2}^{2} \psi_{11}+5 \theta_{1} \theta_{2}^{2} \psi_{11}^{2}+8 \theta_{1} \theta_{2} \psi_{11}^{3}-3 \theta_{2} \psi_{11}^{4}-2 \psi_{11}^{5}\right)\left(a-b c_{2}\right)<0$, and decreases otherwise.

In a procurement problem in which suppliers are not decision-makers, the retailer prefers the suppliers to be negatively correlated to obtain the diversification benefits. However, in our procurement problem with strategic suppliers, we show that the retailer prefers the suppliers to have highly correlated default events despite the loss of diversification benefits (see Theorem 17 result (1)). The reason is that a higher default correlation induces more intense price competition between the two suppliers, leading to lower wholesale prices (see Theorem 17 result (3)). Because the competition effect dominates the diversification effect, a high disruption correlation thus benefits the retailer. This result is similar to the findings of Babich et al. (2007), who study a price-taking retailer's procurement problem with strategic suppliers. Therefore, this result is robust with respect to the retailer's pricing power.

It is shown in Babich et al. (2007) that with a price-taking retailer, the suppliers prefer defaults that are negatively correlated because of the dampened competition. We show that this result continues to hold with a responsive-pricing retailer when the two suppliers' production costs are relatively close. Interestingly, we find that when a supplier's production cost is sufficiently lower than his rival's, he can benefit from a high disruption correlation (see Theorem 17 result (2)). As the disruption correlation increases, the two suppliers become more substitutable with each other in terms of their reliabilities, which leads to a more intense price competition between them (see Theorem 17 result (3)). When a supplier's production cost is sufficiently lower than his rival's, his inherent cost advantage becomes more pronounced as the competition intensity increases, which results in a larger order quantity from the retailer (see Theorem 17 result (4)). The increased order quantity may
more than compensate for the loss of the profit margin, and thus increase the supplier's expected profit.

In contrast to Theorem 3 in Li et al. (2013), we find that the retailer's total order quantity may increase in the disruption correlation (see Theorem 17 result (5)). The incorporation of strategic suppliers is the fundamental reason behind this drastic difference. The intensified price competition between the two suppliers caused by an increased disruption correlation gives the low-cost supplier an edge to win more orders (see Theorem 17 result (4)). Consequently, the retailer's total order quantity may increase when the two suppliers' costs differ significantly. This result complements that of Li et al. (2013) by capturing the potential competition effect between the suppliers.

To better understand the impact and importance of responsive pricing, we next provide a thorough explanation of Theorem 17 result (2). Note that in equilibrium, supplier $i$ 's expected profit function can be deemed to be a function of $\psi_{11}$. Define $\Pi_{i}^{*}\left(\psi_{11}\right) \equiv$ $\Pi_{i}^{*}\left(\psi_{11}, w_{1}^{*}\left(\psi_{11}\right), w_{2}^{*}\left(\psi_{11}\right)\right)=$ $\theta_{i}\left(w_{i}^{*}\left(\psi_{11}\right)-c_{i}\right) \widetilde{Q}_{i}\left(\psi_{11}, w_{1}^{*}\left(\psi_{11}\right), w_{2}^{*}\left(\psi_{11}\right)\right)$. By taking the derivative of $\Pi_{i}^{*}\left(\psi_{11}\right)$ with respect to $\psi_{11}$, we have the following result:

$$
\frac{d \Pi_{i}^{*}\left(\psi_{11}\right)}{d \psi_{11}}=\frac{\partial \Pi_{i}^{*}\left(\psi_{11}\right)}{\partial \psi_{11}}+\frac{\partial \Pi_{i}^{*}\left(\psi_{11}\right)}{\partial w_{i}^{*}} \cdot \frac{d w_{i}^{*}}{d \psi_{11}}+\frac{\partial \Pi_{i}^{*}\left(\psi_{11}\right)}{\partial w_{3-i}^{*}} \frac{d w_{3-i}^{*}}{d \psi_{11}} .
$$

Since supplier $i$ always chooses his optimal wholesale price that achieves a zero profit margin, we have $\frac{\partial \Pi_{i}^{*}\left(\psi_{11}\right)}{\partial w_{i}^{*}}=0$ according to the Envelope theorem. Therefore, $\frac{d \Pi_{i}^{*}\left(\psi_{11}\right)}{d \psi_{11}}$ is composed of two parts: $\frac{\partial \Pi_{i}^{*}\left(\psi_{11}\right)}{\partial \psi_{11}}$ and $\frac{\partial \Pi_{i}^{*}\left(\psi_{11}\right)}{\partial w_{3-i}^{*}} \frac{d w_{3-i}^{*}}{d \psi_{11}}$. We define $\frac{\partial \Pi_{i}^{*}\left(\psi_{11}\right)}{\partial \psi_{11}}$ as the first-order effect of the correlation (we term this the correlation effect) and $\frac{\partial \Pi_{i}^{*}\left(\psi_{11}\right)}{\partial w_{3-i}^{*}} \frac{d w_{3-i}^{*}}{d \psi_{11}}$ as the second-order effect of the correlation (we term this the competition effect of the correlation).

The correlation effect, $\frac{\partial \Pi_{i}^{*}\left(\psi_{11}\right)}{\partial \psi_{11}}=\theta_{i}\left(w_{i}^{*}\left(\psi_{11}\right)-c_{i}\right) \frac{\partial \widetilde{Q}_{i}}{\partial \psi_{11}}$, may have a positive or a negative influence on a supplier's profit depending on how the retailer's order quantity responds to the disruption correlation (i.e., $\frac{\partial \widetilde{Q}_{i}}{\partial \psi_{11}}$ ). As shown in Corollary 18 , this depends on the difference between the two suppliers' wholesale prices.

Corollary 18. When the retailer diversifies for clearance, then for $i=1,2, \widetilde{Q}_{i}$ increases in $\psi_{11}$ if $b\left(\theta_{1} \theta_{2}+\psi_{11}^{2}\right) w_{3-i}-2 b \theta_{i} \psi_{11} w_{i} \geq a \theta_{1} \theta_{2}+a \psi_{11}^{2}-2 a \theta_{i} \psi_{11}$, and decreases otherwise.

As the disruption correlation increases, the benefit of diversification reduces. Therefore, the retailer's order quantity decreases when the two suppliers' wholesale prices are relatively close. However, when the suppliers' wholesale prices differ significantly, the retailer finds it more beneficial to order more from the supplier with a lower wholesale price owing to the reduction in the purchase cost. Although Li et al. (2013) provide a similar comment, we establish this result explicitly. The ultimate driver behind Corollary 18 is responsive pricing. Responsive pricing happens in the third stage and this enables the retailer to set the retail price to optimize her profit after delivery, and its impact emerges originally from Lemma 8. Compared with the problem studied by Babich et al. (2007), where the price-taking retailer's order quantities remain constant, the responsive-pricing retailer's second-stage order quantities are dependent on the suppliers' reliabilities and the disruption correlation.

The competition effect of the correlation, $\frac{\partial \Pi_{i}^{*}\left(\psi_{11}\right)}{\partial w_{3-i}^{*}} \frac{d w_{3-i}^{*}}{d \psi_{11}}=\theta_{i}\left(w_{i}^{*}\left(\psi_{11}\right)-c_{i}\right) \frac{\partial \widetilde{Q}_{i}}{\partial w_{3-i}^{*}} \frac{d w_{3-i}^{*}}{d \psi_{11}}$, is always negative because $\frac{\partial \widetilde{Q}_{i}}{\partial w_{3-i}^{*}}>0$ (from Lemma 8 ) and $\frac{d w_{3-i}^{*}}{d \psi_{11}}<0$ (from Theorem 17). That is, an increase in the correlation intensifies competition and consequently drives the rival's wholesale price lower. Because of responsive pricing, this leads to a lower order quantity from the retailer. As a result, supplier $i$ is always harmed by the competition induced by the increased correlation.

In summary, supplier $i$ benefits from an increase in the supplier disruption correlation when the correlation effect is positive and it dominates the competition effect. Theorem 17 has some important managerial implications. With a responsive-pricing retailer, the supplier with a significant inherent production cost advantage may have an incentive to create a positively correlated supply network by building plants in the same geographic location as his rival or choosing the same tier-2 supplier to form a diamond-shaped supply
chain strategically (Ang et al. 2016). Without responsive pricing, however, this result no longer holds, as demonstrated by Corollary 19 .

Corollary 19. As $b \rightarrow 0$, supplier $i$ 's profit tends to always decrease in $\psi_{11}, i=1,2$.

As explained earlier, the demand becomes less sensitive to the retail price as $b$ decreases. Moreover, as $b \rightarrow 0$, our problem tends to reduce to the problem studied by Babich et al. (2007), who find that with a price-taking retailer, a supplier always becomes worse off when the disruption correlation increases. Again, Corollary 19 serves as a linkage between our work and the literature, highlighting this study's contribution toward extending the literature.

To understand the thresholds used in Theorem 17, we compare them in Corollary 20. This comparison reveals that in the diversification zone, the region in which supplier $i$ 's expected profit increases in the disruption correlation is a subset of the region in which supplier $i$ 's received order quantity increases in the disruption correlation.

Corollary 20. $\tilde{C}_{i} \leq \check{C}_{i} \leq \bar{C}_{i}, i=1,2$.

### 3.7 Impact of Competition Mode

In this section, we consider a different competition mode in which the responsive-pricing retailer commits to ordering from only one of the two suppliers, to emphasize the importance of the suppliers' Nash game in the benchmark model. We call the chosen supplier the exclusive supplier of the retailer. An exclusive supplier exists in practice when the retailer is prohibited from sourcing from more than one supplier at the same time for various reasons (see Yang et al. 2012, Demirel et al. 2017, and the references therein).

Without loss of generality, let supplier 1 be more reliable than supplier 2 , namely $\theta_{1} \geq \theta_{2}$. When the two suppliers provide an equivalent source for the retailer, we assume that she chooses supplier 1. The game sequence is as follows. In the first stage of the game, the two suppliers announce their wholesale prices, $w_{1}$ and $w_{2}$, simultaneously. Second, the retailer
chooses one supplier to order from and decides the order quantity. In the third stage, the retailer decides the retail price according to the delivery quantity. The third-stage problem is solved by equation (4.4) and we know that the retailer always uses clearance when she sources from one supplier only.

As shown in Theorem 21, which supplier will be chosen as the exclusive supplier depends on both suppliers' costs and reliabilities. Reducing cost or improving reliability can help a supplier become the exclusive supplier. Interestingly, the exclusive supplier's equilibrium wholesale price does not depend on his production cost because his goal is to stay in the market while setting the highest possible wholesale price, which is related to the other supplier's cost.

In the second stage, the retailer decides from which supplier to order and the corresponding order quantity. Based on the wholesale prices announced in the first stage, the retailer compares her profits when sourcing from these different suppliers. Because the suppliers' reliabilities differ, the retailer may decide to order from the more reliable supplier that quotes a higher wholesale price. As a result, the retailer orders from supplier 1 only when his wholesale price is sufficiently low (specifically, $w_{1} \leq \sqrt{\frac{\theta_{2}}{\theta_{1}}} w_{2}+\frac{a}{b}\left(1-\sqrt{\frac{\theta_{2}}{\theta_{1}}}\right)$ ) and orders from supplier 2 otherwise. This exclusive-supplier competition mode leads to an extremely intense "in-or-out" price competition that results in the following equilibrium between the two suppliers.

Theorem 21. (1) The equilibrium wholesale prices: $\left\{w_{1}^{*}, w_{2}^{*}\right\}=\left\{c_{1}, \sqrt{\frac{\theta_{1}}{\theta_{2}}} c_{1}-\frac{a}{b}\left(\sqrt{\frac{\theta_{1}}{\theta_{2}}}-1\right)\right\}$ if $c_{1}>\sqrt{\frac{\theta_{2}}{\theta_{1}}} c_{2}+\frac{a}{b}\left(1-\sqrt{\frac{\theta_{2}}{\theta_{1}}}\right)$, and $\left\{w_{1}^{*}, w_{2}^{*}\right\}=\left\{\sqrt{\frac{\theta_{2}}{\theta_{1}}} c_{2}+\frac{a}{b}\left(1-\sqrt{\frac{\theta_{2}}{\theta_{1}}}\right), c_{2}\right\}$ otherwise; (2) The profit of the exclusive supplier always increases in his reliability.

The exclusive supplier's profit is always higher when his reliability is higher. This result is different from the result obtained in the benchmark model where the retailer can dual source. Because the competition is extremely severe and one supplier is entirely excluded from the
market, the excluded supplier cannot have any further effect on the increased reliability of the other supplier. Therefore, the excluded supplier is not able to harm the supplier that receives the order by lowering his wholesale price. As a result, the exclusive supplier always benefits from a higher reliability. This finding shows that the impact of reliability on the suppliers is highly dependent on the retailer's sourcing strategy.

### 3.8 Multiple Suppliers

In this section, we consider the problem where the retailer faces more than two competing suppliers and investigate their wholesale prices in equilibrium. Owing to the complexity involved with $N$ suppliers, we analyze separately the case of independent disruption risks in Section 8.1 and the case of correlated disruption risks in Section 8.2 by making some simplifying assumptions. We demonstrate that the key results developed in the benchmark model are still borne out with multiple suppliers.

### 3.8.1 Independent Disruption Risks

Owing to analytical intractability, the insights to be obtained from a general model, in which there are $n$ independent suppliers differing from one another in terms of reliability and cost are limited. In this subsection, therefore, we focus on the problem with $n$ independent and identical suppliers.

Suppose there are $n \geq 2$ independent suppliers with reliability $\theta$ and production cost c. The game sequence is similar to that in the benchmark model. In the first stage, the $n$ suppliers announce their wholesale prices simultaneously to compete with each other. Second, the retailer responds with quantity $\widetilde{Q}_{i}$ to order from supplier $i$ at his wholesale price denoted by $w_{i}$; In the third stage, the retailer decides the responsive retail price to react to the total delivered quantity. In this subsection, we assume that the retailer uses the clearance sale in the third stage. This assumption has been adopted extensively in the literature for
mathematical tractability. Our analysis of the two-supplier setting also confirms that the retailer always chooses clearance instead of holdback in equilibrium (Corollary 11).

In the third stage of the game, from equation (4.4), the retailer's optimal clearance price is $\widetilde{p}=\frac{a-S}{b}$ for any given $S$. In the second stage, the retailer's objective is to determine the order quantities $Q_{1}, \ldots, Q_{n}$ to maximize her expected profit $\Pi\left(Q_{1}, \ldots, Q_{n}\right)$, which is equal to the expected optimal third-stage profit minus the expected purchase cost in the second stage. In summary, the retailer's problem is:

$$
\max _{Q_{1}, \ldots, Q_{n} \geq 0}\left\{\Pi\left(Q_{1}, \ldots, Q_{n}\right)=E\left[\widetilde{p} \cdot S\left(Q_{1}, \ldots, Q_{n}\right)\right]-E\left[\sum_{i=1}^{n} w_{i} S_{i}\left(Q_{i}\right)\right]\right\}
$$

where $S_{i}\left(Q_{i}\right)$ is the delivered quantity from supplier $i, i=1,2, \ldots, n$.

Lemma 22. For $i=1,2, \ldots, n$, the retailer's optimal order quantity from supplier $i$ is

$$
\widetilde{Q}_{i}=\frac{(1-\theta) a-[(n-2) \theta+1] b w_{i}+b \theta\left(\sum_{j=1}^{n} w_{j}-w_{i}\right)}{2(1-\theta)[1+(n-1) \theta]} .
$$

As we can see, the retailer's order quantity from a supplier always decreases in his wholesale price and increases in the wholesale prices of his competitors.

In the first stage of the game, the $n$ suppliers decide their wholesale prices simultaneously playing a Nash game. As a Stackelberg leader, by knowing the retailer's best sourcing decisions $\widetilde{Q}_{i}$ for $i=1,2, \ldots, n$, supplier $i$ 's objective is to maximize his expected profit $\Pi_{i}\left(w_{i} \mid \mathbf{w}_{-i}\right)$ by choosing his wholesale price $w_{i}$ given his rivals' wholesale price vector $\mathbf{w}_{-i}$. We use backward induction to obtain the equilibrium of the Stackelberg-Nash game. The result is encapsulated in the following theorem.

Theorem 23. For $i=1, \ldots, n$, (1) the equilibrium wholesale price of supplier $i$ is $w_{i}^{*}=$ $\frac{(1-\theta) a+b c[1+\theta(n-2)]}{b[2+(n-3) \theta]}$; (2) $w_{i}^{*}$ decreases in $n$; (3) $w_{i}^{*} \rightarrow c$ as $n \rightarrow+\infty$.

The equilibrium wholesale prices depend on the number of suppliers. Specifically, the equilibrium wholesale price decreases as the number of suppliers increases because of the
intensified competition. The number of suppliers can be considered to be a proxy of the competition level here. As the number of suppliers or reliability increases, each supplier quotes a lower wholesale price to be more competitive. When the number of suppliers in the market tends to infinity, competition in the market tends to be complete. This forces the suppliers to reduce their wholesale prices all the way down to the production cost, and earn zero profit.

Theorem 24. For $i=1, \ldots, n$, supplier $i$ 's profit decreases in $\theta$ if $\theta \geq F(n)$, and increases otherwise, where $F(n)=\frac{1-5 n+8 n^{2}-5 n^{3}+n^{4}+(4-2 n) h(n)+h^{2}(n)}{\left(5-2 n-2 n^{2}+n^{3}\right) h(n)}$ and $h(n)=\left(-1-5 n+23 n^{2}-\right.$ $\left.32 n^{3}+21 n^{4}-7 n^{5}+n^{6}+\sqrt{(n-1)^{4} n\left(5-2 n-2 n^{2}+n^{3}\right)}\right)^{\frac{1}{3}}$.

We see that the counterintuitive result, namely that a supplier's profit may decrease in his reliability, is also borne out when there are $n$ independent suppliers.

### 3.8.2 Correlated Disruption Risks

To investigate the impact of correlation under multiple suppliers and retain mathematical tractability at the same time, we establish the following structure that is connected to the benchmark model and the related literature (Li et al. 2013).

Consider two groups of suppliers, groups 1 and 2. Assume that there are $m$ suppliers in group 1 , where each supplier has production $\operatorname{cost} c_{1}$ and reliability $\theta_{1}$. In group 2 , there are $n$ suppliers that have production cost $c_{2}$ and reliability $\theta_{2}$. Any two suppliers from the same group are perfectly positively correlated and any two suppliers from different groups are correlated with both groups delivering success probability $\psi_{11}$, as defined in Section 3.4. To be consistent with those results and for analytical simplicity, in this section, we only consider the equilibrium in the dual-sourcing clearance region where the total order quantity is lower than the abundant supply.

Let $Q_{j}^{i}$ denote the order quantity from supplier $j$ in group $i$, where $j=1,2, \ldots, m$ (or $n$ ) if $i=1$ (or 2 ). The total order quantity from group $i$ is denoted by $Q^{i}$, where $i=1,2$. Since
the suppliers in the same group are identical and perfectly positively correlated, then by symmetry, their received order quantities and wholesale prices are the same. Let $w^{i}$ denote the wholesale price of a supplier in group $i, i=1,2$.

The game sequence is as follows. First, $w^{i}$ is announced by the suppliers in group $i$ simultaneously, for $i=1,2$. Second, the retailer decides $Q^{1}$ and $Q^{2}$ in response to $w^{1}$ and $w^{2}$. Third, the retailer chooses the optimal retail price after delivery.

In the third stage of the game, the total delivered quantity is denoted as $S\left(Q_{1}^{1}, \ldots, Q_{m}^{1}, Q_{1}^{2}, \ldots, Q_{n}^{2}\right)$ (abbreviated as $S$ ) whose probability mass function is given below:

$$
S\left(Q_{1}^{1}, \ldots, Q_{m}^{1}, Q_{1}^{2}, \ldots, Q_{n}^{2}\right)=\left\{\begin{array}{lc}
Q_{1}^{1}+\ldots+Q_{m}^{1}, & \text { w. p. } \theta_{1}-\psi_{11} \\
Q_{1}^{2}+\ldots+Q_{n}^{2}, & \text { w. p. } \theta_{2}-\psi_{11} \\
Q_{1}^{1}+\ldots+Q_{m}^{1}+Q_{1}^{2}+\ldots+Q_{n}^{2}, \quad \text { w. p. } \psi_{11} \\
0, & \text { w. p. } 1-\theta_{1}-\theta_{2}+\psi_{11}
\end{array}\right.
$$

The optimal retail price for any given $S$ is given by equation (4.4). In the second stage of the game, the retailer's objective is to determine $Q^{1}$ and $Q^{2}$ in order to maximize her expected profit. According to Lemma 8, the retailer's optimal order quantity from group $i$ is

$$
\begin{equation*}
\widetilde{Q}^{i}=\frac{\theta_{3-i}\left(a \psi_{i 0}-b \theta_{i} w^{i}+b \psi_{11} w^{3-i}\right)}{2\left(\theta_{1} \theta_{2}-\psi_{11}^{2}\right)}, i=1,2 . \tag{3.6}
\end{equation*}
$$

In the first stage of the game, $w^{i}$ is decided by the suppliers in group $i$, for $i=1,2$. According to equation 3.6 , we have $w^{i}=\frac{a \theta_{i}-2 \widetilde{Q}^{i} \theta_{i}-2 \widetilde{Q}^{3-i} \psi_{11}}{b \theta_{i}}$ for $i=1,2$, where $\widetilde{Q}^{1}=$ $Q_{1}^{1}+\ldots+Q_{m}^{1}$ and $\widetilde{Q}^{2}=Q_{1}^{2}+\ldots+Q_{n}^{2}$. Thus, we can rewrite the problem of supplier $j$ in group $i$ as:

$$
\max _{Q_{j}^{i} \geq 0}\left\{\theta_{i} Q_{j}^{i}\left\{\frac{a \theta_{i}-2\left[Q_{j}^{i}+\left(\widetilde{Q}^{i}-Q_{j}^{i}\right)\right] \theta_{i}-2 \widetilde{Q}^{3-i} \psi_{11}}{b \theta_{i}}-c_{i}\right\}\right\}
$$

We introduce a Cournot-like competition inside each group to obtain the explicit equilibrium results since the problem loses tractability under the original game sequence. By solving this problem for all suppliers we obtain the following result.

Theorem 25. (1) The equilibrium wholesale prices and order quantities are

$$
\left\{\begin{array}{l}
\left(w^{1}\right)^{*}=\frac{a \theta_{1} \theta_{2}+b m c_{1} \theta_{1} \theta_{2}+a n \theta_{1} \theta_{2}+b m n c_{1} \theta_{1} \theta_{2}-a n \theta_{2} \psi_{11}+b n c_{2} \theta_{2} \psi_{11}-b m n c_{1} \psi_{11}^{2}}{b\left(\theta_{1} \theta_{2}+m \theta_{1} \theta_{2}+n \theta_{1} \theta_{2}+m n \theta_{1} \theta_{2}-m n \psi_{11}^{2}\right)} \\
\left(w^{2}\right)^{*}=\frac{a \theta_{1} \theta_{2}+b n c_{2} \theta_{1} \theta_{2}+a m \theta_{1} \theta_{2}+b m n c_{2} \theta_{1} \theta_{2}-a m \theta_{1} \psi_{11}+b m c_{1} \theta_{1} \psi_{11}-b m n c_{2} \psi_{11}^{2}}{b\left(\theta_{1} \theta_{2}+m \theta_{1} \theta_{2}+n \theta_{1} \theta_{2}+m n \theta_{1} \theta_{2}-m n \psi_{11}^{2}\right)} \\
\left(Q_{k}^{1}\right)^{*}=\frac{\theta_{2}\left(a \theta_{1}-b c_{1} \theta_{1}+a n \theta_{1}-b n c_{1} \theta_{1}-a n \psi_{11}+b n c_{2} \psi_{11}\right)}{2\left(\theta_{1} \theta_{2}+m \theta_{1} \theta_{2}+n \theta_{1} \theta_{2}+m n \theta_{1} \theta_{2}-m n \psi_{11}^{2}\right)} \\
\left(Q_{h}^{2}\right)^{*}=\frac{\theta_{1}\left(a \theta_{2}-b c_{2} \theta_{2}+a m \theta_{2}-b m c_{2} \theta_{2}-a m \psi_{11}+b m c_{1} \psi_{11}\right)}{2\left(\theta_{1} \theta_{2}+m \theta_{1} \theta_{2}+n \theta_{1} \theta_{2}+m n \theta_{1} \theta_{2}-m n \psi_{11}^{2}\right)}
\end{array}\right.
$$

where $k=1, \ldots, m$ and $h=1, \ldots, n$. (2) When $m \rightarrow+\infty$ and $n \rightarrow+\infty,\left(w^{1}\right)^{*} \rightarrow c_{1}$, $\left(w^{2}\right)^{*} \rightarrow c_{2},\left(Q^{1}\right)^{*} \rightarrow \frac{\theta_{2}\left(a \theta_{1}-a \psi_{11}-b \theta_{1} c_{1}+b \psi_{11} c_{2}\right)}{2\left(\theta_{1} \theta_{2}-\psi_{11}^{2}\right)},\left(Q^{2}\right)^{*} \rightarrow \frac{\theta_{1}\left(a \theta_{2}-a \psi_{11}-b \theta_{2} c_{2}+b \psi_{11} c_{1}\right)}{2\left(\theta_{1} \theta_{2}-\psi_{11}^{21}\right)}$, where $\left(Q^{1}\right)^{*}=$ $\sum_{k=1}^{m}\left(Q_{k}^{1}\right)^{*}$ and $\left(Q^{2}\right)^{*}=\sum_{h=1}^{n}\left(Q_{h}^{2}\right)^{*}$.

When $m \rightarrow+\infty$ and $n \rightarrow+\infty$, the optimal wholesale price in each group tends to be the production cost in that group and the total order quantities in groups 1 and 2 tend to be $\widetilde{Q}_{1}$ and $\widetilde{Q}_{2}$ in Lemma 8 by replacing $w_{1}$ with $c_{1}$ and $w_{2}$ with $c_{2}$, respectively. Thus, this limiting case is akin to the problem of Li et al. (2013) where responsive pricing and non-strategic suppliers are considered.

Theorem 26. The profit of supplier 1 in group 1 increases in $\psi_{11}$ if $\left(a \theta_{1}-b c_{1} \theta_{1}+a n \theta_{1}-\right.$ $\left.b n c_{1} \theta_{1}-a n \psi_{11}+b n c_{2} \psi_{11}\right)\left(-a \theta_{1} \theta_{2}+b c_{2} \theta_{1} \theta_{2}-a m \theta_{1} \theta_{2}+b m c_{2} \theta_{1} \theta_{2}-a n \theta_{1} \theta_{2}+b n c_{2} \theta_{1} \theta_{2}-a m n \theta_{1} \theta_{2}+\right.$ $\left.b m n c_{2} \theta_{1} \theta_{2}+2 m a \theta_{1} \psi_{11}-2 m b c_{1} \theta_{1} \psi_{11}+2 m n a \theta_{1} \psi_{11}-2 m n b c_{1} \theta_{1} \psi_{11}-a m n \psi_{11}^{2}+b m n c_{2} \psi_{11}^{2}\right) \geq 0$, and decreases otherwise.

We examine the impact of the disruption correlation between the two suppliers on the game equilibrium we obtained above. We see, as in Section 3.6 for the case of two suppliers, that a supplier may benefit from an increase in the supplier disruption correlation when the correlation effect is positive and it dominates the competition effect. Without loss of generality, we only illustrate the conditions for supplier 1 in group 1 in Theorem 26. The conditions for the other suppliers can be easily obtained.

### 3.9 Conclusion

It is widely believed that high reliability and low correlation with other suppliers should benefit an unreliable supplier. By studying a responsive-pricing retailer's sourcing and pricing problem involving competing strategic suppliers subject to independent or correlated disruptions, we show that while this belief is true in most cases, there exist conditions under which the belief is not borne out. The problem is formulated as a Stackelberg-Nash game in which the suppliers act as the Stackelberg leaders when they set wholesale prices simultaneously in a Nash game. The retailer is the Stackelberg follower by deciding its respective order quantities ex-ante from the suppliers and the retail price ex-post. We identify the reliability effect and the competition effect of the reliability, and show that a higher reliability can hurt an unreliable supplier when the competition effect of the reliability dominates the reliability effect. We also identify the correlation and competition effects of the correlation, and demonstrate that a low-cost supplier can benefit from a higher disruption correlation when the correlation effect is positive and outweighs the competition effect of correlation. Furthermore, we show that the responsive-pricing retailer's total order quantity may increase as the suppliers' disruption risks correlation. Without responsive pricing, the reliability effect always dominates the competition effect of the reliability, and the correlation effect is always negative, therefore the aforementioned belief continues to hold. In conclusion, the competing unreliable suppliers' strategies are highly dependent on the retailer's pricing power.

In this chapter, to keep the analysis explicit, we have considered unreliable suppliers that face random disruptions. In practice, there are also situations where the unreliable suppliers are facing random capacity/yield. Even though we conjecture that the insights continue to hold qualitatively under different assumptions of supply reliability, because the reliability, correlation, and competition effects of the reliability and correlation do not depend on how supply reliability is modeled. It is nevertheless worthwhile to rigorously examine the robustness of the current results with respect to the type of supply disruptions. To
focus on the impact of the suppliers' reliability and correlation on their pricing as well as on the retailer's pricing and sourcing decisions, we have assumed that the demand is deterministic. As a future research topic, it would thus be of interest to examine whether the main insights of this study continue to hold with a stochastic demand. Then one can also study how the demand uncertainty affects the interaction between the retailer and her unreliable suppliers. Other future research directions include examining the impact of the firm's risk-averse behavior (Kazaz and Webster 2011) and investigating strategies regarding asymmetric information (Yang et al. 2009, Yang and Babich 2015).

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# CHAPTER 4 CREATIVE DELINQUENCY OR DESTRUCTIVE SELFISHNESS? ${ }^{2}$ 

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### 4.1 Introduction

Suppose a person, say John, is waiting to get on a bus in a crowded bus station. If everyone is waiting in a queue, then staying in the queue may be Johns only choice. But if some people are jumping the queue, what will John do? What will be Johns choice many others are jumping the queue and there are not enough available seats? Furthermore, if the driver announces that he would allow standing passengers, how would it affect Johns decision? We face such decision problems in our daily lives as well as in the work place. A well-known example is that of firms located on a lake, which use water from the lake in their production processes and then discharge the used dirty water back into the lake; see for example Sethi (1977) or a Homework problem (www.chegg.com/homework-help/questions-and-answers/two-firms-b-situated-next-lake-costs-firm-1-500-per-period-use-filters-avoid-polluting-lak-q17449589). A practical instance of such a firm is a Chloralkali plant given in Thompson et al. (1974).

A government regulation requires that the dirty water be purified before discharging it into the lake. However, since there is a cost associated with the purification process, the firms have an economic incentive to disobey the rule. Moreover, the cost of production may go up with the level of pollution of the lake. While some managers obey the rule because of their higher moral standards, others may not. It is also possible that the obeying managers may lower their standards when they see others polluting the lake. But then the lake becomes quite polluted increasing everyone's production cost. And this may call for an equilibrium solution. This chapter analyzes such problems of noncompliance in operations management to explain why people disobey rules and to what degree, by using a game-theoretic framework. There are two primary reasons for discussing such decision problems. First, whether-or-not-to-obey-a-rule decisions are common in nearly all management processes.

Before any operations decision is made, a manager should have already subconsciously (or intentionally) pre-decided whether or not to play by conventional rules. In traditional decision making models, the usual assumption is that the decision makers always adhere to
rules in obtaining their optimal solution. However, this optimal solution might not be the most attractive one because disregarding the option of not to obey the rules could be an unrealistic assumption. Second, an increasing number of disobedience cases are observed universally in operations management and other domains. The milk adulteration case in China and the Enron scandal in the US are well-known examples. A greater perceived benefit seems to invite disobedience. Regulatory authorities and managers are hard pressed to understand, control, or change this behavior because the traditional financial penalty and moral deterrent seem ineffectual. We also see that individual disobedience occurs in daily life and work, such as speeding violations or personal use of company supplies. It is of interest to note that some actions of noncompliance can improve efficiency. For example, during periods of heavy traffic or at times of highway accidents, many drivers use hard shoulders, high occupancy vehicle lanes while not eligible, or even cut across grass to gain access to off ramp or frontage road. This selfish and unlawful behavior often goes unpunished as it is difficult to trace so many violators or it is viewed to alleviate traffic congestion. Disobedience decisions, like all other decisions, are influenced by factors such as economic benefits and costs. With that said, there will be temptation to disobey when the net gain from disobedience is positive. However, real cases do not always happen in this way, as many people do not disobey even when they see a net gain in doing so. This behavior is due to the fact that their moral standard perceives breaking a rule to incur an intangible moral cost. Nevertheless, ones moral standard does not always match the existing rules. If it is below the rules, then one is likely to disobey them in order to obtain potential benefits.

If it is above the rules, one tends to obey the rules unless the economic benefit from disobedience is greater than the moral cost of disobedience. Human behavioral factors, in addition to economic and moral considerations, also influence peoples decisions. Besides unfairness, other behavioral factors such as social preference and group identity play a role in disobedience decisions, leading to the often quoted adage: when in Rome, do as the

Romans do. This chapter develops a game-theoretic model in which two competitive profitmaximizing firms facing a guiding rule in a single industry decide their disobedience degrees to achieve an equilibrium. Examples of guiding rules imposed by a regulatory authority or the government are: sewage should not be discharged without being treated to a maximum allowed level of pollutant concentration, workers should be paid at least the minimum wage, etc. A linear production cost and a convex moral cost are introduced. In addition, we consider behavioral factors such as rules unfairness and group preference for imitation. Moreover, our disobedience analysis can aid the rule-makers in seeking an optimal rule set. The relevant behavioral operations literature also enlightens us in our analysis. An important behavioral factor in this chapter is unfairness. Fehr and Schmidt (1999) model fairness as self-centered inequity aversion, indicating that people resist unfair outcomes. In their model, ones utility equals ones benefit minus unfairness utilities, and the weight given to loss unfairness is larger than that given to surplus unfairness. We use the idea of weights in a fractional form instead of the additive form. Wu et al. (2008) develop a model of a fair process in a principal-agent context, rooted in psychological preferences for autonomy and fairness. Another behavioral factor is social (group) preference. Giorgi and Post (2011) investigate a reference-dependent choice with a stochastic, state-dependent reference point to explain the social reference effect on decision making.

We introduce a new decision variable termed the disobedience level of a given rule by each firm engaged in a Cournot competition along with parameters of moral standard and punishment. Thus the firms choose not only their production quantities but also their levels of disobedience in a Nash equilibrium setting. We solve the game to obtain these decisions and then study the amount of punishment to reduce the disobedience levels and their effect on the firms profits. We identify the situations of Creative delinquency and Destructive Selfishness, respectively, as those with firms disobediences causing an increase in surplus and those with a firms disobedience benefiting the self and hurting the other in
way that decreases surplus. After that, we consider the behavioral factor of unfairness the firms perceive over the rule as well as the disobedience action of a firm induced by rivals disobedience behavior. Finally, we consider obtaining an optimal rule that can be imposed by a regulatory authority. The introduction of a guiding rule, moral standard and associated moral costs in our chapter induces the firms to not deviate too much from the guiding rule, and thus behave in a socially responsible manner to some extent. This is because the guiding rules are imposed by a regulatory authority that is typically responsible for ensuring social welfare. Thus, broadly speaking, our chapter may be viewed to be related to the vast literature on the social responsibility of corporations.

Finally, given that this chapter is invited for the special issue for the IJPR 55th volume anniversary, we would like to mention that International Journal of Production Research has published numerous contributions addressing the issue of the social responsibility of corporations. Indeed, as early as 1974, Fazakerley (1974) suggested the importance of integrating production systems with social values as a topic for future perspectives of industrialized societies. In an opening address to a conference devoted to social issues such as quality of life in factory work, Goppel (1979) emphasized the necessity of exploration of the interfaces between human factors, production technology, and resource management. Recently published papers such as Lu et al. (2013) draw global attention to corporate social responsibility by providing empirical results that illustrate that the investments by US semiconductor firms on social responsibility have positive impact on their bottom line.

### 4.2 The Base Model

We consider the problem of two competitive firms 1 and 2 selling identical products in a market and operating under a single guiding imposed by a regulatory authority. Examples of guiding rules were given in the introduction. A firm may choose to disobey the guiding rule as it may help to lower its production cost, but it may come with some punishment
and/or self-condemnation. Moreover, the amount of punishment may depend on the extent to which the rule is disobeyed. In Sections 2-5, we assume a specified, and in Section 6 we extend it to the case when the guiding rule is a decision variable of the regulatory authority.

Following the classic Cournot duopoly model, we assume full information and full observation and that the firms simultaneously decide their disobedience levels as well as their production quantities.

Firm's disobedience level is denoted as $x_{i}\left({\underset{\rightarrow}{x}}_{i} \leq x_{i} \leq x_{\text {Rule }}\right), i=1,2$, where $x_{i}<x_{\text {Rule }}$ means firm $i$ disobeys the rule and $x_{i}=x_{\text {Rule }}$ means it obeys the rule, and ${\underset{\rightarrow}{~}} \leq 0$ is the most firm $i$ can disobey by. The effect of the disobedience level on the production cost is assumed to be proportional. For example, in the case of minimum wage rule, any violation would obviously result in a proportional decrease in the production cost. Thus, firm $i$ 's unit production cost as

$$
\begin{equation*}
c_{p i}=v_{i}+t_{i} x_{i}, c_{p i} \geq 0, i=1,2 \tag{4.1}
\end{equation*}
$$

where $v_{i}$ is firm $i$ 's unit variable cost with $v_{1} \neq v_{2}$, and $t_{i} \geq 0$ is the decrease in the cost per unit disobedience. We assume $0 \leq t_{i} \leq-v_{i} /{\underset{\rightarrow}{i}}_{i}$, so that the production cost of firm $i$ remains nonnegative.

The firms also face a proportional punishment of $-\left(x_{i}-x_{\text {Rule }}\right) n$, where $n \geq 0$ is the punishment of unit violation. We define $n-t_{i}$ to be the strength of punishment and characterize two punishment regimes: weak punishment if $n-t_{i} \leq 0$ and strong punishment if $n-t_{i}>0$.

Aside from these costs, firm $i$ is also subject to an intangible disobedience cost called the moral cost, which depends on both the disobedience degree $x_{i}$ and the moral standard $m_{i} \leq 0$ of firm $i$ 's manager, with 0 representing high moral standard. A manager's moral standard refers to his resistance to temptation of benefits from disobedience. Based on the experimental results, List (2007) verifies the existence of such a cost through the dictator game. According to the prospect theory of Tversky and Kahneman (1981), people are more
sensitive to losses (negative things) than to gains (positive things), and the value functions of losses are convex. Based on these ideas, we express the moral cost as

$$
c_{m i}\left\{\begin{array}{c}
b_{i}\left(x_{i}-x_{\text {Rule }}-m_{i}\right)^{2}, \text { if } x_{i} \leq x_{\text {Rule }}+m_{i}  \tag{4.2}\\
0, \text { otherwise, } i=1,2
\end{array}\right.
$$

where $b_{i}>0$ is manager $i$ 's personality coefficient of his disobedience tendency, which influences the shape of his moral cost curve. Thus, $x_{\text {Rule }}+m_{i}$ is firm $i$ 's inner rule which can be different from the guiding rule $x_{\text {Rule }}$ imposed by the authority. With a high moral standard $m_{i}=0$, the inner rule equals to the guiding rule, and any from the guiding rule incurs a moral cost.

The personality coefficient $b_{i}$ is a scaling factor relating to the cost. Specifically, $b_{i}>1$ means an upward scaling of the cost caused by moral standard, and $0<b_{i}<1$ means a downward scaling, and $b_{i}=1$ indicates no scaling.

Next we bring in the idea that a noncompliant action by one firm may cause harm to the other, resulting in an external diseconomy. So we define $d_{3-i} \geq 0$ to be the rate of external diseconomy, so that firm $3-i$ 's disobedience $x_{3-i}$ increases firm $i$ 's cost by $-d_{i} x_{3-i}$. Thus, the total cost to firm $i$ when it makes the decision $x_{i}$ and the rival firm makes the decision $x_{3-i}$ is
$C_{i}(x)=\left\{\begin{array}{c}v_{i}-\left(n-t_{i}\right) x_{i}+n x_{\text {Rule }}+b_{i}\left(x_{i}-x_{\text {Rule }}-m_{i}\right)^{2}-d_{i}\left(x_{3-i}-x_{\text {Rule }}\right), \text { otherwise }, \\ v_{i}-\left(n-t_{i}\right) x_{i}+n x_{\text {Rule }}-d_{i}\left(x_{3-i}-x_{\text {Rule }}\right), \text { if } x_{i} \geq x_{\text {Rule }}+m_{i},\end{array}\right.$

### 4.3 Basic Equilibrium Analysis

Let $q_{i}$ denote the production quantity of firm $i$ and let $q=\left(q_{1}, q_{2}\right)$. Let $P(Q)=a-Q$ ( $Q \leq a$ ) be the market-clearing price when the aggregate quantity available in the market is $Q=q_{1}+q_{2}$. The profit of firm $i$ is

$$
\begin{equation*}
\pi_{i}(q, x)=q_{i}\left[a-\left(q_{1}+q_{2}\right)-C_{i}(x)\right] . \tag{4.4}
\end{equation*}
$$

Based on the best responses by the firms to maximize their own profits, we derive the Nash equilibrium disobedience degrees by solving the equations of $\partial \pi_{i} / \partial x_{i}=0$ and $\partial \pi_{i} / \partial q_{i}=0$.

Proposition 27. In equilibrium, the optimal disobedience degree of firm $i$ is:

$$
x_{i}^{*}=\left\{\begin{array}{c}
\frac{n-t_{i}}{2 b_{i}}+m_{i}+x_{\text {Rule }}, \text { if } n \leq t_{i}  \tag{4.5}\\
x_{\text {Rule }}, \text { if } n>t_{i}, i=1,2 .
\end{array}\right.
$$

Proposition 1 says that if one's strength of punishment $n$ is no more than its disobedience benefit $t_{i}$, then the optimal disobedience degree is $\frac{n-t_{i}}{2 b_{i}}+m_{i}+x_{\text {Rule }}$. Otherwise, firm $i$ does not disobey. It is straightforward to see that one tends to behave worse than one's inner rule to get a better profit if the punishment is not strict enough, and one obeys the rule $x_{\text {Rule }}$ if the punishment is strict. Moreover, in the low punishment regime, a firm's disobedience decision is determined by the moral standard $m_{i}$, the guiding rule $x_{\text {Rule }}$, and the economic benefit $\frac{n-t_{i}}{2 b_{i}}$.

Proposition 1 also tells us that no matter the punishment, firm $i$ 's disobedience level will never be in the range $\left(m_{i}+x_{\text {Rule }}, x_{\text {Rule }}\right)$. This range is wider for a manager with a lower moral standard, inducing him to deviate more from $x_{\text {Rule }}$ when the punishment regime changes from $n>t_{i}$ to $n \leq t_{i}$. Therefore, in a lower moral standard society, the harm from disobedience would be greater than in a high moral standard society, if the authority could not maintain a strict punishment regime.

Next we derive the equilibrium production quantities.

Proposition 28. In equilibrium, the optimal production quantity of firm $i$ is:

$$
q_{i}^{*}=\left\{\begin{array}{c}
\frac{a-2 v_{i}+v_{3-i}-2 t_{i} x_{\text {Rule }}+t_{3-i} x_{\text {Rule }}}{3}, \text { if } n>\max \left\{t_{1}, t_{2}\right\}, \\
{\left[\begin{array}{c}
\frac{a-2 v_{i}+v_{3-i}-2 t_{i} x_{\text {Rule }}+t_{3-i} x_{\text {Rule }}}{3} \\
-\frac{m_{i}\left(d_{3-i}-2 n+2 t_{i}\right)}{3}-\frac{\left(n-t_{i}\right)\left(d_{3-i}-n+t_{i}\right)}{6 b_{i}}
\end{array}\right], \text { if } t_{3-i}<n \leq t_{i},} \\
{\left[\begin{array}{c}
\frac{a-2 v_{i}+v_{3-i}-2 t_{i} x_{\text {Rule }}+t_{3-i} x_{\text {Rule }}}{3} \\
+\frac{m_{3-i}\left(2 d_{i}-n+t_{3-i}\right)}{3}+\frac{\left(n-t_{3-i}\right)\left(4 d_{i}-n+t_{3-i}\right)}{12 b_{3-i}}
\end{array}\right], \text { if } t_{i}<n \leq t_{3-i},} \\
{\left[\begin{array}{c}
\frac{a-2 v_{i}+v_{3-i}-2 t_{i} x_{\text {Rule }}+t_{3-i} x_{\text {Rule }}}{3}+\frac{2 d_{i} m_{3-i}-d_{3-i} m_{i}}{3} \\
-\frac{\left(n-t_{i}\right)\left(d_{3-i}-n+t_{i}-4 b_{i} m_{i}\right)}{6 b_{i}}+\frac{\left(n-t_{3-i}\right)\left(4 d_{i}-n+t_{3-i}-4 b_{3-i} m_{3-i}\right)}{12 b_{3-i}}
\end{array}\right], \text { if } n \leq \min \left\{t_{1}, t_{2}\right\} .}
\end{array}\right.
$$

### 4.4 Effects of Economic and Moral Factors

This section studies how economic and moral factors affect the firms' equilibrium disobedience degrees. We prescribe the punishment strategy of a regulatory authority interested in deciding when to punish to deter disobedience and by how much. Finally, we evaluate the effect of disobedience on the total surplus defined to be the sum of the firms' profits. Consideration of the behavioral factors is postponed later.

### 4.4.1 Punishment Strategy

Proposition 29. Firm $i$ obeys the guiding rule when either $n>t_{i}$ or $m_{i}=0$ and $n=t_{i}$ and obeys its inner rule when $m_{i}<0$ and $n=t_{i}$.

Clearly, the punishment level $n$ plays a very important role in regulating firms' disobedience behavior. A strict punishment $n>t_{i}$ makes the moral standard $m_{i}$ of no consequence to firm $i$, since it will obey. When $n=t_{i}$, a high moral standard firm will obey the guiding rule and a low moral standard firm will simply observe its inner rule. Finally, when $n<t_{i}$, firm $i$ will break even its inner rule. A regulatory authority can therefore use punishment to influence the firms' disobedience degrees.

In most cases, disobedience brings a strict positive profit to a firm because the punishment is usually not very strict. This is because the administration of strict punishment and the required monitoring is very costly for the regulatory authority.

A manager of high moral also could disobey a rule, because the profit resulting from disobedience could be positive when $t_{i}>0$. So a way to deter disobedience is to reduce $t_{i}$, which effectively raises the cost of disobedience. For example, in a city with many jaywalkers, installing a barrier at the edges of the lanes will be effective as it would require a jaywalker to climb over the barrier, and thus reducing his and raising his cost of jaywalking.

### 4.4.2 Effects on Total Surplus

The total surplus or the sum of the profits of both firms in equilibrium is

$$
W\left(q^{*}, x^{*}\right)=\pi_{1}\left(q^{*}, x^{*}\right)+\pi_{2}\left(q^{*}, x^{*}\right) .
$$

In the case when firms' disobedience increases the total surplus, then we refer to this as the case of Creative delinquency. The opposite case is that of Destructive Selfishness.

To identify these cases, we start when $m_{1}=m_{2}=0, t_{1}=t_{2}=0$ and $n=0$, when the equilibrium is no disobedience from the guiding rule. Now suppose firm $i$ 's moral standard declines from 0 to $m_{i}<0$ for some external reasons, then firm $i$ would disobey the guiding rule with $x_{i}=m_{i}+x_{\text {Rule }}$. Facing this, should the regulatory authority formulate a punishment policy? Here we take the view that authority in interested in the amount of total surplus. The variation in the total surplus $\Delta W_{i}$ from the moral decline of firm $i$ is

$$
\begin{align*}
\Delta W & =W\left[q_{m_{i}<0}^{*},\left(m_{i}+x_{\text {Rule }}, x_{\text {Rule }}\right)\right]-W\left[q_{m_{i}=0}^{*},\left(x_{\text {Rule }}, x_{\text {Rule }}\right)\right]  \tag{4.6}\\
& =\frac{m_{i} d_{3-i}}{9}\left(2 a+5 m_{i} d_{3-i}+8 v_{i}-10 v_{3-i}\right), i=1,2 .
\end{align*}
$$

In the case when $m_{i}<\frac{-2 a-8 v_{i}+10 v_{3-i}}{5 d_{3-i}}, \Delta W>0$, we have creative delinquency. On the other hand, when $\frac{-2 a-8 v_{i}+10 v_{3-i}}{5 d_{3-i}}<m_{i}<0, \Delta W<0$, and we have the case of destructive Selfishness.

### 4.5 Impact of Behavioral Factors

According to Gino and Pisano (2008), the decision makers in real life may not be fully rational and devoid of emotions, and their decisions may be influenced by various factors such as fairness and social (group) preferences. In Section 5.1, we introduce the concept of Rule Unfairness to explain some disobedience decisions. Imitation and cultural identity that help us better understand the effect of group identity consciousness on disobedience decisions will be discussed in Section 5.2.

### 4.5.1 Rule Unfairness

Rule unfairness refers to the perception of inequity due to a relative difference in the cost incurred by the two firms from obeying a specific rule. Different rules could lead to $C_{i}>C_{3-i}$, $C_{i}<C_{3-i}$ or $C_{i}=C_{3-i}$ when both firms choose obedience $x_{i}=x_{3-i}=x_{\text {Rule }}$. Therefore firms would perceive unfairness if a threshold is crossed. Here we assume the threshold to be $\frac{C_{i}}{v_{i}}=\frac{C_{3-i}}{v_{3-i}}$. Unlike a product's cost parameters such as $t_{i}$ and $v_{i}$, the moral standard $m_{i}$ of a person can change from a perception of this inequity. Specifically, when both firms follow the rule, then firm $i$ 's obedience cost is $C_{i}\left(x_{i}, x_{3-i}\right)=C_{i}\left(x_{\text {Rule }}, x_{\text {Rule }}\right)$, and if $\frac{C_{i}\left(x_{\text {Rule }}, x_{\text {Rule }}\right)}{v_{i}}=$ $\frac{C_{3-i}\left(x_{\text {Rule }}, x_{\text {Rule }}\right)}{v_{3-i}}$, then firm $i$ views the rule as unfair. To incorporate this effect, we let $\gamma_{i}$ denote the coefficient of rule unfairness by which the moral standard of firm $i$ changes from $m_{i}$ to $m_{i} \gamma_{i}$. With this, firm $i$ 's cost function defined in Section 3 can be modified by replacing $m_{i}$ with $m_{i} \gamma_{i}$.

Here we apply the unfairness weights $\alpha_{i}$ and $\beta_{i}$ to the cost ratios rather than the cost differences as in Fehr and Schmidt (1999). Thus, we define

$$
\gamma_{i}=\left\{\begin{array}{c}
\alpha_{i} \frac{C_{i}\left(x_{\text {Rule }}, x_{\text {Rule }}\right) v_{3-i}}{C_{3-i}\left(x_{\text {Rule }}, x_{\text {Rule }} v_{i}\right.}, C_{i}\left(x_{\text {Rule }}, x_{\text {Rule }}\right) v_{3-i}>C_{3-i}\left(x_{\text {Rule }}, x_{\text {Rule }}\right) v_{i}  \tag{4.7}\\
1, C_{i}\left(x_{\text {Rule }}, x_{\text {Rule }}\right) v_{3-i}=C_{3-i}\left(x_{\text {Rule }}, x_{\text {Rule }}\right) v_{i} \\
\beta_{i} \frac{C_{i}\left(x_{\text {Rule }}, x_{\text {Rule }}\right) v_{3-i}}{C_{3-i}\left(x_{\text {Rule } e}, x_{\text {Rule }}\right) v_{i}}, C_{i}\left(x_{\text {Rule }}, x_{\text {Rule }}\right) v_{3-i}<C_{3-i}\left(x_{\text {Rule }}, x_{R u l e}\right) v_{i}
\end{array}\right.
$$

where $0 \leq \beta_{i} \leq 1 \leq \alpha_{i}$. We see from (4.7) that $\gamma_{i}>1$ in the loss case and $\gamma_{i}<1$ in the surplus case, because people usually are more sensitive to relative loss than they are to relative gain. Therefore, firm $i$ 's moral standard decreases when it incurs a loss from the rule and its moral standard increases if it gains from the rule.

Both relative loss and gain from obeying the rule make the firms perceive unfairness of the rule, and affect their moral standards as specified. That is, the new moral standard of firm $i$ will be effectively $m_{i} \gamma_{i}$. The modified moral standards give rise to the following equilibrium disobedience degrees for $i=1,2$.

$$
x_{i}^{*}\left(\gamma_{i}\right)=\left\{\begin{array}{c}
\frac{n-t_{i}}{2 b_{i}}+m_{i} \gamma_{i}+x_{\text {Rule }}, \text { if } n \leq t_{i} \\
x_{\text {Rule }}, \text { otherwise }
\end{array}\right.
$$

We will write it as $x_{i}^{*}\left(\gamma_{i}\right)$ and the resulting cost as $C_{i}\left(x_{1}^{*}\left(\gamma_{1}\right), x_{2}^{*}\left(\gamma_{2}\right)\right)$ in the analysis to follow. We next do the sensitivity analysis to see how the change of the rule unfairness coefficients affect the equilibrium.

Lemma 30. For a firm with $n<t_{i}$, and a strictly negative moral standard (i.e., $m_{i}<0$ ), its equilibrium disobedience degree is inversely related to its rule unfairness coefficient $\gamma_{i}$. Furthermore, a higher $\gamma_{i}$ increases firm i's production quantity.

Proposition 31. In the presence of rule unfairness we have the following results: 1) when $x_{i}^{*}>x_{3-i}^{*}$, a rule unfair to firm brings the firms' disobedience degrees closer, i.e., $x_{i}^{*}\left(\gamma_{i}\right)-$ $x_{3-i}^{*}\left(\gamma_{3-i}\right)<x_{i}^{*}-x_{3-i}^{*}$. 2) Under the conditions of $m_{i}<0$ and $n \leq t_{i}$, there is a selfregulation mechanism of rule unfairness, which means that firm i prefers to a fairer rule when the rule is unfair to firm $3-i$.

Proposition 4 shows the effects of rule unfairness on the firms' disobedience degrees and production quantities. Since we have $\gamma_{i}=\alpha_{i} \frac{C_{i}\left(x_{\text {Rul }}, x_{R u l e}\right) v_{3-i}}{C_{3-i}\left(x_{\text {Rul }}, x_{R u l e}\right) v_{i}}=\alpha_{i} \frac{\beta_{3-i}}{\gamma_{3-i}}$, we can easily see that $\gamma_{i}$ is inversely related to $\gamma_{3-i}$. And from Lemma 1 , we know that a rule, unfair to firm $i$, will decrease its $x_{i}^{*}\left(\gamma_{i}\right)$ and increase his rival's $x_{3-i}^{*}\left(\gamma_{3-i}\right)$. The above influences provide some guidelines for the regulatory authority to formulate the guiding rules. In section 4.6, we will discuss the effect of changing the rule $x_{\text {Rule }}$ on $\gamma_{i}$.

### 4.5.2 Forced disobedience

Since rule unfairness also influences the firms' decisions as indicated in the previous section, it also affects the firms' profits differently. Note that the following situation can happen. When a firm perceives inequity due to a relative difference in their profits when it obeys the rule and its rival does not, then the firm may be tempted to break the rule by lowering its moral standard, even when its disobedient behavior may not increase its profit as long as the difference in their profits decreases on account of a decrease in the rivals' profit. In other words, a firm's noncompliance confers a competitive disadvantage to its rival. We call this behavior forced disobedience.

To illustrate this concept, we make the simplifying assumption that our two firms are identical in all aspects except their moral standards. In particular, we assume the firms have the same personality coefficients $b_{1}=b_{2}=b$, the same external diseconomies $d_{1}=$ $d_{2}=d$, the same unit variable costs $v_{1}=v_{2}=v$, but their moral standards are $m_{i}<\underline{m}_{i}$ and $m_{3-i}=0$, where the thresholds is given as follows: (1) When $n \leq t_{i}=t_{3-i}, \underline{m}_{i}=$ $-\frac{\left(d+n-t_{i}\right)^{2}-d^{2}+4 b\left(a-v-t_{i} x_{R u l e}\right)}{4 b\left(2 d-n+t_{i}\right)} ;$ (2) $t_{3-i}<n<t_{i}, \underline{m}_{i}=-\frac{1}{4 b\left(2 d-n+t_{i}\right)}\left(4 d^{2}-\left(2 d-n+t_{i}\right)^{2}+\right.$ $\left.4 b\left(a-v+t_{i} x_{\text {Rule }}-2 t_{3-i} x_{\text {Rule }}\right)\right)$.

A decrease in firm $i$ moral standard $m_{i} \leq \underline{m}_{i}$ has a self-serving effect (increasing firm $i$ 's market share) and a schadenfreude effect (inducing a decrease in the firm $3-i$ 's market share). When $m_{i}$ is at or below the threshold $\underline{m}_{i}$, firm $3-i$ loses its entire market share.

For its survival or reaction to the perceived inequity, the best response of firm $3-i$ is also to decrease its moral standard $m_{3-i}$ and thereby decrease $x_{3-i}$ as shown in the following: (1) if $n \leq t_{i}=t_{3-i}$, then $m_{3-i}<0, m_{3-i}=m_{i}, x_{3-i}=m_{3-i}+\frac{n-t_{i}}{2 b}+x_{\text {Rule }}$, and $x_{3-i}=$ $m_{3-i}+\frac{n-t_{i}}{2 b}+x_{\text {Rule }} ;(2)$ if $t_{3-i}<n<t_{i}$, then $m_{3-i}$ has nothing to do with firm $3-i$ 's $x_{3-i}$ nor its profit since its $x_{3-i}=x_{\text {Rule }}$ when $t_{3-i}<n$.

Proposition 5 states that if firm $i$ 's moral standard is at or below the threshold $\underline{m}_{i}$, then firm $3-i$ is completely excluded from the market. This puts firm $3-i$ in a forced disobedience situation, and it responds as follows: (1) if $n \leq t_{i}=t_{3-i}$, then firm $3-i$ decreases its moral standard from zero to a negative value in order to survive, i.e., gain a market share, or the firm set it moral standard to $m_{3-i}=m_{i}$ for achieving fairness. As a consequence, firm $3-i$ 's disobedience degree decreases; (2) if $t_{3-i}<n<t_{i}$, then $x_{3-i}=x_{\text {Rule }}$, and thus firm $3-i$ achieves nothing from decreasing it moral standard since $m_{3-i}$ influences neither its $x_{3-i}$ nor its profit.

### 4.6 Optimal Rules

In this section, we consider the problem of a regulatory authority is to come up with an optimal rule to so as to maximize total surplus. For this, we revert back to the general guiding rule $x_{\text {Rule }}$, which now becomes a decision variable of the authority, who obtains the resulting equilibrium disobedience degrees in the base model.

### 4.6.1 Disobedience under the Base Model

For ease of exposition, without changing the essential characteristics of the outcomes, we simplify the model by setting $b_{1}=b_{2}=1, d_{1}=d_{2}=d$, and $t_{1}=t_{2}=t$. Now the cost
function for firm $i$ is

$$
C_{i}(x)=\left\{\begin{array}{c}
v_{i}-(n-t) x_{i}+n x_{\text {Rule }}+\left(x_{i}-x_{\text {Rule }}-m_{i}\right)^{2}-d\left(x_{3-i}-x_{\text {Rule }}\right), \text { otherwise }  \tag{4.8}\\
v_{i}-(n-t) x_{i}+n x_{\text {Rule }}-d\left(x_{3-i}-x_{\text {Rule }}\right), \text { if } x_{i} \geq m_{i}+x_{\text {Rule }}
\end{array}\right.
$$

where $x_{\text {Rule }}$ is the guiding rule and $\left(x_{i}-x_{\text {Rule }}\right)$ measures the degree of disobedience. Now the equilibrium optimal disobedience degrees are:

$$
x_{i}^{*}=\left\{\begin{array}{l}
\frac{n-t}{2 b_{i}}+m_{i}+x_{\text {Rule }}, \text { if } n \leq t, \\
x_{\text {Rule }}, \text { otherwise, } i=1,2
\end{array}\right.
$$

On substituting $x_{i}^{*}$ into the total surplus function $\Pi=\pi_{1}+\pi_{2}$ and then maximizing it over the rules gives us the optimal rule $x_{\text {Rule }}^{*}$ as below. We also maximize $\pi_{i}$ in some cases when the regulatory authority wants to support a specific firm. An example could be a firm that uses the environment friendly solar energy in its production process.

Proposition 32. For $t \neq 0$, the optimal rule $x_{\text {Rule }}^{*}$ for maximizing the total surplus $\Pi$ (resp., the firm i's profit $\pi_{i}$ ) is given below, respectively: (1) For $\Pi$, $x_{\text {Rule }}^{*}=\frac{2 a-v_{i}-v_{3-i}}{2 t}+$ $\frac{(d+n-t)\left(m_{1}+m_{2}\right)}{2 t}+\frac{(d+n-t)^{2}-d^{2}}{4 t}$, if $n \leq t ; \frac{2 a-v_{i}-v_{3-i}}{2 t}$, if $n>t$. (2) For $\pi_{i}, x_{\text {Rule }}^{*}=\frac{a-2 v_{i}+v_{3-i}}{t}+$ $\frac{(n-t)\left(2 m_{i}-m_{3-i}\right)}{t}-\frac{d\left(m_{i}-2 m_{3-i}\right)}{t}+\frac{(d+n-t)^{2}-d^{2}}{4 t}$, if $n \leq t ; x_{\text {Rule }}^{*}=\frac{a-2 v_{i}+v_{3-i}}{t}$, if $n>t$.

To distinguish the two different optimal rules, we introduce the notations $x_{\text {Rule }}^{*}(\Pi)$ and $x_{\text {Rule }}^{*}\left(\pi_{i}\right)$ as the optimal rule $x_{\text {Rule }}$ maximizing $\Pi$ and $\pi_{i}$, respectively.

Proposition 6 shows that if the punishment $n$ satisfies $-d<(n-t) \leq 0$, then $\partial x_{\text {Rule }}^{*}(\Pi) / \partial m_{i}=\partial x_{\text {Rule }}^{*}(\Pi) / \partial m_{3-i}=(d+n-t) / 2 t$ is positive, which indicates that the optimal rule $x_{\text {Rule }}^{*}(\Pi)$ is an increasing function in $m_{1}$ and $m_{2}$. Therefore, the regulatory authority need to adjust the rule $x_{\text {Rule }}$ upward, while increasing the firms' moral standards $m_{i}$ to achieve higher surplus. Furthermore, the regulatory authority needs to coordinate two management strategies $x_{\text {Rule }}$ and $n$ simultaneously. For example, if $-d>n-t$, the
government may consider adjusting the punishment level $n$ to ensure that $d+n-t>0$, and then keep the same strategy as that for $-d<(n-t) \leq d / 2$.

Considering that $x_{\text {Rule }}^{*}(\Pi)$ and $x_{\text {Rule }}^{*}\left(\pi_{i}\right)$ are functions of $n$, a coordinated punishment strategy, which helps us to simultaneously achieve the maximum $\Pi$ and maximum $\pi_{i}$, with the same rule $x_{\text {Rule }}^{*}$, can be derived as follows:

There exists a coordinated punishment strategy which maximizes $\Pi$ and $\pi_{i}$, simultaneously when $n \leq t$ and

$$
\begin{equation*}
n=\frac{v_{i}-v_{3-i}}{m_{i}-m_{3-i}}+t+d, m_{i} \neq m_{3-i}, i=1,2 . \tag{4.9}
\end{equation*}
$$

Proposition 7 provides a coordinated strategy to punish noncompliant firms. Above equation implies that only in the situation where $v_{i}>v_{3-i}, m_{i}<m_{3-i}$ or $v_{i}<v_{3-i}$, $m_{i}>m_{3-i}$, the coordinated punishment strategy have a chance to exist, and the precondition is that $\frac{v_{i}-v_{3-i}}{m_{i}-m_{3-i}} \leq-d$. Otherwise, firms obey the rule, and both $x_{\text {Rule }}^{*}(\Pi)$ and $x_{\text {Rule }}^{*}\left(\pi_{i}\right)$ are unrelated to the punishment level $n$. Under this situation, the firm with a higher production cost gets a lower moral standard, and thus it will disobey more to reduce the cost.

Furthermore, Proposition 7 implies that $x_{\text {Rule }}^{*}(\Pi)$ is always between $x_{\text {Rule }}^{*}\left(\pi_{i}\right)$ and $x_{\text {Rule }}^{*}\left(\pi_{3-i}\right)$ if $n$ does not satisfy that. When $n<\frac{v_{i}-v_{3-i}}{m_{i}-m_{3-i}}+t+d$ and $m_{i}<m_{3-i}$, or $\frac{v_{i}-v_{3-i}}{m_{i}-m_{3-i}}+t+d<n<t$ and $m_{i}>m_{3-i}$, the optimal rule $x_{\text {Rule }}^{*}\left(\pi_{i}\right)$ is greater than $x_{\text {Rule }}^{*}(\Pi)$ and $x_{\text {Rule }}^{*}(\Pi)$ is greater than $x_{\text {Rule }}^{*}\left(\pi_{3-i}\right)$. In the cases where $n<\frac{v_{i}-v_{3-i}}{m_{i}-m_{3-i}}+t+d$ and $m_{i}>m_{3-i}$, or $\frac{v_{i}-v_{3-i}}{m_{i}-m_{3-i}}+t+d<n<t$ and $m_{i}<m_{3-i}$, the optimal rule $x_{R u l e}^{*}\left(\pi_{3-i}\right)$ is greater than $x_{\text {Rule }}^{*}(\Pi)$.

### 4.6.2 With Rule Unfairness

Clearly, the regulatory authority can also influence the value of $\gamma_{i}$ by setting its rule. The more unfair a rule is perceived by a firm, the higher its coefficient of rule unfairness is.

Rule unfairness affects the optimal rule in the way we combine the moral standard and the unfairness coefficient $\gamma_{i}$.

By substituting $m_{i}$ and $m_{3-i}$ with $m_{i} \gamma_{i}$ and $m_{3-i} \gamma_{3-i}$, respectively, in Propositions 6 and 7, the effect of unfairness can be obtained as described below.

Proposition 33. Considering the effect of rule unfairness, the change of optimal rule $x_{\text {Rule }}^{*}(\Pi)$ is $\frac{\left(m_{i} \gamma_{i}+m_{3-i} \gamma_{3-i}-m_{i}-m_{3-i}\right)(d-t+n)}{2 t}$, when $n \leq t$. Furthermore, $x_{\text {Rule }}^{*}(\Pi)$ increases when $\left(1-\gamma_{3-i}\right) m_{3-i}<\left(\gamma_{i}-1\right) m_{i}$ and $-d<n-t<0$, and it decreases when $\left(\gamma_{i}-1\right) m_{i}<$ $\left(1-\gamma_{3-i}\right) m_{3-i}$ and $n-t<-d$.

Proposition 8 implies that if $\left(1-\gamma_{3-i}\right) m_{3-i}<\left(\gamma_{i}-1\right) m_{i}$, then the regulatory authority increases the optimal rule $x_{\text {Rule }}^{*}(\Pi)$ when it uses the punishment level $n$ such that $-d<$ $n-t<0$ and decreases $x_{\text {Rule }}^{*}(\Pi)$ when it uses a relatively lower punishment level $n$ such that $n-t<-d$; and if $\left(\gamma_{i}-1\right) m_{i}<\left(1-\gamma_{3-i}\right) m_{3-i}$, then the authority sets policies opposite to the above two punishment schemes, respectively. However, in the special case of $\left(\gamma_{i}-1\right) m_{i}=\left(1-\gamma_{3-i}\right) m_{3-i}$, the authority does not need to revise the optimal $x_{\text {Rule }}^{*}(\Pi)$ when it changes the punishment policy.

The results obtained here can be used to balance the strength of the rule $x_{\text {Rule }}^{*}(\Pi)$, the firms' costs of obedience, and the strength of punishment, so as to make come up with the best policy for a given specific objective.

### 4.7 Conclusion

This chapter studies a disobedience problem of firms that consider economic, moral, as well as behavioral factors in their decisions. We use a game theoretic model with two players where we obtain firms' decision strategies on their disobedience degrees and production quantities. In this way, we are able to analyze the effects of moral and behavioral factors, and derive an optimal rule strategy for regulatory authorities.

We provide some guidelines for firms in deciding to disobey. First, managers need to know the consequences of their disobedience resulting in punishment, external diseconomy, moral loss, rule unfairness, etc. Second, firms should recognize the situations of destructive selfishness and creative delinquency. Noncompliance in a creative delinquency situation may be tolerated since it increases total surplus. Third, reducing the production cost can increase the profit of a firm and avoid the situation of disobedience. Finally, firms need to pay attention to the challenges of imitation and cultural identity when they enter a new market.

We also provide some guidelines to the regulatory authorities. First, they should consider the firms' disobedience actions in formulating their punishment policies. It is particularly important to well understand the differences between destructive selfishness and creative delinquency situations and then respond to them respectively. Second, we suggest the authorities to improve the effect of supervision on disobedience actions with the help of behavioral factors. In some situations, it is possible for the authorities to create rule unfairness to nudge the firms to go closer to their optimal moral standards in order to raise total surplus. Third, our study provides guidelines for optimal rule making. To increase total surplus, the authorities should seek an optimal rule, as well as a coordinated punishment strategy to maximize total surplus. Besides, they could use perceived unfairness to influence the optimal rule. This result provides themwith a good approach to balance the firms' moral standards and the optimal rule.

To our knowledge, this chapter takes an advanced step in a game theoretic formulation of the firms' disobedience behavior. Here we conclude the chapter by pointing out its limitations. Beside the factors considered in this chapter, there are other factors, such as imitation, group identification, habituation, emotion and overconfidence, that may influence a firm's disobedience decision. Furthermore, our chapter only deals with a duopoly setting, without considering the case of many firms or the case in which there is a dominant firm. Introducing
the other behavioral factors mentioned above and exploring the implications of more general game settings are interesting directions for future research. Also important would be to test the implications of such studies in experimental settings.

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## CHAPTER 5

## CONCLUSION

In the first problem, the effect of penalties is different between allowing reliability improvement and not allowing reliability improvement. When reliability improvement is not allowed, the penalty is set to influence the production activity of the supplier. When reliability improvement is allowed, the penalty is mainly used to motivate the efforts of the supplier. Only under extreme cases in which the low-type supplier is excluded from the market is the penalty set to realize that objective. In this way, the case allowing reliability improvement is shown to be a special case. In the second problem, it is widely believed that high reliability and low correlation with other suppliers should benefit an unreliable supplier. By studying a responsive-pricing retailer's sourcing and pricing problem involving competing strategic suppliers subject to independent or correlated disruptions, we show that while this belief is true in most cases, there exist conditions under which the belief is not borne out. The problem is formulated as a Stackelberg-Nash game in which the suppliers act as the Stackelberg leaders when they set wholesale prices simultaneously in a Nash game. The retailer is the Stackelberg follower by deciding its respective order quantities ex-ante from the suppliers and the retail price ex-post. We identify the reliability effect and the competition effect of the reliability, and show that a higher reliability can hurt an unreliable supplier when the competition effect of the reliability dominates the reliability effect. In the third problem, we provide some guidelines for firms in deciding to disobey. First, managers need to know the consequences of their disobedience resulting in punishment, external diseconomy, moral loss, rule unfairness, etc. Second, firms should recognize the situations of destructive selfishness and creative delinquency. Noncompliance in a creative delinquency situation may be tolerated since it increases total surplus. Third, reducing the production cost can increase the profit of a firm and avoid the situation of disobedience. Finally, firms need to pay attention to the challenges of imitation and cultural identity when they enter a new market.

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