Observation of the Helicity-Selection-Rule Suppressed Decay of the χ_{c2} Charmonium State–Supplement

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Supplementary material for the partial wave analysis of $\chi_{c2} \to K\overline{K}\pi$

1 Amplitude

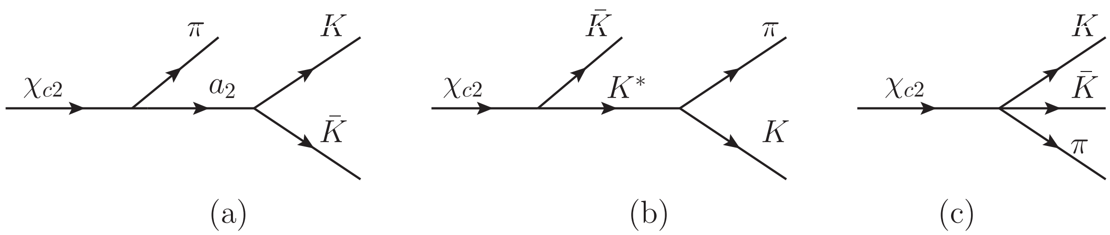

Figure 1: The diagrams of the quasi-two body decays and the direct three-body decay for $\chi_{c2} \to K\overline{K}\pi$

In this analysis, three processes shown in Fig. 1 are considered in the partial wave analysis (PWA). For the quasi-two body decays $\chi_{c2}(\lambda_0) \to a_2(\lambda_1)\pi$, $a_2 \to K\overline{K}$ (Figure 1 (a)), where $\lambda_i (i = 0, 1)$ indicate helicities for the χ_{c2}, and a_2, respectively. The spin indexes for pion and koans are suppressed due to the 0-spin values they have. The helicity-coupling amplitude is given by:

$$A_1(\lambda_0) = \sum_{\lambda_1} F^{\chi_{c2}}_{\lambda_1,0}(r_1) D^{2^*}_{\lambda_0,\lambda_1}(\phi_0, \theta_0, 0) BW(m_{KK}) F^{a_2}_{0,0}(r_2) D^{2^*}_{\lambda_1,0}(\phi_1, \theta_1, 0) \tag{1}$$

where $r_1(r_2)$ is the momentum differences between a_2 and π (two kaons) in the rest frame of χ_{c2} (a_2), and $\theta_0(\phi_0)$, $\theta_1(\phi_1)$ are the polar (azimuthal) helicity angles of the momentum vector of π (kaon) in the helicity system of $\chi_{c2}(a_2)$. For the decay $a_2 \to K^+K^-$, the z-axis of helicity system is taken along the direction of kaon momentum ($\mathbf{p_K}$) in the a_2 rest frame, and x-axis is taken in the plane formed by the momentum of a_2 ($\mathbf{p_a}$) in the χ_{c2} rest frame and the momentum ($\mathbf{p_K}$). The y-axis, together with the z, x-axes forms a right hand system. The $BW(m)$ denotes the Breit-Wigner for the resonance a_2, $BW(m) = \frac{1}{m^2 - m_0^2 - im\Gamma}$, with mass m and width Γ. The helicity-coupling amplitudes $F^X_{\lambda_i,\lambda_j}$ is calculated in the LS-coupling scheme [1]. For a decay

$a(J,\eta_J) \to b(s,\eta_s) + c(\sigma,\eta_\sigma)$, where the quantum number (J,η_J) denotes (spin,parity), one has:

$$F^J_{\lambda,\nu} = \sum_{ls} g_{ls} \left(\frac{2l+1}{2J+1}\right)^{1/2} \langle l0S\delta|J\delta\rangle\langle\, s\lambda\sigma-\nu|S\delta\rangle\; r^l \frac{B_l(r)}{B_l(r_0)}, \tag{2}$$

where g_{ls} is a coupling constant taken as complex number, $r = p_b - p_c$, and r_0 corresponds to the $(p_b+p_c)^2 = m_a^2$, where m_a is the mass of parent particle a. The conservation of CP parity is implied in the above equation. $B_l(p)$ is the Blatt-Weisskopf factor, and taken as:

$$L = 0 : B(p) = 1,$$

$$L = 1 : B(p) = \frac{1}{\sqrt{1+(qp)^2}},$$

$$L = 2 : B(p) = \frac{1}{\sqrt{9+3(qp)^2+(qp)^4}},$$

$$L = 3 : B(p) = \frac{1}{\sqrt{225+45(qp)^2+6(qp)^4+(qp)^6}},$$

$$L = 4 : B(p) = \frac{1}{\sqrt{11025+1575(qp)^2+135(qp)^4+10(qp)^6+(qp)^8}},$$

where q is constant fixed to 3 GeV^{-1} for the meson final states.

For the quasi-two-body decay $\chi_{c2}(J,\lambda_0) \to K^*(R,\lambda_1)\overline{K}$, $K^*(R,\lambda_1) \to K\pi$ (Fig. 1(b)), The $\lambda_i (i = 0,1)$ indicate helicities for the χ_{c2} and K^*, respectively, The helicity amplitude is given by:

$$A_2(\lambda_0) = \sum_{\lambda_1} F^{\chi_{c2}}_{\lambda_1,0}(r_1) D^{2^*}_{\lambda_0,\lambda_1}(\phi_0,\theta_0,0) BW(m_{K\pi}) F^{K^*}_{0,0}(r_2) D^{R^*}_{\lambda_1,0}(\phi_1,\theta_1,0) \tag{3}$$

where $r_1(r_2)$ is the momentum differences between K^* and $\overline{K}$ (K and π) in the rest frame of $\chi_{c2}(K^*)$, and $\theta_0(\phi_0)$, $\theta_1(\phi_1)$ are the polar (azimuthal) helicity angles of the momentum vector of $K^*(\pi)$ in the helicity-system of $\chi_{c2}(K^*)$. The helicity-coupling amplitudes $F^X_{\lambda_i,\lambda_j}$ is calculated using Eq. 2. The $BW(m)$ denotes the Breit-Wigner for the resonance K^*.

For the amplitude (A_3) of direct three-body decay $\chi_{c2} \to K\overline{K}\pi$ (Fig. 1 (c)), we model the pair of $K\overline{K}$ or $K\pi$ to be 2^+ system.

For the charged conjugate modes, we assume that they have the same decay rate due to the $SU(3)$ symmetry. Hence, the magnitudes and the phase of their coupling constants are taken as the same value in the amplitude.

The total amplitude is obtained by adding these three processes coherently:

$$A(\lambda_0) = \sum_{i=1}^{3} A_i(\lambda_0). \tag{4}$$

The decay rate is given by:

$$\frac{d\sigma}{d\phi_3} = \left(\frac{3}{8\pi^2}\right) \sum_{\lambda_0,\lambda_0'} \rho(\lambda_0,\lambda_0') A(\lambda_0) A^*(\lambda_0'), \tag{5}$$

where the $d\phi_3$ is the element of standard three-body phase space, and the spin density matrix $\rho(\lambda_0,\lambda_0')$ describing the χ_{c2} production can be estimated from the $e^+e^- \to \psi' \to \gamma\chi_{c2}$ process, which is determined by

$$\rho(\lambda_0,\lambda_0') = \int d\cos\theta_0 d\phi_0 \sum_{M,\lambda_\gamma=\pm 1} D^1_{M,\lambda_\gamma-\lambda_0}(\theta_0,\phi_0) D^{1*}_{M,\lambda_\gamma-\lambda_0'}(\theta_0,\phi_0) A^{(J)}_{\lambda_\gamma,\lambda_0} A^{(J)*}_{\lambda_\gamma,\lambda_0'}, \tag{6}$$

where M, λ_γ and λ_0 are the helicity values for ψ', photon and χ_{c2} states, respectively, and $A_{\lambda_\gamma,\lambda_0}$ is the helicity amplitude for $\psi' \to \gamma\chi_{c2}$ with the helicity angle $\Omega_0(\theta_0,\phi_0)$. The sum over M takes $M = \pm 1$ since the ψ' is produced from the e^+e^- annihilation. Recent measurement shows that the contribution of high magnetic- and electric-multipole to the χ_{c2} production is negligible, and $E1-$transition dominates this process [2]. Hence, components of helicity amplitude are chosen to satisfy the $E1$-relation [3], namely, $A_{1,2} = \sqrt{2}A_{1,1} = \sqrt{6}A_{1,0}$. Further considering the requirement of parity conservation, one has a relation $A_{-\lambda_\gamma,-m_1} = (-1)^J A_{\lambda_\gamma,m_1}$. Thus the spin densities are determined to be

$$\rho = \mathrm{diag}\{y^2, x^2, 2, x^2, y^2\}, \text{ with } x = A_{1,1}/A_{1,0},\ y = A_{1,2}/A_{1,0}, \tag{7}$$

with the $E1$-relations, one has $\rho \propto \mathrm{diag}\{2, 1, 2/3, 1, 2\}$ [4].

2 Fit strategy

The relative magnitudes and phases for coupling constants are determined by an unbinned maximum likelihood fit. The joint probability density for observing the N events in the data sample is

$$\mathcal{L} = \prod_{i=1}^{N} P(x_i), \tag{8}$$

where $P(x_i)$ is a probability to produce event i with four-vector momentum $x_i = (p_K, p_{\bar{K}}, p_\pi)_i$.

The normalized $P(x_i)$ is calculated from the differential cross section

$$P(x_i) = \frac{(d\sigma/d\phi_3)_i}{\sigma_{MC}}, \tag{9}$$

where the normalization factor σ_{MC} is calculated from a MC sample with N_{MC} accepted events, which are generated with a phase space model and then subject to the detector simulation, and are passed through the same event selection criteria as applied to the data analysis. With an MC sample of sufficiently large size, the σ_{MC} is evaluated with

$$\sigma_{MC} = \frac{1}{N_{MC}} \sum_{i=1}^{N_{MC}} \left(\frac{d\sigma}{d\phi_3}\right)_i. \tag{10}$$

For technical reasons, rather than maximizing $\mathcal{L}$, $S = -\ln \mathcal{L}$ is minimized using the package MINUIT. The backgrounds are subtracted from the likelihood:

$$ln\mathcal{L} = \ln \mathcal{L}_{data} - \ln \mathcal{L}_{bg}. \tag{11}$$

3 Estimation of signal yield and statistical uncertainty

After the parameters are determined in the fit, the signal yields of a given resonance can be estimated by scaling its cross section ratio R_i to the net event numbers, i.e..

$$N_i = R_i * (N_{\rm obs} - N_{\rm bg}), \text{ with } R_i = \frac{\sigma_i}{\sigma_{\rm tot}}, \tag{12}$$

where σ_i is the cross section for the i-th resonance, and $\sigma_{\rm tot}$ is the total cross section, and $N_{\rm obs}$ and $N_{\rm bg}$ are the numbers of observed events and background events.

The statistical error, δN_i, associated with signal yields N_i is estimated based on the covariance matrix, V, obtained in the fit according to:

$$\delta N_i^2 = \sum_{m=1}^{N_{\rm pars}} \sum_{n=1}^{N_{\rm pars}} \left(\frac{\partial N_i}{\partial X_m} \frac{\partial N_i}{\partial X_n} \right)_{\mathbf{X}=\mu} V_{mn}(\mathbf{X}), \tag{13}$$

where $\mathbf{X}$ is a vector containing parameters, and μ contains the fitted values for all parameters. The sum runs over all $N_{\rm pars}$ parameters.

4 Input/Output check of the PWA

A pseudo data is generated with inclusion of all intermediate states in the baseline solution ($K^*(892)$, $K_2^*(1430)$, $K_3^*(1780)$, $a_2(1320)$, and include the PHSP), and coupling constants are fixed to the PWA solution, and then the events are subject to the detector simulation. The candidate events of the input are obtained by applying the same selection criteria to the pseudo data events. To compare with the data sample, we take the same size of input events as that we selected in the data, and the same PWA fit procedure is performed to the input events. Table 1 shows the output results compared to input numbers, and they are consistent with each other within the statistical errors.

Table 1: I/O full check for $\chi_{c2} \to K^{*\pm}K^{\mp} \to K^+K^-\pi^0$ channel

	Input	Output
$K^*(892)$	140.4 ± 14.5	148.7 ± 25.3
$K_2^*(1430)$	547.0 ± 26.8	536.3 ± 26.5
$K_3^*(1780)$	35.3 ± 9.2	37.2 ± 9.5
$a_2(1320)$	115.5 ± 16.6	116.2 ± 16.9
PHSP	163.6 ± 22.3	164.9 ± 21.2

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