ESSAYS ON RETAIL OPERATIONS

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ESSAYS ON RETAIL OPERATIONS

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This dissertation consists of two essays, each focusing on an important topic in retail operations. These topics are each summarized below.

In the first essay, we investigate the optimal pricing and package size decisions of a retailer selling a perishable product either in packages or in bulk. Bulk sale is defined as selling the product in a container (instead of packages) that allows customers to buy as much or as many as they want. We analyze how adding a bulk sale option affects the optimal decisions, when it is optimal to offer the product in package/bulk and when it is optimal to offer both at the same time. We also investigate implications of the pricing and package size decisions as well as the bulk sale option on the food waste at the consumer level. According to our results, when the market is homogeneous, selling the product in packages instead of bulk gives twice more profit to the retailer but it also gives twice more relative waste. When the market is heterogeneous with two consumer segments, adding a bulk sale option could increase expected profit up to 12 percent and could decrease relative waste up to 8 percent.

In the second essay, we work on the multi-period assortment problem for a retailer with variety-seeking/avoiding consumers. If consumers are variety-seeker, they are not likely to buy the same product in two subsequent periods. If customers are variety-avoidant, their probability of repeat purchase is high. We assume that each consumer‘s variety seek-

ing/avoiding tendency is characterized by a parameter $V \in [-1, 1]$. In our analysis, we consider two different firm types which we call the Dynamic and the Static firms. The Dynamic firm optimizes the profit over the entire horizon by changing (if necessary) the assortment offered in each period dynamically, whereas the static firm has to decide one assortment to offer throughout the horizon. We provide some structural results for the finite and infinite horizon versions of the problem. We show that the existence of variety seeking/avoiding behavior decreases the retailer's profit. For the infinite horizon problem, we show that the static firm's optimal assortment is a popular-eccentric set and for some cases repeating to cycle between two assortment yields more profit than offering the same in each period.

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CHAPTER 1

INTRODUCTION

This dissertation consists of two essays, each focusing on an important topic in retail operations. These topics are each summarized below.

In the first essay, we investigate the optimal pricing and package size decisions of a retailer selling a perishable product either in packages or in bulk. Bulk sale is defined as selling the product in a container (instead of packages) that allows customers as much or as many as they want. We analyze how adding a bulk sale option effects the optimal decisions and when it is optimal to offer the product in package/bulk and when it is optimal to offer both at the same time. Our second objective is to find out implications of the pricing and package size decisions as well as the bulk sale option on the customer food waste. Food waste is a very important problem of the modern world. According to the United Nations Food and Agriculture Organization (UNFAO, 2017), about one third of all food produced in the world ends up as waste. According to a recent report by ReFED (ReFED, 2017), which is a non-profit organization dedicated to reduce food waste in the United States, adjusting packaging size and design alone could result in 208,000 tons of diverted food waste. We investigated three versions of the problem: homogeneous market, heterogeneous market with two customers and infinitely heterogeneous market. According to our results, when the market is homogeneous, selling the product in packages instead of bulk gives twice more profit to the retailer but it also gives twice more relative waste at the consumer level. When the market is heterogeneous with two consumer segments, it is never optimal to sell the product only in bulk and depending on the proportion of consumers in the market and their relative consumption amounts it could be optimal to introduce bulk sale besides package sale. When bulk and package sale co-exist expected profit could increase up to 12 percent and relative waste could decrease up to 8 percent.

In the second essay, we work on the multi-period assortment problem for a retailer with variety-seeking/avoiding consumers. In product categories where consumers have large consideration sets and make many choices over time, there may be some desire for more variety because of satiation or curiosity. This is called variety-seeking behavior (Kahn et al. (1986) and others). For example in the yogurt category, consumers tend to purchase some flavors they like and some they just want to try. In other products categories such as paper products, consumers tend to be loyal towards one brand and buy the same products constantly over time. This is called variety-avoiding behavior. If customers are variety-seeking, they are not likely to buy the same product in two subsequent periods. If customers are variety-avoidant, their probability of repeat purchase is high. In order to capture variety seeking/avoidance behavior we borrow the model from Givon (1984) and adopt it to the assortment planning problem. As Givon (1984) does, we assume that each consumer is characterized by a parameter $V \in [-1, 1]$ which measures the consumer's attitude towards variety such that: (i) $V = 1$ corresponds to the extreme search for variety case, (ii) $V = 0$ means consumer has no utility (positive or negative) from active search of variety and (iii) $V = -1$ means consumer tries to avoid variety at all costs. In our analysis, we consider two different firm types which we call the Dynamic and the Static firms. The Dynamic firm optimizes the profit over the entire horizon by changing (if necessary) the assortment offered in each period dynamically, whereas the static firm has to decide one assortment to offer throughout the horizon. We consider the assortment problem both under finite and infinite time horizons. According to our results, the existence of variety seeking/avoiding behavior decreases the retailers profit compared with non-existing case. For the infinite horizon problem, we show that the static firm's optimal assortment is a popular-eccentric set and for some cases repeating to cycle between two assortment yields more profit than offering the same in each period.

CHAPTER 2

PACKAGE SIZE AND PRICING DECISIONS WITH A BULK SALE OPTION

2.1 Introduction

The very thin margins in today's retail industry force retail companies to search for innovative strategies to extract more profit from their customers. Packaging is one such strategy, which, according to Kuvykaite et al. (2009) has an important role in marketing communications and is one of the most important factors influencing consumers' purchase decisions. Their empirical research on purchase decisions of milk and washing powder shows that package size and material are the most important visual determinants of a customer's purchasing decision.

In this study, we consider a perishable product, in the sense that it expires in the amount of time between two customer shopping trips to the store. When doing their shopping, customers often do not know precisely how much of a product they will consume until their next visit to the store. Since the purchase decision is made before the consumption occurs, there can be a mismatch between the two quantities: buying more than their consumption needs will result in waste as the excess product perishes; buying less will result in a shortage (and possibly a trip to the convenience store where the product is sold a higher price). We assume that customers take into account the relative costs of waste and shortage when deciding how much of the product to purchase. When the product is offered in packages of a pre-set size, the customer's purchase quantity is restricted to be a multiple of the package size. For example, if the consumer predicts her consumption of the product to be 3 units and the product is offered in packages of 2, she has to decide between buying less (1 package of 2 units) or more (2 packages of 2 units) than her desired quantity.

Buying more than the desired purchase quantity will likely lead to some wastage as the excess product will expire before the next shopping trip. In the grocery industry, large package sizes are known to contribute to food waste which is a very important problem, worldwide. A number of recent studies estimate the scale of global food waste and show how devastating it is. According to the United Nations Food and Agriculture Organization, about one third of all food produced in the world ends up as waste and the value of this waste is estimated at $750 billion, at producer prices. Buzby et al. (2014) estimated that, in the United States, 31 percent of the 430 billion pounds of available food supply went uneaten in 2011, with losses at the retail level representing 10 percent (43 billion pounds) and losses at consumer level representing to 21 percent (90 billion pounds) of the available food supply. The estimated total value of food loss at the retail and consumer levels in the United States was $161.6 billion in 2010. Beyond these economic considerations, food waste also has a important environmental cost as rotting food in landfills is the largest contributor to methane emissions which cause global warming.

According to a recent report by ReFED, (ReFED, 2017), which is a non-profit organization dedicated to reduce food waste in the United States, optimizing package size and design to ensure complete consumption by consumers can help to reduce food waste. According to their estimates, adjusting packaging size and design alone could result in 208,000 tons of diverted food waste.

In recent years, some retailers have introduced a new way to sell products to consumers called *bulk sale*. In practice, the term *bulk sale* has two different meanings: the more common one refers to the selling of a product in large packages at a substantial price discount, typically at big-box retailers such as Costco and Sam's Club in the United States. The second meaning, which is the one we use in our study, refers to the selling of a product in large bins from which customers can help themselves to the exact quantity they wish to purchase. This type of bulk sale is most commonly found in premium grocery stores such as Whole Foods Market and is available for products such as flour, nuts or peanut butter. Other examples, which also exist in many traditional supermarkets, include the selling of fruits such as kiwi and apples individually, as opposed to in packs of 6 and the selling of eggs by the unit instead of in cartons of a dozen or half a dozen units. From a consumer's perspective, the bulk sale option has the advantage of offering the flexibility to purchase exactly the desired quantity, which should lead to a lower amount of product waste. For this reason, bulk sale has been presented as a potential way to reduce the amount of food that is wasted due to packaging. From a retailer's perspective, selling the product in packages may have the advantage of forcing consumers to buy more than their desired purchase quantity, leading to more sales and more profit. However, we hope to show that offering the bulk sale option, at the appropriate price, could be a profit-enhancing decision for the retailer. Hence, the bulk sale option can be beneficial to both society and the retailer's bottom line.

In this paper, we study a retailer's optimal package size and price decisions when a perishable product is sold in packages and/or in bulk and investigate the implications of these decisions on waste. In doing so we answer questions such as: How does the existence of a bulk sale option affect the optimal package size and price? How effective is a bulk sale option at reducing consumption waste? How does customer heterogeneity affect the retailer‘s optimal decisions?

Literature Review:

Finding the optimal package size and price without the bulk sale option is already a difficult problem and there is only limited theoretical work on the topic. Gerstner and Hess (1987) study a version of the problem where a monopolist tries to sort consumers in the most profitable way by using different package sizes and prices for different Segment of consumers. In their model, they assume that there are two segments of customers in the market and the consumption rates of the customers are fixed. They show that, by offering several package sizes at different unit prices, customers are automatically sorted into market segments and this allows the seller to extract more consumer surplus and to earn higher profits. Koenigsberg et al. (2010) focus on package size and price decision for a perishable product assuming heterogeneity in the consumer usage rate and allowing consumers to buy more than one package. They show that the size of the package should be as small as possible when product cost increases as a linear or convex function of package size. Their results also indicate that small package sizes reduce product waste by allowing consumers to match the purchase quantity with their requirements. One important difference between our study and these two studies is in the way we model the consumer purchase behavior: we take into account the uncertainty in the consumption amount faced by the consumers using a newsvendor model formulation.

There are also a number of empirical studies which investigate different behavioral issues related to package size. Granger and Billson (1972) show that displaying the unit price of the product to the consumers (as opposed to just the package price), leads to significant changes in the chosen package size. Wansink (1996) studies the issue of whether larger package sizes lead to more usage and shows that large packages of familiar, branded products encourage more use than small packages. Allenby et al. (2004) investigate the effect of a price reduction for a package size for a particular brand in terms of substitution of demand across brands. Jain (2012) studies the impact of the package size of so-called vice goods (i.e., goods for which a moderate consumption is not harmful but excessive consumption has long-term harmful effects such as potato chips, cookies, ice cream, alcohol etc.) on consumers' self-control problems and investigate the conditions under which firms will offer small packages to help consumers combat excessive consumption.

To the best of our knowledge, there is no study that considers package sale and bulk sale simultaneously.

The paper is organized as follows. In §2.2, we develop our model. In §2.2.1 we present how consumer choice change depending on package sale and bulk sale options. In §2.3, §2.4 and §2.5 we investigate optimal solutions when the market is homogeneous, heterogeneous with two segments and full heterogeneous respectively. In §2.6 we discuss three extension to our model. In §2.7, we conclude and discuss further research directions. Unless otherwise stated, all proofs are in the Appendix.

2.2 Model

A retailer is selling a unique perishable product to a population of heterogeneous customers. We assume that the product's expiration date coincides with the timing of the customers' next shopping trip and refer to this length of time as a *period*; in other words, we are focusing on perishable products which are bought regularly (e.g., once a week) such as fruits, vegetables, juices, eggs, dairy products, meat, etc. At the time of purchase, consumers do not know precisely how much of the product they will wish to consume until the end of the period.

There exist K different customer segments in the population; let α_k denote the proportion of customers from segment k in the population, for $k = 1, ..., K$ such that $\sum_{k=1}^{K} \alpha_k = 1$. Customers differ may in how much they value the product, how much they wish to consume as well as in their cost of wasting excess product and running out of it. Let v_k denote the product valuation for customers in segment $k = 1, ..., K$. Let D_k be a random variable denoting the quantity which consumers from segment k would like to consume over a period, which we refer to as their *consumption needs*, and let F_k and μ_k denote its cdf and mean respectively. Any quantity which has not been consumed by the end of the period has to be discarded, for which the consumer incurs a cost. Running out of the product before the end of the period also comes at a positive cost. Let w_k and r_k respectively denote the waste cost and run-out cost for customers from segment k. In practice, the waste cost could corresponds to the cost of composting the excess product or the mental anguish that some waste-conscious consumers may incur when throwing away food which they can no longer consume, while the run-out cost could correspond to the incremental cost of buying the extra quantities at the corner store where products are typically sold at a higher price than at the supermarket as well as the opportunity cost of such "emergency" shopping trip.

Let $U_k(Q)$ denote the expected *gross* utility a customer from segment k derives from purchasing Q units of the product, which is calculated as:

$$U_k(Q) = v_k E[\min(Q, D_k)] - r_k E[D_k - Q]^+ - w_k E[(Q - D_k]^+ \tag{2.1}$$

where $E[D_k - Q]^+$ is the expected shortage quantity and $E[(Q - D_k]^+$ is the expected waste.

If the distribution of consumers' consumption needs D_k has a uniform distribution on $[0, B_k]$ for $k = 1, ..., K$. In this case, the expected gross utility of a consumer as a function of purchase quantity Q is:

$$U_k(Q) = \begin{cases} v\frac{B_k}{2} - \frac{(v+r)(B_k-Q)^2+wQ^2}{2B_k} & for\ Q \leq B_k \\ \\ v\frac{B_k}{2} - w\left(Q - \frac{B_k}{2}\right) & for\ Q \geq B_k \end{cases} \tag{2.2}$$

Note that the second part of the equation corresponds to cases where the purchase amount is larger than the maximum value of the customer segment's consumption needs. As mentioned earlier, the utility of buying nothing is negative and specifically equal to $u_k(0; s, P^p) = -\frac{B_k}{2}r$. Also we have $u(B_k; s, P^p) = \frac{v_k - w_k - P^p}{2}$. In what follows, we assume that $c > \frac{v_k + r_k - w_k}{2}$ for $k = 1, ..., K$. While all of our results hold when this condition does not hold, imposing this lower bound on the unit variable cost greatly simplifies the exposition of our results as it guarantees that no customer will buy more than their maximum possible consumption needs B_k. This is because it implies that $u_k(0; s, P^p) > u(B_k; s, P^p)$ for $P^p > c$; therefore all consumers prefer buying nothing to buying a quantity larger or equal to B_k.

The retailer can decide to sell the product in packages of a fixed size s or in *bulk*, which means that customers can buy the exact quantity they wish. The product unit price is P^p when sold in package form (so that the price of one package is sP^p) and P^b when sold in bulk. We assume that customers either buy a (discrete) number of packages or use the bulk sale option. Let $u_k^p(i, s, P^p)$ denote the expected *net* utility a customer from segment k derives from purchasing i packages of size s at a unit price of P^p and let $u_k^b(Q^b, P^b)$ denote the expected *net* utility a customer from segment k derives from purchasing Q^b units of bulk at a unit price of P^b. We have:

$$u_k^p(i; s, P^p) = U_k(is) - P^p is \quad \text{for } i \in \mathbb{N}$$
$$u_k^b(Q^b; P^b) = U_k(Q^b) - P^b Q^b \quad \text{for } Q^b \in \mathbb{R}^+$$

Note that purchasing nothing from the retailer leads to a negative utility (equal to $-r_k\mu_k$). We argue that this makes sense given our focus on products which are purchased regularly, that is, that are staples of the consumers' diet and therefore, for which an insufficient (or zero) quantity purchased leads to an intolerable shortage - forcing the customer to make a trip to the more expensive corner store to purchase the item after the her consumption needs are revealed.

Customers are expected utility maximizers; they choose the quantity of packages $i \in \mathbb{N}$ or the quantity of bulk $Q^b \in \mathbb{R}^+$ so as to maximize their net utility given the package size s and the unit price of package P^p and bulk P^b. As a result their expected net utility from purchase is given by:

$$\max \left\{ \max_{i \in \mathbb{N}} u_k^p(i, s, P^p), \max_{Q^b \in \mathbb{R}^+} u_k^b(Q^b, P^p) \right\}$$

Note that, when $P^p > P^b$, every consumer (weakly) prefers to buy in bulk since it is cheaper and allows them to buy the exact quantity they desire. In case of a tie for the maximum expected net utility, we assume that customers buy the largest number of packages which gives them maximum utility.

Let $\hat{i}_k(s, P^p, P^b)$ and $\hat{Q}_k^b(s, P^p, P^b)$ respectively denote the number of packages and bulk quantity purchased by customers from segment k as a function of package size, package unit price and bulk price. Because we assume that customers will either choose the package or bulk sale option, at most one of these two quantities is positive.

The retailer chooses the package size s, package price P^p and bulk price P^b so as to maximize his profit denoted by π. Formally, we have:

$$\max_{s,P^p,P^b} \pi(s, P^p, P^b) = \sum_{k=1}^{K} \alpha_k \left[(P^p - c)\hat{\imath}_k(s, P^p, P^b)s + (P^b - c)\hat{Q}_k^b(s, P^p, P^b) \right] \tag{2.3}$$

where c is the retailer's unit variable cost of selling the product. Note that we focus our analysis on prices which satisfy $c < P^p \leq P^b$. The first condition guarantees that the profit margin on the product is positive. The second condition comes from the fact that no customer would buy a package if $P^p > P^b$, as argued above. Also note that the retailer does not give out quantity discounts for buying large quantities in bulk or multiple packages.

Discussion of the modeling assumptions

In this section we discuss some of our modeling assumptions. First we make a number of simplifying assumptions regarding the different parameters which are included in our model. We assume that the retailer's unit variable cost is the same for selling the product in packages or in bulk. In practice there may be a difference, such as an extra cost from packaging the units together. Also we assume that customers value the product equally whether it is sold in packages or in bulk when, in real life, some customers may have a lower valuation for the product when it is sold in bulk due to hygiene concerns; others may value the products bought in bulk more because they were able to hand-pick them. Similarly we assume that the waste and run-out costs are the same whether the product was bought in bulk or in package at the start of the period, which we argue makes sense given on interpretation of the run-out cost as the cost of buying extra units of the products at the more expensive corner store and given that the waste cost captures only the un-consumed product itself, not its packaging. We also do not include the inconvenience cost to the customer from buying products in bulk, that is, grabbing them one by one or using a scoop, putting them into a bag or container and possibly having to weigh them in order to generate a price label.

Regarding our consumer choice model, we have assumed that consumers either buy the product in bulk or in packages (or not at all), that is, we do not allow customers to purchase a combination of packages and bulk. This assumption is motivated by practical concerns as we observe that in practice, most customers choose between the two options.

Further, we assume there is a unique package size s. While the multi-package size problem is certainly interesting, our goal is to compare the bulk and package sale options and specifically, we want to focus our analysis on the tradeoffs faced by the retailer when designing a package size for multiple consumer segments. With multiple package sizes and multiple consumer segments the problem at hand becomes one of allocating package sizes to consumer segments and doing so involves carefully pricing the products so as to meet the incentive-compatibility constraints. In practice, due to space and handling constraints, the number of possible package sizes which can be offered in a store is generally limited and typically significantly much smaller than the number distinct segments in the population. Hence, we believe that the single-package size problem we consider constitutes a relevant first step in understanding the impact of bulk on a retailer's profit and on consumer waste. We also note that a typical grocery store contains multiple examples of products which are offered only in bulk and in a unique package size. For example, many stores propose one-size bags of pre-cut cold meats and cheese which can also be purchased at the store's deli counter. Whole Foods Market sells its in-store-prepared peanut butter in a large jar from which consumers can help themselves to fill up plastic containers as well as in pre-filled containers of a unique size which are available for grab next to the jar. Target sells kiwis by the unit or in pre-packaged boxes of eight and oranges in bulk next to bags of six.

Note that the bulk option can be viewed as a package of the smallest possible size: for products which can be purchased in continuous quantities, such as flour or maple syrup, the size of the package is an infinitesimal quantity; for products which are by nature discrete, such as eggs or kiwis, the bulk option is a package of size one. Smaller packages are typically offered at higher prices, therefore this angle supports our assumption that the unit price of the product in bulk is higher than the unit price of the product in packages.

In our model, the retailer has perfect knowledge of the consumers' utility function parameters and consumption needs distribution and therefore can compute exact purchase quantities for each segment given his decisions on package size and prices. As a result the retailer does not hold any excess inventory and therefore does not incur any waste. Our waste analysis is therefore focused solely on product waste at the consumer level. We argue that this is relevant for many products such as dairy items for which studies report wastage percentages of about 17% at the consumer level versus only 0.25% at the distribution and retail level ((Gunders, 2012)).

Finally note that in our model, it is the retailer who makes the package size decision, not the manufacturer. In practice this applies with products such as the above mentioned peanut butter at Whole Foods Market and buns or cookies which are baked locally from the retail store's bakery section. In those instances the product is exactly the same when offered in bulk or in package because it is either delivered in bulk form from the same supplier or is prepared on site, then packaged right at the store. This situation further justifies our assumption of equal valuation for the product when offered in bulk or in packages.

Our notation is summarized in Table 2.1.

In what follows, let $\lfloor x \rfloor$ denote x rounded down to the nearest integer (floor) and $\lceil x \rceil$ is x rounded up to the nearest integer (ceiling). Also let $\lfloor x \rceil$ denote the number x rounded to the nearest integer.

2.2.1 Consumer Choice

In this section we show how to compute the consumer purchase quantity. We consider three settings: (i) only the bulk sale option, (ii) only the package sale option, (iii) the bulk and package sale options coexist and consumers choose between the two.

Table 2.1: Notation

Symbol	Definition
Decision variables	
P^p :	Package price (per unit)
P^b :	Bulk price (per unit)
s :	Package size
Parameters	
v_k :	Product valuation of customers from segment k (per unit)
r_k :	Run-out cost of customers from segment k (per unit)
w_k :	Waste cost of customers from segment k (per unit)
F_k :	Cdf of consumption needs distribution for segment k
μ_k :	Expected consumption needs for segment k
α_k :	Proportion of customers from segment k
c :	Product variable cost (per unit)
Other variables	
i :	Number of packages purchased by customer
Q^b :	Quantity purchased in bulk
$\hat{i}_k(s, P^p, P^b)$:	Ideal number of packages for customers from segment k
$U_k(Q)$:	Expected Gross Utility of customers from segment k
$u_k^p(i)$:	Expected Net Utility of customers from segment k from i packages
$u_k^b(Q^b)$:	Expected Net Utility of customers from segment k from bulk quantity Q_b
Q_k^{*b} :	Utility maximizing quantity for bulk purchase for segment k
Q_k^{*p} :	Utility maximizing quantity for package purchase if the package size was infinitesimally small for segment k
$W_k^{abs}(Q)$:	Expected absolute waste of customers from segment k
$W_k^{rel}(Q)$:	Expected relative waste of customers from segment k
$\pi(s, P^p, P^b)$:	Expected Profit of the Retailer

Consumer choice with only bulk sale option

When the product is only offered in bulk, customers can buy the exact quantity which maximizes their expected net utility $u_k^b(Q^b; P^b) = U_k(Q^b) - P^b Q^b$. This function is mathematically equivalent to the expected profit function of a classical Newsvendor model and is a concave function. Let $Q_k^{*b} = \arg\max_{Q^b \in \mathbb{R}^+} u_k^b(Q^b)$ denote the quantity purchased in bulk by customers from segment k, given a bulk unit price P^{*b} which is given by the well-known newsvendor critical fractile:

$$Q_k^{*b} = \begin{cases} F_k^{-1}\left(\theta_k^b\right) & \text{if } P^b \leq v_k + r_k \\ 0 & \text{otw.} \end{cases} \tag{2.4}$$

where $\theta_k^b = \frac{v_k + r_k - P^b}{v_k + r_k + w_k}$. We refer to this quantity as segment k's *ideal bulk purchase quantity* given a unit bulk price of P^b. If Segment k consumer's consumption needs follow a uniform distribution assumption, we have $Q_k^{*b} = \theta_k^b B_k$.

Consumer choice with only package option

When the product is only offered in packages of size s, the customers' purchase quantity is restricted to be a multiple of the package size and the optimal number of packages is the one which maximizes their expected net utility $u_k^p(i; s, P^p) = U_k(is) - P^p is$ for $i \in \mathbb{N}$.

Let Q_k^{*p} be the quantity which maximizes $U_k(Q) - P^p Q$, which is segment k customers' net utility ignoring the integrality constraint on i (it also corresponds to the expected net utility from buying bulk at a unit price P^p). We have:

$$Q_k^{*p} = \begin{cases} F_k^{-1}\left(\theta_k^p\right) & \text{if } P^p \leq v_k + r_k \\ 0 & \text{otw.} \end{cases}$$

where $\theta_k^p = \frac{v_k + r_k - P^p}{v_k + r_k + w_k}$. We refer to this quantity as segment k's *ideal package purchase quantity* given a unit package price of P^p. If Segment k consumer's consumption needs follow a uniform distribution assumption, we have $Q_k^{*p} = \theta_k^p B_k$.

However, customers may not be able to purchase exactly Q_k^{*p} because they can only buy a discrete number of packages of size s. Because the function $U_k(Q) - P^p Q$ is concave in Q and achieves a unique maximum at Q_k^{*p}, the number of packages purchased by segment k customers, i.e., $\hat{i}_k(s, P^p)$, is either $\left\lfloor \frac{Q_k^{*p}}{s} \right\rfloor$ or $\left\lceil \frac{Q_k^{*p}}{s} \right\rceil$. depending on which value achieves the highest net utility. Note that $\left\lfloor \frac{Q_k^{*p}}{s} \right\rfloor = 0$ when $s > Q_k^{*p}$. We provide a simple numerical example for illustration.

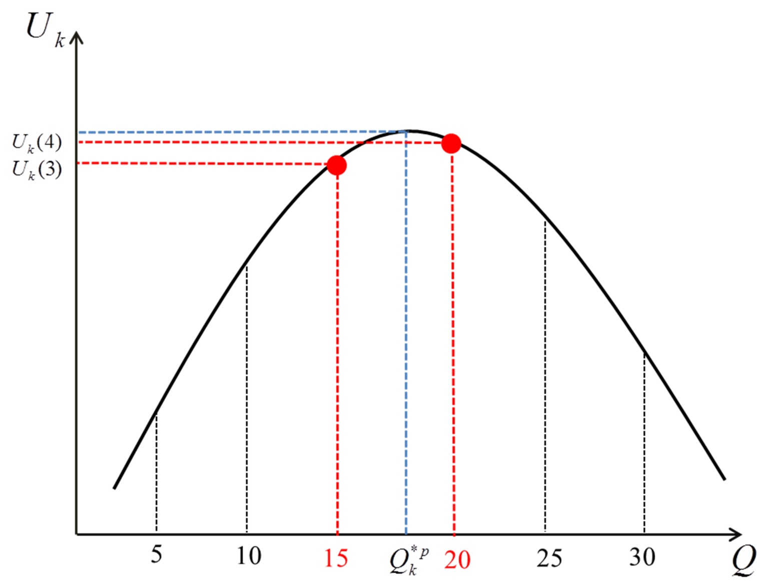


Figure 2.1: An example of consumer decision in only package sale case

Example 2.2.1.1. *Suppose* $Q_k^{*p} = 18$ *and* $s = 5$. *The consumer can only buy quantities which are multiple of 5 (5, 10, 15, 20 etc.) as shown in Figure 2.1. Among these alternatives buying 3 and 4 packages (15 units or 20 units of product) gives the highest utility. Notice that these points correspond to* $\left\lfloor \frac{Q_k^{*p}}{s} \right\rfloor = \left\lfloor \frac{18}{5} \right\rfloor = 3$ *and* $\left\lceil \frac{Q_k^{*p}}{s} \right\rceil = \left\lceil \frac{18}{5} \right\rceil = 4$. *Since buying* 4 *packages gives more utility, consumer prefers to buy* $i_k^R = 4$ *packages of the product (it could be* $\left\lfloor \frac{Q_k^{*p}}{s} \right\rfloor$ *for some other values of* s*).*

When P^p is fixed, the number of packages purchased by customers, i.e., $\hat{i}_k(s, P^p)$, is a piecewise constant, non-increasing function of s, as customers buy fewer packages as they get larger. As a result, the quantity purchased by customers, i.e., $s\hat{i}_k(s, P^p)$ is a piecewise increasing function of s. Both functions are illustrated on Figure 2.2 for one customer segment.

When the package size is very small, customers are able to purchase a quantity which is very close to their ideal purchase quantity Q_k^{*p}. Points which are above this quantity corresponds to cases where the purchase quantity is rounded up, i.e., $\hat{i}_k(s, P^p) = \left\lceil \frac{Q_k^{*p}}{s} \right\rceil$ and points which are below this quantity corresponds to cases where the purchase quantity is rounded down, $\hat{i}_k(s, P^p) = \left\lfloor \frac{Q_k^{*p}}{s} \right\rfloor$. The jump points on both curves are values at which the customers receive the same expected utility from buying two successive numbers of packages. More formally, let γ_k^n denote the value of the package size s such that k-segment customers receive the same expected utility from consuming n or $n+1$ packages for $n = 0, 1, ...,$, that is, such that

$$u_k^p(n, s, P^p) = u_k^p((n+1), s, P^p) \tag{2.5}$$

Remember that we have assumed that, in case of a tie in the maximum expected net utility, customers buy the largest number of packages which gives them this maximum value. Therefore k-customers buy $n+1$ packages when the package size is γ_k^n. In particular, they buy one package if the package size is γ_k^0 as they receive the same expected net utility from buying one package or buying nothing.

Also, let $\gamma_k^{-1} = +\infty$.

The γ_k^n values can be used to express the number of packages bought by customers of segment k, i.e., $\hat{i}_k$ as a function of the package size s, as shown in the following lemma.

Lemma 2.2.1.2. *$\hat{i}_k(s, P^p)$ is a non-increasing piecewise constant function of s for fixed P^p with thresholds γ_k^n such that $\hat{i}_k(s, P^p) = n$ for $s \in [\gamma_k^n, \gamma_k^{n-1})$, for $n = 0, 1, 2, ...$ Also, $\hat{i}_k(s, P^p)s$ is a piecewise increasing function of s for fixed P^p with the same thresholds. Finally, $\hat{i}_k(\gamma_k^n, P^p)\gamma_k^n$ is decreasing in n.*

The following lemma shows that, when the distribution of consumption needs is uniformly distributed, the optimal number of packages purchased can be obtained by rounding $\frac{Q_k^{*p}}{s}$ to the nearest integer.

Lemma 2.2.1.3. *For every given package unit price P^p and package size s, under a uniform distribution for Segment k consumer needs on $[0, B_k]$, we have $\hat{i}_k(s, P^p) = \left\lfloor \frac{Q_k^{*p}}{s} \right\rceil$.*

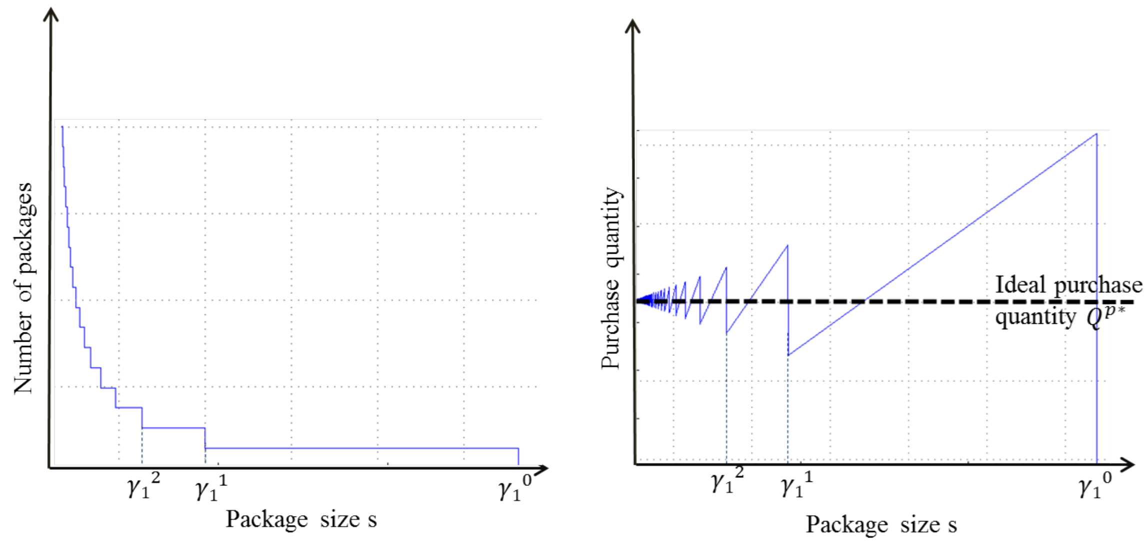


Figure 2.2: Number of packages bought and total purchase quantity as a function of package size when unit package price is fixed

Consumer choice with both bulk and package sales options

We now assume that the retailer provides both bulk and package sale options and the consumers choose between these two options (or nothing). As mentioned above, we assume that prices are set such that $c < P^p \leq P^b$, so that consumers face a tradeoff between price and quantity.

Given the results from the previous two sections, customers from segment k who buy in bulk buy Q_k^{*b} units and customers who buy in packages buy either i_k^L or i_k^R packages. As a result, their expected net utility is given by

$$\max\left\{u_k^b(Q_k^{*b}; P^b), u_k^p(i_k^L; s, P^p), u_k^p(i_k^R; s, P^p)\right\}$$

In case of a tie between the utility from the bulk sale option and the maximum utility from the package size option, we assume that consumers buy the product in packages.

The following Lemma provides a sufficient condition for customers of segment k to select the package sale option.

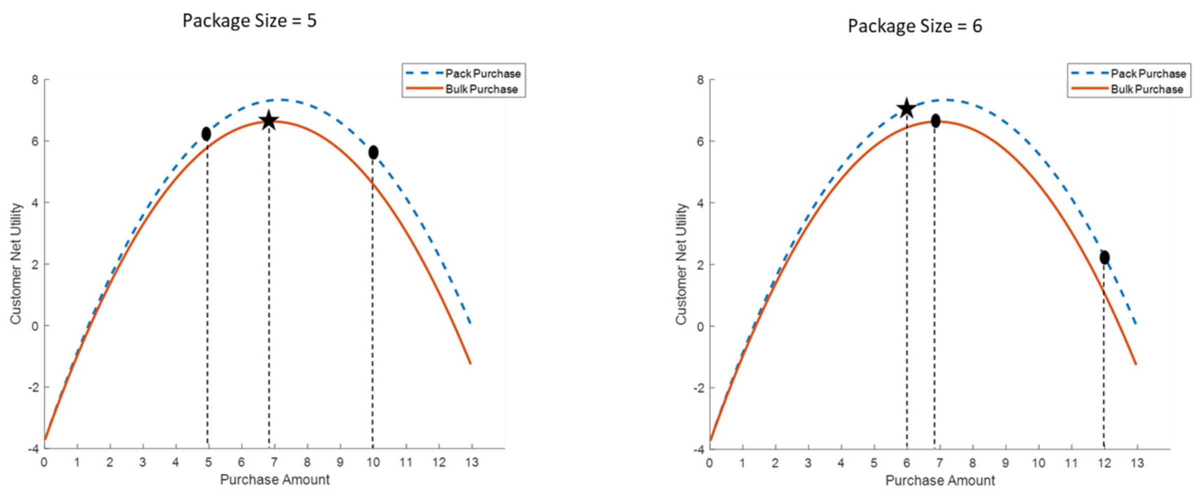


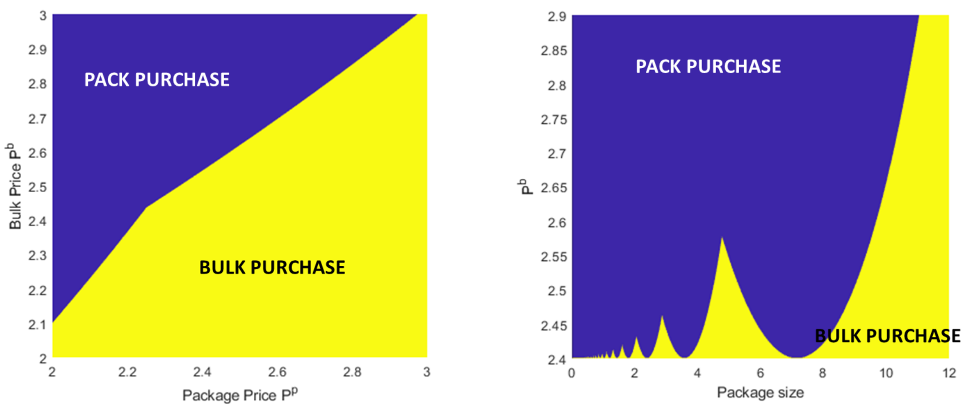
Figure 2.3: An example of change of consumer choice by package size

Lemma 2.2.1.4. *Assume $P^p \leq P^b$. If $Q_k^{*b} \leq si_k^L$ then customers from segment k choose the package sale option.*

Example 2.2.1.5. *Let $K = 1, v = 5, r = 0.5, w = 1, B_L = 15, P^p = 2.4, P^b = 2.5$ and $c = 1$. In Figure-2.3 we show a consumer's purchase decision for two different package size options. When the package size is 5 units, the utility of buying 6.9 units of product in bulk gives higher utility than buying 1 or 2 packages of product (or buying 5 or 10 units). Hence, consumer prefers to buy product in bulk. On the other hand, when package size is 6 units, consumer prefers to buy 1 pack of product to buying in bulk (or buying 2 pack) since this gives more utility.*

The parameters that the retailer has control over are bulk/pack price and package size. In the following example we show how consumers decision change depending on these parameters.

Example 2.2.1.6. *Let $K = 1, v = 5, r = 0.5, w = 1, B_L = 15$ and $c = 1$. In Figure-2.4 (a) we show how consumer's purchase decision change by P^b and P^p when $s = 5$. Due to the selection of high (relatively) package size, consumer choose bulk purchase in the majority of*



(a) Change By P^p and P^b ($s = 5$)

(b) Change By s and P^b ($P^p = 2.4$)

Figure 2.4: Change of consumer‘s choice by P^b, P^b and s (Example 2.2.1.6)

the area. As can be inferred from Figure-2.4 (b), the area of bulk purchase is going to shrink as s gets smaller because consumer is better off buying in small packages since it enables consumer to better match her demand and purchase amount along with paying less per unit.

2.2.2 Retailer's Optimal Decisions

In this section we investigate retailer‘s optimal decisions under the assumption that segment k's consumption needs D_k has a uniform distribution on $[0, B_k]$ for $k = 1, ..., K$.

Besides reporting the retailer's optimal decisions, we also investigate their impact on the expected waste quantities at the consumer level. Let Q_k be the total product quantity purchased by Segment k consumers (either in bulk or in packages). We calculate both expected *absolute* waste, calculated as $W_k^{abs}(Q_k) = E[Q_k - D_k]^+$ and expected *relative* waste, calculated as $W_k^{rel}(Q_k) = \frac{W_k^{abs}(Q_k)}{Q_k}$. In practice absolute waste measure the total physical quantity of product which is thrown away (a relevant metric for estimating how much is potentially sent to a landfill) and relative waste measure the percentage of product purchased by the consumer which is wasted (a relative metric when comparing different settings such as with and without the bulk sale option). When D_k is uniformly distributed on $[0, B_k]$, the absolute waste from Segment k consumers is equal to:

$$W_k^{abs}(Q_k) = \begin{cases} \frac{Q_k^2}{2B_k} & for\ Q_k \leq B_k \\ \\ Q_k - \frac{B_k}{2} & for\ Q_k \geq B_k \end{cases} \tag{2.6}$$

We also report total expected absolute and relative waste across all K segments as $W^{abs}(Q_k) = \sum_{k=1}^{K} \alpha_k W_k^{abs}(Q_k)$ and $W^{rel}(Q_k) = \frac{\sum_{k=1}^{K} \alpha_k W_k^{abs}(Q_k)}{\sum_{k=1}^{K} \alpha_k Q_k}$.

In what follows we study the retailer's optimal package size and pricing decisions in a number of different settings. We first consider the case of an homogeneous market, that is, when there is only one consumer segment. Next, we consider the heterogeneous market case with two different consumer segments who differ in their maximum consumption needs quantity B_k, but have the same the same product valuation, waste cost and run-out cost.

2.3 Homogeneous Market

In this section we consider the case of a unique customer segment. We let $B_k = B$, $v_k = v, w_k = w$ and $r_k = r$ and set $\theta^p = \frac{v+r-P^p}{v+r+w}$. In this case, since all customers behave identically and purchase either only in packages or in bulk, it is optimal for the retailer to offer only one sale option, that is, either only packages or bulk.

2.3.1 Offering Only the Bulk Sale Option

The following lemma summarizes our results when the retailer offers only the bulk sale option. Let $\eta = \frac{v+r-c}{v+r+w}$ and $\delta = \frac{(v+r-c)^2}{v+r+w}$.

Lemma 2.3.1.1. *If the retailer only offers the product in bulk to a homogeneous market, the optimal bulk unit price is* $P^{*b} = \frac{v+r+c}{2}$ *and the optimal retailer profit is* $\frac{\delta}{4}B$. *The customers' purchase quantity is equal to* $\frac{\eta}{2}B$ *which gives them an expected net utility of* $\left(\frac{\delta}{8} - \frac{r}{2}\right)B$. *Finally, absolute expected waste is equal to* $\frac{\eta^2}{8}B$ *and relative expected waste is* $\frac{\eta}{4}$.

Note that at the optimal solution, the consumers receive the maximum utility given the the bulk price as they are able to purchase their ideal bulk purchase quantity Q^{*b} given the unit bulk price P^{*b}. Also, they generate a positive amount of expected waste both in absolute and relative terms, which is due to the random nature of their consumption needs.

2.3.2 Offering Only the Package Sale Option

Next we consider the case where the retailer only offers the product in packages. In this case, the retailer optimizes both the package size s and the product unit price P^p such that $P^p > c$. Our first result provides a formula for the optimal package size as a function of a fixed unit price P^p.

Lemma 2.3.2.1. *If the retailer only offers the product in packages to a homogeneous market, and the product unit price is fixed at P^p, then the optimal package size $s^*(P^p) = \frac{B}{2}\left(\frac{v+r+w}{P^p+w}\right) = \frac{B}{2}\frac{1}{1-\theta^p}$ for $P^p \leq \frac{v+r-w}{2}$ and $s^*(P^p) = 2B\left(\frac{v+r-P^p}{v+r+w}\right) = 2B\theta^p$ for $P^p \geq \frac{v+r-w}{2}$. Customers buy one package and receive a negative expected net utility of $-\frac{r}{2}B$, which is the expected net utility of buying nothing.*

Intuitively, the retailer offers the largest possible package size such that all consumers buy one package. At this value the consumers receive a negative utility which matches the utility they receive from buying nothing.

Next we provide the optimal value for the size and unit price of the package. As in the previous section we let $\eta = \frac{v+r-c}{v+r+w}$ and $\delta = \frac{(v+r-c)^2}{v+r+w}$.

Lemma 2.3.2.2. *If the retailer only offers the product in packages to a homogeneous market, the optimal package unit price is $P^{*p} = \frac{v+r+c}{2}$, the optimal package size is ηB and the optimal retailer profit is $\frac{\delta}{2}B$. The customers purchase one package, therefore their purchase quantity is equal to ηB, which gives them an expected net utility of $-\frac{r}{2}B$. Finally, expected absolute waste is equal to $\frac{\eta^2}{2}B$ and relative expected waste is $\frac{\eta}{2}$.*

2.3.3 Comparison Between Bulk Only and Package Only

By comparing the optimal values in Lemmas 2.3.1.1 and 2.3.2.2, we see that the optimal product unit price is the same in both cases. It is interesting to note that this optimal price does not depend on the waste cost w but depends on the run-out cost r. Interestingly, the retailer makes twice as much profit when selling the product in packages versus in bulk. This is because he forces the consumers to buy twice as much when selling in packages, up to the point where they are indifferent between buying the product and buying nothing. As a result the customers are expected to waste four times more in absolute terms and twice more in relative terms when the product is sold in packages versus in bulk. We summarize these findings in the following theorem (proof is omitted).

Theorem 2.3.3.1. *If the retailer sells the product to a homogeneous market, it is optimal to sell it in packages. However doing so leads to four (two) times the amount of absolute (relative) waste compared to optimally selling in bulk.*

2.4 Heterogeneous Market with Two Consumer Segments

In this section we assume there are two customer segments, i.e. $K = 2$, who differ only in their maximum consumption needs quantity B_k. As in the previous section, we let $v_k = v, w_k = w$ and $r_k = r$ and set $\theta^p = \frac{v+r-P^p}{v+r+w}$. Without lost of generality we let $B_1 > B_2$ so that Segment 1 consumers have higher consumption needs than Segment 2 consumers or in others words, Segment 1 customers have low consumption needs and Segment 2 consumers have high consumption needs. For the ease of readability we use B_L instead of B_1 and B_H instead of B_2. Let $\alpha_1 = \alpha$ denote the proportion of segment 1 (low consumption needs) customers so that $1 - \alpha$ is the proportion of segment 2 (high consumption needs) customers.

2.4.1 Offering Only the Bulk Sale Option

First we analyze the case where the retailer offers only the bulk sale option. Lemma 2.4.1.1 summarizes the optimal price and the other results of interest. Note that the result applies to any number of segments, that is, it holds for $K \geq 1$.

Lemma 2.4.1.1. *If the retailer only offers the product in bulk to two customer segments who differ only in their maximum consumption needs quantity, the optimal bulk unit price is* $P^{*b} = \frac{v+r+c}{2}$ *and the optimal retailer profit is* $\frac{\delta}{4}\sum_{k=1}^{K} \alpha_k B_k$*. Segment k consumers' purchase quantity is equal to* $\frac{\eta}{2} B_k$ *which gives them an expected net utility of* $\left(\frac{\delta}{8} - \frac{r}{2}\right) B_k$*. Finally, expected absolute waste from Segment k consumers is equal to* $\frac{\eta^2}{8} B_k$ *and expected relative waste is* $\frac{\eta}{4}$*.*

Notice that the optimal unit price is the same as in the homogeneous market case.

2.4.2 Offering Only the Package Sale Option

In this section we analyze the case where retailer offers only packages of a fixed size s. Under the uniform distribution assumption for the consumption needs, we use (2.2) to obtain that $\gamma_k^n = \frac{2B_k\theta^p}{1+2n}$ for $n = 0, 1, ...$ Note that, under our assumption that $B_L < B_H$, we have $\gamma_1^n < \gamma_2^n$ for all $n = 0, 1, ...$

First we fix the unit prices P^p and show that the optimal package size can only take one of three possible values.

Theorem 2.4.2.1. *For a fixed package price, the optimal package size is either* γ_1^0, γ_2^0 *or* $\gamma_2^{\bar{n}}$ *where* $\bar{n} = \left\lfloor \frac{B_H}{2B_L} \right\rceil$.

where $\lfloor x \rceil$ denotes the rounding of x to the nearest integer value.

When the optimal package size is γ_1^0, Segment 1 customers are indifferent between buying one package and buying nothing, hence, given our tie-breaking rule, they buy one package

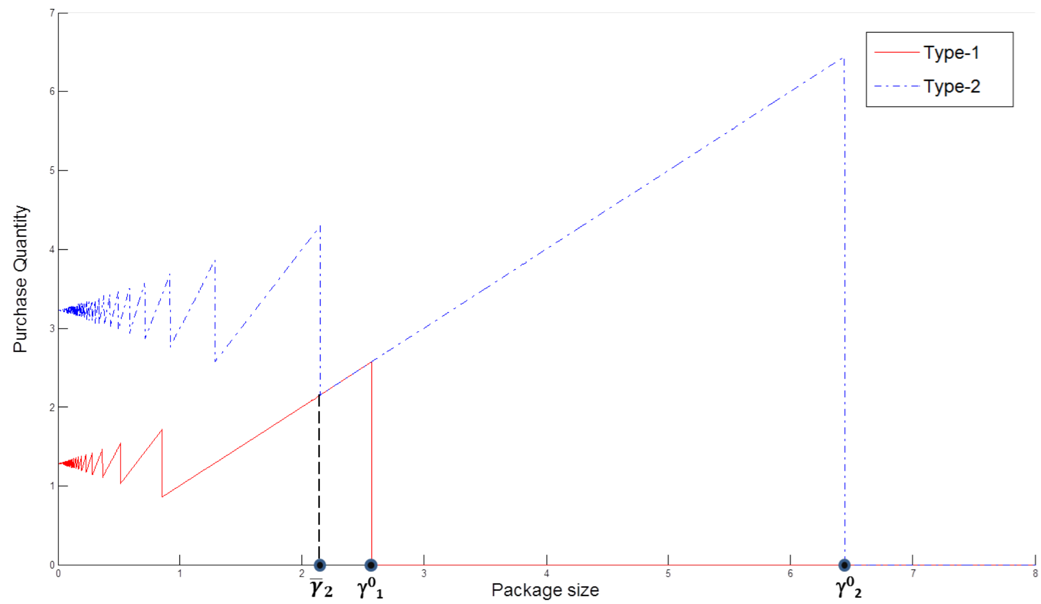


Figure 2.5: Possible optimal values for the package size in a 2-segment market when price is fixed

and Segment 2 customers buy $\bar{n}$ packages. When the optimal package size is γ_2^0, Segment 2 customers are indifferent between buying one package and buying nothing, hence they buy one package, and Segment 2 customers buy nothing. When the optimal package size is $\gamma_2^{\bar{n}}$, Segment 2 customers are indifferent between buying $\bar{n}$ and $\bar{n}+1$ packages, hence they buy $\bar{n}+1$ packages and segment-1 customers buy one package. Figure 2.5 represents the three possible values for the optimal package size given a fixed P^p. On this picture $\bar{n} = 1$.

We now provide the optimal solution, that is, the optimal package size and unit price.

Theorem 2.4.2.2. *In a heterogeneous market with two consumer segments, the optimal package unit price is* $P^{*p} = \frac{v+r+c}{2}$. *When* $n = \left\lfloor \frac{B_H}{2B_L} \right\rceil < \frac{B_H}{2B_L}$, *the optimal package size is:*

$$s^* = \begin{cases} \eta B_H & \text{for } \alpha \leq \frac{n}{1+n} \\ \eta \frac{B_H}{1+2n} & \text{for } \frac{n}{1+n} \leq \alpha \leq \frac{B_H + n(B_H - B_L - 2B_L n)}{B_L + n(B_H + B_L - 2B_L n)} \\ \eta B_L & \text{for } \alpha \geq \frac{B_H + n(B_H - B_L - 2B_L n)}{B_L + n(B_H + B_L - 2B_L n)} \end{cases}$$

When $n = \left\lfloor \frac{B_H}{2B_L} \right\rceil > \frac{B_H}{2B_L}$, *the optimal package size is:*

$$s^* = \begin{cases} \eta B_H & \text{for } \alpha \leq \frac{B_H - nB_L}{B_L + B_H - nB_L} \\ \eta B_L & \text{for } \alpha \geq \frac{B_H - nB_L}{B_L + B_H - nB_L} \end{cases}$$

where $\eta = \frac{v+r-c}{v+r+w}$.

We see that the optimal price is same as in the homogeneous market case and the bulk sale case. The optimal package size can take three different values, which correspond to three solutions which we refer to as the "Tailor to Segment 1", "Tailor to Segment 2" and "Compromise" solutions. We say that the retailer chooses to "tailor to Segment 1" when the packages size is set equal to ηB_L, which makes Segment 1 customers indifferent between buying one package and buying nothing. As a result Segment 1 customers buy one package and Segment 2 customers buy n packages where $n = \left\lfloor \frac{B_H}{2B_L} \right\rceil$, which is equal to one or more. We say the retailer chooses to "tailor to Segment 2" when the package size is set equal to ηB_H, which makes Segment 2 customers receive the same utility from buying one package and buying nothing. As a result Segment 2 customers buy one package and Segment 1 customers buy nothing. We say the retailer chooses the "compromise" solution when the package size is set equal to $\eta \frac{B_H}{1+2n}$, which is such that Segment 2 customers receive the same net utility from buying n and $n+1$ packages. As a result customers from segment 1 buy one package and customers from Segment 2 buy $n+1$ packages.

As expected, when the proportion of Segment 1 customers is low (high), it is optimal to for the retailer to tailor to Segment 2 (Segment 1). For intermediate values of α it is possible that the retailer needs to strike a *compromise* by choosing another value of the package size. Table 2.2 provides the value of the retailer's profit, consumers' purchase quantity, utility and waste for each possible solution.

Example 2.4.2.3. *Let* $K = 2$, $v = 20, r = 4, w = 1$, $B_L = 2$, $B_H = 13$, $c = 16$.

Table 2.2: Retailer's profit and consumers' purchase quantity, utility and waste when the retailer offers only packages.

Solution	Tailor to Segment 1	Tailor to Segment 2	Compromise
Package size	ηB_L	ηB_H	$\eta \frac{B_H}{1+2n}$
Retailer's profit	$\frac{\delta}{2}(n(1-\alpha)+\alpha)B_L$	$\frac{\delta}{2}(1-\alpha)B_H$	$\frac{\delta}{2}\frac{n(1-\alpha)+1}{1+2n}B_H$
Purchase quantity			
Segment 1	ηB_L (one pack)	0	$\eta\frac{1}{1+2n}B_H$ (one pack)
Segment 2	$n\eta B_L$ (n packs)	ηB_H (one pack)	$\eta\frac{n+1}{1+2n}B_H$ ($n+1$ packs)
Consumer utility			
Segment 1	$-\frac{r}{2}B_L$	$-\frac{r}{2}B_L$	$\delta\frac{B_H(B_L(1+2n)-B_H)}{2B_L(1+2n)^2}-\frac{r}{2}B_L$
Segment 2	$\frac{\delta n}{2}\left(1-\frac{nB_L}{B_H}\right)B_L-\frac{r}{2}B_H$	$-\frac{r}{2}B_H$	$\delta\frac{B_H}{2}\frac{n(1+n)}{(1+2n)^2}-\frac{r}{2}B_H$
Absolute waste			
Segment 1	$\frac{\eta^2}{2}B_L$	0	$\frac{\eta^2}{2}\frac{(B_H)^2}{B_L(1+2n)^2}$
Segment 2	$\frac{n^2\eta^2}{2}\frac{B_L^2}{B_H}$	$\frac{\eta^2}{2}B_H$	$\frac{\eta^2}{2}\frac{B_H(1+n)^2}{(1+2n)^2}$
Relative waste			
Segment 1	$\frac{\eta}{2}$	0	$\frac{\eta}{2}\frac{B_H}{B_L(1+2n)}$
Segment 2	$\frac{n\eta}{2}\frac{B_L}{B_H}$	$\frac{\eta}{2}$	$\frac{\eta}{2}\frac{(1+n)}{(1+2n)}$

where $n=\left\lfloor\frac{B_H}{2B_L}\right\rfloor$, $\eta=\frac{v+r-c}{v+r+w}$ and $\delta=\frac{(v+r-c)^2}{v+r+w}$.

The optimal product unit price is 20. For $\alpha \in [0, 0.75]$, the optimal package size is 4.16, which is tailored to Segment 2 customers: Segment 1 customers do not buy anything and Segment 2 customers buy one package. For $\alpha \in [0.75.0.91]$, the optimal packages size is 0.59; this is the compromise solution: Segment 1 customers buy one package and Segment 2 customers buy 4 packages. Finally for $\alpha \in [0.91, 1]$, the optimal packages size is 0.64, which is tailored to Segment 1 customers: Segment 1 customers buy one package and Segment 2 customers buy 3 packages. Note that the optimal package size is not monotone in α as it is smallest when the retailer chooses the comprise solution.

The conditions which determine which package size value is optimal only depend on the values of α, B_L and B_H and not on the cost parameters. Figure 2.6 shows how the optimal solution varies with the $\frac{B_L}{2B_H}$ ratio and the proportion of Segment 1 customers α when the retailer offers only packages. Note that we see the Compromise region appear only for values of $\frac{B_L}{2B_H}$ such that the decimal numbers are between 0 and 0.5, which is what the condition $n=\left\lfloor\frac{B_H}{2B_L}\right\rfloor<\frac{B_H}{2B_L}$ directly implies.

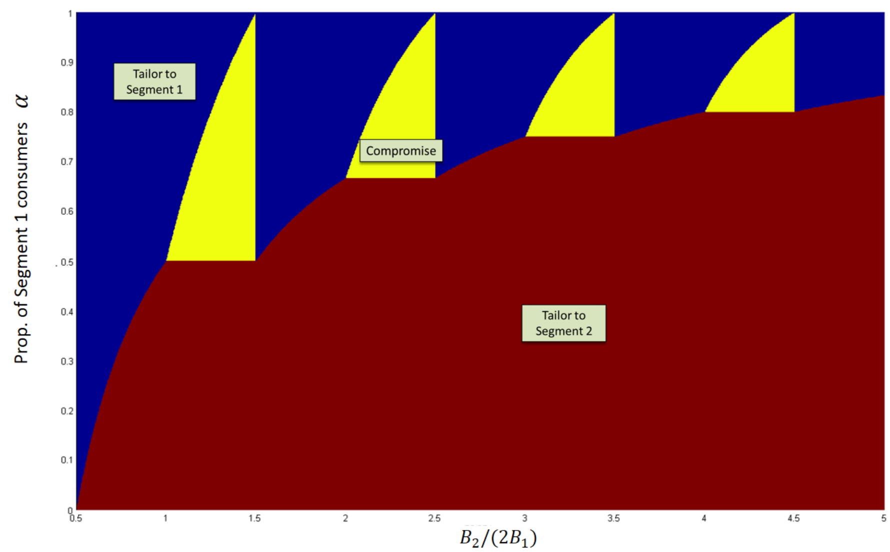


Figure 2.6: Optimal solution when the retailer offers only packages

2.4.3 Offering Both Bulk and Package Sale Options

In this section we analyze the case where retailer is able to offer the product both in bulk and in packages. As profit maximizer the retailer is choosing between three options: offering the product only in bulk, only in packages or at the same time in bulk and packages. Given that there are two consumer segments, the last option implies that consumers from one segment buy the product in bulk while consumers from the other segment buy packages.

Our first result eliminates one of the above options as a possible solution to the retailer's profit maximization problem.

Theorem 2.4.3.1. *It is never optimal for the retailer to only offer the product in bulk. Depending on the demand and cost parameters, it is either optimal to (i) offer the product only in packages to both segments or (ii) offer the products in bulk to Segment 1 customers and in packages to Segment 2 customers. In Case (i) the retailer chooses either the tailor to Segment 1 or compromise solution. In Case (ii), the optimal bulk unit price is* $P^{b*} = \frac{B_H(v+r)(1-\alpha)+\alpha B_L(v+r+c)}{B_H(1-\alpha)+2\alpha B_L}$, *the optimal package unit price is* $\frac{v+r+c}{2} - \frac{\alpha^2 B_L^2(v+r-c)}{(B_H(1-\alpha)+2\alpha B_L)^2}$ *and the optimal package size is* $s^* = \eta B_H$.

Interestingly, when it is optimal for the retailer to offer bulk and packages selling options, the package size is designed so that Segment 2 customers are indifferent between buying in bulk or buying one package, which means they buy one package. Also Segment 1 buy in bulk, choosing the quantity which maximizes their net utility given the bulk unit price. Table 2.3 provides the value of the retailer's profit, consumers' purchase quantity, utility and waste for this case.

Table 2.3: Retailer's profit and consumers' purchase quantity, utility and waste when it is optimal to offer both bulk and packages.

Solution	Bulk & package
Package size	ηB_H
Retailer's profit	$\frac{\delta}{2}\frac{(\alpha B_L+(1-\alpha)B_H)^2}{2\alpha B_L+(1-\alpha)B_H}$
Purchase quantity	
Segment 1	$\eta\frac{B_L^2\alpha}{B_H(1-\alpha)+2B_L\alpha}$ (in bulk)
Segment 2	ηB_H (one pack)
Consumer utility	
Segment 1	$\frac{B_L\left((B_L\alpha)^2(\delta-4r)-rB_H(1-\alpha)(B_H(1-\alpha)+4B_L\alpha)\right)}{2(B_H(1-\alpha)+2B_L)^2}$
Segment 2	$\frac{B_H}{2}\left(\frac{\delta(B_L\alpha)^2}{(B_H(1-\alpha)+2B_L\alpha)^2}-r\right)$
Absolute waste	
Segment 1	$\frac{\eta^2}{2}\frac{B_L^3\alpha^2}{(2B_L\alpha+B_H(1-\alpha))^2}$
Segment 2	$\frac{\eta^2}{2}B_H$
Relative waste	
Segment 1	$\frac{\eta}{2}\frac{B_L\alpha}{(2B_L\alpha+B_H(1-\alpha))}$
Segment 2	$\frac{\eta}{2}$

where $n=\left\lfloor\frac{B_H}{2B_L}\right\rceil$, $\eta=\frac{v+r-c}{v+r+w}$ and $\delta=\frac{(v+r-c)^2}{v+r+w}$.

Table 2.4 in the Appendix provides the necessary and sufficient conditions for each possible solution to be optimal.

Figure 2.7 shows how the optimal solution varies with the $\frac{B_L}{2B_H}$ ratio and the proportion of Segment 1 customers α when the retailer is able to offer both bulk and packages.

Figure 2.8 represents the percentage increase in the retailer's profit from being able to include a bulk sale option as a function of $\frac{B_L}{2B_H}$ ratio and the proportion of Segment 1 customers α, which are the only parameters which have an impact (i.e., the values v, r, w

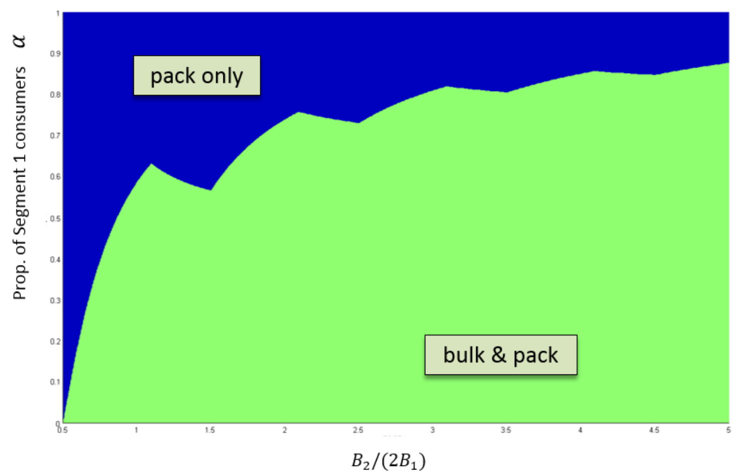


Figure 2.7: Optimal solution when the retailer can offer bulk and packages

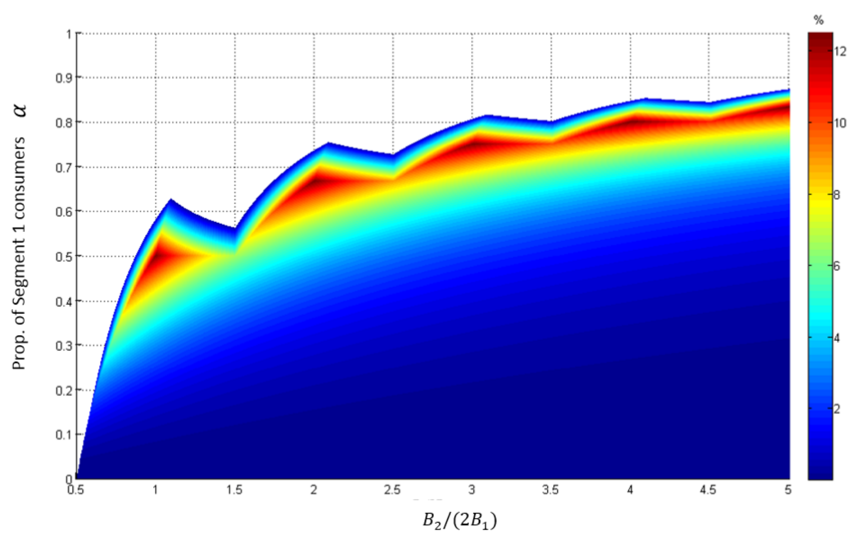


Figure 2.8: Percentage increase in the retailer's profit from the ability to offer bulk

and c do not matter here). The white regions on Figure 2.8 corresponds to parameter values for which the retailer optimally choose to offer only packages. The maximum value of the percentage increase in profit is equal to 12.5%, which is also the theoretical upper bound as stated in Theorem-2.4.3.2, and averages out to 1.66% across all values considered on the graph (i.e,. $\frac{B_L}{2B_H} \in [0, 5]$ and $\alpha \in [0, 1]$).

Theorem 2.4.3.2. *Consider a retailer who initially offers the product only in packages. Having the ability to also offer the product in bulk increases the profit up to 12.5%.*

Another scenario could be considering a retailer who is selling the product only in bulk. We already know from Theorem-2.4.3.1 that selling only bulk is never optimal and the optimal strategy is as shown in Figure-2.7. But percent increase in the profits as a consequence of being able to include a package sale option will be different. In Figure-2.9 we show the percentage increase in the retailer's profit in that case as a function of $\frac{B_L}{2B_H}$ ratio and the proportion of Segment 1 customers α. The maximum value of the percentage increase in profit is equal to 100%, which is also the theoretical upper bound as stated in Theorem-2.4.3.3, and averages out to 75.45% across all values considered on the graph.

Theorem 2.4.3.3. *Consider a retailer who initially offers the product only in bulk. Having the ability to also offer the product in packages increases the profit up to 100%.*

2.4.4 Impact on Waste

In this section we study the impact on waste at the consumer level from adding a new sale option.

First we consider the case of a retailer who initially offers the product only in packages then is able to add the bulk sale option. Figure 2.10 represents the change in absolute and relative expected waste from the retailer being able to include a bulk sale option as a

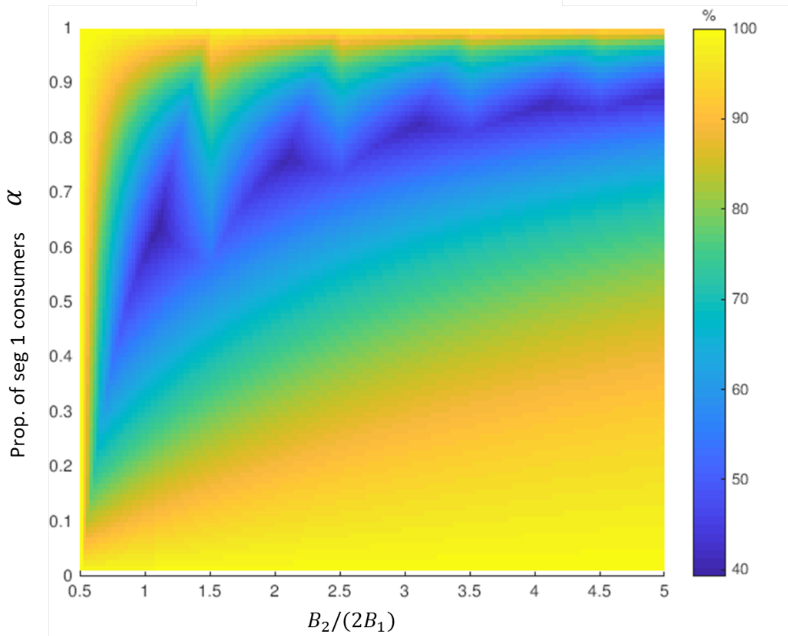
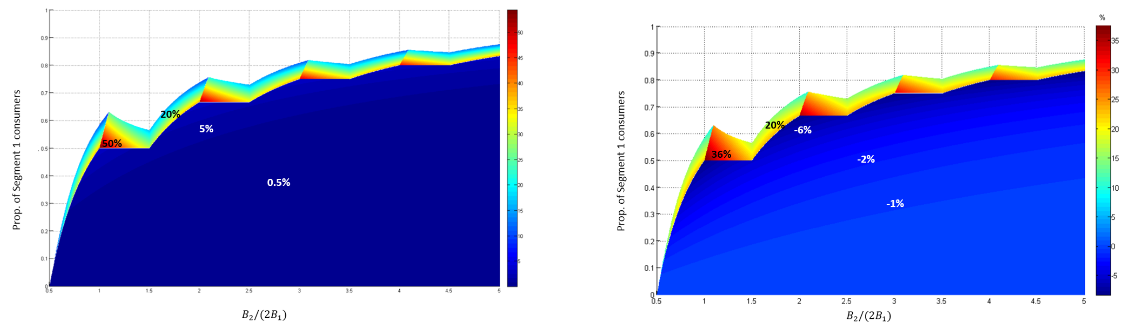


Figure 2.9: Percentage increase in the retailer's profit from the ability to offer pack

function of $\frac{B_L}{2B_H}$ ratio and the proportion of Segment 1 customers α, which once again, are the only parameters which have an impact (i.e., the values v, r, w and c do not matter here). According to the numerical results the maximum change in relative expected waste is 54.45% and the minimum change is 0% with an average of 1.95%. For the relative expected waste we observe the maximum change as 37.4% and minimum change as -8.33% (decrease) with an average of 0.26%.

Intuitively, the impact on absolute waste is an increase because, with the introduction of bulk, the retailer increases the total quantity purchased by both customers segments, which in turns leads to an increase in the amount of absolute waste. It is notable that adding bulk has in many cases a negative impact on waste in our model. This is due to the fact that the retailer is able to use the bulk sale to force Segment 2 customers to buy more products.

Second, we consider the case of a retailer who initially offers the product only in bulk then is able to add the package sale option. Figure-2.11 shows the change in absolute and relative expected waste from the retailer being able to include the package sale option as



(a) Change in absolute expected waste

(b) Change in relative expected waste

Figure 2.10: Change in consumers' absolute and relative expected waste by adding bulk option

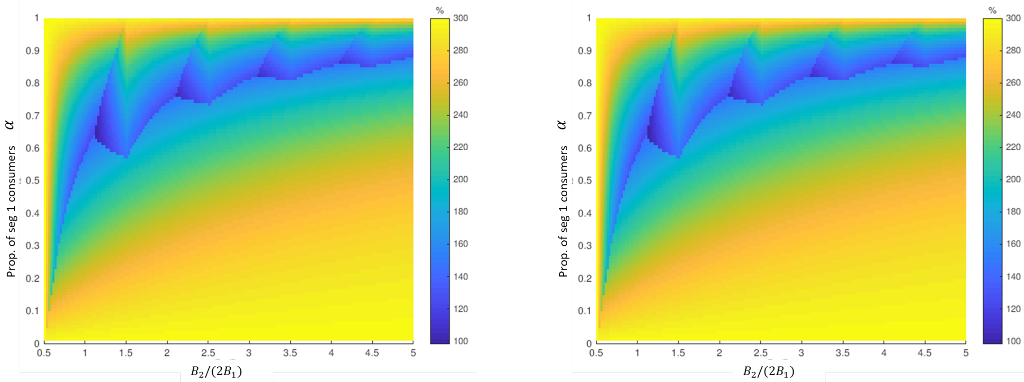
a function of $\frac{B_L}{2B_H}$ ratio and the proportion of Segment 1 customers α. According to the numerical results the maximum change in relative expected waste is 300% and the minimum change is 100% with an average of 235%. For the relative expected waste we observe the maximum change as 100% and minimum change as 40% with an average of 90.4%. In fact, we have the following theorem:

Theorem 2.4.4.1. *Consider a retailer who initially offers the product only in bulk. Having the ability to also offer the product in packages will always increase absolute and relative waste.*

2.5 Full Heterogeneity

In this section we assume that consumers from Segment k follows a uniform distribution on $[0, B]$ where B follows a continuous distribution with cdf G and pdf g on support $[0, \overline{B}]$. All the other assumptions are the same. As we did before, we analyze the different strategies, i.e., only bulk sale, only package sale and both, and determine the best strategy for the retailer.

Let $u^p(i, s, P^p; B)$ denote the expected *net* utility a customer with maximum consumption needs B derives from purchasing i packages of size s at a unit price of P^p and let $u^b(Q^b, P^b)$



(a) Change in absolute expected waste (b) Change in relative expected waste

Figure 2.11: Change in consumers' absolute and relative expected waste by adding package option

denote the expected *net* utility a customer with maximum consumption needs B derives from purchasing Q^b units of bulk at a unit price of P^b. Also let $\hat{\imath}(s, P^p, P^b; B)$ and $\hat{Q}^b(s, P^p, P^b; B)$ respectively denote the number of packages and bulk quantity purchased by consumers with maximum consumption needs B In this case the retailer's profit can be written as:

$$\max_{s,P^p,P^b} \pi(s, P^p, P^b) = \int_0^{\overline{B}} \left[(P^p - c)\hat{\imath}(s, P^p, P^b; B)s + (P^b - c)\hat{Q}^b(s, P^p, P^b; B)\right] g(B)dB \tag{2.7}$$

2.5.1 Offering Only the Bulk Sale Option

When the retailer offers only bulk for a unit price $P^b \le v + r$, consumers with maximum consumption needs B buy a quantity $\hat{Q}^b(s, P^p, P^b; B) = B\frac{v+r-P^b}{v+r+w}$. Therefore we have:

$$\pi(P^b) = \int_0^{\overline{B}} (P^b - c)B\frac{v + r - P^b}{v + r + w} g(B)dB = (P^b - c)\frac{v + r - P^b}{v + r + w} E[B]$$

As before, let $\eta = \frac{v+r-c}{v+r+w}$ and $\delta = \frac{(v+r-c)^2}{v+r+w}$.

Lemma 2.5.1.1. *When the retailer offers only bulk, the optimal bulk unit price is* $\frac{v+r+c}{2}$, *optimal profit is* $\frac{\delta}{4}E[B]$. *Also, total absolute waste at consumer level is* $\frac{\eta^2}{8}E[B]$ *and the total relative waste is* $\frac{\eta}{4}$.

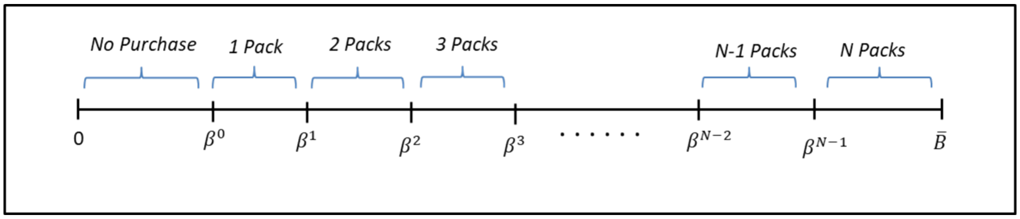


Figure 2.12: Full heterogeneity case purchase structure for only package sale

Notice that the optimal price is the same as in the homogeneous case and heterogeneous case with two consumer segments, when the retailer offers only bulk.

2.5.2 Offering Only the Package Sale Option

Next we consider the case when the retailer offers only the package sale option. We first show that the number of packages purchased given a a price P^p and a package size s is a piecewise constant increasing function.

Theorem 2.5.2.1. *Given a package unit price P^p and package size s let $\beta^n(P^p, s) = \frac{1}{2}\frac{v+r+w}{v+r-P^p}(2n+1)s$ and let $\beta^{-1} = 0$. Customers with $B \in (\beta^{n-1}, \beta^n]$ buy n packages for $n = 0, 1, ...$*

The maximum number of packages bought by consumers for a given package unit price P^p and package size s is the smallest value of n such that $\beta^n > \overline{B}$, which is equal to $\bar{n} \equiv \left\lceil \overline{B}\frac{v+r-P^p}{s(v+r+w)} \right\rfloor$. Hence, we can write the retailer's profit function as:

$$
\begin{aligned}
\pi(P^p, s) &= (P^p - c)\int_0^{\overline{B}} \hat{\imath}(s, P^p, P^b; B)\ s\ g(B)dB \\
&= (P^p - c)\sum_{n=1}^{\left\lceil \overline{B}\frac{v+r-P^p}{s(v+r+w)} \right\rfloor} ns\left[G(\beta^n(P^p, s)) - G(\beta^{n-1}(P^p, s))\right] \\
&= (P^p - c)\sum_{n=1}^{\left\lceil \overline{B}\frac{v+r-P^p}{s(v+r+w)} \right\rfloor - 1} ns\left[G(\beta^n(P^p, s)) - G(\beta^{n-1}(P^p, s))\right] \\
&\quad + \left\lceil \overline{B}\frac{v+r+w}{s(v+r-P^p)} \right\rfloor s\left[1 - G(\beta^{\left\lceil \overline{B}\frac{v+r-P^p}{s(v+r+w)} \right\rfloor - 1}(P^p, s)\right]
\end{aligned}
$$

Lemma 2.5.2.2. *If G is uniformly distributed on $[0, \bar{B}]$, the optimal package unit price is $P^{p*} = \frac{v+r+c}{2}$ and the optimal package size is any value $s^* = \eta\frac{\bar{B}}{2N}$ for $N \in \{1, 2, ...\}$. The optimal profit does not depend on N and is equal to $\pi^* = \delta\frac{\bar{B}}{8}$. Total absolute waste at consumer level depends on N and is equal to*

$$W^{abs} \quad = \quad \eta^2\frac{\bar{B}}{8}\left[\sum_{n=1}^{N-1}\frac{n^2}{N^2}\ln\left(\frac{2n+1}{2n-1}\right) + \ln\left(\frac{2N}{2N-1}\right)\right] \tag{2.8}$$

and the total relative waste is

$$W^{rel} \quad = \quad \frac{\eta}{2}\left[\sum_{n=1}^{N-1}\frac{n^2}{N^2}\ln\left(\frac{2n+1}{2n-1}\right) + \ln\left(\frac{2N}{2N-1}\right)\right] \tag{2.9}$$

We see that the package unit price and the retailer's profit is the same as the bulk unit price when it is the only available option. Also note that there is an infinite number of optimal package sizes. The largest possible packages size is $\eta\frac{\bar{B}}{2}$ (when $N = 1$); in this case half the consumer population buys one package and the other half buys nothing. As N gets larger, the packages size gets smaller which reduces consumer waste without affecting the retailer's profit. The term in brackets in the absolute waste expression in (2.8) and (2.9) tends to 1/2 as N tends to infinity, so that, for infinitesimally small package size, the absolute waste becomes equal to $\eta^2\frac{\bar{B}}{16}$ and the relative waste becomes equal to $\frac{\eta}{4}$, which are the corresponding values under bulk sale option only. In contrast, the absolute (relative) waste is maximized when $N = 1$, in which case it is equal to $2\ln(2) \equiv 138\%$ of the absolute (relative) waste under the bulk sale option only.

2.5.3 Offering Both Bulk and Package

When bulk and package are offered at the same time, depending on the prices and package size, some consumers may prefer to buy package, some may prefer to buy in bulk and some may not buy the product at all. As long as $P^b < v + r$ consumers prefer to by some amount of product to buying nothing and for $P^p \geq P^b$ consumers prefer to buy in bulk. So, for

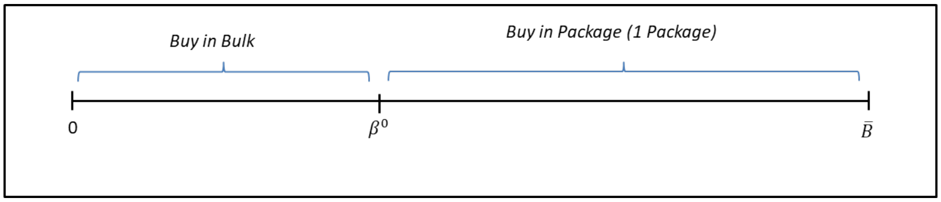


Figure 2.13: A feasible purchase structure for bulk and package coexist case (Full Heterogeneity)

$P^p \leq P^b < v+r$ Figure-2.12 will change such that there will be regions with bulk purchase and package purchase with different number of packages. But it is difficult to show how the structure will be (Intuitively we expect no-purchase region to be replaced by bulk purchase). For this reason, we try to find a lower bound for the optimal profit and the corresponding optimal decisions.

In order to find a lower bound we assume a purchase structure illustrated in Figure-2.13 where β^0 is the B value where consumer's utility of buying in bulk is equal to utility of buying 1 package. We assume that consumers can buy only one package.

Under this assumption, for a given P^b, P^p and s the aggregate sales from bulk sales is $\int_0^{\beta^0} B\frac{v+r-P^b}{v+r+w}\frac{1}{\bar{B}}dB$ and the aggregate sales from package sales is $\int_{\beta^0}^{\bar{B}} s\frac{1}{\bar{B}}dB$. Then the retailers optimization problem is:

$$\max_{P^b,P^p,s} \pi = (P^b - c)\int_0^{\beta^0} B\frac{v+r-P^b}{v+r+w}\frac{1}{\bar{B}}dB + (P^p - c)\int_{\beta^0}^{\bar{B}} s\frac{1}{\bar{B}}dB \tag{2.10}$$

If we can show that the structure we assumed is a feasible case of the original problem then, the solution of (2.10) is a lower bound for the original problem.

Lemma 2.5.3.1. *When bulk and package coexist, limiting the maximum number of packages that consumers can buy to one gives a lower bound for optimal profit such that* $\pi^* \geq \frac{4\bar{B}(v+r-c)^2}{27(v+r+w)}$.

We show in the proof of Lemma-2.5.3.1 that the purchase structure we assumed holds for the optimal values we get from (2.10).

2.5.4 Best Strategy for the Retailer

When we compare the three results we get for only bulk sale, only package sale and co-existence cases we get the following theorem:

Theorem 2.5.4.1. *Selling package and bulk at the same time with* $P^{b*} = \frac{2v+2r+c}{3}$, $P^{p^*} = \frac{4v+4r+5c}{9}$ *and* $s^* = \frac{2\bar{B}(v+r-c)}{3(r+v+w)}$ *yields at least 18% more profit than selling only bulk or selling only package.*

As a result, when the market is fully heterogeneous, It is always better to offer bulk and package together in terms of profit.

2.6 Extensions

2.6.1 Different Cost for Bulk and Package Sale

In the previous analyses, we assumed that unit cost of a product is the same, independent of being sold in a package or in bulk. In this section we investigate how the results change if the cost differs.

The total packaging cost of a given product can simply be divided into two; packaging material cost and packaging equipment cost. In our setting, at the retailer level the packaging is a simple process and usually does not require packaging equipment. But of course there is some labor cost associated with it. Since bulk sale also incurs some labor cost and compared with the other activities, labor spend on packaging or bulk sale arrangement is very low, we can ignore the labor cost and assume that the cost difference between package sale and bulk sale comes from the packaging material cost. Let c^b and c^p be bulk and package unit cost respectively. Based on the discussion above we assume $c^p = c^b+$ per unit packaging material cost. The unit cost of packaging material depends on the amount of product in a package. As package size gets larger, unit packaging cost decreases. It could be ignored when the

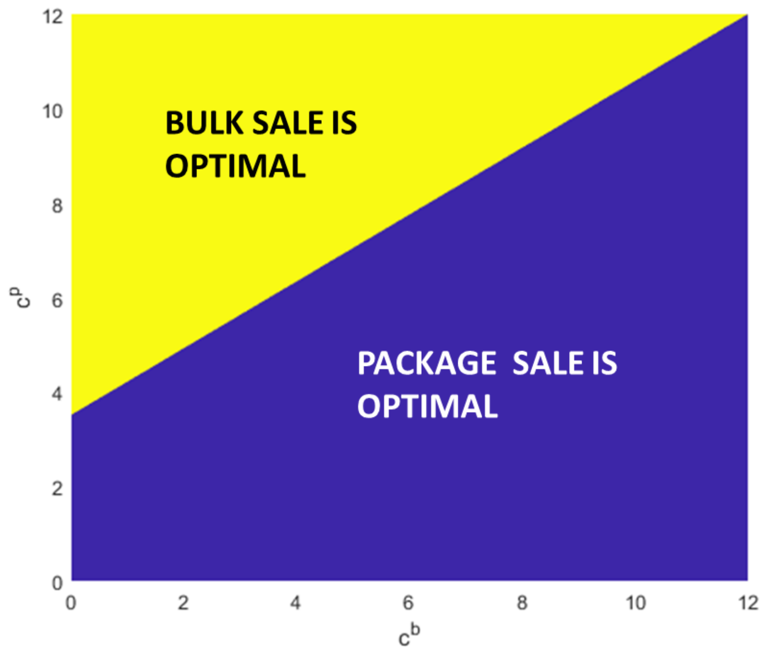


Figure 2.14: Homogeneous market-change of optimal strategy by c^p and c^b ($v = 10$ and $r = 2$

package size is very big.

Homogeneous market:

As discussed earlier, when the market is homogeneous, it is never optimal to sell bulk and package at the same time. Recall that when $c^b = c^p$ retailer earns twice more by selling the product in packages. If $c^b > c^p$ we still have the package sale optimal but retailer earns more than two times compared with bulk sale. However, if $c^b \leq c^p$, it is possible to have bulk sale to be optimal. Namely, package sale is optimal if $c^p < \frac{(\sqrt{2}-1)(v+r)}{\sqrt{2}} + \frac{c^b}{\sqrt{2}}$. Figure-2.14 presents the change of optimal strategy by c^b and c^p for $v = 10$ and $r = 2$. As v or r increases, the area of package sale gets larger. That is because, as v or r increases, the consumer wants to buy more product and the retailer exploits this by selling the product in package.

Heterogeneous Market with 2 consumers:

All the results for the only bulk sale and only package sale when the market is heterogeneous with two consumer segments still holds with only change of c to eitherc^b or c^p. For the

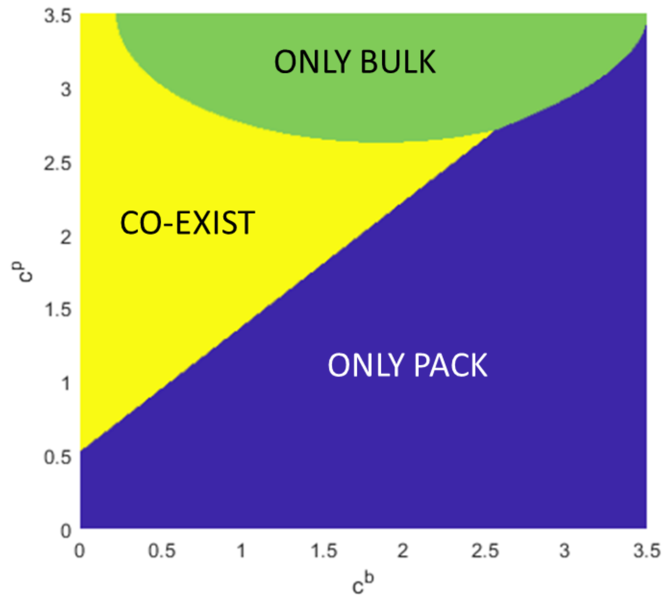


Figure 2.15: Heterogeneous market-change of optimal strategy by c^p and c^b ($v = 10$ and $r = 2$)

package sale, we still have 3 optimal package size candidates as stated in Theorem-2.4.2.1 .On the other hand, when package and bulk sale co-exist, it is still optimal to adjust p^b, p^p and s such that the consumer segment with low expected consumption buys in bulk. The optimal values becomes messier due to not having some cancellations but we still have closed form solutions. The most drastic effect of different cost assumption is on the best strategy of the retailer. Theorem-2.4.3.1 doesn't hold anymore because now bulk sale also can be the optimal strategy. We present an example in Figure-2.15 where we can observe three different optimal strategies based on c^p and c^b. (The parameters are $v = 10, r = 5$, $w = 2$,B1=1 and B2=10)

2.6.2 Normally Distributed Consumer Consumption

One of the most important assumptions in our study is assuming a uniform distribution for consumer consumption in a period. Now we change this assumption and assume that consumer consumption is normally distributed. Namely, we assume that $D_k \sim NORMAL(\mu_k, \sigma_k)$.

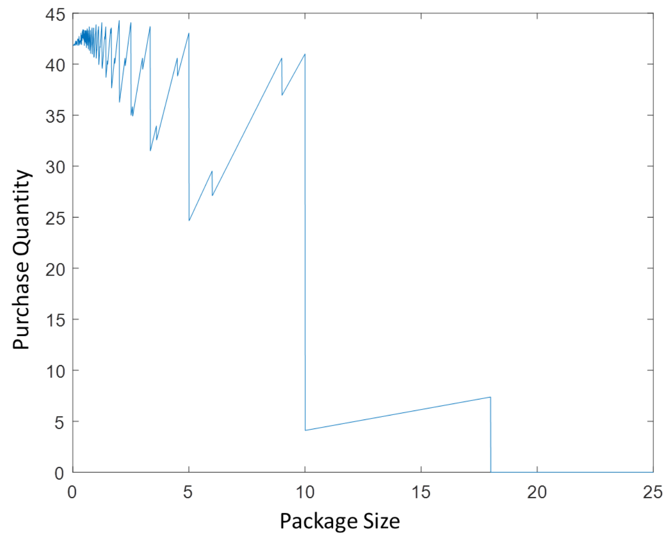


Figure 2.16: Change of purchase quantity by s when $D_k \sim NORMAL(\mu_k, \sigma_k)$

This assumption changes consumer utility function and unfortunately most of our results do not hold in this case. Finding a closed form solution even for the simplest case of homogeneous market bulk sale setting becomes very tedious. Moreover, Theorem-2.4.2.1, which enables us to find closed form solutions for the only package sale strategy doesn't hold any more. As shown Figure-2.16 for a given price, the optimal package size could be any γ_1^n such that $0 < \gamma_k^n \leq \gamma_k^0$ which makes the problem analytically intractable. In Figure we assumed $v = 7, c = 1, r = 0.5, w = 0.3, \mu_k = 10$ and $\sigma_1 = 1$.

2.6.3 Consumers with Different v

Another way to differentiate consumers in our model is assuming that their product valuation v is different. We investigated how does our results change when we allow each consumer to have different v, say v_k. In order to have consumer segments that can be ordered by their purchase quantity (for a given price and package size), we assume that consumers have same consumption rate, that is, $B_k = B$ for all k. Hence, with the uniform distribution assumption, the ideal purchase quantity of a consumer segment with higher v_k will be higher.

Under these assumptions, since consumer choice explained in §2.2.1 remains the same, the optimal results for the homogeneous market also remains the same. For the heterogeneous market with two consumer segments, when there is only bulk sale, the optimal bulk price P^{*b} changes depending on $c/(2v_1 + r)$ where $v_1 \leq v_2$. If $c/(2v_1 + r) \leq 1$ the optimal price is such that both consumer segments buy the product. Otherwise, the optimal price is such that Segment-1 consumers buy nothing. On the other hand, when there is only package sale, Theorem-2.4.2.1 still holds. We easily get the closed form solutions for the optimal package size and price when $\theta_2 \leq 1/2$ but otherwise it becomes tedious due to the calculation of γ_k^n values.

2.7 Conclusion

Package size decision problem has not taken enough interest from the operations management researchers. There are limited numbers of studies focusing on the possible improvements that can be achieved by adjusting package size according to the market variables. Besides, motivated by the environmental concerns, an old sale strategy of selling the product without packages is getting more attention nowadays. Called as bulk sale or package free sale, this old way of retailing the product for sure has the potential to save from packaging materials but it has one other potential benefit; it can reduce food waste, which is one of the most important problems of modern life. Bulk sale allows the consumers to better adjust their purchase amount with their expected consumption amount and thus reduces over purchases due to the package size limitation. With this study we fill the gap in the literature by proposing an appropriate model that takes into account the uncertainty of consumption at the consumer level to find the optimal package size and price decision of a retailer selling a perishable product. Moreover, we investigated the effect of introducing bulk sale to the optimal pricing decisions and profits. We also showed how food waste at the consumer level changes based on the retailers optimal pricing, package size and bulk sale decisions.

We found that, when the market is homogeneous, the retailer adjusts the package size and price such that consumers are indifferent between buying nothing and buying the whole package. Selling product in package is an advantage for the retailer because; he can push consumers to buy more by limiting their purchase option to a certain amount. When we compare the optimal results of bulk sale with package sale, we see that retailer makes two times more profit by using this advantage. But making consumers buy more product, result in more waste; expected relative waste for the package sale case is twice more than bulk sale case.

On the other hand, when the market is heterogeneous with two consumer segments, depending on the ratio of the segment in the market and their average consumption amount, it could be optimal to sell the product both in package and in bulk. If it is, the package size and pricing dictions are designed such that the consumer segment with lower average consumption buys in bulk and the other segment buys in package. According to our numerical analysis, when product is sold in both bulk and package retailer can earn up to 12.5% more profit. On the other hand, relative waste can be reduced by 8%. But on average, coexisting of bulk and package increases profits by 1.66% and increase relative waste around 0.26%. When we extent the problem to infinite number of consumers, which is called full heterogeneity, we see that offering bulk and package sale at the same time is always better than offering only package sale with at least 18% more profit.

As a result, we can conclude that introducing bulk sale is not always beneficial to reduce the food waste at the consumer level but depending on the market structure, it could be a good strategy to create a win-win situation by making the retailers earn more and reducing the waste. In our study, we do not consider the advantage of bulk sale in reduction of package material. Taking into account that effect may result in a better solution with costs savings from packaging and reduction of pollution by less packaging related contamination.

2.8 Proof of Results

2.8.1 Proof of Lemma 2.2.1.2

We have $\hat{i}_k(s, P^p) = \arg\max_{i \in \mathbb{N}} u_k^p(i, s, P^p) = \max\left\{u_k^p\left(i_k^L, s, P^p\right), u_k^p\left(i_k^R, s, P^p\right)\right\}$ where $i_k^L = \left\lfloor \frac{Q_k^{*p}}{s} \right\rfloor$ and $i_k^R = \left\lceil \frac{Q_k^{*p}}{s} \right\rceil$.

Suppose that for a given s, we have $\hat{i}_k(s, P^p) = i_k^L$, which implies that $u_k(i_k^L, s, P^p) > u_k(i_k^R, s, P^p)$. As s increases, $u_k(i_k^L, s, P^p)$ increases and $u(i_k^R, s, P^p)$ decreases so there is no impact on $\hat{i}_k(s, P^p)$. Eventually s reaches a value such that $i_k^L = i_k^R = Q_k^{*p}$. After this value, $\hat{i}_k(s, P^p)$ still does not change but it is now equal to i_k^R. As s further increases, $u_k(i_k^L, s, P^p)$ increases and $u_k(i_k^R, s, P^p)$ decreases but there is still no impact on $\hat{i}_k(s, P^p)$, until the value of s such that $u_k(i_k^R, s, P^p) = u_k(i_k^R, s, P^p)$. At that value, the optimal number of packages drops by one and becomes equal to i_k^L again.

The above argument also shows that $\hat{i}_k(s, P^p)$ only changes (drops by one) at values of s such that $u_k^p(i_k^L, s, P^p) = u_k^p(i_k^R, s, P^p)$. If we let $n = i_k^L$,, this condition becomes $u_k^p(n, s, P^p) = u_k^p(n+1, s, P^p)$, which is the definition of the threshold γ_k^n. As a result $\hat{i}_k(s, P^p)$ is constant, specifically $\hat{i}_k(s, P^p) = n$, for $s \in [\gamma_k^n, \gamma_k^{n-1})$.

The total purchase quantity $\hat{i}_k(s, P^p)s$ is therefore equal to ns for $s \in [\gamma_k^n, \gamma_k^{n-1})$, so that it is a piecewise increasing function of s.

Finally, we prove that $\hat{i}_k(\gamma_k^n, P^p)\gamma_k^n$ is decreasing in n by showing that $\hat{i}_k(\gamma_k^{n+1}, P^p)\gamma_k^{n+1} - \hat{i}_k(\gamma_k^n, P^p)\gamma_k^n \leq 0$. Recall that we have $\hat{i}_k(\gamma_k^n, P^p) = n+1$ at γ_k^n and $\gamma_k^n = \frac{2B_k\theta_k}{1+2n}$.

$$
\begin{aligned}
\hat{i}_k(\gamma_k^{n+1}, P^p)\gamma_k^{n+1} - \hat{i}_k(\gamma_k^n, P^p)\gamma_k^n &= (n+2)\frac{2B_k\theta_k}{1+2(n+1)} - (n+1)\frac{2B_k\theta_k}{1+2n} \\
&= 2B_k\theta_k\left(\frac{-1}{4n^2+8n+3}\right) \\
&\leq 0 \quad \forall n \in \mathbb{N}
\end{aligned}
$$

2.8.2 Proof of Lemma 2.2.1.3

We show that $u_k^p\left(\left\lfloor\frac{Q_k^{*p}}{s}\right\rfloor, s, P^p\right) \geq u_k^p\left(\left\lceil\frac{Q_k^{*p}}{s}\right\rceil, s, P^p\right)$ if and only if $\left\lfloor\frac{Q_k^{*p}}{s}\right\rceil \leq \frac{Q_k^{*p}}{s}$. In what follows, let $n = \left\lceil\frac{Q_k^{*p}}{s}\right\rceil$.

$$
\begin{aligned}
v\frac{B_k}{2} - \frac{(v+r)(B_k-(n-1)s)^2 + w((n-1)s)^2}{2B_k} - P^p(n-1)s &\geq v\frac{B_k}{2} - \frac{(v+r)(B_k-ns)^2 + w(ns)^2}{2B_k} - P^p ns \\
\frac{(v+r)(B_k-(n-1)s)^2 + w((n-1)s)^2}{2B_k} + P^p(n-1)s &\leq \frac{(v+r)(B_k-ns)^2 + w(ns)^2}{2B_k} + P^p ns \\
(v+r)(B_k-(n-1)s)^2 + w((n-1)s)^2 &\leq (v+r)(B_k-ns)^2 + w(ns)^2 + 2P^p sB_k \\
\frac{(v+r-P^p)}{s(v+r+w)}2B_k &\leq (2n-1) \\
\frac{Q_k^{*p}}{s} + \frac{1}{2} &\leq n
\end{aligned}
$$

which is equivalent to $\left\lfloor\frac{Q_k^{*p}}{s}\right\rceil \leq \frac{Q_k^{*p}}{s}$.

2.8.3 Proof of Lemma 2.2.1.4

When $P^b > P^p$, we have $u_k^b(Q_k^{*b}) = U_k(Q_k^{*b}) - P^b Q_k^{*b} \leq U_k(Q_k^{*b}) - P^p Q_k^{*b} \leq U_k(si_k^L) - P^p si_k^L = u_k^p(si_k^L)$ since by definition $si_k^L = s\left\lfloor\frac{Q_k^{*p}}{s}\right\rfloor \leq Q_k^{*p}$ and Q_k^{*p} is the maximum of the concave function $U_k(Q) - P^p Q$. Hence the bulk sales yields a lower expected net utility than consuming i_k^L packages. Customers of segment k will therefore choose to purchase either i_k^L or i_k^R packages.

2.8.4 Proof of Lemma 2.3.1.1

From §2.2.1, if the retailer offers the product only in bulk, the purchase quantity of a consumer is $B\left(\frac{v+r-P^b}{v+r+w}\right)$ so that the optimization problem is:

$$\max_{P^b} \quad \pi(P^b) = (P^b - c)B\left(\frac{v+r-P^b}{v+r+w}\right) \tag{2.11}$$

which is maximized at $P^b = \frac{v+r+c}{2}$. The other values are obtained directly from substitution in the appropriate equations.

2.8.5 Proof of Lemma 2.3.2.1

Since there is only one customer segment we drop the subscript references k, e.g. we write only $\hat{i}(s, P^p)$ in what follows. We first prove that there exists an optimal package size for a fixed price P^p such that the customers buy only one package. Suppose not (contraction), that is, the optimal package size s^* is such that $\hat{\imath}_k(s^*, P^p) > 1$. Now consider alternative package size $\bar{s} = s^*\hat{\imath}_k(s^*, P^p)$. First we show that $\hat{\imath}(\bar{s}, P^p) = 1$. Based on the analysis in §2.2.1, there are two cases: (i) $\hat{\imath}(s^*, P^p) = \left\lfloor \frac{Q^{*p}}{s^*} \right\rfloor$ or (ii) $\hat{\imath}(s^*, P^p) = \left\lceil \frac{Q^{*p}}{s^*} \right\rceil$. In case (i), we have $U(\bar{s}) = U\left(s^*\hat{\imath}(s^*, P^p)\right) = U\left(s^* \left\lfloor \frac{Q^{*p}}{s^*} \right\rfloor\right) \geq U\left(s^* \left\lceil \frac{Q^{*p}}{s^*} \right\rceil\right) > U\left(2\bar{s}\right) > U(3\bar{s}) > ...$, where the second inequality is because $2\bar{s} = 2s^*\hat{\imath}(s^*, P^p) = 2s^* \left\lfloor \frac{Q^{*p}}{s^*} \right\rfloor > s^* \left\lceil \frac{Q^{*p}}{s^*} \right\rceil = s^* \left[\left\lfloor \frac{Q^{*p}}{s^*} \right\rfloor + 1\right] > Q^{*p}$ and $U(Q)$ is decreasing for $Q > Q^{*p}$. In case (ii), we have $U(\bar{s}) = U\left(s^*\hat{\imath}(s^*, P^p)\right) = U\left(s^* \left\lceil \frac{Q^{*p}}{s^*} \right\rceil\right) > U\left(2\bar{s}\right) > U(3\bar{s}) > ...$, where the inequality is because $2\bar{s} > \bar{s} = s^* \left\lceil \frac{Q^{*p}}{s^*} \right\rceil > Q^{*p}$ and $U(Q)$ is decreasing for $Q > Q^{*p}$. Therefore, in both cases (i) and (ii), $\max_{i \in \mathbb{N}} u^p(i, \bar{s}, P^p) = U(i\bar{s}) - P^p i\bar{s} = U(\bar{s}) - P^p\bar{s} = u^p(1, \bar{s}, P^p)$ which implies that $\hat{i}(\bar{s}, P^p) = 1$. Further we have $\pi(\bar{s}, P^p) = (P^p - c)\hat{i}(\bar{s}, P^p)\bar{s} = (P^p - c)\bar{s} = (P^p - c)\hat{i}(s^*, P^p)s^* = \pi(s^*, P^p)$, which proves that package size $\bar{s}$ is also optimal, hence we have a contradiction.

It follows that we can focus on the package size values such that customers buy only one package, that is, the retailer solves $\max_s \pi(S, P^p) = \max_{s:\hat{\imath}(s,P^p)=1} \pi(s, P^p) = (P^p - c)s$. This function is maximized when s is set such as the customers are just indifferent between buying one package and buying nothing, that is, s^* must be such that $u^p(\hat{\imath}(s, P^p)) = u^p(0) = -\frac{B}{2}r$. Solving this equation yields:

$$s^*(P^p) = \begin{cases} \frac{B_L}{2}\left(\frac{v+r+w}{P^p+w}\right) & for\ P^p \leq \frac{v+r-w}{2} \\ \\ 2B_L\left(\frac{v+r-P^p}{v+r+w}\right) & for\ P^p \geq \frac{v+r-w}{2} \end{cases}$$

2.8.6 Proof of Lemma 2.3.2.2

Given Lemma 2.3.2.1,

$$\max_{s} \quad \pi(P^p) = \begin{cases} (P^p - c)\frac{B}{2}\frac{v+r+w}{P^p+w} & \text{if } P^p \leq \frac{v+r-w}{2} \\ (P^p - c)2B\frac{v+r-P^p}{v+r+w} & \text{if } P^p > \frac{v+r-w}{2} \end{cases} \tag{2.12}$$

The first part of this expression is increasing in P^p and the second part is concave in P^p and maximized at $\frac{v+r+c}{2}$. Since $\frac{v+r+c}{2} > \frac{v+r-w}{2}$, it is optimal to set P^p equal to $\frac{v+r+c}{2}$. The other values are obtained directly from substitution in the appropriate equations.

2.8.7 Proof of Lemma 2.4.1.1

From §2.2.1, if the retailer offers the product only in bulk, the purchase quantity of a consumer from segment k is $B_k\left(\frac{v+r-P^b}{v+r+w}\right)$ so that the optimization problem is:

$$\max_{P^b} \quad \pi(P^b) = (P^b - c)\sum_{k=1}^{K} \alpha_k B_k \frac{v + r - P^b}{v + r + w}$$

which is maximized at $P^b = \frac{v+r+c}{2}$. The other values are obtained directly from substitution in the appropriate equations.

2.8.8 Proof of Theorem 2.4.2.1

Before proving Theorem 2.4.2.1, we prove four lemmas.

Lemma 2.8.8.1

Lemma 2.8.8.1. $\forall\ m \in \mathbb{N}^+$, $n \in \mathbb{N}^+$ *and* $n \geq m \geq 2$, $\exists\ i \in \mathbb{N}^+$ *satisfying:*

$$\frac{n}{2m-1} \leq i \leq \frac{n}{m} \tag{2.13}$$

Proof. It is clear that for $n \leq 2m - 1$ we have at least one $i \in \mathbb{N}^+$ such as $i = 1$. For $n > 2m - 1$ we define $\varepsilon = n - m$. Then we have $\varepsilon > 2m - 1 - m \Rightarrow \varepsilon > m - 1 \Rightarrow \varepsilon \geq m$.

Note that $\frac{n}{m} - \frac{n}{2m-1} \geq 1$ is sufficient to have an integer number satisfying (2.13). If we plug in $n = m + \varepsilon$ in this expression, then we get $\varepsilon \geq \frac{m^2}{m-1}$ as a sufficient condition. Besides, we have$\frac{m^2}{m-1} = \frac{m^2}{m-1}\frac{m+2}{m+2} = \frac{m^2}{m^2+m-2}(m+2)$ and for $m \geq 2$ since $\frac{m^2}{m^2+m-2} \leq 1$ we get $\frac{m^2}{m-1} \leq m+2$. Hence, $\varepsilon \geq (m+2)$ is sufficient to satisfy (2.13). Since $\varepsilon \geq m$ and for $\varepsilon \geq m+2$ we can find such an integer, it is enough to check for $\varepsilon = m$ and $\varepsilon = m+1$. For $\varepsilon = m$ we have, $\frac{m+\varepsilon}{2m-1} \leq i \leq \frac{m+\varepsilon}{m} \Rightarrow \frac{2m}{2m-1} \leq i \leq \frac{2m}{m} \Rightarrow \frac{2m}{2m-1} \leq i \leq 2$ and $i = 2$ is a solution for this inequality. For $\varepsilon = m+1$ we have $\frac{m+\varepsilon}{2m-1} \leq i \leq \frac{m+\varepsilon}{m} \Rightarrow \frac{2m+1}{2m-1} \leq i \leq \frac{2m+1}{m} \Rightarrow \frac{2m+1}{2m-1} \leq i \leq 2+\frac{1}{m}$ where $i = 2$ is also a solution □

Lemma 2.8.8.2

Lemma 2.8.8.2. $\gamma_k^0 \geq (2i+1)\gamma_k^i$ *for all* $\theta_k \in [0,1]$.

Proof. γ_k^i has different expressions depending on θ_k. We need to consider three cases.

Case-1 $\theta_k < \frac{1}{2}$: We have $\gamma_k^i = \frac{2B_k\theta_k}{1+2i}$ $\forall i \geq 0$. It is clear that $\gamma_k^0 = (1+2i)\gamma_k^i$

Case-2 $\frac{1}{2} \leq \theta_k \leq \frac{2i+1}{2i+2}$**:** We have $\gamma_k^0 = \frac{B_k}{2(1-\theta_k)}$ and $\gamma_k^i = \frac{2B_k\theta_k}{1+2i}$ $\forall i \geq 1$. After a simple algebra we get $\gamma_k^0 = \frac{2B_k\theta_k}{1+2i}\frac{1+2i}{4(1-\theta_k)\theta_k} = \gamma_k^i\frac{1+2i}{4(1-\theta_k)\theta_k}$. For $\theta_k \geq \frac{1}{2}$ we have $4(1-\theta_k)\theta_k \leq 1$ which yields $(1+2i)\gamma_k^i \leq \gamma_k^i\frac{1+2i}{4(1-\theta_k)\theta_k} = \gamma_k^0$ $\forall i \geq 1$.

Case-3 $\frac{2i+1}{2i+2} \leq \theta_k \leq$ **1:** $\gamma_k^0 = \frac{B_k}{2\overline{\theta_k}}$ and $\gamma_k^i = \frac{B_k}{i^2}(i+\overline{\theta_k}-\sqrt{\overline{\theta_k}^2 + 2i\overline{\theta_k}})$ $\forall i \geq 1$. From $i \geq 1$ we get $\theta_k > \frac{2i+1}{2i+2} \geq \frac{3}{4} \Rightarrow \overline{\theta_k} \leq \frac{1}{4}$. We try to show that $\frac{1}{2\overline{\theta_k}} \geq (2i+1)\frac{1}{i^2}(i+\overline{\theta_k} - \sqrt{\overline{\theta_k}^2 + 2i\overline{\theta_k}})$.The LHS is increasing in $\overline{\theta_k}$ so we have $\frac{1}{2\overline{\theta_k}} \geq \frac{1}{2(1/4)} = 2$.On the other hand, the derivative of RHS with respect to $\overline{\theta_k}$ is $(2i+1)\frac{1}{i^2}1-(i+\overline{\theta_k})/\sqrt{\overline{\theta_k}^2 + 2i\overline{\theta_k}}$ and it less than zero for all $\overline{\theta_k} \leq \frac{1}{4}$,so RHS is decreasing in $\overline{\theta_k}$. This yields

$$(2i+1)\frac{1}{i^2}(i+\overline{\theta_k} - \sqrt{\overline{\theta_k}^2 + 2i\overline{\theta_k}}) \leq (2i+1)\frac{1}{i^2}(i+\frac{1}{4} - \sqrt{\frac{1}{4}^2 + 2i\frac{1}{4}}) = \frac{(2i+1)(1+4i-\sqrt{1+8i})}{4i^2}$$

As a result we get $LHS - RHS \geq 2 - \frac{(2i+1)(1+4i-\sqrt{1+8i})}{4i^2} = \frac{2i\sqrt{1+8i}+\sqrt{1+8i}-(6i+1)}{4i^2} \geq 0 \;\; for\ \forall i \geq 1$ $\square$

2.8.9 Lemma 2.8.9.1

Lemma 2.8.9.1. *For $\gamma_1^1 < s \leq \bar{\gamma}_2$ if the package price P^p is fixed then $arg\ \underset{s}{max}\ \ \pi(s, P^p) = \bar{\gamma}_2$.*

Proof. Note that $\pi(s, P^p) = \sum_{k=1}^{2} \alpha_k (P^p - c)\hat{\imath}_k(s, P^p)s$ and maximizing profit means maximizing total weighted sales. For $\gamma_1^1 < s \leq \bar{\gamma}_2$, since Type-1 customers buy only one package it is clear that $\bar{\gamma}_2$ maximizes Type-1's purchase amount. On the other hand for Type-2, since purchase amount is maximized at the points where $\hat{\imath}_2(s, P^p)$ change, we need to consider $s = \gamma_2^n$ s.t. $\gamma_1^1 < \gamma_2^n \leq \bar{\gamma}_2$. For $\theta_2 \leq \frac{2n+1}{2n+2}$ Type-2 buys $(n+1)\frac{2B_H\theta_2}{1+2n}$ amounts of product when $s = \gamma_2^n$. We have $(n+1)\gamma_2^n - (n+2)\gamma_2^{n+1} = \frac{2B_H\theta_2}{3+8n+4n^2} > 0$, which shows as n increases the amount of product the customer buys decreases. Since $\bar{\gamma}_2$ is the jump point with the lowest n value, purchase amount of Type-2 is also maximum at $\bar{\gamma}_2$. Same logic also applies for $\theta_2 > \frac{2n+1}{2n+2}$. $\square$

2.8.10 Lemma 2.8.10.1

Lemma 2.8.10.1. *For a given package price P^p, customer with lower B_k (Type-1 in our setting) buys only one package or none at optimal solution.*

Proof. Suppose Type-1 buys n_1 packages and Type-2 buys n_2 packages at an optimal solution where $n_1 \geq 2$ and $n_2 \geq n_1$. And suppose optimal package size is s^*. We will show that one can always find a better solution $\bar{s}$ where Type-1 buys one package and Type-2 buys $\bar{n}_2$ packages such that $\frac{n_2}{(2n_1-1)} \leq \bar{n}_2 \leq \frac{n_2}{n_1}$. When Type-1 buys n_1 packages and Type-2 buys n_2 packages, weighted total sales with package size s^* is equal to $\alpha n_1 s^* + (1-\alpha)n_2 s^*$. We know that Type-1 buys the product as long as package size is less than γ_1^0. Thus, Type-1 will still buy the product if we set package size as $\bar{s}$ which satisfies:

$$n_1 s^* \leq \bar{s} \leq \gamma_1^0 \tag{2.14}$$

For such an $\bar{s}$, Type-1 will buy at least as much as he buys when package size is s^* but, we don't know how Type-2's decision will change. He may buy more or less depending on utility he gets. But if we find an $\bar{n}_2$ value which satisfies (2.14) and $\bar{n}_2\bar{s} = n_2 s^*$ then we can guarantee that Type-2 will buy same amount of product when the package size is $\bar{s}$. So we need to find if such an $\bar{n}_2$ exists. From $\bar{n}_2\bar{s} = n_2 s^*$ we have $\bar{n}_2 = \frac{n_2 s^*}{\bar{s}}$ and by using (2.14) we can write

$$\begin{aligned} n_1 s^* \leq \bar{s} \leq \gamma_1^0 &\Rightarrow \frac{n_2 s^*}{\gamma_1^0} \leq \frac{n_2 s^*}{\bar{s}} \leq \frac{n_2 s^*}{n_1 s^*} \\ &\Rightarrow \frac{n_2 s^*}{\gamma_1^0} \leq \bar{n}_2 \leq \frac{n_2}{n_1} \end{aligned} \tag{2.15}$$

Now we are looking for an integer value $\bar{n}_2$ which satisfies (2.15). At the beginning we assumed Type-1 buys n_1 packages with package size s^* which implies $s^* \leq \gamma_1^{(n_1-1)}$ and from Lemma 2.8.9.1 we have $\gamma_k^0 \geq (2i+1)\gamma_k^i$. By combining these two we get:

$$\begin{aligned} s^* \leq \gamma_1^{(n_1-1)} &\Rightarrow (2n_1-1)s^* \leq (2n_1-1)\gamma_1^{(n_1-1)} \\ &\Rightarrow (2n_1-1)s^* \leq \gamma_1^0 \end{aligned} \tag{2.16}$$

This allows us to write (2.14) as $n_1 s^* \leq \bar{s} \leq (2n_1-1)s^*$ which this reduces (2.15) to

$$\frac{n_2}{(2n_1-1)} \leq \bar{n}_2 \leq \frac{n_2}{n_1} \tag{2.17}$$

As shown in Lemma 2.8.9.1 there always exists $\bar{n}_2$ which satisfies (2.17). In fact, $\bar{n}_2 = \hat{\imath}_2(\bar{s})$ because:

$$\begin{aligned} U_k(\bar{n}_2\bar{s}) = U_k(n_2 s^*) > U_k((n_2+1)s^*) = U_k(n_2 s^* + s^*) = U_k(\bar{n}_2\bar{s} + s^*) \\ \Rightarrow U_k(\bar{n}_2\bar{s}) > U_k(\bar{n}_2\bar{s} + s^*) \end{aligned} \tag{2.18}$$

We know that $U_k(Q)$ is concave and $\bar{n}_2\bar{s} < \bar{n}_2\bar{s} + s^* < \bar{n}_2\bar{s} + \bar{s}$. This with (2.18) implies:

$$U_k(\bar{n}_2\bar{s}) > U_k(\bar{n}_2\bar{s} + \bar{s}) = U_k((\bar{n}_2 + 1)\bar{s}) \tag{2.19}$$

With the same logic it can also be shown that $U_k(\bar{n}_2\bar{s}) > U_k((\bar{n}_2 - 1)\bar{s})$. $\square$

2.8.11 Back to the Proof of Theorem 2.4.2.1

First we establish that the optimal package size for a given price P^p must be one of the threshold values γ_k^n for $k = 1, 2$ and $n = 0, 1, \ldots$. If it was not the case, a marginal increase in the package size would lead to customers from both segments buying more total quantity since $\hat{i}_k(s, P^p)s$ is strictly increasing in s between two γ thresholds. As a result the retailer's profit, which is equal to $\pi(s, P^p) = (P^p - c)\alpha\hat{i}_1(s, P^P) + (1 - \alpha)\hat{i}_2(s, P^P)$, would increase. Hence the optimal package size must be equal to one of the γ threshold values.

Next we show that $\bar{n}$ is such that $\gamma_2^{\bar{n}} \leq \gamma_1^0 \leq \gamma_2^{\bar{n}-1}$. This condition is equivalent to:

$$\begin{aligned}
& \frac{2B_H\theta^p}{1 + 2\bar{n}} \leq 2B_L\theta^p \leq \frac{2B_H\theta^p}{2\bar{n} - 1} \\
\Leftrightarrow \quad & \frac{B_H}{2B_L} - \frac{1}{2} \leq \bar{n} \leq \frac{B_H}{2B_L} + \frac{1}{2} \\
\Leftrightarrow \quad & \bar{n} = \left\lfloor \frac{B_H}{2B_L} \right\rceil
\end{aligned}$$

Depending on the parameters, it might be optimal for the retailer to sell only customer Type-2 (since his expected demand is higher he buys more than Type-1). In this case retailer sets package size as γ_2^0 because it gives the highest weighted sales. Notice that Type-1 will not buy any product when $s = \gamma_2^0$.On the other hand, it might be optimal to sell both customer types. In this case from Lemma 2.8.10.1 we know that at optimal solution Type-1 customer will buy one package. So optimal package size is greater than γ_1^1. Besides, from Lemma 2.8.9.1 we know that $\bar{\gamma}_2$ dominates all jump points between γ_1^1 and $\bar{\gamma}_2$. This leaves us two jump points as a candidate of optimal solution $\bar{\gamma}_2$ and γ_1^0.

2.8.12 Proof of Theorem 2.4.2.2

We evaluate the expected profit of the retailer for each of the optimal package size candidates points γ_1^0, γ_2^0 and $\gamma_2^{\bar{n}}$ given in Theorem 2.4.2.1.

When the package size is γ_2^0, Segment 2 customers buy one package and Segment 1 customers buy nothing. Hence the retailer's profit as a function of P^p is given by:

$$\pi(\gamma_2^0, P^p) \quad = \quad (P^p - c)(1-\alpha)2B_H \left(\frac{v+r-P^p}{v+r+w}\right)$$

When the package size is γ_1^0, Segment 1 customers buy one package and Segment 2 customers buy $\bar{n} = \left\lfloor \frac{B_H}{2B_L} \right\rfloor$ as defined in Lemma X. Hence the retailer's profit as a function of P^p is given by:

$$\pi(\gamma_1^0, P^p) = (P^p - c)2B_L \left(\frac{v+r-P^p}{v+r+w}\right) \left(\alpha + (1-\alpha)\left\lfloor \frac{B_H}{2B_L} \right\rfloor\right). \tag{2.20}$$

When package size is $\gamma_2^{\bar{n}}$ with $\bar{n} = \left\lfloor \frac{B_H}{2B_L} \right\rfloor$, Segment 1 customers buy only one package and Segment 2 customers buy $\bar{n}+1$ packages.

Hence, the retailer's profit as a function of P^p is given by:

$$\pi(\gamma_2^{\bar{n}}, P^p) \quad = \quad (P^p - c)2B_H \left(\frac{v+r-P^p}{v+r+w}\right) \frac{\left(\alpha + (1-\alpha)\left(\left\lfloor \frac{B_H}{2B_L} \right\rfloor + 1\right)\right)}{2\left\lfloor \frac{B_H}{2B_L} \right\rfloor + 1}.$$

First note that all three profit expressions are maximized at $P^p = \frac{v+r+c}{2}$.

The corresponding packages sizes are:

$$\begin{aligned}
\gamma_1^0 &= \frac{v+r-c}{v+r+w} B_L \\
\gamma_2^0 &= \frac{v+r-c}{v+r+w} B_H \\
\gamma_2^{\bar{n}} &= \frac{v+r-c}{v+r+w} \frac{B_H}{2\left\lfloor \frac{B_H}{2B_L} \right\rfloor + 1}
\end{aligned}$$

And the profit values simplify to

$$\pi(\gamma_2^0, P^{p*}) = \frac{(v+r-c)^2}{2(v+r+w)} B_H (1-\alpha), \tag{2.21}$$

$$\pi(\gamma_1^0, P^{p*}) = \frac{(v+r-c)^2}{2(v+r+w)} B_L \left(\alpha + (1-\alpha)\left\lfloor \frac{B_H}{2B_L} \right\rceil\right), \tag{2.22}$$

$$\pi(\gamma_2^{\bar{n}}, P^{p*}) = \frac{(v+r-c)^2}{2(v+r+w)} B_H \frac{\left(\alpha + (1-\alpha)\left(\left\lfloor \frac{B_H}{2B_L} \right\rceil + 1\right)\right)}{2\left\lfloor \frac{B_H}{2B_L} \right\rceil + 1}.$$

We have that $\pi(\gamma_2^0, P^{p*}) \geq \pi(\gamma_1^0, P^{p*})$ if and only if $\alpha \leq \frac{B_H - \left\lfloor \frac{B_H}{2B_L} \right\rceil B_L}{B_L + B_H - \left\lfloor \frac{B_H}{2B_L} \right\rceil B_L} = \tilde{\alpha}_1$. Similarly, we have $\pi(\gamma_2^0, P^{p*}) \geq \pi(\gamma_2^{\bar{n}}, P^{p*})$ if and only if $\alpha \leq \frac{\left\lfloor \frac{B_H}{2B_L} \right\rceil}{1 + \left\lfloor \frac{B_H}{2B_L} \right\rceil}$. Finally, $\pi(\gamma_2^{\bar{n}}, P^{p*}) \geq \pi(\gamma_1^0, P^{p*})$ if and only if $\alpha \leq \frac{B_H + \left\lfloor \frac{B_H}{2B_L} \right\rceil \left(B_H - B_L - 2B_L \left\lfloor \frac{B_H}{2B_L} \right\rceil\right)}{B_L + \left\lfloor \frac{B_H}{2B_L} \right\rceil \left(B_H + B_L - 2B_L \left\lfloor \frac{B_H}{2B_L} \right\rceil\right)}$.

If $\left\lfloor \frac{B_H}{2B_L} \right\rceil < \frac{B_H}{2B_L}$ then $\frac{\left\lfloor \frac{B_H}{2B_L} \right\rceil}{1 + \left\lfloor \frac{B_H}{2B_L} \right\rceil} \leq \frac{B_H + \left\lfloor \frac{B_H}{2B_L} \right\rceil \left(B_H - B_L - 2B_L \left\lfloor \frac{B_H}{2B_L} \right\rceil\right)}{B_L + \left\lfloor \frac{B_H}{2B_L} \right\rceil \left(B_H + B_L - 2B_L \left\lfloor \frac{B_H}{2B_L} \right\rceil\right)}$ so that it is possible for the highest value of profit to be obtained at $\pi(\gamma_2^{\bar{n}}, P^{p*})$.

2.8.13 Proof of Theorem 2.4.3.1

To prove this result we take the following five steps (i) we obtain the optimal prices and packages size when the retailer offers bulk to Segment 1 customers and packages to Segment 2 customers, (ii) we show that it is never optimal for the retailer to offer only bulk, (iii) we obtain an upper bound on the profit the retailer can obtain when she offers bulk to Segment 2 customers and packages to Segment 1 customers, (iv) we show that it is never optimal for the retailer to offer bulk to Segment 2 customers and packages to segment 1 customers and (v) we provide necessary and sufficient conditions under which each possible solution is optimal and show that it is never optimal for the retailer to offer only packages which are tailored to Segment 2 customers.

For use in the analysis below, we define $V_k^b(P^b)$ as the utility received by k-segment customers when buying the product in bulk in the quantity which maximizes their utility, that is, $V_k^b(P^b) = U_k(Q_k^{*b}) - P^b Q_k^{*b}$ where $Q_k^{*b} = B_k \frac{v+r-P^b}{v+r+w}$, for $P^b \leq v + r$. Because only one segment of customers buy the product in packages, the packages size can be designed such that they buy only one package (i.e., if the optimal package size was such that the segment which buys them buys $n > 1$ packages of size s, then the retailer could change the packages to ns and earn the same profit as it would not impact the decision of the other customer segment). So let $V_k^p(s, P^p)$ denote the utility received by k-segment customers when buying one package of the product, that is, $V_k^p(s, P^p) = U_k(s) - P^p s$. Finally, let V_k^0 denote the utility segment-k customers get from buying nothing, that is $V_k^0 = -\frac{B_k}{2} r$.

Step (i)

We analyze the case where P^b, P^p and s are set such that Segment 1 customers, buy the product in bulk and Segment 2 customers buy the product in packages. The problem can be formulated as a mechanism design problem as follows:

$$\begin{aligned}
\max_{c \leq P^p \leq P^b, s} \pi(P^b, P^p, s) &= \alpha(P^b - c) B_L \left(\frac{v + r - P^b}{v + r + w} \right) + (1 - \alpha)(P^p - c)s \qquad (2.23) \\
\text{s.t.} \qquad & V_1^b(P^b) \geq V_1^p(s, P^p) \qquad (IC_1) \\
& V_2^p(s, P^p) \geq V_2^b(P^b) \qquad (IC_2) \\
& V_1^b(P_b) \geq V_1^0 \qquad (IR_1) \\
& V_2^p(s, P^p) \geq V_2^0 \qquad (IR_2)
\end{aligned}$$

In this formulation the first two constraints correspond to the incentive compatibility constraints and the last two are the individual rationality constraints. The (IR_1) constraint in (2.23) is equivalent to $P^b \geq v + r$ which is also equivalent to $V_2^b(P^b) \geq V_2^0$, which makes (IR_2) redundant given (IC_2). We argue that (IC_2) must be tight at optimality. Suppose it is not and we increase P^{p*} (keeping s^* same) until (IC_2) becomes binding. This increases the retailer's expected profit. At the same time (IC_1) continues to be satisfied because $V_1^p(s, P^p)$ decreases with P^p. The same is true for (IR_1) since it does not depend on P^p. Therefore by increasing P^p until (IC_2) is binding, the retailer can increase expecting profits without violating constraints, which is a contradiction. Next we use the fact that (IC_2) is binding to express P^p as a function of P^b and s and obtain:

$$P^p(P^b, s) \quad = \quad v + r - \frac{B_H(v + r - P^b)^2}{2s(v + r + w)} - \frac{s(v + r + w)}{2B_H} \tag{2.24}$$

Plugging this expression into the objective function of 2.23, and solving for P^b and s, we obtain:

$$\begin{aligned} P^{b*} &= \frac{(1-\alpha)B_H(v+r) + \alpha B_L(v+r+c)}{(1-\alpha)B_H + 2\alpha B_L} \\ s^* &= B_H\eta \end{aligned}$$

Plugging these values into (2.24), we get:

$$P^{p*} \quad = \quad \frac{v+r+c}{2} - \frac{\alpha^2 B1^2(v+r-c)}{2(B_H(1-\alpha)+2\alpha B1)^2}$$

Since these values satisfy constraints (IC_1) and (IR_1), they must be solution for the constrained problem. By substituting these values into the profit expression we obtain a profit value equal to $\frac{\delta}{2}\frac{(\alpha B_L+(1-\alpha)B_H)^2}{2\alpha B_L+(1-\alpha)B_H}$

Step (ii)

We show that it is never optimal for the retailer to offer only bulk by showing that the profit of offering bulk only, denoted $\pi^*_{ONLYBULK}$, is dominated by the profit obtained when the retailer offers bulk for Segment 1 customers and packages to Segment 2 customers, denoted $\pi^*_{BULK\&PACK}$ (as computed in Step (i)). We have:

$$\begin{aligned} \pi^*_{BULK\&PACK} - \pi^*_{ONLYBULK} &= \frac{\delta}{2}\frac{(\alpha B_L + (1-\alpha)B_H)^2}{2\alpha B_L + (1-\alpha)B_H} - \frac{\delta}{2}\frac{(\alpha B_L + (1-\alpha)B_H)}{2} \\ &= \frac{\delta}{2}\frac{B_H(1-\alpha)(B_H(1-\alpha)+B_L\alpha))}{2(B_H(1-\alpha)+2B_L\alpha)} \geq 0 \end{aligned}$$

Step (iii)

We analyze the case where the retailer sets P^b, P^p and s such that Segment 1 consumers buys packages and Segment 2 consumers buy bulk. The problem can be formulated as:

$$\max_{P^b,P^p,s} \quad \pi(P^b, P^p, s) = \alpha(P^p - c)s + (1-\alpha)(P^b - c)B_H\left(\tfrac{v+r-P^b}{v+r+w}\right)$$

$$s.t.$$

$$\begin{aligned} V_1^p(s, P^p) &\geq V_1^b\ (P^b) && (IC_1) \\ V_1^p(s, P^p) &\geq\ V_1^0 && (IR_1) \\ V_2^b(P^b) &\geq\ V_2^p\ (ns, P^p) \quad \forall n \in \mathbb{N}^+ && (IC_2) \\ V_2^b(P^b) &\geq\ V_2^0 && (IR_2) \end{aligned}$$

Instead of solving this problem, we find an upper bound on the optimal profit by relaxing IR_1, IC_2, and IR_2. The relaxed problem is:

$$\max_{P^b,P^p,s} \quad \pi(P^b, P^p, s) = \alpha(P^p - c)s + (1-\alpha)(P^b - c)B_H\left(\tfrac{v+r-P^b}{v+r+w}\right)$$
$$s.t. \qquad V_1^p(s, P^p) \geq V_1^b\ (P^b) \qquad (IC_1) \tag{2.25}$$

We observe that the constraint must be binding in the optimal solution. This is because otherwise one could increase P^{p*} without violating constraints and get more profit. Hence, we can write P^p as a function of P^b and s as:

$$P^p(P^b, s) = v + r - \frac{B_L(v + r - P^b)^2}{2s(v + r + w)} - \frac{s(v + r + w)}{2B_L} \tag{2.26}$$

Note that, because we have assumed that $c \geq \frac{v+r-w}{2}$, it is enough to consider the case of $s \leq B_L$. Plugging this into the profit function and solving for P^b and s we obtain $P^{b*} =$

$\frac{B_H(c+r+v)(1-\alpha)+B_L(r+v)\alpha}{2B_H(1-\alpha)+B_L\alpha}$ and $s = B_L\frac{(v+r-c)}{(r+v+w)}$ and the optimal profit as: $\pi_R^* = \frac{\delta}{2}\frac{(B_H(1-\alpha)+B_L\alpha)^2}{(2B_H(1-\alpha)+B_L\alpha)}$, which the solution to the relaxed problem and therefore an upper bound on the profit the retailer can obtain when offering bulk to Segment 2 customers and packages to Segment 1 customers.

Step (iv)

Next we show that this upper bound is dominated by the profit obtained from selling only packages or by the profit obtained when offering bulk to Segment 1 customers and packages to Segment 2 customers. More precisely, we show that for $B_H \geq B_L\frac{\alpha}{(1-\alpha)}$ we have $\pi^*_{BULK\&PACK} \geq \pi_R^*$ and for $B_H \leq B_L\frac{\alpha}{(1-\alpha)}$ we have $\pi^*_{ONLYPACK} \geq \pi_R^*$ (The package size is assumed to be γ_1^0 for only package sale case). As a reminder we have:

$$\pi^*_{BULK\&PACK} = \frac{\delta}{2}\frac{(B_H(1-\alpha)+B_L\alpha)^2}{B_H(1-\alpha)+2B_L\alpha}$$

$$\pi^*_{ONLYPACK} = \frac{\delta}{2}B_L(n(1-\alpha)+\alpha)$$

where $n = \left\lfloor \frac{B_H}{2B_L} \right\rfloor$

(i) $B_H \geq B_L\frac{\alpha}{(1-\alpha)}$: We consider $\pi^*_{BULK\&PACK} - \pi_R^*$:

$$\pi^*_{BULK\&PACK} - \pi_R^* = (B_H(1-\alpha) - B_L\alpha)\frac{(v+r-c)^2}{2(r+v+w)}\frac{(B_H(1-\alpha)+B_L\alpha)^2}{(2B_H(1-\alpha)+B_L\alpha)B_H(1-\alpha)+2B_L\alpha}$$
$$\geq 0 \quad \text{for} \quad B_H(1-\alpha) - B_L\alpha \geq 0 \quad \text{ or } \quad B_H \geq B_L\frac{\alpha}{(1-\alpha)}$$

(ii) $B_H \leq B_L\frac{\alpha}{(1-\alpha)}$: We consider $\pi^*_{ONLYPACK} - \pi_R^*$:

$$\pi^*_{ONLYPACK} - \pi_R^* = \left(B_L^2 n\alpha + 2B_LB_Hn(1-\alpha) - B_H^2(1-\alpha)\right)\frac{(v+r-c)^2}{2(r+v+w)}\frac{(1-\alpha)}{2B_H(1-\alpha)+B_L\alpha}$$

We want to show that $B_L^2 n\alpha + 2B_L B_H n(1-\alpha) - B_H^2(1-\alpha) \geq 0$ for $B_H \leq B_L \frac{\alpha}{(1-\alpha)}$. We consider $\alpha \geq \frac{3}{4}$ and $\alpha \geq \frac{3}{4}$ cases separately.

For $\alpha \geq \frac{3}{4}$: Notice that $n = \left\lfloor \frac{B_H}{2B_L} \right\rceil \geq \frac{B_H}{2B_L} - \frac{1}{2}$ and the function is increasing in n. So we have:

$$
\begin{aligned}
B_L^2 n\alpha + 2B_L B_H n(1-\alpha) - B_H^2(1-\alpha) &\geq B_L^2 \left(\frac{B_H}{2B_L} - \frac{1}{2}\right)\alpha + 2B_L B_H \left(\frac{B_H}{2B_L} - \frac{1}{2}\right)(1-\alpha) - B_H^2(1-\alpha) \\
&= \frac{B_L}{2}(B_H(3\alpha-2) - B_L\alpha) \\
&\geq \frac{B_L}{2}(B_H(3\alpha-2) - B_H\frac{(1-\alpha)}{\alpha}\alpha) \\
&= \frac{B_L}{2}B_H(4\alpha-3) \\
&\geq 0 \text{ for } \alpha \geq \frac{3}{4}
\end{aligned}
$$

For $\alpha \leq \frac{3}{4}$: Notice that $\frac{\alpha}{(1-\alpha)} \leq 3$ and this implies $B_H \leq B_L \frac{\alpha}{(1-\alpha)} \leq 3B_L$.We have $n = \left\lfloor \frac{B_H}{2B_L} \right\rceil = 1$ for $B_L < B_H \leq 3B_L$. For $n = 1$ we get:

$$
\begin{aligned}
B_L^2 n\alpha + 2B_L B_H n(1-\alpha) - B_H^2(1-\alpha) &= B_L^2\alpha + 2B_L B_H(1-\alpha) - B_H^2(1-\alpha) \\
&\geq \left(B_H\frac{(1-\alpha)}{\alpha}\right)^2\alpha + 2\left(B_H\frac{(1-\alpha)}{\alpha}\right)B_H(1-\alpha) - B_H^2(1-\alpha) \\
&= \frac{B_H^2}{\alpha}(3 - 7\alpha + 4\alpha^2) \\
&\geq 0 \text{ for } \alpha \leq \frac{3}{4}
\end{aligned}
$$

Step (iv)

From the above analysis there are four possible solutions to consider for the retailer: (a) offer only packages and **tailor to Segment 1** customers, earning a profit of $\frac{\delta}{2}B_L(n(1-\alpha)+\alpha)$; (b) offer only packages and **tailor to Segment 2** customers, earning a profit of $\frac{\delta}{2}(1-\alpha)B_H$; (c) offer only packages and **compromise**, earning a profit of $\frac{\delta}{2}\frac{B_H(n(1-\alpha)+1)}{(1+2n)}$ and (d) offer both **bulk and packages** with bulk aimed at Segment 1 customers and packages at Segment 2 customers, earning a profit of $\frac{\delta}{2}\frac{(\alpha B_L+(1-\alpha)B_H)^2}{2\alpha B_L+(1-\alpha)B_H}$. By comparing the profit expressions in those four cases we obtain the following full characterization of the optimal solution for the retailer who is able to offer both bulk and packages, which is given in Table below.

Table 2.2 provides the value of the retailer's profit, consumers' purchase quantity, utility and waste for each possible solution.

Table 2.4: Retailer's profit and consumers' purchase quantity, utility and waste when the retailer can offer bulk and packages.

Solution	Tailor to Segment 1	Compromise	Bulk&Pack
Conditions	$K \geq 1$ or $M \geq 1$ $\alpha > \frac{B_H+n(B_H-B_L-2B_Ln)}{B_L+n(B_H+B_L-2B_Ln)}$	$K \geq 1$ or $M \geq 1$ $\alpha \leq \frac{B_H+n(B_H-B_L-2B_Ln)}{B_L+n(B_H+B_L-2B_Ln)}$	$K > 1$ or $M > 1$
Bulk unit price	NA	NA	$\frac{B_H(v+r)(1-\alpha)+\alpha B_L(v+r+c)}{B_H(1-\alpha)+2\alpha B_L}$
Package unit price	$\frac{v+r+c}{2}$	$\frac{v+r+c}{2}$	$\frac{v+r+c}{2} - \frac{\alpha^2 B_L^2(v+r-c)}{(B_H(1-\alpha)+2\alpha B_L)^2}$
Package size	ηB_L	$\eta \frac{B_H}{1+2n}$	ηB_H
Retailer's profit	$\frac{\delta}{2}(n(1-\alpha)+\alpha)B_L$	$\frac{\delta}{2}\frac{n(1-\alpha)+1}{1+2n}B_H$	$\frac{\delta}{2}\frac{(\alpha B_L+(1-\alpha)B_H)^2}{2\alpha B_L+(1-\alpha)B_H}$
Purchase quantity			
Segment 1	ηB_L (one pack)	$\eta\frac{1}{1+2n}B_H$ (one pack)	$\eta\frac{B_L^2\alpha}{(B_H(1-\alpha)+2B_L\alpha)^2}$ (in bulk)
Segment 2	$n\eta B_L$ (n packs)	$\eta\frac{n+1}{1+2n}B_H$ ($n+1$ packs)	ηB_H (one pack)
Consumer utility			
Segment 1	$-\frac{r}{2}B_L$	$\delta\frac{B_H(B_L(1+2n)-B_H)}{2B_L(1+2n)^2} - \frac{r}{2}B_L$	$\frac{B_L\left((B_L\alpha)^2(\delta-4r)-rB_H(1-\alpha)(B_H(1-\alpha)+4B_L\alpha)\right)}{2(B_H(1-\alpha)+2B_L)^2}$
Segment 2	$\frac{\delta n}{2}\left(1-\frac{nB_L}{B_H}\right)B_L - \frac{r}{2}B_H$	$\delta\frac{B_H}{2}\frac{n(1+n)}{(1+2n)^2} - \frac{r}{2}B_H$	$\frac{B_H}{2}\left(\frac{\delta(B_L\alpha)^2}{(B_H(1-\alpha)+2B_L\alpha)^2} - r\right)$
Absolute waste			
Segment 1	$\frac{\eta^2}{2}B_L$	$\frac{\eta^2}{2}\frac{(B_H)^2}{B_L(1+2n)^2}$	$\frac{\eta^2}{2}\frac{B_L^3\alpha^2}{(B_H(1-\alpha)+2B_L\alpha)^2}$
Segment 2	$\frac{n^2\eta^2}{2}\frac{B_L^2}{B_H}$	$\frac{\eta^2}{2}\frac{B_H(1+n)^2}{(1+2n)^2}$	$\frac{\eta^2}{2}B_H$
Relative waste			
Segment 1	$\frac{\eta}{2}$	$\frac{\eta}{2}\frac{B_H}{B_L(1+2n)}$	$\frac{\eta}{2}\frac{B_L\alpha}{(B_H(1-\alpha)+2B_L\alpha)}$
Segment 2	$\frac{n\eta}{2}\frac{B_L}{B_H}$	$\frac{\eta}{2}\frac{(1+n)}{(1+2n)}$	$\frac{\eta}{2}$

where $n = \left\lfloor \frac{B_H}{2B_L} \right\rceil$, $\eta = \frac{v+r-c}{v+r+w}$, $\delta = \frac{(v+r-c)^2}{v+r+w}$, $K = B_L(n(1-\alpha)+\alpha)\frac{(2B_L\alpha+B_H(1-\alpha))}{(B_L\alpha+B_H(1-\alpha))^2}$ and $M = B_H\frac{n(1-\alpha)+1}{1+2n}\frac{(2B_L\alpha+B_H(1-\alpha))}{(B_L\alpha+B_H(1-\alpha))^2}$.

Note that it is never optimal to offer only packages tailored to Segment 2 customers when the retailer is able to offer bulk. This is because $\frac{\delta}{2}(1-\alpha)B_H \leq \frac{\delta}{2}\frac{(\alpha B_L+(1-\alpha)B_H)^2}{2\alpha B_L+(1-\alpha)B_H}$ for all values of the parameters. In other words, the retailer can always increase profits by offering bulk to Segment 1 customers.

2.8.14 Proof of Theorem 2.4.3.2

In order to get the bound, we investigate profit increase where selling bulk and pack at the same time becomes optimal. The percent increase in profit is calculated by $\left(\frac{\pi_{BP}-\pi_P}{\pi_P}\right)\times 100$. It is enough to find a bound on $\frac{\pi_P}{\pi_B}$. We have:

$$\frac{\pi_{BP}}{\pi_P} = \begin{cases} \frac{(\alpha B_L+(1-\alpha)B_H)^2}{(n(1-\alpha)+\alpha)B_L(2\alpha B_L+(1-\alpha)B_H)} & ,if \;\; \text{Tailor To Segment-1} \\ \frac{(\alpha B_L+(1-\alpha)B_H)^2}{(1-\alpha)B_H(2\alpha B_L+(1-\alpha)B_H)} & ,if \;\; \text{Tailor To Segment-2} \\ \frac{(\alpha B_L+(1-\alpha)B_H)^2(1+2n)}{B_H(n(1-\alpha)+1)(2\alpha B_L+(1-\alpha)B_H)} & ,if \;\; \text{Compromise} \end{cases}$$

We consider 3 cases separately.

CASE-1: Tailor to Segment-1: We want to find a bound on:

$$\frac{(\alpha B_L + (1-\alpha)B_H)^2}{(n(1-\alpha)+\alpha)B_L\left(2\alpha B_L + (1-\alpha)B_H\right)} \tag{2.27}$$

under the conditions stated in Table-2.4 and Theorem-2.4.2.2. For the first condition of $n < \frac{B_H}{2B_L}$ and $\alpha \geq \frac{B_H+n(B_H-B_L-2B_Ln)}{B_L+n(B_H+B_L-2B_Ln)}$ we have the first derivative of (2.27) with respect to α is equal to $\frac{(B_H+B_L\alpha-B_H\alpha)(B_H^2(\alpha-1)-3B_LB_H\alpha+2B_L^2n\alpha)}{B_L(B_H+2B_L\alpha-B_H\alpha)^2(n+\alpha-n\alpha)^2}$ and it is negative for $n \leq \frac{B_H}{2B_L}$ which means (2.27) is decreasing in α for $n \leq \frac{B_H}{2B_L}$. We have $\alpha \geq \frac{B_H+n(B_H-B_L-2B_Ln)}{B_L+n(B_H+B_L-2B_Ln)}$ so we plug in $\alpha = \frac{B_H+n(B_H-B_L-2B_Ln)}{B_L+n(B_H+B_L-2B_Ln)}$ (2.27) and get $\frac{(B_H^2+B_L^2(1+2n)-B_LB_H(2+3n))^2}{B_LB_H\left(B_LB_H(3+4n)-B_H^2-2B_L^2n(1+2n)\right)}$ which is also increasing in n. We plug in $n = \frac{B_H}{2B_L}$ and finally get $\frac{(\alpha B_L+(1-\alpha)B_H)^2}{(n(1-\alpha)+\alpha)B_L(2\alpha B_L+(1-\alpha)B_H)} \leq \frac{9}{8}$.By similar analysis we get the same upper bound for the second condition of $n > \frac{B_H}{2B_L}$ and $\alpha \geq \frac{B_H-nB_L}{B_L+B_H-nB_L}$

CASE-2: Tailor to Segment-2: We want to find a bound on:

$$\frac{(\alpha B_L + (1-\alpha)B_H)^2}{(1-\alpha)B_H\left(2\alpha B_L + (1-\alpha)B_H\right)} \tag{2.28}$$

under the conditions stated in Table-2.4 and Theorem-2.4.2.2. For the first condition of $n < \frac{B_H}{2B_L}$ and $\alpha \leq \frac{n}{1+n}$ we have the first derivative of (2.28) with respect to α is equal to $\frac{2B_L^2\alpha(B_H+B_L\alpha-B_H\alpha)}{B_H(-1+\alpha)^2(B_H(-1+\alpha)-2B_L\alpha)^2}$ and it is positive which means (2.28) is increasing in α. We have $\alpha \leq \frac{n}{1+n}$ so we plug in $\alpha = \frac{n}{1+n}$ (2.28) and get $\frac{(B_H+B_Ln)^2}{B_H(B_H+2B_Ln)}$ which is also increasing in n. We plug in $n = \frac{B_H}{2B_L}$ and finally get $\frac{(\alpha B_L+(1-\alpha)B_H)^2}{(1-\alpha)B_H(2\alpha B_L+(1-\alpha)B_H)} \leq \frac{9}{8}$. By similar analysis we get the same upper bound for the second condition of $n > \frac{B_H}{2B_L}$ and $\alpha \leq \frac{B_H-nB_L}{B_L+B_H-nB_L}$.

CASE-3: Compromise: We want to find a bound on:

$$\frac{(\alpha B_L + (1-\alpha)B_H)^2(1+2n)}{B_H\left(n(1-\alpha)+1\right)\left(2\alpha B_L + (1-\alpha)B_H\right)} \tag{2.29}$$

under the conditions $n < \frac{B_H}{2B_L}$ and $\frac{n}{1+n} \leq \alpha \leq \frac{B_H+n(B_H-B_L-2B_L n)}{B_L+n(B_H+B_L-2B_L n)}$.(2.29) is decreasing in α. We have $\alpha \geq \frac{n}{1+n}$ so we plug in $\alpha = \frac{n}{1+n}$ into (2.29) and get $\frac{(B_H+B_L n)^2}{B_H(B_H+2B_L n)}$ which is increasing in n. We plug in $n = \frac{B_H}{2B_L}$ and finally get $\frac{(\alpha B_L+(1-\alpha)B_H)^2(1+2n)}{B_H(n(1-\alpha)+1)(2\alpha B_L+(1-\alpha)B_H)} \leq \frac{9}{8}$.

2.8.15 Proof of Theorem 2.4.3.3

In order to get bounds, we first investigate profit increase where only package sale is optimal. Then, we investigate the profit difference where selling bulk and package at the same time is optimal. **Comparison of Only Bulk Sale and Only Pack Sale:** The percent increase in profit is calculated by $\left(\frac{\pi_P-\pi_B}{\pi_B}\right) \times 100$. It is enough to find a bound on $\frac{\pi_P}{\pi_B}$. We have

$$\frac{\pi_P}{\pi_B} = \begin{cases} \frac{2B_L(n(1-\alpha)+\alpha)}{\alpha B_L+(1-\alpha)B_H} & , if \text{ Tailor To Segment-1} \\ \frac{2B_H(1-\alpha)}{\alpha B_L+(1-\alpha)B_H} & , if \text{ Tailor To Segment-2} \\ \frac{2B_H(n(1-\alpha)+1)}{(\alpha B_L+(1-\alpha)B_H)(1+2n)} & , if \text{ Compromise} \end{cases}$$

We consider 3 cases separately.

CASE-1: Tailor to Segment-1: We want to find a bound on:

$$\frac{2B_L\left(n(1-\alpha)+\alpha\right)}{\alpha B_L+(1-\alpha)B_H} \tag{2.30}$$

under the conditions stated in Table-2.4 and Theorem-2.4.2.2. For the first condition of $n < \frac{B_H}{2B_L}$ and $\alpha \geq \frac{B_H+n(B_H-B_L-2B_L n)}{B_L+n(B_H+B_L-2B_L n)}$ (2.30) is decreasing in α. We have $\frac{B_H+n(B_H-B_L-2B_L n)}{B_L+n(B_H+B_L-2B_L n)} \leq \alpha \leq 1$ so we plug in $\alpha = 1$ into (2.30) and get $\frac{2B_L(n(1-\alpha)+\alpha)}{\alpha B_L+(1-\alpha)B_H} \leq 2$. By similar analysis we get the same upper bound for the second condition of $n > \frac{B_H}{2B_L}$ and $\alpha \geq \frac{B_H-nB_L}{B_L+B_H-nB_L}$

CASE-2: Tailor to Segment-2: We want to find a bound on:

$$\frac{2B_H(1-\alpha)}{\alpha B_L+(1-\alpha)B_H} \tag{2.31}$$

under the conditions stated in Table-2.4 and Theorem-2.4.2.2. For the first condition of $n < \frac{B_H}{2B_L}$ and $\alpha \leq \frac{n}{1+n}$ (2.31) is decreasing in α. We have $0 \leq \alpha \leq \frac{n}{1+n}$ so we plug in $\alpha = 0$

(2.31) and get $\frac{2B_H(1-\alpha)}{\alpha B_L+(1-\alpha)B_H} \leq 2$. By similar analysis we get the same upper bound for the second condition of $n > \frac{B_H}{2B_L}$ and $\alpha \leq \frac{B_H - nB_L}{B_L+B_H-nB_L}$.

CASE-3: Compromise: We want to find a bound on:

$$\frac{2B_H\left(n(1-\alpha)+1\right)}{\left(\alpha B_L+(1-\alpha)B_H\right)(1+2n)} \tag{2.32}$$

under the conditions $n < \frac{B_H}{2B_L}$ and $\frac{n}{1+n} \leq \alpha \leq \frac{B_H+n(B_H-B_L-2B_Ln)}{B_L+n(B_H+B_L-2B_Ln)}$.(2.32) is increasing in α. We plug in $\alpha = \frac{B_H+n(B_H-B_L-2B_Ln)}{B_L+n(B_H+B_L-2B_Ln)}$ in (2.32) $\Rightarrow \frac{2B_H(n(1-\alpha)+1)}{(\alpha B_L+(1-\alpha)B_H)(1+2n)} \leq 2$.

Comparison of Only Bulk Sale and Bulk-Pack Sale: We want to find a bound on $\frac{\pi_{BP}}{\pi_B} = \frac{2(\alpha B_L+(1-\alpha)B_H)}{(2\alpha B_L+(1-\alpha)B_H)}$ which is is decreasing in α. We plug in $\alpha = 0$ and get $\frac{2(\alpha B_L+(1-\alpha)B_H)}{(2\alpha B_L+(1-\alpha)B_H)} \leq 2$.

2.8.16 Proof of Theorem 2.4.4.1

We first consider the absolute waste and then the relative waste.

Absolute Waste: We first compare Only Bulk Sale and Bulk-Pack sale and show that $W_{BP}^{Abs} \geq W_B^{Abs}$.Then we compare Only Bulk Sale and Only Package sale to show that $W_P^{Abs} \geq W_B^{Abs}$.

Comparison of Only Bulk Sale and Bulk-Pack Sale: It is enough to show that $\frac{W_{BP}^{Abs}}{W_B^{Abs}} \geq 1$. We have $\frac{W_{BP}^{Abs}}{W_B^{Abs}} = \frac{4(B_H^2(1-\alpha)^2+3B_LB_H(1-\alpha)\alpha+B_L^2\alpha^2)}{(B_H(-1+\alpha)-2B_L\alpha)^2}$ and it is decreasing in α.We plug in $\alpha = 1$ and get $\frac{W_{BP}^{Abs}}{W_B^{Abs}} \geq 1$

Comparison of Only Bulk Sale and Only Pack Sale: It is enough to show that $\frac{W_P^{Abs}}{W_B^{Abs}} \geq 1$. We have

$$\frac{W_P^{Abs}}{W_B^{Abs}} = \begin{cases} \frac{4B_L(B_Ln^2(1-\alpha)+B_H\alpha)}{B_H(B_H(1-\alpha)+B_L\alpha)} & ,if \text{ Tailor To Segment-1} \\ \frac{4(B_H^2(1-\alpha)^2+3B_LB_H(1-\alpha)\alpha+B_L^2\alpha^2)}{(B_H(-1+\alpha)-2B_L\alpha)^2} & ,if \text{ Tailor To Segment-2} \\ \frac{4B_H(B_L(1+n)^2(1-\alpha)+B_H\alpha)}{B_L(1+2n)^2(B_H+B_L\alpha-B_H\alpha)} & ,if \text{ Compromise} \end{cases}$$

We consider 3 cases separately. For the first case of Tailor to Segment-1, we want to show that $\frac{4B_L(B_Ln^2(1-\alpha)+B_H\alpha)}{B_H(B_H(1-\alpha)+B_L\alpha)} \geq 1$ under the conditions stated in Theorem-2.4.2.2. For the first condition of $n < \frac{B_H}{2B_L}$ and $\alpha \geq \frac{B_H+n(B_H-B_L-2B_Ln)}{B_L+n(B_H+B_L-2B_Ln)}$ the expression is increasing in α. We plug in $\alpha = \frac{B_H+n(B_H-B_L-2B_Ln)}{B_L+n(B_H+B_L-2B_Ln)}$ and get another expression which is increasing in n. We plug in $n = \frac{B_H}{2B_L} - \frac{1}{2}$ and get $\frac{W_P^{Abs}}{W_B^{Abs}} \geq 1$.By similar analysis we get the same result for the second condition of $n > \frac{B_H}{2B_L}$ and $\alpha \geq \frac{B_H-nB_L}{B_L+B_H-nB_L}$. For the second case of tailor to Segment-2 we have Bulk-Pack sale optimal which we considered above. The third case of compromise is similar to the first case.

Relative Waste: We first compare Only Bulk Sale and Bulk-Pack sale and show that $W_{BP}^{\mathrm{Re}l} \geq W_B^{\mathrm{Re}l}$.Then we compare Only Bulk Sale and Only Package sale to show that $W_P^{\mathrm{Re}l} \geq W_B^{\mathrm{Re}l}$.

Comparison of Only Bulk Sale and Bulk-Pack Sale: It is enough to show that $\frac{W_{BP}^{\mathrm{Re}l}}{W_B^{\mathrm{Re}l}} \geq 1$. We have $\frac{W_{BP}^{\mathrm{Re}l}}{W_B^{\mathrm{Re}l}} = \frac{2(B_H^2(-1+\alpha)^2-3B_LB_H(-1+\alpha)\alpha+B_L^2\alpha^2)}{(B_H(-1+\alpha)-2B_L\alpha)(B_H(-1+\alpha)-B_L\alpha)}$. It is decreasing in α.We plug in $\alpha = \not 1$ and get $\frac{W_{BP}^{Abs}}{W_B^{Abs}} \geq 1$

Comparison of Only Bulk Sale and Only Pack Sale: We have the same cases as shown for the absolute waste case. For the first case of tailor to Segment-1, we want to show that $\frac{W_P^{Abs}}{W_B^{Abs}} = \frac{2(B_Ln^2(1-\alpha)+B_H\alpha)}{B_H(n(1-\alpha)+\alpha)} / \geq 1$ under the conditions stated in Theorem-2.4.2.2.For the first condition of $n < \frac{B_H}{2B_L}$ we have $\frac{W_P^{Abs}}{W_B^{Abs}}$ is increasing in α.We plug in $\alpha = 0$ and get an expression which is increasing in n. After plugging in $n = \frac{B_H}{2B_L} - \frac{1}{2}$ we get $\frac{W_P^{Abs}}{W_B^{Abs}} \geq 1$.By similar analysis we get the same result for the second condition of $n > \frac{B_H}{2B_L}$ and $\alpha \geq \frac{B_H-nB_L}{B_L+B_H-nB_L}$. For the second case of tailor to Segment-2, we have Bulk-Pack sale optimal which we considered above. The third case of compromise is similar to the first case.

2.8.17 Proof of Theorem 2.5.2.1

The threshold corresponds to values of B such that the customer is indifferent between buying $n-1$ and n packages, and can be obtained by solving $u^p(n-1, s, P^p; B) = u^p(n, s, P^P; B)$ for B.

2.8.18 Proof of Lemma 2.5.2.2

If G has a uniform distribution on $[0, \overline{B}]$ we have:

$$\begin{aligned}
\pi(P^p, s) &= \frac{1}{\overline{B}}(P^p - c) \sum_{n=1}^{\lceil B\frac{v+r-P^p}{s(v+r+w)}\rceil - 1} ns \left(\frac{1}{2}\frac{v+r+w}{v+r-P^p}(2n+1)s - \frac{1}{2}\frac{v+r+w}{v+r-P^p}(2n-1)s \right) \\
&\quad + (P^p - c)\left\lceil \overline{B}\frac{v+r-P^p}{s(v+r+w)} \right\rceil s \left[1 - \frac{1}{\overline{B}}\frac{1}{2}\frac{v+r+w}{v+r-P^p}\left(2\left\lceil \overline{B}\frac{v+r-P^p}{s(v+r+w)} \right\rceil - 1 \right) s \right] \\
&= (P^p - c)\left\lceil \overline{B}\frac{v+r-P^p}{s(v+r+w)} \right\rceil s - \frac{1}{2\overline{B}} s^2 \frac{v+r+w}{v+r-P^p}(P^p - c)\left\lceil \overline{B}\frac{v+r-P^p}{s(v+r+w)} \right\rceil^2
\end{aligned}$$

For a given P^p it is to set s so that $\left\lceil \overline{B}\frac{v+r-P^p}{s(v+r+w)} \right\rceil$ is a integer greater or equal to one. If it was not the case, one can increase s until this value becomes an integer and it will increase the retailer's profit. So let $N = \overline{B}\frac{v+r-P^p}{s(v+r+w)}$ so that $s = \overline{B}\frac{v+r-P^p}{N(v+r+w)}$ where $N =\in \{1, 2, ...\}$. From this, we get:

$$\pi(P^p, s) = (P^p - c)\frac{\overline{B}}{2}\frac{v+r-P^p}{v+r+w}$$

We see that the profit does not depend on s. The optimal package price is $P^p = \frac{v+r+c}{2}$. The optimal package size is then equal to $\eta\frac{\overline{B}}{2N}$ where $N \in \{1, 2, ...\}$.
Given the optimal price and packages size, we have: $\beta^n = (2n+1)\frac{\bar{B}}{2N}$. Note that $\beta^N > \overline{B}$. Using (**??**), we calculate absolute waste as:

$$\begin{aligned}
W^{abs} &= \frac{1}{\bar{\overline{B}}}\sum_{n=1}^{N-1}\int_{\beta^{n-1}}^{\beta^n}\frac{(ns)^2}{2B}dB + \frac{1}{\bar{\overline{B}}}\int_{\beta^{N-1}}^{\overline{B}}\frac{(Ns)^2}{2B}dB \\
&= \eta^2\frac{\bar{B}}{8}\left[\sum_{n=1}^{N-1}\frac{n^2}{N^2}\ln\left(\frac{2n+1}{2n-1}\right) + \ln\left(\frac{2N}{2N-1}\right)\right]
\end{aligned}$$

$$\begin{aligned}
W^{rel} &= \frac{W^{abs}}{TotalPurchase} = \frac{W^{abs}}{\eta\frac{\bar{B}}{4}} \\
&= \frac{\eta}{2}\left[\sum_{n=1}^{N-1}\frac{n^2}{N^2}\ln\left(\frac{2n+1}{2n-1}\right) + \ln\left(\frac{2N}{2N-1}\right)\right]
\end{aligned}$$

2.8.19 Proof of Lemma 2.5.3.1

We will first solve (2.10) for a given β^0 and then optimize over β^0. We assume $s \leq \beta^0$ in order to simplify the problem. (Otherwise we need to work with piecewise functions which make the problem more complex. There could be other solutions with $s > \beta^0$ but since we are looking for a lower bound, working $s \leq \beta^0$ is fine as long as the the result satisfies $s \leq \beta^0$). For a given β^0 we need to have $u^{bulk}(Q^{*bulk}, \beta^0) = u^{pack}(1, \beta^0)$. From this equation we derive package price as a function of bulk price and package size as:

$$P^p(P^b, s) = v + r - \frac{\beta^0(v + r - P^b)^2}{2s(v + r + w)} - \frac{s(v + r + w)}{2\beta^0} \tag{2.33}$$

After plugging in (2.33) in (2.10) the problem becomes:

$$\max_{P^b, s} \pi = (P^b - c)\frac{(\beta^0)^2}{2\bar{B}}\frac{v + r - P^b}{v + r + w} + (P^p(P^b, s) - c)s\frac{\bar{B} - \beta^0}{\bar{B}}$$

which is a concave optimization problem for a given b and the first order conditions give $P^{b*} = \frac{2\bar{B}(r+v) - \beta^0(v+r-c)}{2\bar{B}}$ and $s^* = \frac{\beta^0(v+r-c)}{(r+v+w)}$. From (2.33) we get $P^{p*} = \frac{4\bar{B}^2(v+r+c) - (\beta^0)^2(v+r-c)}{8\bar{B}^2}$. Now we plug in these values in the original problem (2.10) and optimize over b. The problem (2.10) reduces to:

$$\max_{\beta^0} \pi = \frac{\beta^0(2B - b1)^2(v + r - c)^2}{8\bar{B}^2(r + v + w)}$$

which is a simple optimization problem with $(\beta^0)^* = \frac{2\bar{B}}{3}$ with $\pi^* = \frac{4\bar{B}(v+r-c)^2}{27(v+r+w)}$. Besides we have $P^{b*} = \frac{2v+2r+c}{3}$, $s^* = \frac{2\bar{B}(v+r-c)}{3(r+v+w)}$ and $P^{p^*} = \frac{4v+4r+5c}{9}$. Notice that we have $s^* \leq \beta^0$ as we assumed.

Does the structure holds?

Since $P^{b*} < v + r$ all the consumers prefer to buy bulk to buying nothing. We need to show that consumers with $B \geq \beta^0$ prefer to buy at least 1 package to buying bulk. Formally, we need to show that $u^{pack}(1,B) \geq u^{bulk}(Q^{*bulk},B)$ or $\Delta = u^{pack}(1,B) - u^{bulk}(Q^{*bulk},B) \geq 0$ for $B \geq \beta^0$. With P^{p^*}, $P^{b^*}$and s^* found above we have $\Delta = -\frac{3(6B^2-25B\bar{B}+24\bar{B}^2)(v+r-c)^2}{64B(r+v+w))}$ and $\frac{d\Delta}{dB} = \frac{(36\bar{B}^2-9b^2)(v+r-c)^2}{32B^2(r+v+w)} > 0$, hence Δ is increasing in B. Recall that $\Delta = 0$ for $B = \beta^0$. Since Δ is increasing in B we have $\Delta \geq 0$ for $B \geq \beta^0$.

2.8.20 Proof of Theorem 2.5.4.1

The result follows from Lemma-2.5.3.1. Comparison of optimal profit from only bulk sale (or only package sale since they are the same) with the lower bound for co-existence case gives 18% difference.

CHAPTER 3
ASSORTMENT PLANNING STRATEGIES FOR VARIETY-SEEKING CONSUMERS

3.1 Introduction

Retail operations consist of several important decisions ranging from inventory management to promotions. One of these important decisions is *Assortment Planning*, which is defined by choosing which products to offer and how to stock them. Which products does a retailer offer among alternatives affects the consumers' purchase decision. When a retailer offers a few alternative there is a risk that consumers will go to another store with more selection of a specific type of product. Carrying a high variety of items thus increases the chance of a purchase but it also brings together more cost and more problems like inventory management of high variety of products, shelf space allocation, cannibalization of high margin products etc. The purpose of assortment planning is finding the product combination that maximizes the profit by considering the associated costs and problems.The assortment a retailer carries has an enormous impact on sales and gross margin, and hence assortment planning has received high priority from retailers, consultants and software providers.

Consumers' purchase decision making process involves various external or internal guides. One of the interval driver for a consumer that affects purchase decisions is the attitude towards escaping from or searching variety in consumption. In product categories where consumers have large consideration sets and make many choices over time, there may be some desire for more variety because of satiation or curiosity. This is called variety-seeking behavior. For example in the yogurt category, consumers tend to purchase some flavors they like and some they just want to try. In other products categories such as paper products, consumers tend to be loyal towards one brand and buy the same products constantly over time. This is called variety-avoiding behavior. There is vast literature, especially in marketing field, about the existence and causes of variety seeking/avoiding behavior. One of the earliest studies is by McAlister and Pessemier (1982) where they define variety as "the proverbial spice of life". They make a conceptual analysis on the topic and offer a structural taxonomy of causes of varied behavior which has been adopted by most of the proceeding research on variety seeking related studies. According to their study, the explicable causes of variety seeking can be divided into two. One is derived variation and the other one is direct variation. Derived variation is a result of external or internal forces that have nothing to do with the gain from change of preference itself. For example buying various brand or type of product due to the different taste of the family members. On the other hand, a variation in purchase motivated by a utility gained by the change itself is called direct variation. The reasons for direct variation could be desire for an unfamiliar alternative, desire for the information and alternation among the familiar.

There are studies in the marketing literature, especially on brand loyalty, that incorporates variety seeking/avoiding behavior into the consumer choice models. Givon (1984) proposes a dynamic stochastic choice model for the variety seeking/avoiding behavior motivated by switching among familiar brands. The main idea of this model is that consumer choice behavior is determined not only by the mean inherent utility of the different brands, but also by the utility (or disutility) derived by the consumer from switching brands from one period to the next. An individual consumer's utility (or disutility) from the switching action depends on the change itself, irrespective of the brands she switches to and from. The tendency of the consumer to seek/avoid variety is captured by a continuous variable that extends from the extreme tendency to avoid variety to the extreme tendency to vary consumption.

Although there is a substantial evidence about the existence of variety seeking/avoiding behavior, the previous research on retailers assortment planning problem has not incorporated variety seeking behavior of customers into the choice models. In this essay, we work on the multi-period assortment problem for a retailer with variety-seeking/ avoiding consumers.

We extend the Givon (1984) consumer choice model to make it possible to study the effect of customers' attitude towards variety on the optimal assortments to offer in a multi-period setting. We assume that each consumer is characterized by a parameter $V \in [-1, 1]$ which measures the consumer's attitude towards variety such that: (i) $V = 1$ corresponds to the extreme search for variety case, (ii) $V = 0$ means consumer has no utility (positive or negative) from active search of variety and (iii) $V = -1$ means consumer tries to avoid variety at all costs. In our analysis, we consider two different firm types which we call the Dynamic and the Static firms. The Dynamic firm optimizes the profit over the entire horizon by changing (if necessary) the assortment offered in each period dynamically, whereas, the static firm has to decide one assortment to offer throughout the horizon. We consider the assortment problem both under finite and infinite time horizons. We compare the retailer's profitability under each scenario. We show that the retailer profitability increases with the dynamic assortment strategy. We also show that it is possible to generate a high level of satisfaction at the customer level by having a mixed assortment strategy, e.g., assortment 1 (products 1,2,3,4,5) in some periods and assortment 2 (products 1,2,3,6,7) in other periods. Under this strategy, customers are still exposed to a large number of products over time. At the same time, the retailer gains economies of scale for the products it carries every period because the product line is less fragmented. This assortment strategy can be achieved quite easily by asking the vendors to change the product mix they ship according to some schedule or a probabilistic structure. This is an example of joint value creation in the retail and consumer-packaged goods industries. All three parties involved benefit from the application of this dynamic assortment strategy: The retailers benefit from increased profitability, the consumers from access to a higher level of variety over time, and the manufacturers from increased consumption of their products. We also provide some structural properties of the optimal assortments such as, when is it a popular set, when is it decreasing/increasing in size over time, when does it have a cycling nature, etc. We also numerically investigate the profit loss from not varying the assortment or ignoring the consumers' variety seeking/avoiding behavior.

Literature Review:

We divide the related literature into three parts. The first part is about the existence and causes of variety seeking/avoiding behavior. The second part includes the studies in the marketing literature with consumer choice models based on variety seeking/avoiding behavior. And the third part is (dynamic) assortment planning studies.

The existing studies form a rich literature about the existence and causes of variety seeking behavior. One of the most important studies about of the causes of variety seeking behavior is McAlister and Pessemier (1982). They define variety as the proverbial spice of life. They propose a conceptual analysis and offer a structural taxonomy of causes of varied behavior which has been adopted by most of the proceeding research on variety seeking related studies. According to their study, the explicable causes of variety seeking can be divided into two groups. One is derived variation and the other one is direct variation. Direct variation is caused by satisfaction from the changing behavior itself whereas derived variation is caused by the forces that are not related with a preference for change in and of itself. van Trijp (1995) presents an extensive study on the theory on variety seeking behavior especially in the food sector. They discuss the intrinsic and extrinsic motivations for switching behavior. They also study the effect of variety seeking behavior on brand loyalty, its effect on consumer choice and threats /opportunities to marketing management. Ratner et al. (1999) studies the pleasure effect of varied behavior on consumers. That is, the level of enjoyment from a switched product compared to a repeated purchase. Their study is different from other studies in the sense that they try to analyze the enjoyment from switch instead of utility. Their results show that people get less pleasure when they switch from their favorite product. They propose two different explanations about what is behind this behavior. Another study about the causes of variety seeking is Ju (2015).

The paper investigates how the information presented to the customer about the product affects the variety seeking behavior. They conducted experimental studies with university students and show that if the information is low, degree of variety seeking is high. Besides, they show that novice customers tend to seek variety more than expert customers when the information is limited. Kahn (1995) provide a literature review on the subject and discuss the motivating factors of variety seeking behavior among the consumers. Different from McAlister and Pessemier (1982), they provide one other reason for variety seeking which they call future preference uncertainty. It is defined as seeking variety in order to decrease the risk from future uncertainties. They also provide literature review of the measurement tools and predictive models.

Variety seeking behavior has received considerable attention in the marketing literature. One of the earliest studies is Jeuland (1979) which study the variety avoiding behavior of consumers to the brand loyalty. It is the first paper to integrate variety related behavior to MNL choice model. They add a parameter to the MNL choice model to capture variety avoidance of the consumers. They use a maximum likelihood estimation procedure and use an individual data on cooking oil purchases in France. We also use MNL choice model in our study. But the formulation of our consumer choice model is an extension of Givon (1984). It is one of the fundamental papers in variety seeking brand loyalty area. He proposes a first order markowian stochastic choice model based on variety seeking/avoiding behavior. He assumes that there is utility/disutility for a consumer derived from the change of the last purchased brand itself which is independent of the brands she switches to or from. The tendency to seek or avoid variety is measured with a parameter called VS that varies from -1 to +1 where -1 refers to extreme inertia and +1 refers to extreme variety seeking. Givon (1984) and most other studies assume that an individual is either variety seeker or avoider but Bawa (1990) considers the coexistence of variety seeking and avoiding behavior within the same individual. They empirically estimate their proposed model parameters by a household level panel data. They show that according to the empirical results, half of the households reveal hybrid behavior instead of pure inertia or pure variety seeking. There are also studies in the marketing literature which investigate the effect of variety seeking/avoiding behavior on marketing variables, such as price, promotions etc. Seetharaman and Chintagunta (1998) works on the effects of price and promotions on variety avoiding and variety-seeking behavior. They use the term habit-persistence instead of variety avoidance. They extend stochastic model of Givon (1984) to include marketing variables such as price and promotions. They empirically test their model with panel data from six different brands of canned tuna fish and show that customers strongly avoid variety and are price sensitive for this specific product. Kahn and Louie (1990) studies a stochastic variety seeking model where the uncertainty comes from the unknown promotion times. They assume that at an arbitrary time there is a probability of product being in promotion. They extend Kahn et al. (1986) model and incorporate price promotions to the stochastic variety seeking model along with intrinsic utility of switching itself. They derive steady state choice probabilities for different price discount cases and show how variety seeking or avoiding customers react to the promotions. Kahn and Louie (1990) also investigates price promotions with variety seeking customers similar to Kahn and Raju (1991) but they focus on effects on market share. They show how the brand choice probabilities change depending on the price promotions and number of promoted products. They show that, if consumers are variety avoider, a promoted brands share decreases after the removal of promotion if it is the only brand to be promoted. On the other hand, if consumers are variety seeker, the brands share does not decrease after the removal of promotions. Another study on market sharing is Feinberg et al. (1992). They study the implications of variety seeking magnitude on market share. According to their results, as variety seeking increases, the most unpopular brand gains market share and the most popular product loses share. Chintagunta (1998) try to incorporate the variety avoiding/seeking behavior into a brand purchase timing model. They claim that if a consumer is variety avoider, the next item she switches will be close to his last choice with some attributes and vice versa for variety seekers. Hence, they propose a choice model where variety seeking/avoiding behavior is captured at the attribute level of the brands. They propose a failure time model and test their model with an empirical study. A relatively new study, Sajeesh and Raju (2010), investigates variety seeking effect on competitive positioning and pricing strategies. The variety seeking effect on consumer choice is captured by the decrease of willingness to pay for the last purchased product. They show that product differentiation reduces when there are variety seeking consumers in the market. They also show that, in a two period problem, prices drop in the second period due to the incentive to prevent losing variety seeking customers and that existence of variety seeking reduce the firm profits. Zeithammer and Thomadsen (2013) studies the price and quality competition in a vertically differentiated duopoly market. The consumers are variety seekers and variety seeking is modeled as the diminishing marginal utility for repeated purchase of same product. They find that when the qualities are similar, variety seeking increases both prices and profits. On the other hand, if qualities are different, price competition is sharper and profits are lower. The most recent study in the marketing literature is Xiong et al. (2017). Similar to Zeithammer and Thomadsen (2013), this paper also studies price and quality competition in a duopoly market but they also consider variety avoiders. They use the term *habit formation* instead of variety avoiding. They assume that there are two consumers segments in the market, one segment is the variety seekers and the other is variety avoiders. With a two period problem of Hotelling type setting, they show how firms‘ optimal price and quality decisions change when variety seeking and variety avoiding exists among consumers.

There is a growing body of literature that considers dynamic assortments. In these papers, the rationale for changing the assortment through time is either (i) the need to learn about consumer preferences (see for example Caro and Gallien (2007), Chen and Plambeck (2008) and Ulu et al. (2012)) or (ii) the need to adapt to changing customer preferences (see for example Caro and Martinez-de Albeniz (2009), Caldentey and Caro (2010) and Saure and Zeevi (2009)). We refer the reader to Kök et al. (2015) and Mou et al. (2017) for the detailed (dynamic) assortment planning literature. To the best of our knowledge, no paper has incorporated the consumers' attitude towards variety as a motivation for changing assortments.

This essay is organized as follows. In §3.2, we develop our model. In §3.3 and §3.4 we work on the finite and infinite horizon problems respectively and show some structural results. In §3.5 we show some numerical examples.

3.2 Model

We consider a product category where the set of potential products to offer is $N = \{0, 1, ..., n\}$ and 0 denotes the outside option. Let $S_t \subseteq N$ denote the assortment offered by the retailer in period t and S denote a generic assortment. We assume that $0 \in S_t$ for all t.[1]

We represent consumer choice using the model from Givon (1984). As discussed in the introduction, the main idea of this model is that consumer choice behavior is determined not only by the mean inherent utility of the different products, but also by the utility (or disutility) derived by the consumer from switching products from one period to the next.[2]

Mean inherent utilities for the products are given by $\mathbf{u} = (u_0, u_1, ..., u_n)$ such that $u_1 \geq u_2 \geq ... \geq u_n$. We define $\theta_j(S_t) = \frac{u_j}{\sum_{k \in S_t} u_k}$ for $j \in S_t$ (zero otherwise), which corresponds to the probability of choosing product j from assortment S_t when consumer preferences are only driven by the mean inherent utility of the products (as in the classic Multinomial Logit (MNL) model. Also we define $\boldsymbol{\gamma}(S) = (\gamma_0, \gamma_1, ..., \gamma_n)$ such that $\gamma_j(S) = \frac{1}{|S|}$ for $j \in S$

[1] In what follows the outside option is, unless otherwise stated, treated like a product with zero selling price and cost.

[2] While Givon (1984) assumes that all n products are available in each period, we have modified his model to allow the firm to offer $S_t \subseteq N$ in period t.

(zero otherwise), which corresponds to the consumer making a random choice amongst the products offered in the assortment, with each product being equally likely to be chosen. Let $\boldsymbol{\theta} = (\theta_0, \theta_1, ..., \theta_n)$ and $\boldsymbol{\gamma} = (\gamma_0, ..., \gamma_n)$.

As in Givon (1984), we let $V \in [-1, 1]$ be a parameter which measures the consumer's attitude towards variety such that positive (negative) values correspond to the case of a variety seeking (avoidance) behavior. Let $P_{ij}(S_t)$ be the probability that a consumer with variety attitude parameter V who bought product $i \in S_{t-1}$ in period t buys product j in period t:

$$P_{ij}(S_t) = \begin{cases} \frac{|V|-V}{2} + (1-|V|)\theta_j(S_t) & , j \in S_t, j = i \\ \frac{|V|+V}{2}\gamma_j(S_t \backslash \{i\}) + (1-|V|)\theta_j(S_t) & , j \in S_t, i \in S_t, j \neq i \\ |V|\gamma_j(S_t) + (1-|V|)\theta_j(S_t) & , j \in S_t, i \notin S_t, j \neq i \\ 0 & , j \notin S_t \end{cases} \tag{3.1}$$

$$= \begin{cases} \frac{|V|-V}{2} + (1-|V|)\theta_j(S_t) & , j \in S_t, j = i \\ \frac{|V|+V}{2|S_t|-1} + (1-|V|)\theta_j(S_t) & , j \in S_t, i \in S_t, j \neq i \\ \frac{|V|}{|S_t|} + (1-|V|)\theta_j(S_t) & , j \in S_t, i \notin S_t, j \neq i \\ 0 & , j \notin S_t \end{cases} \tag{3.2}$$

In this expression, the first row corresponds to the probability of buying the same product in periods $t-1$ and t. The second row is the probability of switching to a different product, even though the product purchased in period $t-1$ is still available in period t. Finally the third row is the probability of switching to a different product when the product chosen in period $t-1$ is no longer available in period t.

In particular, the extreme case of $V = -1$ corresponds to a behavior of extreme rejection of variety with no regard for the mean inherent utility of products: the consumer is always looking to buy the same product which he purchased in the previous period (referred to as perfect product inertia). In this case, (3.2) simplifies to

$$P_{ij}(S_t) = \begin{cases} 1 & j \in S_t, j = i \\ 0 & j \in S_t, i \in S_t, j \neq i \\ \frac{1}{|S_t|} & j \in S_t, i \notin S_t, j \neq i \\ 0 & j \notin S_t \end{cases} \tag{3.3}$$

We see that the consumer with $V = -1$ always chooses the same product as in the previous period, unless it is no longer available, in which case he chooses randomly amongst all products in the assortment (and the outside option).

The case of $V = 1$ corresponds to a behavior of extreme search for variety with no regard for the mean inherent utility of products: the consumer is always looking to buy a different product from the one he purchased in the previous period. In this case, (3.2) simplifies to:

$$P_{ij}(S_t) = \begin{cases} 0 & j \in S_t, j = i \\ \frac{1}{|S_t - 1|} & j \in S_t, i \in S_t, j \neq i \\ \frac{1}{|S_t|} & j \in S_t, i \notin S_t, j \neq i \\ 0 & j \notin S_t \end{cases} \tag{3.4}$$

We see that the consumer with $V = 1$ always buys a product which is different from the one he purchased in the previous period, with all other products being equally likely to be chosen.

Finally the case of $V = 0$ corresponds to the classical multinomial logit model behavior, wherein the consumer's choice only depends on the mean inherent utilities of the products. In this case, (3.2) simplifies to $P_{ij}(S_t) = \theta_j(S_t)$ for all $j \in S_t$ and 0 otherwise, that is the choice does not depend on the product purchased in period $t - 1$.

Let $\mathbf{P}(S_t)$ denote the consumer choice transition probability matrix $(P_{ij}(S_t))_{i,j=1,\ldots,n}$ from period $t - 1$ to period t.[3]

[3]Note that the following formulas calculate P_{ij} for $i \notin S_{t-1}$ but these values will not be used.

Model discussion

Inherent to Givon (1984)'s model are two key assumptions. First, it is assumed that only the previous period's purchase matters in the consumer's choice decision. Second, the probability of switching to brand j does not depend on the brand switched from, i.e., $P_{ij}(S_t) = P_{kj}(S_t)$ for all $i, k \neq j$. See Givon (1984) on page 4 for more detail about the consumer choice model.

We write down P for $V \in \{-1, 0, 1\}$ for a particular example below:

Example 3.2.0.1. *Let $n = 2$ and $S = \{0, 1\}$. If $V = 1$, we have:*

$$P = \begin{pmatrix} 0 & 1 & 0 \\ 1 & 0 & 0 \\ 1/2 & 1/2 & 0 \end{pmatrix}.$$

where the products are listed in the following order in rows and columns: 0, 1, 2.

If $V = -1$, we have:

$$P = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 1/2 & 1/2 & 0 \end{pmatrix}.$$

If $V = 0$ and $u_0 = 1, u_1 = 2, u_2 = 1$ then $\boldsymbol{\theta}(S) = (1/3, 2/3, 0)$ and

$$P = \begin{pmatrix} 1/3 & 2/3 & 0 \\ 1/3 & 2/3 & 0 \\ 1/3 & 2/3 & 0 \end{pmatrix}.$$

As we see from the example, since product-2 is not in the assortment, probability to switch to product-2 is zero in all cases. On the other hand, we see that in both V=1 and V=-1 cases, the transition probability from product-2 to other products is allocated equally. If an individual buys product-1 or buys from outside, then for V=1, the transition to the other product (extreme search for variety) is for certain , and for V=-1, purchasing the same product (extreme avoidance from variety) is for certain. For V=0 case, since utility from switching itself is zero, the transition probabilities are equal to $\theta_j(S)$

In Table-3.1 and Table-3.2 we present the change of switching probabilities from product i to j by θ_j and V where the assortment size is 5. The difference between the two tables is in Table-3.1 we have $i \in S_t$ and in Table-3.2 we have $i \notin S_t$ (in other words, product i is dropped from the assortment). When we compare the two tables, we see that for $V < 0$, the probability of switching increases when product i is dropped from the assortment and it decreases for $V > 0$. The reason is: when $V < 0$ and $i \notin S_t$, the avoiders not being able to find product i will look for other options (including the outside option) and this will increase the switching probabilities to the products that are in the assortment. For $V > 0$, when the previously chosen product is in the assortment, the share from variety seekers will be distributed among the other n-1 (which is 4 in the example) products. But when the product is dropped from the assortment (and the assortment size is kept the same by adding another product) it will be shared among n (which is 5 in the example) products. We also see from Table-3.2 that when the product is dropped from the assortment the switching probabilities are symmetric over $V = 0$.

In Table-3.3 we present the probability of purchasing the same product in a row when it is offered in both periods. Note that probability of buying the same product is decreasing in V for a given θ_j which is very straightforward because as V increases people become more variety seeker and thus more inclined to change their choice. We should also emphasize that these probabilities do not depend on the size of the assortment set.

Let λ denote the market size, which we assume is constant over the time horizon. We assume that all consumers in the market have the same variety attitude parameter V for the given product category. Let $q_{t,j}(S_t)$ be the probability that a customer buys product j in period t given assortment S_t. We refer to $\mathbf{q}_t = (q_{t,0}, q_{t,1}, ..., q_{t,n})$ as the *demand probability*

Table 3.1: Probability of switching from product i to j where $i \in S_t$ and $|S_t| = 5$

θ_j	$V = -1$	$V = -0.5$	$V = 0$	$V = 0.5$	$V = 1$
0	0	0	0	0.125	0.25
0.2	0	0.1	0.2	0.225	0.25
0.25	0	0.125	0.25	0.25	0.25
0.5	0	0.25	0.5	0.375	0.25
0.75	0	0.375	0.75	0.5	0.25
1	0	0.5	1	0.625	0.25

Table 3.2: Probability of switching from product i to j where $i \notin S_t$ and $|S_t| = 5$

θ_j	$V = -1$	$V = -0.5$	$V = 0$	$V = 0.5$	$V = 1$
0	0.2	0.1	0	0.1	0.2
0.2	0.2	0.2	0.2	0.2	0.2
0.25	0.2	0.225	0.25	0.225	0.2
0.5	0.2	0.35	0.5	0.35	0.2
0.75	0.2	0.475	0.75	0.475	0.2
1	0.2	0.6	1	0.6	0.2

Table 3.3: Probability of purchasing the same product i in a row where $i \in S_{t-1}$ and $i \in S_t$

θ_j	$V = -1$	$V = -0.5$	$V = 0$	$V = 0.5$	$V = 1$
0	1	0.5	0	0	0
0.2	1	0.6	0.2	0.1	0
0.25	1	0.625	0.25	0.125	0
0.5	1	0.75	0.5	0.25	0
0.75	1	0.875	0.75	0.375	0
1	1	1	1	0.5	0

vector and $\mathbf{q}_t\lambda$ as the *demand volume* in period t. In the very first period ($t = 1$), because the product has has never been sold before, we assume that $\mathbf{q}_1(S_1) = |V|\boldsymbol{\gamma}(S_1) + (1 - |V|)\boldsymbol{\theta}(S_1)$, that is, we assume that the consumer's choice in period 1 is a weighted average of a choice driven solely by the mean inherent utilities of the product (i.e., $\boldsymbol{\theta}(S_1)$) and an equally likely choice amongst all the offered products (i.e., $\boldsymbol{\gamma}(S_1)$), with the weight being the absolute value of the consumer's variety attitude parameter V.

In subsequent periods, i.e., $t = 2, 3, ...$, we have $\mathbf{q}_t = f(\mathbf{q}_{t-1}, S_t)$ where f is the demand probability update function defined a

$$f(\mathbf{q}, S) = \mathbf{q}P(S) \tag{3.5}$$

We have $\mathbf{q}_t = \mathbf{q}_{t-1}\mathbf{P}(S_t) = \mathbf{q}_1 \prod_{\tau=1}^{t} \mathbf{P}(S_\tau)$.

The retailer receives a revenue of r per unit sold and incurs an inventory stocking cost according to a concave function of the demand volume given by σx^β where $\sigma \geq 0$, $\beta \in [0, 1]$ and x is the demand volume. This function is used by Ryzin and Mahajan (1999). Like them, we assume that $\lambda^{1-\beta} q >> \sigma q^\beta$ which implies that the probability of having negative demand is small. (For a detailed informations refer to Ryzin and Mahajan (1999)). The retailer's profit in period $t \geq 1$ is given by:

$$\pi_t(S_t) = \sum_{j=1}^{n} \left\{ r\lambda q_{t,j}(S_t) - \sigma \left[\lambda q_{t,j}(S_t)\right]^\beta \right\}$$

Let $\Pi(S_1, ..., S_T) = \sum_{t=1}^{T} \pi_t(S_t)$ denote the total expected profit the retailer receives from offering assortments $S_1, ..., S_T$ in periods 1 to T.

We consider three versions of the retailer's problem: the dynamic firm, the static firm and the myopic firm.

The dynamic firm

The dynamic firm is able to select a different assortment in each period to maximize its expected profit over the entire horizon. This problem can be formulated as a dynamic program where the state space is the $(1 \times n + 1)$ demand probability vector at the end of the previous period. Let $V_t(\mathbf{q}_{t-1})$ denote the optimal expected profits from periods t to T. We have:

$$V_t(\mathbf{q}_{t-1}) = \max_{S_t \subseteq N} \pi_t(S_t; \mathbf{q}_{t-1}) + V_{t+1}(f(\mathbf{q}_{t-1}, S_t)), \text{ for } 1 \leq t < T$$

and $V_{T+1} = 0$.

Let S_t^* denote the optimal assortment in period t for the dynamic firm.

The static firm

The static firm is unable to vary the assortment from period to period but chooses the assortment which, under this constraint, maximizes expected profit over the entire horizon, that is, the retailer solves $\max_{S \subseteq N} \sum_{t=1}^{T} \pi_t(S)$.

Let S^s be the optimal assortment for the static firm.

The myopic firm

The myopic firm maximizes the one-period expected profit function in each period, with no regard to the impact the assortment has on future profits. Let $S_t^m(\mathbf{q}_{t-1})$ be the assortment chosen in period t, given the demand probability vector in period $t-1$. We have:

$$S_t^m = \arg\max_{S \subseteq N} \pi_t(S; \mathbf{q}_{t-1}). \tag{3.6}$$

We now define two important sets, namely $S_{\boldsymbol{\theta}}$ and $S_{\boldsymbol{\gamma}}$, which are important for the later analyses. Let S_θ be the set that optimizes one period expected profit with demand probability vector $\boldsymbol{\theta}(S)$, that is,

$$\pi_t(S) = \sum_{\substack{j \in S \\ j \neq 0}} \left\{ r\lambda\theta_j(S) - \sigma\lambda^\beta \left(\theta_j(S)\right)^\beta \right\}$$

This is the classical assortment planning problem with the MNL model studied by Ryzin and Mahajan (1999). The optimal assortment is known to be a *popular set*, that is, $S^* = \{1, ..., k\}$ for $k \in \{1, ..., n\}$ with $u_1 \geq u_2 \geq ... \geq u_k \geq ... \geq u_n$.

Let S_γ be the set that optimizes one period expected profit with demand probability vector $\boldsymbol{\gamma}(S)$, that is,

$$\begin{aligned}\pi_t(S) &= \sum_{\substack{j \in S \\ j \neq 0}} \left\{ r\lambda\gamma_j(S) - \sigma\lambda^\beta \left(\gamma_j(S)\right)^\beta \right\} \\ &= \sum_{\substack{j \in S \\ j \neq 0}} \left\{ r\lambda\frac{1}{|S|} - \sigma\lambda^\beta \left(\frac{1}{|S|}\right)^\beta \right\}\end{aligned}$$

Table 3.4: Notation

Symbol	Definition
Decision variables	
S_t :	Assortment in period t
Parameters	
N :	Set of potential products $N = \{0, 1, ..., n\}$
$\mathbf{u}$:	Product mean inherent utilities $\mathbf{u} = (u_0, u_1, ...u_n)$ $u_1 \geq u_2.... \geq u_n$
$\boldsymbol{\theta}$:	Variety attitude-neutral choice probabilities given assortment S $\boldsymbol{\theta} = (\theta_0, \theta_1, ..., \theta_n)$
V :	Consumer variety attitude parameter $-1 \leq V \leq 1$
$P_{ij}(S)$:	Probability that a consumer who bought i buys product j in the next period with assortment S
$q_{t,j}(S)$:	Probability that a customer buys product j in period t given assortment S.
$q_\infty(S)$:	The stationary distribution for $P(S)$
Other variables	
r :	Selling price
λ :	Market size i
σ :	Cost function magnitude parameter
β :	Cost function concavity parameter
S_t^*	Dynamic Firm Optimal Assortment
S_t^m	Myopic Firm Optimal Assortment
S^s	Static Firm Optimal Assortment
S_γ	The set that optimizes one period expected profit with demand probability vector $\gamma(S)$
S_θ	The set that optimizes one period expected profit with demand probability vector $\theta(S)$
$S_0(V)$	Optimal assortment with q_1
$S_\infty^+(V)$	Optimal assortment with with q_∞ for $0 \leq V \leq 1$
$S_\infty^-(V)$	Optimal assortment with with q_∞ for $-1 \leq V < 0$

Let $m = |S|$, we have:

$$\pi_t(S) = r\lambda \frac{m-1}{m} - \sigma(m-1)\lambda^\beta \left(\frac{1}{m}\right)^\beta \tag{3.7}$$

The second derivative of 3.7 with respect to m is less than zero. Thus, π_t is concave in m. Let $\hat{m}$ denote the value which optimizes the FOC. The optimal value m^* is obtained by rounding this value either up or down, up to a maximum of n. In other words, S_γ is any set of of size m^*. In particular we can set $S_\gamma = \{1, ..., m^*\}$. This is summarized in Lemma-3.2.0.2.

Lemma 3.2.0.2. *S_γ is any set of of size m^* where* $m^* = \underset{m \in \mathbb{Z}, m \leq N}{\arg\max}\ r\lambda\frac{1}{m^2} - \sigma\lambda^\beta \left(\frac{1}{m}\right)^\beta - \sigma\lambda^\beta\beta(m-1)\left(\frac{1}{m}\right)^{\beta-1}$

3.3 Finite Horizon Problem

We first study the finite horizon problem where assortment decision is made for some $T < \infty$. We first investigate the optimal solution when the firm wants to maximize the revenue.

By our transition probability definition, the process is Markovian such that the current period purchase decision only depends on the last purchased product. The demand probability of product $j \in S_t$ in period t as a function of its value in period $t-1$ is equal to:

$$q_{t,j}(S_t; \mathbf{q}_{t-1}) = \begin{cases} q_{t-1,j} + \frac{1}{|S_t|}\sum_{i \notin S_t} q_{t-1,i} & V = -1 \\ (1+V)\theta_j(S_t) - Vq_{t-1,j} - \frac{V}{|S_t|}\sum_{i \notin S_t} q_{t-1,i} & -1 \leq V \leq 0 \\ \theta_j(S_t) & V = 0 \\ (1-V)\theta_j(S_t) + \frac{V}{|S_t-1|}\sum_{\substack{i \in S_t \\ i \neq j}} q_{t-1,i} + \frac{V}{|S_t|}\sum_{i \notin S_t} q_{t-1,i} & 0 \leq V \leq 1 \\ \frac{1}{|S_t-1|}\sum_{\substack{i \in S_t \\ i \neq j}} q_{t-1,i} + \frac{1}{|S_t|}\sum_{i \notin S} q_{t-1,i} & V = 1 \end{cases} \tag{3.8}$$

and $q_{t,j} = 0$ for $j \notin S$.

In the special case of $S_{t-1} \subseteq S_t$ (i.e. no products are dropped from period $t-1$ to period t), 3.8 reduces to:

$$q_{t,j}(S_t) = \begin{cases} q_{t-1,j} & V = -1 \\ (1+V)\theta_j(S_t) - Vq_{t-1,j} & -1 \leq V \leq 0 \\ \theta_j(S_t) & V = 0 \\ (1-V)\theta_j(S_t) + \frac{V}{|S_t-1|}\sum_{\substack{i \in S_t \\ i \neq j}} q_{t-1,i} & 0 \leq V \leq 1 \\ \frac{1}{|S_t-1|}\sum_{\substack{i \in S_t \\ i \neq j}} q_{t-1,i} & V = 1 \end{cases} \tag{3.9}$$

3.3.1 Revenue Maximization Problem

Suppose objective of the firm is to maximize the revenue. It is mathematically equivalent to assume that $\beta = 1$ in 3.6. The following lemma shows that offering all the products is optimal when the firm's objective is maximizing the revenue.

Lemma 3.3.1.1. *[Revenue maximization] If $\beta = 1$, then $S^m(\mathbf{q}_{t-1}) = S_t^* = S^s = N$ for all $\mathbf{q}$.*

Proof. The retailer's problem can be written as $\max_{S\subseteq N} r\lambda \sum_{j=1}^{n} q_{t,j}(S)$ which is equivalent to $\min_{S\subseteq N} r\lambda q_{t,0}(S)$. Hence, the problem reduces to minimizing no-purchase probability.

When $0 \leq V \leq 1$, we have:

$$q_{t,0}(S) \quad = \quad (1-V)\theta_0(S) + \frac{V}{|S|-1}\sum_{\substack{i\epsilon S\\ i\neq 0}} q_{t-1,i} + \frac{V}{|S|}\sum_{i\notin S} q_{t-1,i}$$

For $k \notin S$ let $S' = S \cup \{k\}$. We have:

$$q_{t,0}(S') \quad = \quad (1-V)\theta_0(S') + \frac{V}{|S|}\sum_{\substack{i\epsilon S\\ i\neq 0}} q_{t-1,i} + \frac{V}{|S|}q_{t-1,k} + \frac{V}{|S|+1}\sum_{\substack{i\notin S\\ i\neq k}} q_{t-1,i}$$

And therefore:

$$\begin{aligned} q_{t,0}(S) - q_{t,0}(S') = (1-V)\left(\theta_0(S) - \theta_0(S')\right) + \left(\frac{V}{|S|-1} - \frac{V}{|S|}\right)\sum_{\substack{i\epsilon S\\ i\neq 0}} q_{t-1,i} \\ + \left(\frac{V}{|S|} - \frac{V}{|S|+1}\right)\sum_{\substack{i\notin S\\ i\neq k}} q_{t-1,i} \\ > 0 \end{aligned}$$

since $\theta_0(S) \geq \theta_0(S')$ and $V \geq 0$.

When $-1 \leq V \leq 0$, we have:

$$\begin{aligned}
q_{t,0}(S) &= -V + (1+V)\theta_0(S) + V\sum_{\substack{j\in S\\ j\neq 0}} q_{t-1,j} + V\frac{|S|-1}{|S|}\sum_{i\notin S} q_{t-1,i} \\
q_{t,0}(S') &= -V + (1+V)\theta_0(S') + V\sum_{\substack{j\in S\\ j\neq 0}} q_{t-1,j} + Vq_{t-1,k} + V\frac{|S|}{|S|+1}\sum_{\substack{i\notin S\\ i\neq k}} q_{t-1,i} \\
q_{t,0}(S) - q_{t,0}(S') &= (1+V)\left(\theta_0(S) - \theta_0(S')\right) - \frac{V}{|S|}q_{t-1,k} - \frac{V}{|S|(|S|+1)}\sum_{\substack{i\notin S\\ i\neq k}} q_{t-1,i} > 0
\end{aligned}$$

since $\theta_0(S) \geq \theta_0(S')$ and $V \leq 0$.

Since adding a product to S decreases the probability of outside option for all values of $\mathbf{q}_{t-1}$, the myopically optimal assortment always includes all products. □

In general, when the objective is revenue maximization, offering all the products is not surprising for assortment planning problems. Because, when the revenue is maximized, the problem reduces to minimizing the outside option (no purchase) probability and in a classical setting, adding a product usually decreases that probability. But when there is variety seeking behavior, it is not straightforward to see if adding a product will decrease the outside option. In fact, we proved that adding a product to S decreases the probability of outside option.

3.3.2 Structural Results

In this section we provide some structural results for the benchmark case of $V = 0$ and extreme cases of V=1 and V=-1. We show the tradeoffs between myopic, static and dynamic firm with a 2 period problem example. Besides, we show that the static firm

The benchmark case of $V = 0$

When there is no variety seeking behavior among the consumers, the inherent utilities from the products determines the consumer purchase probabilities and there is no change in the transition probability since the inherent utilities remain the same throughout the horizon.

Lemma 3.3.2.1. *If $V = 0$, then $S^m(\mathbf{q}_{t-1}) = S_t^* = S^s = S_\theta$ for all $\mathbf{q}_{t-1}$*

Proof. When $V = 0$, we have $\mathbf{q}_t(S_t; \mathbf{q}_{t-1}) = \theta_j(S_t)$ for all $\mathbf{q}_{t-1}$. Therefore $\pi_t(S_t)$ is maximized at S_θ. $\square$

This result also means that the optimal assortment is a popular set as shown by Ryzin and Mahajan (1999).

Extreme Inertia $V = -1$

Now assume that all the customers in the market extremely variety avoider. They want to buy the same product in every period as long as the product is offered. Now we go back to the original problem of profit maximizing and investigate what happens in the extreme case of $V = -1$. When $V = -1$ we have

$$q_{t,j}(S_t; q_{t-1}) = q_{t-1,j} + \frac{1}{|S_t|} \sum_{i \notin S_t} q_{t-1,j}$$

Let the first period optimal be S_1^*, with $q_{1,j} = \frac{1}{|S_1^*|}$ for all $j \in S_1^*$

Now consider the second period. Since $V = -1$, we know from 3.9 that adding a product to S_1^* won't change $q_{2,j}$ and we will get $q_{2,j} = \frac{1}{|S_1^*|}$ for $j \in S_1^*$ and $q_{2,j} = 0$ for $j \notin S_1^*$. Hence, profit will be the same.

Now consider the case where we remove k products from S_1^*. Let the new assortment be $\bar{S}$. Notice that $\left|\bar{S}\right| = |S_1^* - k|$. For all $j \in \bar{S}$ we get:

$$\begin{aligned} q_{2,j}(\bar{S}; q_1) &= \frac{1}{|S_1^*|} + \frac{1}{|S_1^* - k|} \frac{k}{|S_1^*|} \\ &= \frac{|S_1^* - k| + k}{|S_1^* - k|\,|S_1^*|} \\ &= \frac{1}{|S_1^* - k|} \end{aligned}$$

Which is the same with $q_{1,j}(S)$ such that $|S| = |S_1^* - k|$. Since we know that $\pi(S_1^*, q_{1,j}(S_1^*)) > \pi(S; q_{1,j}(S))$ we get $\pi(S_1^*, q_{1,j}(S_1^*)) > \pi(\bar{S}; q_{2,j}(\bar{S}))$.

For $t > 2$, same logic applies and we get the following lemma for the static and dynamic firm.

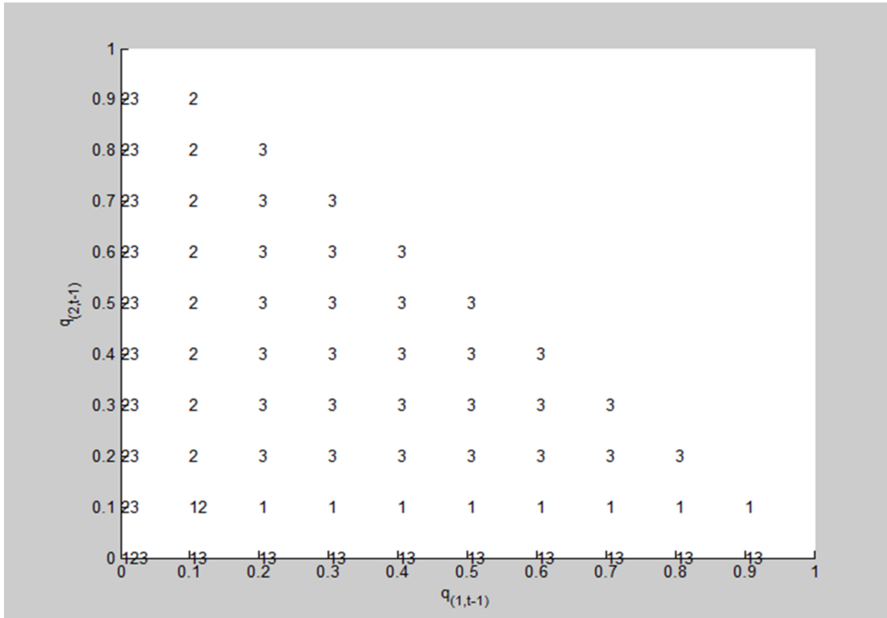


Figure 3.1: An example of myopic firm optimal assortment for a given q when $V = -1$

Lemma 3.3.2.2. *If $V = -1$, then $S_t^* = S^s = S_\gamma$, for $t = 1, ..., T$.*

Lemma 3.3.2.2 doesn't apply for the myopic firm because the starting period is important in order to get S_γ. By our definition, myopic firm solution is for any given q_{t-1} (even if it is not reachable). In Figure-3.1 we present the optimal assortment for an example of 2 product case depending on q_{t-1} ($V = -1, r = 2.1, \sigma = 2, \lambda = 20, \beta = 0.6$). In the figure x-axis represents $q_{1,t-1}$ and y-axis represents $q_{2,t-1}$. We see the optimal solution in period t as a combination of this two values. Notice that, $q_{1,t-1}$=0 corresponds to the case where only product-2 is offered in the previous period and vice versa. We denote the assortments as Assortment 1= {Product 1}, Assortment-2={Product 2} and Assortment-3={Product 1, Product 2}. For the given parameters we have S_γ={Product 1, Product 2} but we for $q_{1,t-1} = 0.5$ and $q_{2,t-1} = 0.1$ the optimal assortment is offering Product 1. Which is a counter example of Lemma 3.3.2.2 for the myopic firm (We have ties for some combinations which is also shown in the figure by 13, 23 or 123).

Extreme variety seeking $V = 1$

When the consumers are extremely variety seeker, they won't buy the same product they purchased in the last period even if it is the only product offered. We have the following lemma for static firm.

Lemma 3.3.2.3. *When $V = 1$, we have $S^* = S_\gamma$.*

Proof. When $V = 1$ we have $\mathbf{q}_1 = \boldsymbol{\gamma}(S)$ and from 3.9, we can see that $\mathbf{q}_t = \boldsymbol{\gamma}(S)$ for all $t = 2, ..., T$. □

According to our numerical analysis, the optimal solution for myopic and dynamic firm is not necessarily S_γ.

Two-period Dynamic Firm Problem

Finite horizon dynamic firm problem is the most challenging case in our study. As discussed earlier, dynamic firm tries to optimize profit by changing the assortment in each period. Because of the variety seeking/avoiding behavior, customer choice is changing in every period and not taking into account this change will definitely hurt the retailer. For example, when customers are highly variety seekers, offering a popular product in a two consecutive period can decrease the profit from the popular product in the second period. Adding or removing a product is going to change consumers purchase decisions and taking the advantage of this change is possible by dynamically changing the assortment. The following example illustrates the trade-offs for a 2-period 2-product problem.

Example 3.3.2.4. *Let $n = 2$, $T = 2$ $r = 10, \lambda = 100, \sigma = 9, \beta = 0.9$ $\mathbf{u} = (1, 40, 8)$.*

When $V = 0.75$ we have:

S_1	$\mathbf{q}_1(S_1)$	$\pi_1(S_1)$	$S_2 = S^m(\mathbf{q}_1)$	$\mathbf{q}_2(S^2)$	$\pi_2(S^2)$	Π
$\{1\}$	$(0.38, 0.62, 0)$	250.18	$\{2\}$	$(0.26, 0, 0.74)$	307.00	557.18
$\{2\}$	$(0.40, 0, 0.60)$	240.14	$\{1\}$	$(0.23, 0.77, 0)$	321.14	561.28
$\{1, 2\}$	$(0.26, 0.45, 0.29)$	279.01	$\{1, 2\}$	$(0.29, 0.41, 0.31)$	265.65	544.66

We see that it is optimal to offer $\{2\}$ *in the first period, followed by* $\{1\}$ *in the second period. However, notice that the myopically optimal assortment in period* 1 *is* $\{1\}$. *The intuition is as follows: by offering only product 2 in the first period and only product 1 in the second period, the retailer is able to maximize sales and save on inventory costs. This is because of the surge in the purchase probabilities of the popular product in the second period caused by the variety seeking customers who bought less popular product in period 1. By not offering the popular product in the first period, retailer loses some demand but the increase of demand of the popular product in the second period outweighs the loss in the first period.*

When $V = -0.4$ *we have:*

S_1	$\mathbf{q}_1(S_1)$	$\pi_1(S_1)$	$S_2 = S^m(\mathbf{q}_1)$	$\mathbf{q}_2(S^2)$	$\pi_2(S^2)$	Π
$\{1\}$	$(0.21, 0.79, 0)$	328.48	$\{1\}$	$(0.10, 0.90, 0)$	383.27	711.76
$\{2\}$	$(0.27, 0, 0.73)$	303.78	$\{2\}$	$((0.17, 0, 0.83)$	348.21	653.00
$\{1, 2\}$	$(0.15, 0.62, 0.23)$	331.38	$\{1\}$	$(0.12, 0.88, 0)$	374.28	705.66

We see that it is optimal to offer $\{1\}$ *in the first and second periods. Again, notice that the myopically optimal assortment in period* 1 *is* $\{1, 2\}$, *which is different from the optimal assortment of the dynamic firm in period 1.*

In general, as the previous example illustrates, it is not necessarily optimal for the dynamic firm to offer the myopically optimal assortment in each period.

3.4 Infinite Horizon Problem

For most of the products in food retail, there is a chain of recurring transactions over time. Consider the yogurt example which has high variety seeking potential. It is a repeat purchase product and retailer-consumer interaction goes on for a long time period. Thus, a steady state analysis of the system could be more appropriate for this type of products. In this section we consider the infinite horizon problem where $T = +\infty$.

3.4.1 Static Firm

In order to study the infinite horizon problem we need the steady state demand probability distributions. Let $\mathbf{q}_\infty(S)$ denote the steady state probability, obtained by solving $\mathbf{q} = \mathbf{q}P(S)$. It is obvious that when $j \notin S$ we have $q_{\infty,j}(S) = 0$. For $V \in \{-1, 1\}$ from Lemma-3.3.2.2 and Lemma-3.3.2.3 we have $q_{\infty,j}(S) = \gamma(S)$. For $V \in [0, 1)$ we solve the equation:

$$
\begin{aligned}
q_{\infty,j}(S) &= \sum_{k=1}^{n} q_{\infty,k}(S) P_{ij}(S) \\
&= q_{\infty,j}(S)(1-V)\theta_j(S) + \sum_{\substack{k \neq j \\ k \in S}} \left(\frac{V}{|S-1|} + (1-V)\theta_j(S) \right) q_{\infty,k}(S)
\end{aligned}
$$

Note that have $\sum_{k \in S} q_{\infty,k}(S) = 1$ which implies $\sum_{\substack{k \neq j \\ k \in S}} q_{\infty,k}(S) = 1 - q_{\infty,j}(S)$. After plugging this expression in the above equation we get :

$$
q_{\infty,j}(S) = q_{\infty,j}(S)(1-V)\theta_i(S) + \left(\frac{V}{|S-1|} + (1-V)\theta_j(S) \right) (1 - q_{\infty,j}(S))
$$

which has a straightforward solution. For $V \in (-1, 0)$ we use the similar approach and the final result is:

$$
q_{\infty,j}(S) = \begin{cases} \boldsymbol{\gamma}(S) & V = -1 \\ \boldsymbol{\theta}(S) & -1 < V \leq 0 \\ (1-V)\theta_j(S)\frac{|S|-1}{|S|-1+V} + \frac{V}{|S|-1+V} & 0 \leq V \leq 1 \\ \boldsymbol{\gamma}(S) & V = 1 \end{cases} \tag{3.10}
$$

and $q_{\infty,j} = 0$ for $j \notin S$.

Note that interestingly, the steady state distribution jumps from $\boldsymbol{\gamma}(S)$ when $V = -1$ to $\theta(S)$ for $V = -1 + \epsilon$. This is because even a very small chance of buying a different product enables consumers preferences to converge to the popular products when the horizon is infinitely long.

We have the following result as a direct consequence of 3.10.

Lemma 3.4.1.1. *When $-1 < V \leq 0$ and $T = \infty$, $S^* = S_\theta$. When $V \in \{-1, 1\}$, then $S^* = S_\gamma$.*

This is an interesting result, because we see that in the long run, except extreme variety avoiders, having variety-avoiding consumers, do not have any affect on the optimal assortment. The static firm is safe when he disregards the existing variety avoiding behavior among the consumers because optimal solution in both cases are the same. The following theorem summarizes this result.

Theorem 3.4.1.2. *In the long-run, except extreme variety avoidance, ignoring existing variety-avoiding behavior among the consumers does not hurt the static firm.*

Lemma-3.4.1.1 also says that the optimal assortment is a popular set for variety avoiding case $V \in (-1, 0]$. What about variety-seeking case? We define a type of assortment set below, which also appears in Alptekinoğlu and Grasas (2014), to explain the structure of optimal assortment in that case.

Definition 3.4.1.3. *Suppose there are n different products with $u_1 \geq u_2 \geq .... \geq u_n$. The set S is* "popular-eccentric" *if, (i) the most popular product is in- b-* S and (ii) when product k is in S then either products $\{1, 2, .., k-1\}$ or $\{k+1, k+2, .., n\}$ are in S.*

Notice that popular set is also a popular-eccentric set with only popular products being in the set.

Lemma 3.4.1.4. *For $V \in [0, 1)$ the optimal assortment for the static firm infinite horizon problem is a popular-eccentric Set.*

We provide the proof in §3.6.1. The Proof is similar to Mahajan-van Ryzin (1999) popular set proof. The difference comes from adding a product with $u_k = 0$ to the assortment. In Mahajan-van Ryzin,1999 if you add a product with $u_k = 0$ to the assortment, the profit

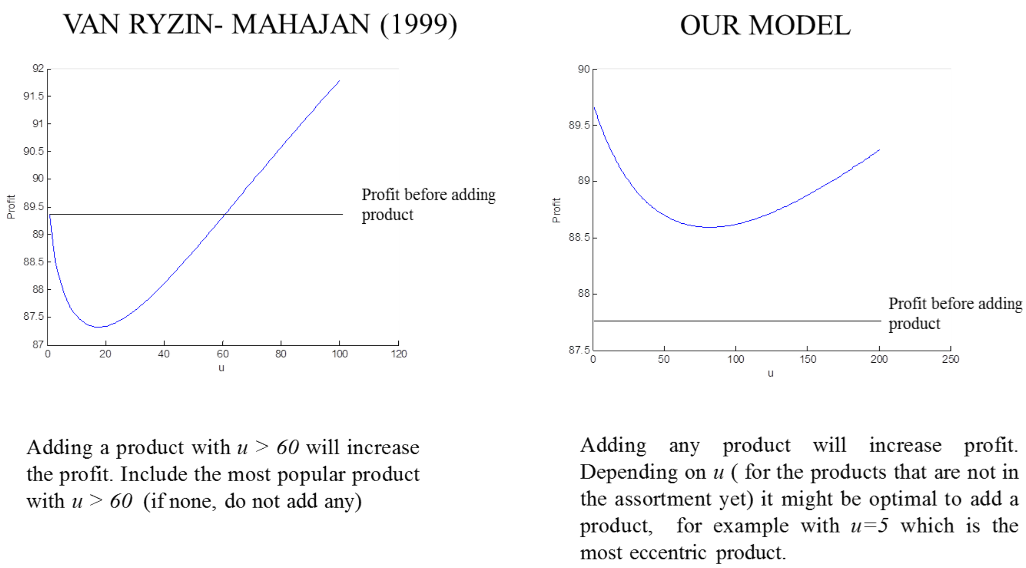


Figure 3.2: The explanation of proof for Lemma-3.4.1.4 - Ryzin and Mahajan (1999) vs our model comparison

doesn't change. But if you add a product with $u_k = 0$ in our model, for $V > 0$, some consumers will buy that product and this will change the profit (can be negative or positive)

The following is an example of an optimal assortment being popular-eccentric set.

Example 3.4.1.5. *Let* $n = 3$, $r = 4.2, \lambda = 50, \sigma = 4, \beta = 0.8$ $\mathbf{u} = (5, 250, 60, 10)$. *In this case the optimal assortment is* $\{1, 3\}$ *and the retailer's profit is 80.7265.*

Up to now, we have shown the structure of the optimal assortment for different values of V. Now we provide a result on the structure of the optimal profit. In order to prove our next result, we use the following definition and result from Ryzin and Mahajan (1999).

Definition 3.4.1.6. *We say vector* $\vec{x} = (x_1, ..., x_m)$ majorizes *vector* $\vec{y} = (y_1, ..., y_m)$*, denoted by* $\vec{x} \succeq_M \vec{y}$*, if* $\sum_{i=1}^{m} x_i = \sum_{i=1}^{m} y_i'$ *and* $\sum_{i=1}^{j} x_{[i]} \geq \sum_{i=1}^{j} y_{[i]}'$ *for* $j = 1, ..., m-1$*, where* $x_{[i]}$ *denotes the* i*-th highest component of vector* $\vec{x}$*.*

Example 3.4.1.7. *Consider* $\vec{x} = (3,1,5)$ *and* $\vec{y} = (3,3,3)$*. We have* $\vec{x} \succeq_M \vec{y}$ *because after sorting each vector in decreasing order we have* $(5,3,1)$ *and* $(3,3,3)$ *and* $5 > 3$, $5+3 > 3+3$ *and* $5+3+1 = 3+3+3$.

Lemma 3.4.1.8. *If* $g(\cdot)$ *is a convex function and* $\vec{x} \succeq_M \vec{y}$ *then* $\sum_{j=1}^{m} g(x_i) \geq \sum_{j=1}^{m} g(y_i)$.

Proof. See Lemma 2 in (Ryzin and Mahajan, 1999) □

Let $\pi_\infty(S;V)$ denote the per-period retailer's profit in an infinite horizon problem with static inventory S.

Theorem 3.4.1.9. *For any* S, $\pi_\infty(S;V)$ *is decreasing in* V *for* $V \in [0,1]$.

Proof. Without loss of generality let $S = \{1,2,...,n\}$ and assume that $u_1 \geq u_2 \geq ... \geq u_n$. First note that $q_{\infty,1} \geq q_{\infty,2} \geq ... \geq q_{\infty,n}$ so that the i-th largest element of $\vec{q}_\infty$ is the i-th element. For $j = 1,...,n$, we have:

$$\sum_{i=1}^{j} q_{\infty,i}(S;V) = (1-V)\frac{\sum_{i=1}^{j} u_i}{\sum_{k=1}^{n} u_k}\frac{|S-1|}{|S-1|+V} + \frac{jV}{|S-1|+V}$$

Taking the first derivative:

$$\frac{\partial \sum_{i=1}^{j} q_{\infty,i}(S;V)}{\partial V} = \frac{|S-1|}{(|S-1|+V)^2}\left(j - \frac{\sum_{i=1}^{j} u_i}{\sum_{k=1}^{n} u_k}|S|\right)$$

which is negative because we have $\frac{\sum_{i=1}^{j} u_i}{\sum_{k=1}^{n} u_k} \geq \frac{j}{|S|}$ given that $u_1 \geq u_2 \geq ... \geq u_n$. Therefore it follows that $q_{\infty,j}(S;V) \succeq_M q_{\infty,j}(S;V')$ for $V' > V$.

Let $g(q) = mq - \sigma(\lambda q)^\beta$. As Van Ryzin and Mahajan (1999) does, we assume $\lambda^{1-\beta} q >> \sigma q^\beta$. This implies that g is convex. Also we have $\pi_\infty(S;V) = \sum_{j=1}^{n} g(q_{\infty,j})$. Using Lemma 3.4.1.8, we conclude that $\pi_\infty(S;V) \geq \pi_\infty(S;V')$; therefore π_∞ is decreasing in V for $V \in [0,1]$. □

3.4.2 Cycling Firm

In this section we consider a retailer which cycles through a series of assortment over the infinite horizon. We refer to a *k-cycle* with sets $S^0, ..., S^{k-1}$ as an assortment policy such that $S_t = S^{\mod (t,k)}$ for $t = 1, ...,$ where $\mod (x, y)$ denotes the remainder when x is divided by y.

For example a 2-cycle policy with S^0 and S^1 is such that $S_t = S^1$ for $t = 1, 3, 5, ...$ and $S_t = S^0$ for $t = 2, 4, ...$

As we have shown in the previous section, we need stationary distributions in order to solve the assortment problem. Because of the dependency of the transition matrix on the previously offered assortment, a general solution of stationary distribution for an arbitrary k-cycle is intractable. To simplify the calculations we limit our study to 2-cycle assortments.

Steady State distribution for 2-cycle

Let $\mathring{q}_{\infty,j}(S_1|S_2)$ be the infinite horizon purchase probability of product j when assortment S_1 is offered and the preceding assortment was S_2. We have the following formula (derivation of the formula is in §3.6.2).

$$\mathring{q}_{\infty,j}(S_1|S_2) = \begin{cases} \frac{H^2(h^1-g^1)-H^1g^1(h^2-g^2)+g^1(1-g^2-(1-V)\theta_k(S^2))+(1-V)\theta_k(S^1)}{1-g^2g^1} & j \in S_1 \text{ and } j \in S_2 \\ h^1H^2 + g^1(1-H^2) + (1-V)\theta_k(S^1) & j \in S_1 \text{ and } j \notin S_2 \\ 0 & else \end{cases}$$

where

$$\begin{aligned} H^1 = &\ \frac{(m_2D_2D_1 - D_1m_2(m_2-1)(m_1-W_2) - m_1(m_1-1)m_2(m_2-1)W_1)}{D_2D_1 - m_1(m_1-1)m_2(m_2-1)} \\ m_1 = &\ |S_1|, \quad D_1 = V|S_1 \backslash S_2|, \quad h^1 = \frac{V}{m_1}, \quad g^1 = \frac{V}{m_1-1} \\ W_1 = &\ (1-V) \sum_{\substack{i\in S_1 \\ i\notin S_2}} \theta_i(S_1) \end{aligned}$$

Notice that the expression reduces to 3.10

Two-product analysis

Suppose there are only two products: product 1 and 2 with $u_1 > u_2 > u_0$. The retailer has three options. Let assortment $A = \{1\}$, assortment $B = \{2\}$ and assortment $C = \{1, 2\}$. We want to find out when it is optimal to have a 2-cycle solution. By using the notation for cycling case (We drop ∞ for the ease of reading) the 1-cycle (or non-cycle) probabilities can be written as:

$$\mathring{q}_1(A|A) = (1-V)\theta_1(A)\frac{1}{1+V} + \frac{V}{1+V} \qquad \mathring{q}_2(B|B) = (1-V)\theta_2(B)\frac{1}{1+V} + \frac{V}{1+V}$$

$$\mathring{q}_1(C|C) = (1-V)\theta_1(C)\frac{2}{2+V} + \frac{V}{2+V} \qquad \mathring{q}_2(C|C) = (1-V)\theta_2(C)\frac{2}{2+V} + \frac{V}{2+V}$$

When there are only two products, the 2-cycle options are AB, AC and BC. When the assortment is cycling between A and B we have $\mathring{q}_1(B|A) = 0$, $\mathring{q}_2(A|B) = 0$ and

$$\begin{aligned}
\mathring{q}_1(A|B) &= \frac{2V}{(2+V)} + \frac{2(1-V)}{4-V^2}(2\theta_1(A) - V\theta_2(B)) \\
\mathring{q}_2(B|A) &= \frac{2V}{(2+V)} + \frac{2(1-V)}{4-V^2}(2\theta_2(B) - V\theta_1(A))
\end{aligned}$$

When the assortment is cycling between A and C we have $\mathring{q}_1(A|C) = 0$ and

$$\begin{aligned}
\mathring{q}_1(A|C) &= \frac{V(4-3V)}{2(2-V^2)} - \frac{2(1-V)}{2-V^2}V\theta_1(C) + \frac{(1-V)}{2-V^2}(2\theta_1(A) - V\theta_2(C)) \\
\mathring{q}_1(C|A) &= \frac{V(2-V)^2}{4(2-V^2)} + \frac{2(1-V)}{2-V^2}\theta_1(C) - \frac{(1-V)}{2(2-V^2)}(2\theta_1(A) - V\theta_2(C)) \\
\mathring{q}_2(C|A) &= \frac{V}{2} + (1-V)\theta_2(C)
\end{aligned}$$

When the assortment is cycling between B and C we have $\mathring{q}_1(B|C) = 0$ and

$$\begin{aligned}
\mathring{q}_2(B|C) &= \frac{V(4-3V)}{2(2-V^2)} - \frac{2(1-V)}{2-V^2}V\theta_2(C) + \frac{(1-V)}{2-V^2}(2\theta_2(B) - V\theta_1(C)) \\
\mathring{q}_2(C|B) &= \frac{V(2-V)^2}{4(2-V^2)} + \frac{2(1-V)}{2-V^2}\theta_2(C) - \frac{(1-V)}{2(2-V^2)}(2\theta_2(B) - V\theta_1(C)) \\
\mathring{q}_1(C|B) &= \frac{V}{2} + (1-V)\theta_1(C)
\end{aligned}$$

By using the above expressions for purchase probabilities in different cycle combinations, we can compare non-cycle and 2-cycle profits. We compare profit in a period, for example $\pi(A|A)$ with $\frac{\pi(A|C)+\pi(C|A)}{2}$. We have the following two lemma regarding the optimal solutions for $V=0$ and $V=-1$ cases.

Lemma 3.4.2.1. *When $V=0$, k-cycle, s.t. $k \geq 2$ is never optimal.*

Proof. It is a direct result from Lemma-3.3.2.1 □

Lemma 3.4.2.2. *When $V=1$;*

(a) Offering only A or Only B yields the same profit. (which is also true for $k>2$)

(b) For $\beta<1$ cycling between assortments A and B always gives more profit than offering only assortment C. For $\beta=1$ cycling between assortment A and B and offering only assortment C yields the same profit. (both are optimal)

(c) For $m \geq \sigma\lambda^{\beta-1}$, the optimal assortment is cycling between assortment A and B.

Proof. (a) We have $\mathring{q}_1(A|A)=\mathring{q}_2(B|B)=\frac{1}{2}$ which implies that it offering either products yields the same profit.

(b) $\pi(C|C)-\frac{\pi(A|B)+\pi(B|A)}{2}=-\frac{\sigma\lambda^\beta}{3^\beta}(2-2^\beta)<0$ for $\beta<1$ which implies cycling between assortment A and B always yields more profit than offering only assortment C.

(c) We compare offering only A with cycling between A and B.

$\pi(A|A)-\frac{\pi(A|B)+\pi(B|A)}{2}=2^{-\beta}\left(-1+(4/3)^\beta\right)\sigma\lambda^\beta-\frac{\lambda m}{6}$.

We have $2^{-\beta}\left(-1+(4/3)^\beta\right)\leq\frac{1}{6}$ which implies if $m\geq\sigma\lambda^{\beta-1}$ (or we can say $m\geq\sigma$) then cycling between assortment A and B yields more profit than offering only assortment A.

Then we compare cycling A and B and cycling A and C.

$\frac{\pi(A|C)+\pi(C|A)}{2}-\frac{\pi(A|B)+\pi(B|A)}{2}=\left(\left(\frac{2}{3}\right)\beta-\frac{1}{2^{(1+2\beta)}}-\frac{1}{2^\beta}\right)\sigma\lambda^\beta-\frac{\lambda m}{24}$. Which is negative for $m\geq\sigma\lambda^{\beta-1}$ implying, cycling between assortment A and B yields more profit than cycling between assortment A and C .

We have $\frac{\pi(A|B)+\pi(B|A)}{2}>\frac{\pi(A|C)+\pi(C|A)}{2}$ and $\frac{\pi(A|B)+\pi(B|A)}{2}>\pi(A|A)=\pi(B|B)$ and from (b)

$\frac{\pi(A|B)+\pi(B|A)}{2} > \pi(C|C)$. □

3.4.3 Dynamic Firm

In this section we study the optimal policy in the infinite horizon for a dynamic firm, that is a firm which is free to vary the assortment from period to period. In particular we study in which case a static or k-cycle policy is optimal .

Lemma 3.4.3.1. *A static (1-cycle) policy is optimal in the infinite horizon problem when $V = \{-1, 0\}$. For $V = -1$, we have $S_t^* = S_{\boldsymbol{\gamma}}$ for $t = 1, ..., \infty$. For $V = 0$, we have $S_t^* = S_{\boldsymbol{\theta}}$ for $t = 1, ..., \infty$.*

Proof. Follows directly from Lemmas 3.3.2.2 and 3.3.2.1. □

3.5 Numerical Analysis

Our numerical study is divided into three parts. The objective of part I is to compare the performance of the dynamic firm with that of the static firm. In the second part we investigate the value of cycling the assortment for the infinite horizon problem. And in the last part, we show numerical examples about the magnitude of profit loss when the retailer ignores the variety seeking/avoiding behavior in the market.

3.5.1 Value of Dynamic Assortment

In this section we study the value of varying the assortment from period to period. We numerically compare the retailer's profit under (i) the optimal dynamic firm policy (ii) the optimal static firm. We try to see how two different firm types differ in terms of profit depending on V and T. We assume there are 3 products (Product 1 is the most popular and product 3 is the eccentric product) that can be offered. We use the parameters $r = 5$, $\sigma = 4.8$, $\beta = 0.9$, $\lambda = 50$ and $u = \{10, 60, 20, 5\}$. In table 3.5 we present the profit

Table 3.5: Static and Dynamic Firm profit comparison by T

	Static Firm		Dynamic Firm		
T	Assortment	Total Profit	Assortment	Total Profit	%Profit Difference
1	$\{1\}$	62.2236	$\{1\}$	62.2236	0
2	$\{1,3\}$	119.6562	$\{1\}$-$\{1,2\}$	121.1195	1.29
3	$\{1,3\}$	178.5402	$\{1\}$-$\{2\}$-$\{1\}$	182.4124	2.17
4	$\{1,3\}$	237.3652	$\{1\}$-$\{1,3\}$-$\{2\}$-$\{1\}$	240.7544	1.43
5	$\{1,3\}$	296.1990	$\{1\}$-$\{2\}$-$\{1\}$-$\{2\}$-$\{1\}$	301.6901	1.95

difference between dynamic and static firm by T when $V = 0.3$. For the one period problem the optimal assortments for the two type of firms are the same and it is offering the most popular product. When $T = 2$, the optimal assortment for the static firm becomes the most popular product and the eccentric product. As we discussed before, having popular and eccentric product at the same time in the assortment is a result of variety seeking behavior. The Dynamic firm optimal assortment is offering the popular product in period-1 and offering the most popular two products in period-2. We see that while static firm offers the eccentric product, dynamic firm does not. The optimal assortment does not change for static firm as T increases after $T = 2$. For the dynamic firm we observe the cyclic pattern when $T = 5$. The average profit difference is around 2%.

In Table 3.6 we present the profit difference between dynamic and static firm by V when $T = 4$. We observe that the difference between dynamic firm and static firm is very low when consumer are variety avoiders ($V < 0$), whereas it is close to 9% when consumers are highly variety seekers. We also see that optimal profit decreases as consumers become variety seeker/avoider (as V deviates from zero). As seen when $V = -0.9$ case, in the last period, dynamic firm drops product 3 from the assortment. As a general numerical observation we observe that, when V is less than zero the dynamic firm never adds a product to the previous period's assortment.

Table 3.6: Static and Dynamic Firm profit comparison by V

V	Static Firm Assortment	Static Firm Total Profit	Dynamic Firm Assortment	Dynamic Firm Total Profit	%Profit Difference $\left(\frac{Dynamic-Static}{Static} \times 100\right)$
-0.9	$\{1,2,3\}$	202.8351	$\{1,2,3\}$-$\{1,2,3\}$-$\{1,2,3\}$-$\{1,2\}$	203.0012	0.08
-0.5	$\{1\}$	258.4016	$\{1\}$-$\{1\}$-$\{1\}$-$\{1\}$	258.4016	0
0	$\{1\}$	292.047	$\{1\}$-$\{1\}$-$\{1\}$-$\{1\}$	292.047	0
0.5	$\{1,2\}$	217.9938	$\{2,3\}$-$\{1\}$-$\{2\}$-$\{1\}$	227.1293	4.19
0.9	$\{1,2,3\}$	194.7385	$\{2,3\}$-$\{1\}$-$\{2\}$-$\{1\}$	211.9995	8.86

3.5.2 Value of a Cyclic Assortment

As we discussed in section 3.4 , for infinite horizon problem when consumers are variety seekers, it might be better to cycle between two assortments instead of offering 1 assortment. In this section we provide some numerical examples about the magnitude of profit increase when cycling is better, namely we compare infinite static and infinite cycling firms. We assume there are 3 products and use the same parameters as section. Notice that 1 cycle also can be a solution for cycling firm, which is a special case of cycling between the same same assortment. The results are presented in Table 3.7 The profit values in the table are per period. According to the results, for $V \leq 0$ infinite static and infinite-cycling firms has the same solution. But for V>0, the optimal assortment for the cycling firm is offering product 1 in one period and product 2 in the other period. Compared with infinite static firm, this strategy yields 12.35% more profit when the consumers are highly variety seekers. In this example the optimal assortments are different for the two firm types when V>0 but it is not always the case. We may have infinite static and cycling firms have the same optimal assortment when V>0, depending on the other parameters.

3.5.3 Value of Variety-Seeking Behavior

In this section we discuss the impact of the retailer ignoring the consumers' attitude towards variety. Specifically we assume that the retailer considers that $V = 0$ and optimizes the assortment accordingly, offering $S = S_{\boldsymbol{\theta}}$ in each period.

Table 3.7: Infinite Static and Cycling Firm profit comparison by V

V	Infinite-Static Firm		Infinite-Cycling Firm		%Profit Difference
	Assortment	Profit	Assortment	Profit	$\left(\frac{Cycling-Static}{Static} \times 100\right)$
-0.9	$\{1\}$	73.0118	$\{1\}$	73.0118	0
-0.5	$\{1\}$	73.0118	$\{1\}$	73.0118	0
0	$\{1\}$	73.0118	$\{1\}$	73.0118	0
0.5	$\{1, 2\}$	54.0666	$\{1\} \longleftrightarrow \{2\}$	57.7791	6.87
0.9	$\{1, 2, 3\}$	48.6273	$\{1\} \longleftrightarrow \{2\}$	54.6373	12.35

Table 3.8: Finite Horizon Ignorant Firm profit loss by V

V	Ignorant Firm		% Profit Difference	
	Assortment	Total Profit	Static Firm	Dynamic Firm
-0.9	$\{1\}$	182.7487	10.99	11.08
-0.5	$\{1\}$	258.4016	0	0
0	$\{1\}$	292.047	0	0
0.5	$\{1\}$	201.0796	8.41	12.95
0.9	$\{1\}$	159.4545	22.13	32.95

Finite Horizon:

In Table 3.8 we compare the ignorant firm profit to the static and dynamic firm of finite horizon with $T = 4$. We use the same parameters as before. The optimal assortment when V=0 is offering only product 1 as seen in the table, the total profit corresponding the Ignorant Firm is the profit from offering only product 1 with purchase probabilities calculated by using the real V values. The results show that not taking into account the variety seeking behavior can result in a profit loss of about 33% when there exist a significant variety seeking tendency.

Infinite Horizon:

We do similar analysis with the same parameters for the infinite horizon case and compare ignorant firm profit with infinite static and infinite cycling firms. The results are in Table 3.9 According to the results ignorant firm is losing about 37% percent profit when the variety

Table 3.9: Infinite Horizon Ignorant Firm profit loss by V

	Ignorant Firm		% Profit Difference	
V	Assortment	Total Profit	Infinite-Static Firm	Infinite-Cycling Firm
-0.9	$\{1\}$	73.0118	0	0
-0.5	$\{1\}$	73.0118	0	0
0	$\{1\}$	73.0118	0	0
0.5	$\{1\}$	49.3558	9.54	17.06
0.9	$\{1\}$	39.7884	22.21	37.31

seeking tendency is very high. Even when it is moderate, the loss is around 17% which is a big amount especially in the retail market.

In the infinite horizon problem with static firm, for $V \in (-1, 0]$, the optimal assortment is to set $S^* = S_{\boldsymbol{\theta}}$ therefore the retailer who ignores consumers attitude towards variety will not suffer any profit lost. However, if $V \in (0, 1)$, he might. In particular if $V = 1$, the optimal assortment is $S^* = S_{\boldsymbol{\gamma}}$ but the retailer who ignores variety will set $S_{\boldsymbol{\theta}}$.

The profit difference in this case is given by:

$$\begin{aligned}\pi_\infty(S_{\boldsymbol{\gamma}}) - \pi_\infty(S_{\boldsymbol{\theta}}) = &\sum_{j=1}^{n} \left\{ r\lambda q_{\infty,j}(S_{\boldsymbol{\gamma}}) - \sigma \left[\lambda q_{\infty,j}(S_{\boldsymbol{\gamma}})\right]^\beta \right\} \\ &- \sum_{j=1}^{n} \left\{ r\lambda q_{\infty,j}(S_{\boldsymbol{\theta}}) - \sigma \left[\lambda q_{\infty,j}(S_{\boldsymbol{\theta}})\right]^\beta \right\}\end{aligned}$$

When $V = 1$ we have $\mathbf{q}_\infty(S) = \boldsymbol{\gamma}(S)$ for all S. Therefore,

$$\begin{aligned}\pi_\infty(S_{\boldsymbol{\gamma}}) - \pi_\infty(S_{\boldsymbol{\theta}}) &= \sum_{j \in S_{\boldsymbol{\gamma}}} \left\{ r\lambda \frac{1}{|S_{\boldsymbol{\gamma}}|} - \sigma \left[\lambda \frac{1}{|S_{\boldsymbol{\gamma}}|}\right]^\beta \right\} - \sum_{j \in S_{\boldsymbol{\theta}}} \left\{ r\lambda \frac{1}{|S_{\boldsymbol{\theta}}|} - \sigma \left[\lambda \frac{1}{|S_{\boldsymbol{\theta}}|}\right]^\beta \right\} \\ &= r\lambda \left(\frac{|S_{\boldsymbol{\gamma}}| - 1}{|S_{\boldsymbol{\gamma}}|} - \frac{|S_{\boldsymbol{\theta}}| - 1}{|S_{\boldsymbol{\theta}}|} \right) - \sigma \begin{pmatrix} (|S_{\boldsymbol{\gamma}}| - 1) \left[\lambda \frac{1}{|S_{\boldsymbol{\gamma}}|}\right]^\beta \\ -(|S_{\boldsymbol{\theta}}| - 1) \left[\lambda \frac{1}{|S_{\boldsymbol{\theta}}|}\right]^\beta \end{pmatrix}\end{aligned}$$

As a result we see that the difference in profits only depends on the difference in size of the two assortments. In table 3.10 we show the magnitude of this difference depending on different parameters and product numbers. According to the results, when there are

Table 3.10: S_γ and S_θ comparison when $V = 1$

Parameters	u	S_θ	$\pi(S_\theta)$	S_γ	$\pi(S_\gamma)$	% Difference
$r = 4.8,\ \sigma = 4$ $\lambda = 50,\ \beta = 0.8$	$(10, 50, 20, 9, 70$	$\{1\}$	67.46	$\{1, 2, 3, 4\}$	91.04	25.89
	$(60, 50, 20, 9, 7)$	$\{1, 2, 3\}$	89.48	$\{1, 2, 3, 4\}$	91.04	1.71
	$(600, 50, 20, 9, 7)$	$\{1, 2, 3\}$	89.48	$\{1, 2, 3, 4\}$	91.04	1.71
$r = 4.8,\ \sigma = 4$ $\lambda = 50,\ \beta = 0.95$	$(5, 100, 60, 40, 30, 15, 10)$	$\{1\}$	34.86	$\{1, 2, 3, 4, 5, 6\}$	50.33	30.73
$r = 12,\ \sigma = 4$ $\lambda = 50,\ \beta = 0.9$	$(5, 100, 60, 50, 45, 44, 43, 42, 41, 40, 39)$	$\{1\}$	227.52	$\{1, 2, 3, 4, 5, 6, 7, 8, 9, 10\}$	441.99	48.52

4 products the highest difference occurs when outside option is not very popular. As it becomes more popular, the difference increases. When there are 6 and 10 products, we see bigger differences. In those cases, while S_θ is only the most popular product, S_γ includes all the products. For 10 product case, S_γ yields 48.52% more profit.

3.6 Proof of Some Results

3.6.1 Proof of Lemma 3.4.1.4

Proof: Proof is very similar to Ryzin and Mahajan (1999). We show that profit associated with adding a product is quasi-convex and thus optimal can be achieved in extremes of the function. In Ryzin and Mahajan (1999) adding a product with u=0 makes the profit same with offering only S. But in our case, due to the variety seeking behavior, even a product with $u = 0$ can increase the profit. Let $\frac{|S|}{|S|+V} = \omega$ and $\frac{V}{|S|+V} = \varphi$.When we add product k to assortment S the purchase probability of product j becomes:

$$q_{\infty,j}(S \cup \{k\}) = \frac{u_j(1-V)\omega}{\sum_{j \epsilon S} u_j + u_k} + \varphi$$

Then the profit becomes:

$$\begin{aligned}
\pi(S\cup\{k\}) &= \sum_{j\in S\cup\{k\}} m\lambda q_{\infty,j}(S\cup\{k\}) - \sigma c(\lambda q_{\infty,j}(S\cup\{k\}))^{\beta} \\
&= m\lambda \sum_{j\in S\cup\{k\}} \left(\frac{u_j(1-V)\omega}{\sum_{j\epsilon S} u_j + u_k} + \varphi\right) - \sigma c\lambda^{\beta} \sum_{j\in S\cup\{k\}} \left(\frac{u_j(1-V)\omega}{\sum_{j\epsilon S} u_j + u_k} + \varphi\right)^{\beta} \\
&= \frac{m\lambda\left(|S+1|\varphi + (1-V)\omega\left[\sum_{j\in S} u_j + u_k\right]\right)}{\sum_{j\epsilon S} u_j + u_k} \\
&\quad - \frac{\sigma c\lambda^{\beta}}{\left(\sum_{j\epsilon S} u_j + u_k\right)^{\beta}} \left[\begin{array}{c} \sum_{j\epsilon S}\left(u_j(1-V)\omega + \varphi\sum_{j\epsilon S} u_j + \varphi u_k\right)^{\beta} \\ + \left(u_k\left((1-V)\omega + \varphi\right) + \varphi\sum_{j\epsilon S} u_j\right)^{\beta} \end{array}\right]
\end{aligned}$$

Let $0 \le \delta \le u_1$. and define the following functions:

$$f(\delta) = \left(\sum_{j\epsilon S} u_j + \delta\right) \tag{3.11}$$

$$g(\delta) = m\lambda\left(|S+1|\varphi + (1-V)\omega\left[\sum_{j\in S\cup\{k\}} u_j + \delta\right]\right)$$

$$- \sigma c\lambda^{\beta}\left[\sum_{j\in S}\left(u_j(1-V)\omega + \varphi\sum_{j\epsilon S} u_j + \varphi\delta\right)^{\beta} + \left(\delta(1-V)\omega + \varphi\left(\sum_{j\epsilon S} u_j + \delta\right)\right)^{\beta}\right] f^{1-\beta}$$

We will show that $h(\delta) = \frac{g(\delta)}{f(\delta)}$ is quasi-convex. We use the following result from Mangasarian (1969): "The function $g(\cdot)/f(\cdot)$ is quasi-convex on X if (i) $g(\cdot)$ is convex and $f(\cdot) > 0$ for all $v \in X$ and (ii) $f(\cdot)$ is linear on X."

The function $f(\cdot)$ is linear in δ. We need to show that $g(\cdot)$ is convex in δ. Notice that x^{β} is concave for $\beta \le 1$. Then, $\sum_{j\in S}\left(u_j(1-V)\omega + \varphi\sum_{j\epsilon S} u_j + \varphi\delta\right)^{\beta}$ and $\left(\delta(1-V)\omega + \varphi\left(\sum_{j\epsilon S} u_j + \delta\right)\right)^{\beta}$ are increasing concave in δ and summation of two increasing concave is also increasing concave. Besides, $f^{1-\beta}$ is increasing concave and multiplication of increasing concave functions is concave. And finally, linear minus concave is convex.

The function $h(\delta) = \frac{g(\delta)}{f(\delta)}$ represents the profit associated with adding a variant with preference δ to the existing set S. In Ryzin and Mahajan (1999) when $\delta = 0$ the function reduces to $\pi(S)$ but in our case it is not. But if you add a product with $\delta = 0$ in our model, for $V > 0$, some consumers will buy that product due to variety seeking behavior and this will change the profit (can be negative or positive) Which means even adding a product with preference $\delta = 0$ will change the profit (can be negative of positive).

3.6.2 Derivation of Steady State Distribution for 2-Cycle

Suppose the retailer cycles between assortments S_i and S_j. We try to find $\mathring{q}_{\infty,k}(S_i|S_j)$ which is the stationary probability of purchasing product k in assortment S_i given that previous assortment was S_j. Assuming that stationary distribution exists, we should have:

$$\begin{aligned} \mathring{q}_k(S_i|S_j) &= \sum_{\xi \in S_i} \mathring{q}_\xi(S_i|S_j) P_{\xi k}(S_i|S_j) \\ &= \sum_{\xi \in S_i} \mathring{q}_\xi(S_i|S_j) \sum_{\omega \in S_j} \mathring{q}_\omega(S_i|S_j) P_{\omega k}(S_i) \end{aligned}$$

For $0 \leq V \leq 1$ we get:

$$\mathring{q}_k(S_i|S_j) = (1-V)\theta_k(S_i) + \frac{V}{|S_i|} \sum_{\substack{\xi \in S_i \\ \xi \notin S_j \\ \xi \neq k}} \mathring{q}_\xi(S_j|S_i) + \frac{V}{|S_i - 1|} \sum_{\substack{\xi \in S_i \\ \xi \in S_j \\ \xi \neq k}} \mathring{q}_\xi(S_j|S_i)$$

Let $|S_i - S_j| = \Delta_i$ and $|S_i \cap S_j| = \Gamma$. Also define: $H^i = \sum_{\substack{k \in S_i \\ k \notin S_j}} \mathring{q}_k(S_i|S_j)$ which is the sum of stationary purchase probabilities of products that are in S_i but not in S_j (Distinct products) and $G^i = \sum_{\substack{k \in S_i \\ k \in S_j}} \mathring{q}_k(S_i|S_j)$ which is the sum of stationary purchase probabilities of products that are in S_i and in S_j (Common Products).

Notice that $H^i + G^i = 1$ and we can write $G^i = 1 - H^i$. We first derive H^i and then $\mathring{q}_k(S_i|S_j)$.

Derivation of H^i :

(i) Consider product j where $j \in S_j$ and $j \notin S_i$, We have:

$\sum\limits_{\substack{k \in S_i \\ k \notin S_j \\ k \neq j}} \mathring{q}_k(S_i|S_j) = \sum\limits_{\substack{k \in S_i \\ k \notin S_j}} \mathring{q}_k(S_i|S_j) = H^i$ and $\sum\limits_{\substack{k \in S_i \\ k \in S_j \\ k \neq j}} \mathring{q}_k(S_i|S_j) = \sum\limits_{\substack{k \in S_i \\ k \in S_j}} \mathring{q}_k(S_i|S_j) - \mathring{q}_j(S_i|S_j)$

Since $j \notin S_i \Rightarrow \mathring{q}_j(S_i|S_j) = 0$ and thus $\sum\limits_{\substack{k \in S_i \\ k \in S_j \\ k \neq j}} \mathring{q}_k(S_i|S_j) = G^i$. Define $h^i = \frac{V}{|S_i|}$ and $g^i = \frac{V}{|S_i - 1|}$.

For the product j with $j \in S_j$ and $j \notin S_i$, we have:

$$\begin{aligned} \mathring{q}_j(S_j|S_i) &= h^j \sum_{\substack{k \in S_i \\ k \notin S_j \\ k \neq j}} \mathring{q}_k(S_i|S_j) + g^j \sum_{\substack{k \in S_i \\ k \in S_j \\ k \neq j}} \mathring{q}_k(S_i|S_j) + (1 - V)\theta_j(S_j) \\ &= h^j H^i + g^j G^i + (1 - V)\theta_j(S_j) \end{aligned}$$

Then we can write:

$$\begin{aligned} H^j = \sum_{\substack{j \in S_j \\ j \notin S_i}} \mathring{q}_j(S_j|S_i) &= \sum_{\substack{j \in S_j \\ j \notin S_i}} \left[h^j H^i + g^j G^i + (1 - V)\theta_j(S_j) \right] \\ &= \Delta_2 h^j H^i + \Delta_2 g^j G^i + (1 - V) \sum_{\substack{j \in S_j \\ j \notin S_i}} \theta_j(S_j) \end{aligned}$$

As a result we get:

$$H^j = \Delta_2 \left[h^j H^i + g^j G^i \right] + (1 - V) \sum_{\substack{j \in S_j \\ j \notin S_i}} \theta_j(S_j)$$

and by using similar calculations we can easily get:

$$H^i = \Delta_1 \left[h^i H^j + g^i G^j \right] + (1 - V) \sum_{\substack{J \in S_i \\ J \notin S_j}} \theta_j(S_i)$$

(ii) Now consider product j such that $j \in S_j$ and $j \in S_i$. We have $\sum\limits_{\substack{k \in S_i \\ k \notin S_j \\ k \neq j}} \mathring{q}_k(S_i|S_j) = \sum\limits_{\substack{k \in S_i \\ k \notin S_j}} \mathring{q}_k(S_i|S_j) = H^i$. So we can write $\mathring{q}_j(S_j|S_i)$ as:

$$\begin{aligned}
\mathring{q}_j(S_j|S_i) &= h^j \sum_{\substack{k \in S_i \\ k \notin S_j \\ k \neq j}} \mathring{q}_k(S_i|S_j) + g^j \sum_{\substack{k \in S_i \\ k \in S_j \\ k \neq j}} \mathring{q}_k(S_i|S_j) + (1-V)\theta_j(S_j) \\
&= h^j H^i + g^j \left(G^i - \mathring{q}_j(S_i|S_j)\right) + (1-V)\theta_j(S_j)
\end{aligned}$$

We use the same trick and sum $\mathring{q}_j(S_j|S_i)$ for all $j \in S_j$ and $j \in S_i$ to get G^j.

$$\begin{aligned}
G^j = \sum_{\substack{j \in S_j \\ j \in S_i}} \mathring{q}_j(S_j|S_i) &= \sum_{\substack{j \in S_j \\ j \in S_i}} \left[h^j H^i + g^j \left(G^i - \mathring{q}_j(S_i|S_j)\right) + (1-V)\theta_j(S_j)\right] \\
&= \sum_{\substack{j \in S_j \\ j \in S_i}} h^j H^i + \sum_{\substack{j \in S_j \\ j \in S_i}} g^j \left(G^i - \mathring{q}_j(S_i|S_j)\right) + \sum_{\substack{j \in S_j \\ j \in S_i}} (1-V)\theta_j(S_j) \\
&= \Gamma \left[h^j H^i + g^j \left(G^i\right)\right] - g^j \sum_{\substack{j \in S_j \\ j \in S_i}} \mathring{q}_j(S_i|S_j) + \sum_{\substack{j \in S_j \\ j \in S_i}} (1-V)\theta_j(S_j) \\
&= \Gamma h^j H^i + (\Gamma - 1) g^j G^i + \sum_{\substack{j \in S_j \\ j \in S_i}} (1-V)\theta_j(S_j)
\end{aligned}$$

As a result we have:

$$G^j = \Gamma h^j H^i + (\Gamma - 1) g^j G^i + \sum_{\substack{j \in S_j \\ j \in S_i}} (1-V)\theta_j(S_j)$$

(iii) Now we go back and reconsider $j \in S_j$ and $j \notin S_i$ case.

$$\begin{aligned}
\mathring{q}_j(S_j|S_i) &= h^j H^i + g^j G^i + (1-V)\theta_j(S_j) \\
&= h^j \left[\Delta_1 \left[h^i H^j + g^i G^j \right] + (1-V) \sum_{\substack{J \in S_i \\ J \notin S_j}} \theta_j(S_i) \right] \\
&+ g^j \left[\Gamma h^i H^j + (\Gamma - 1) g^i G^j + (1-V) \sum_{\substack{J \in S_i \\ J \in S_j}} \theta_j(S_i) \right] + (1-V)\theta_j(S_j) \\
&= h^j \Delta_1 h^i H^j + h^j \Delta_1 g^i G^j + h^j (1-V) \sum_{\substack{J \in S_i \\ J \notin S_j}} \theta_j(S_i) \\
&+ g^j \Gamma h^i H^j + g^j (\Gamma - 1) g^i G^j + (1-V) \sum_{\substack{J \in S_i \\ J \in S_j}} \theta_j(S_i) + (1-V)\theta_j(S_j) \\
&= h^i H^j \left[h^j \Delta_1 + g^j \Gamma \right] + g^i G^j \left[h^j \Delta_1 + g^j (\Gamma - 1) \right] + K3 + (1-V)\theta_j(S_j)
\end{aligned}$$

Define: $K_1 = h^i \left[h^j \Delta_1 + g^j \Gamma \right]$, $K_2 = g^i \left[h^j \Delta_1 + g^j (\Gamma - 1) \right]$

and $K_3 = h^j \sum_{\substack{J \in S_i \\ J \notin S_j}} M_j(S_i) + g^j \sum_{\substack{j \in S_i \\ j \in S_j}} M_j(S_i)$

Then we get:

$$\begin{aligned}
\mathring{q}_j(S_j|S_i) &= H^j K_1 + G^j K_2 + K_3 + (1-V)\theta_j(S_j) \\
&= H^j K_1 + (1 - H^j) K_2 + K_3 + (1-V)\theta_j(S_j) \\
&\Rightarrow \sum_{\substack{j \in S_j \\ j \notin S_i}} \mathring{q}_j(S_j|S_i) = \Delta_2 \left[H^j K_1 + (1 - H^j) K_2 + K_3 \right] + \sum_{\substack{j \in S_j \\ j \notin S_i}} (1-V)\theta_j(S_j) \\
&\Rightarrow H^j = \Delta_2 \left[K_2 + H^j (K_1 - K_2) + K_3 \right] + \sum_{\substack{j \in S_j \\ j \notin S_i}} (1-V)\theta_j(S_j) \\
&\Rightarrow H^j = \frac{\Delta_2 \left[K_2 + K_3 \right] + \sum_{\substack{j \in S_j \\ j \notin S_i}} (1-V)\theta_j(S_j)}{1 - \Delta_2 (K_1 - K_2)}
\end{aligned}$$

After simplifications, for $0 \leq V \leq 1$ we get:

$$H^i = \frac{(Y_i D_i D_j - D_j \bar{Y}_i (Y_j - W_i) - \bar{Y}_j \bar{Y}_i W_j)}{D_i D_j - \bar{Y}_j \bar{Y}_i}$$

Where :

$$\begin{aligned} Y_i = &\ |S_i| \\ \bar{Y}_i = &\ Y_i(Y_i - 1) \\ D_i = &\ V|S_i - S_j| \\ W_i = &\ (1-V) \sum_{\substack{J \in S_i \\ J \notin S_j}} \theta_j(S_i) \end{aligned}$$

Now we are ready to derive $\mathring{q}_k(S_i|S_j)$.

Derivation of $\mathring{q}_k(S_i|S_j)$:

Consider $k \in S_i$ and $k \notin S_j$. We have:

$$\mathring{q}_k(S_i|S_j) = h^i H^j + g^i(1 - H^j) + (1-V)\theta_k(S_i)$$

$$\mathring{q}_k(S_j|S_i) = 0$$

Now consider $k \in S_j$ and $k \in S_i$. We have:

$$\mathring{q}_k(S_j|S_i) = h^j H^i + g^j \left(1 - H^i - \mathring{q}_j(S_i|S_j)\right) + (1-V)\theta_j(S_j)$$

$$\mathring{q}_k(S_i|S_j) = h^i H^j + g^i \left(1 - H^j - \mathring{q}_k(S_j|S_i)\right) + (1-V)\theta_k(S_i)$$

We plug in $\mathring{q}_k(S_i|S_j)$ in $\mathring{q}_k(S_j|S_i)$ and solve the equation. After some algebra we get:

$$\mathring{q}_k(S_j|S_i) = \frac{H^i(h^j - g^j) - H^j g^j (h^i - g^i) + g^j(1 - g^i - (1-V)\theta_k(S_i)) + (1-V)\theta_k(S_j)}{1 - g^i g^j}$$

REFERENCES

Allenby, G. M., Shively, T. S., Yang, S., and Garratt, M. J. (2004). A choice model for packaged goods: Dealing with discrete quantities and quantity discounts. *Marketing Science*, 23(1):95–108.

Alptekinoğlu, A. and Grasas, A. (2014). When to carry eccentric products? optimal retail assortment under consumer returns. *Production and Operations Management*, 23(5):877–892.

Bawa, K. (1990). Modeling inertia and variety seeking tendencies in brand choice behavior. *Marketing Science*, 9(3):263–278.

Caldentey, R. and Caro, F. (2010). Dynamic assortment planning. Technical report, Working paper, UCLA.

Caro, F. and Gallien, J. (2007). Dynamic assortment with demand learning for seasonal consumer goods. *Management Science*, 53(2):276–292.

Caro, F. and Martinez-de Albeniz, V. (2009). Strategic assortment rotation.

Chen, L. and Plambeck, E. L. (2008). Dynamic inventory management with learning about the demand distribution and substitution probability. *Manufacturing & Service Operations Management*, 10(2):236–256.

Chintagunta, P. K. (1998). Inertia and variety seeking in a model of brand-purchase timing. *Marketing Science*, 17(3):253–270.

Feinberg, F. M., Kahn, B. E., and McAlister, L. (1992). Market share response when consumers seek variety. *Journal of Marketing Research*, 29(2):227.

Gerstner, E. and Hess, J. D. (1987). Why do hot dogs come in packs of 10 and buns in 8s or 12s? a demand-side investigation. *Journal of business*, 60(4):491–517.

Givon, M. (1984). Variety seeking through brand switching. *Marketing Science*, 3(1):1–22.

Granger, C. W. and Billson, A. (1972). Consumers' attitudes toward package size and price. *Journal of Marketing Research*, 9(3):239–248.

Gunders, D. (2012). Wasted: How america is losing up to 40 percent of its food from farm to fork to landfill. *Natural Resources Defense Council*, pages 1–26.

Jain, S. (2012). Marketing of vice goods: A strategic analysis of the package size decision. *Marketing Science*, 31(1):36–51.

Jeuland, A. P. (1979). Brand choice inertia as one aspect of the notion of brand loyalty. *Management Science*, 25(7):671–682.

Ju, X. (2015). Consumers variety seeking: from an information perspective. *International Business Research*, 8(3):42.

Kahn, B. E. (1995). Consumer variety-seeking among goods and services: An integrative review. *Journal of Retailing and Consumer Services*, 2(3):139–148.

Kahn, B. E., Kalwani, M. U., and Morrison, D. G. (1986). Measuring variety-seeking and reinforcement behaviors using panel data. *Journal of Marketing Research*, 23(2):89–100.

Kahn, B. E. and Louie, T. A. (1990). Effects of retraction of price promotions on brand choice behavior for variety-seeking and last-purchase-loyal consumers. *Journal of Marketing Research*, 27(3):279–289.

Kahn, B. E. and Raju, J. S. (1991). Effects of price promotions on variety-seeking and reinforcement behavior. *Marketing Science*, 10(4):316–337.

Koenigsberg, O., Kohli, R., and Montoya, R. (2010). Package size decisions. *Management Science*, 56(3):485–494.

Kök, A. G., Fisher, M. L., and Vaidyanathan, R. (2015). Assortment planning: Review of literature and industry practice. In Agrawal, N. and Smith, S. A., editors, *Retail supply chain management*, pages 175–236. Springer.

Kuvykaite, R., Dovaliene, A., and Navickiene, L. (2009). Impact of package elements on consumers purchase decision. *Economics and management*, (14):441–447.

McAlister, L. and Pessemier, E. (1982). Variety seeking behavior: An interdisciplinary review. *Journal of Consumer research*, 9(3):311–322.

Mou, S., Robb, D. J., and DeHoratius, N. (2017). Retail store operations: Literature review and research directions. *European Journal of Operational Research*, 265(2):399–422.

Ratner, R. K., Kahn, B. E., and Kahneman, D. (1999). Choosing less-preferred experiences for the sake of variety. *Journal of Consumer Research*, 26(1):1–15.

ReFED (2016 (accessed November 2, 2017)). *A Roadmap To Reduce U.S. Food Waste By 20 Percent*. `http://www.refed.com/analysis?sort=economic-value-per-ton`.

Ryzin, G. v. and Mahajan, S. (1999). On the relationship between inventory costs and variety benefits in retail assortments. *Management Science*, 45(11):1496–1509.

Sajeesh, S. and Raju, J. S. (2010). Positioning and pricing in a variety seeking market. *Management science*, 56(6):949–961.

Saure, D. and Zeevi, A. (2009). Optimal dynamic assortment planning. Technical report, Columbia GSB Working Paper.

Seetharaman, P. and Chintagunta, P. (1998). A model of inertia and variety-seeking with marketing variables. *International Journal of Research in Marketing*, 15(1):1–17.

Ulu, C., Honhon, D., and Alptekinoğlu, A. (2012). Learning consumer tastes through dynamic assortments. *Operations Research*, 60(4):833–849.

UNFAO (2013 (accessed November 2, 2017)). *Food Wastage Footprint: Impact on Natural Resources*. `http://www.fao.org/docrep/018/i3347e/i3347e.pdf`.

van Trijp, J. C. M. (1995). *Variety-seeking in product choice behavior. Theory with applications in the food domain.* Van Trijp.

Wansink, B. (1996). Can package size accelerate usage volume? *The Journal of Marketing*, 60(3):1–14.

Xiong, L., Liu, G., and Jiang, S. (2017). Competition with variety seeking and habitual consumption: Price commitment or quality commitment? *Mathematical Problems in Engineering*, 2017(2017):1–14.

Zeithammer, R. and Thomadsen, R. (2013). Vertical differentiation with variety-seeking consumers. *Management Science*, 59(2):390–401.

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