THREE ESSAYS FOR THE RETAIL PLANNER: SPATIALIZING BASS TEMPORALIZING HUFF AND VISUALIZING THE ENSEMBLE

by

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I dedicate this work to my family and my two brothers for their continuous and unconditional support, over our lifetimes, as nurtured by our parents.

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by

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THREE ESSAYS FOR THE RETAIL PLANNER: SPATIALIZING BASS

TEMPORALIZING HUFF AND VISUALIZING THE ENSEMBLE

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This dissertation integrates and extends an ensemble of classic spatial and temporal models seeking to contribute to retail trade area theory. The manuscript represents an integrated approach to location intelligence across retail trade areas. It is divided into three studies. The first chapter sets up the theoretical framework and mechanisms, demonstrating how the Bass model of retail innovation diffusion can be extended with Bayesian analysis to generate trade areas. The second chapter focuses on operationalizing the spatial interactions and generating processes of the Huff model to capture incremental periodic end state market area equilibria. The third chapter presents a novel and innovative three-way factor node analysis with a visualization called "Avatar". It can be either a 3D printed object or a 3D software visualization operating in a special 3D Excel add-on.

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THREE ESSAYS FOR THE RETAIL PLANNER: SPATIALIZING BASS, TEMPORALIZING HUFF AND VISUALIZING AN ENSEMBLE

INTRODUCTION

This dissertation integrates an ensemble of classic spatial and temporal innovation diffusion models while seeking to contribute to retail trade area theory. It comprises three chapters that present, operationalize and validate a novel approach to location intelligence across a retail trade space theoretically, empirically utilizing Bayesian inference.

REVIEW

Chapter two (i.e., "Spatializing Bass: From Retail Innovation to Market Area) is in response to a problem of *"insufficient predictive mechanisms for the development of...the... basics for a theory of consumer demand at the micro-analytic level" (Mason, 1975), and* sets out a theoretical framework for store level trade areas via a "Bass-Bayes Spatial Extension" (BBSE). The BBSE mechanism seeks to better understand high spatial and temporal resolution innovation diffusion processes at the micro-analytic store level. This is of critical importance to functional operations within the firm, such as sales, marketing, forecasting, supply chain management. As the amount and complexity of dynamic spatial-temporal digital data and information produced by sensor technology continues to overwhelm human-scale abilities to acquire knowledge, understand it and apply it, new methods are required in geospatial information science to simplify and factor key indicators from this deluge.

One problem in retail trade area analysis is that "gross decision parameters at the level of the firm" (Mason, 1975) for trade area and are not sufficiently predictive mechanisms for the development of “the basics for a theory of consumer demand at the micro-analytic level" (Ibid).

An alternative Bayesian spatial-temporal ensemble analysis appears to provide this dynamic capability while providing a means of monitoring and simulating the expected developing trade area, dynamically at the micro-level, in the presence of developing economic uncertainty and competition.

Chapter three (i.e., "Temporalizing Huff: Pathways to Equilibrium") begins with the end state trade area equilibria patterned by Huff and uses the BBSE methodology to determine how those equilibria emerge over time as consumers make choices to purchases innovative new retail offerings from single and then from networks of stores.

The first problem is to establish the scale and scope of the micro-temporal approach. Second is to develop a method for establishing shape and structure across the retail trade area also of importance is to discover whether this methodology can inform parsimonious proactive steps and actions the retailer can take to optimize market share.

Chapter four (i.e., "Space-Time Diffusion Visualization using Bayesian Inference") develops an integrated "Avatar" approach at the center of which is a visual model i.e., a (semaphore object) that can visually “telegraph” information by the observer recognizing its simple and familiar shapes and thus provides an example of how such visualizations can beneficially function. The Avatar visualization of the Bayesian spacetime ensemble model can be either a 3D printed semaphoric object or a 3D software visualization operating in a special 3D Excel add-on, as shown in Figure 1.

CONCLUSION

In these three chapters it is shown that disparate and mutually exclusive models of forecasted temporal pathways on one hand and shape/structure spatial interactions on the other can be co-mingled within a high spatial and temporal resolution store level trade area ecological environment for ensemble modeling and analysis of innovation diffusion scenarios.

The two models of interest are the Bass diffusion model, a temporal model, which has no spatial context and the Huff gravity model which has no temporal context other than a vague notion of eventual trade area market equilibrium. The Bass model is dynamic and the Huff model is static. Chapter one uses Bayesian procedures to spatialize the Bass model. Chapter two uses the Bass spatialization to temporalize the Huff model. The third chapter attempts to develop a visualization of the entire ensemble process.

Why is this important? Bass and Huff are recognized in the literature as two of the most important models in marketing and management science. The Bass model predicts the quantity and timing of adoptions of a new innovative product, service or idea. The Huff model enables the proportion of neighborhood trade directed to competitive alternatives to be estimated as the prime output. The dissertation makes a contribution by offering a spatial-temporal integration of the two.

To solve the problem the chapters link retail marketing, economic geography and geospatial information science concepts over store level trade areas (SLTAs) to identify likely evidenced-based areal units (e.g., census blocks) where Bass-forecasted "Innovators" and "Imitators" are located. Applied business demography concepts are utilized to correlate key demographic "Innovator" attributes with census data at the census block levels. As census blocks are qualified by empirical sales they are aggregated into a qualified innovation trade area shape and structure for the innovation within the store level trade area.

Marketing insight can thus be obtained regarding how specific processes lead to or do not lead to eventual market equilibrium outcomes. Traditional logical deductions about diffusionary processes from spatial-interaction, eventual market-equilibrium assumptions (i.e., gravity models) face the risk of ecological fallacy. Logical inductions made about the derivative end-state conditions of a retail market based on innovator processes are themselves subject to an exception fallacy.

The study details the steps taken to mitigate such fallacies while supporting the notion of non-intuitive decision-making based on modeling and empirical evidence utilizing binary classification methods, sensitivity/specificity analysis, likelihood ratios and Type I and II errors. In addition intervention opportunities for a retail innovation propagator (Allaway et al. 1994) and the outcomes in terms of corporate goals are summarized to simplify procedures for the everyday retail practitioner. A separate spreadsheet analysis manual has been prepared for this proprietary software product.

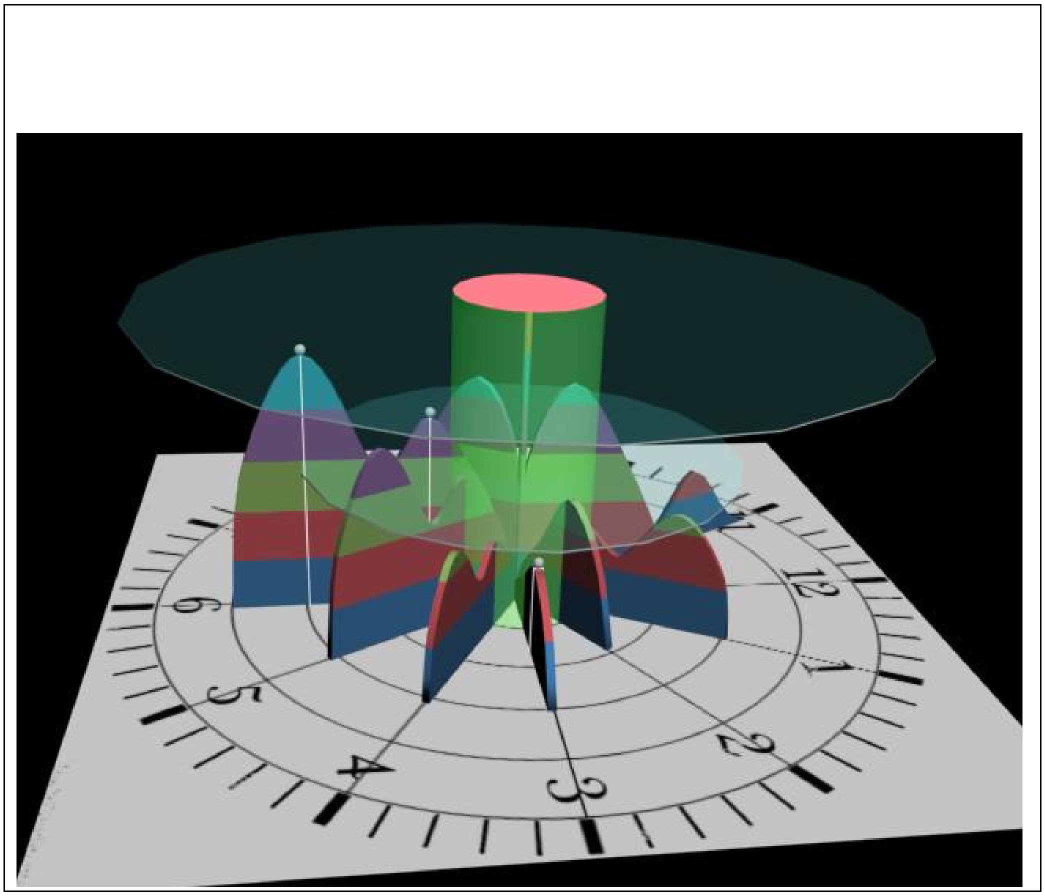

Figure 1. "Avatar", C. Franklin 2012

CHAPTER 2

SPATIALIZING BASS: FROM RETAIL INNOVATION TO MARKET AREA

This chapter presents a procedure for determining "where" (i.e., spatial) and "when" (i.e., temporal) Adopters of new innovations appear within the store level trade area (SLTA). Written for the retail practitioner, it is a novel and innovative methodology for allocating aspatial Bass diffusion model Adopters (i.e., "innovators and imitators") to specific spatial locations (i.e., qualified census blocks or QCBs). The shape of the innovation diffusion's trade area is a subset of the SLTA. A census block is "qualified" when it contains at least one empirical adoption of the innovation. The REA (random empirical adoption) activity is utilized for its geo-coded address location (e.g., home improvement product innovations like exterior windows require installation for Do-It-For-Me or DIFM customers). Each QCB is a Bayesian A_i event. QCBs accumulate into the eventual innovation trade area shape. The qualified census blocks are the statistical demographic areal units-of-analysis of the innovation diffusion trade area. Bayes' theorem updates all of the QCB priors and posteriors each Bas period, utilizing the law of total probability. The methodology is referred to as "Bass-Bayes Spatial Extension" or BBSE. An example is presented and results are extensively reviewed.

INTRODUCTION

One of the principle foundations in marketing science is the Bass diffusion model. The model is one in a series of contributions Frank M. Bass laid out, charting the time-path of an innovation from initial launch to market saturation and eventual equilibrium (Bass 1969). The utility of the Bass model has been demonstrated many times: Bass (1969) Norton and Bass (1987), Bass, Krishnan and Jain (1994) and Mahajan, Muller and Bass (1995).

The Bass model however has been of little use to those retail practitioners, who require connecting forecasts to actual locations within the store level trade area. There are many retail practitioners who need more. Specifically the rate and extent of spatial spread of innovative products and services, that evolves across the store level trade area, and the "shape" (Mason 1975) of an innovation that maps into an evolving retail "store level trade area" (SLTA) market area before eventually leading to a structure (Mason 1975) of market equilibrium.

The goal of every retail-marketing practitioner is to maximize shareholder wealth through the use of differentiating and sustainable marketing strategies and ongoing merchandising programs. The objective is to penetrate, grow and sustain market share. One potential and powerful spatial differentiating strategy is to monitor the evolving product trade area "shape" map and be in a position to conduct appropriate intervention strategies to optimize results. Appropriate intervention tactics can maximize and sustain the innovation diffusion spread among Adopters with particular focus on the adoption influence of Innovators through word-of-mouth influence on Imitators. Thus a managed innovation diffusion process has the potential for optimizing market penetration both in total percentage and spatial patterning.

The goal of this chapter is to develop a simple procedure that provides a spatial extension to the Bass model to overcome the aspatial challenge arising with the Bass diffusion model. Since it provides no spatial information on Adopter (Imitator and/or Innovator) it cannot take advantage of the census data associated with areal units-of-analysis, appropriate for mapping dynamic spatial interactions within a dynamic store level trade area. The elegant simplicity of Bayes' theorem and its non-intuitive "inverse probability" feature provides the solution.

Since Bayesian calculations use only arithmetic functions, the Bass-Bayes Spatial Extension (BBSE) is parsimonious and well suited to retail practitioners seeking simple and easy to use methods. BBSE has the ability to detect, monitor and react to spatial-temporal generating processes at each Bass temporal step, across the store level trade area (SLTA). Such an approach yields spatial-temporal mapping opportunities of the evolving retail innovation market area.

A novel and innovative methodology is developed for allocating Bass diffusion model "innovators and imitators" to specific qualified census blocks (QCBs); that defines the innovation's "trade area" as derived from the SLTA. A QCB is defined as a census block containing at least one empirical adoption of the innovation. The empirical adoption is defined by its geo-coded address and sales transaction value, as a hypothetical home improvement product innovation, requiring delivery and installation. Bayes' theorem is utilized to calculate the priors and posteriors of each QCB (i.e., Bayesian A_i event) at each Bass time step. QCBs accumulate into the eventual equilibrium innovation trade area shape (which may or may not be identical to the SLTA).

Using census blocks as the statistical demographic areal unit of analysis, Bayes' theorem is utilized to calculate and update all of the QCBs priors and posteriors utilizing the law of total probability. As new QCBs are added to the Pr (A_i) events list, they form an aggregating contiguous or non-contiguous trade area shape for that particular innovative product.

The spatial probability of Adopter's QCB location follows the classic Three-Machine/Defect problem Bayesian procedures (Kalbfleisch 2012) The probability an Adopter is also an Innovator emerges algorithmically based on the heteroskedastic ratio between Innovators and Imitators, which changes for each Bass time period. Hierarchical QCBs for ranking is achieved utilizing demographic attribute values of interest (e.g., population, occupied housing units and so on) for empirically weighting priors.

THE BASS DIFFUSION MODEL

Early diffusion research in the marketing discipline was rare (Rogers 2004). Bass built upon the ideas of Fourt, Woodlock (1960), Mansfield (1961), and initial discussions with Rogers (1962), as well as transdisciplinary work from epidemiology (BBRI 2010) and his impact was such that by 2003, approximately 16% of 5,200 diffusion publications were focused exclusively on marketing innovation diffusion studies, as a result of the Bass diffusion model (Rogers 2004).

"The Bass Model is the most widely applied new-product diffusion model. It has been tested in many industries and with many new products (including services) and technologies." (BBRI 2010).

The Bass diffusion model calculates the timing and quantity of Adopters/Adoptions occurring during the diffusion of an innovative product (BBRI 2010). The Bass model has several alternate forms (Ibid.). This chapter utilizes the Robinson, Lakhani (1975) formulation:

$$\boldsymbol{n(t)} = \left[\boldsymbol{p} + \left(\frac{\boldsymbol{q}}{\boldsymbol{M}}\right)\boldsymbol{N(t-1)}\right][\boldsymbol{M} - \boldsymbol{N(t-1)}] \qquad (1)$$

Where:

- "t" - refers to the time step (including zero) of adoption activity, a succession of one-time acts of acquiring the innovation under study

- "M" - potential market refers to "total adoptions". Estimated by management through surveys, subjective evaluation or a combination of techniques (BBRI 2010).
- "p" - the "coefficient of innovation"
- "q" - the "coefficient of imitation" - determined through experimentation, curve fitting and/or by studying analogous products
- "n(t)" - refers to either "Adopters" or "adoptions" (where "Adopter" makes one and only one acquisition)
- *"N*(t-1)" - cumulative adoption/adopter function

"The theory stems mathematically from the contagion models which have found such widespread applications in epidemiology." (Bass 1967). Also the Bass model is aspatial and presents only temporal forecasts of adopters/adoptions (BBRI 2010).

8The model relies on a number of assumptions:

1. It is delimited mathematically by its parameters: i.e., "p", "q" and "M"
2. The ultimate number of Adopters "M" must be known beforehand or estimable for proper calibration of the model. Thus M's consists of those who have adopted and those who have not yet adopted.
3. The model is limited to first time purchases of new innovative products and *assumes that sales of a new product are primarily driven by word-of-mouth from satisfied customers"* (BBRI 2010).

4. *"...the probability of adopting by those who have not yet adopted is a linear function of those who had previously adopted."* (BBRI 2010) i.e., the model assumes "*the timing of a consumer's initial purchase is related to the number of previous buyers"* (Bass 1969).
5. The model assumes constant pricing with inelastic demand although the "innovators" demand curve begins highly inelastic and gradually transitions to one that is highly elastic.
6. The model lacks any spatial connection to assist in spatial analytics of the shape and structure of market area development.

The following example illustrates how the model works. Table 1 and Figure 1 represent a Bass diffusion forecast for an innovation over 14 Bass time intervals, from initial Bass time period t_0 (with a positive Y-intercept value) to t_{13}.

The three parameters used to obtain this hypothetical Bass model forecast are the "coefficient of innovation", (p=0.08); the "coefficient of imitation," (q=0.5) and the "potential adopter market", (M=1000) typically estimated from empirical surveys or subjective retail analyst evaluations (BBRI 2010).The model distinguishes between two types of Adopters: "innovators" and "imitators". Innovators are defined as those who have a high personal trait of "innovativeness" and are first movers in a market to purchase innovations (Hägerstrand 1968; Rogers 1958). "Innovators" also, and importantly, have characteristically low risk aversion usually correlated to high socioeconomic means and status (Ibid). An additional key trait is that "Innovators" do not rely on the purchase choices of others (unlike imitators or "followers", who are greatly influenced by the opinions of others through word-of-mouth because of a generally very high level of risk aversion (Ibid).

Table 1. Fourteen Bass Periods C. Franklin 2018

Period	Time	Innovators	Imitators	Adopters
t_0	0	80	0	80
t_1	1	74	37	110
t_2	2	65	77	142
t_3	3	53	111	164
t_4	4	40	125	165
t_5	5	24	112	139
t_6	6	16	80	96
t_7	7	8	46	55
t_8	8	4	23	27
t_9	9	2	11	12
t_{10}	10	1	5	5
t_{11}	11	0	2	2
t_{12}	12	0	1	1
t_{13}	13	0	0	0
	Total	**370**	**630**	**1000**

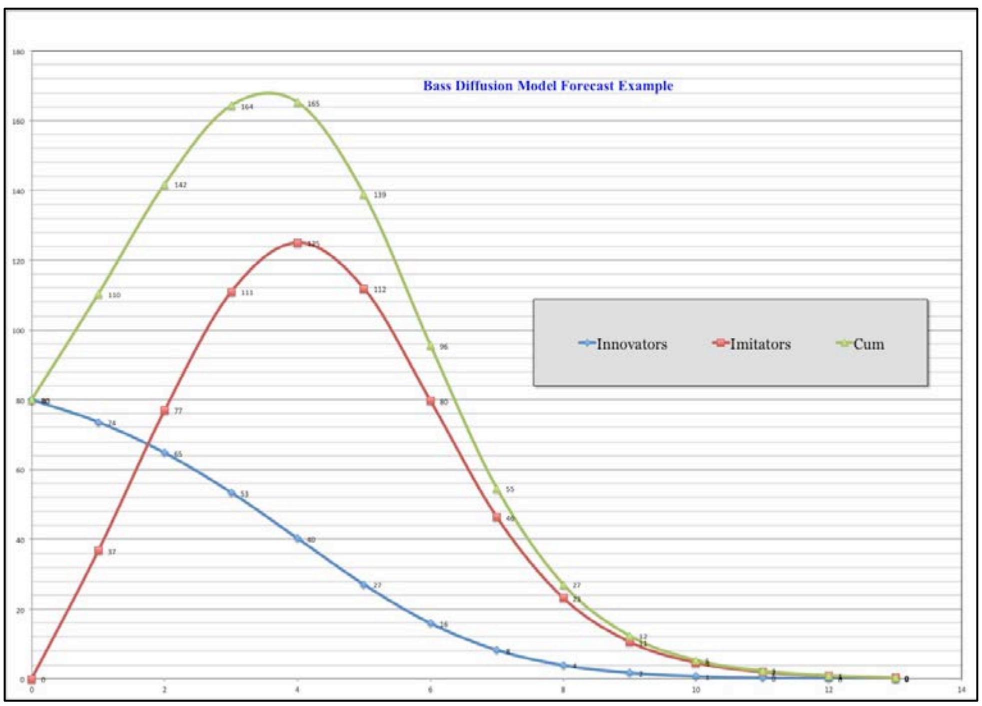

Figure 2. Time Series Adopter Graph C. Franklin 2018

BAYES' THEOREM

> "*There is no dispute over the mathematical validity of Bayes' Theorem, which is a direct consequence of the definition of conditional probability. It would also be generally agreed that Bayes' Theorem is applicable in the...", classic three-machine factory example* (Kalbfleisch, J. G. 2012).

Bayes' theorem has been used to solve many search and rescue problems as well as recovery e.g., location of the sunken Titanic and missing US Navy submarines. (McGrayne 2011). In fact, Alan Turing "broke" the German Nazi "enigma code" machine during WWII using the non-intuitive "inverse probability" of Bayes' theorem, which was still top secret until relatively recently (Stone 2016).

Bayesian principles in terms of psychology have been long understood in areas like Brain science, confirming the human brain utilizes Bayesian principles in all of its pattern/problem solving and recognition (Seth 2012).

Bayes' theorem has emerged *"...as a powerful tool with a wide range of applications...machine learning, epidemiology, psychology, forensic science, human object recognition, evolution, visual perception, ecology..."* (Stone 2016).

To obtain a high spatial-temporal resolution, Bayes' theorem is applied dynamically to Bass model outputs, at each Bass time step, in combination with new empirical "evidence" (i.e., geocoded sales of innovation adoption) to adjust posteriors distributions. The Bass model output is spatialized, simply by allocating Adopters to qualified census blocks (QCBs). A posterior probability distribution is calculated while implementing the law of total probability. Repetition of the interactions from each Bass period provide an evolving trade area map from which Imitators and

Innovators can be selected using likelihood ratios developed from the heteroskedastic variability found in the numbers of Imitators and Innovators.

"As Bayesians, we start with a belief, called a prior. Then we obtain some data and use it to update our belief. The outcome is called a posterior. Should we obtain even more data, the old posterior becomes a new prior and the cycle repeats." (Zajac 2016).

Spatial distribution of Bass adopters begins after empirical sales have initiated. The empirical sales (or Adoptions) create "qualify census blocks" (QCBs), where at least one adopter's innovation purchases has occurred.

A "prior" is updated with new sales evidence (and by definition the spatial information associated with the census block is then available also). Each new empirical sale of the innovation to an Adopter changes the priors and thus the posteriors on which successive generations of the Bass spatial allocation is based; while providing *"new and improved beliefs"* about the spatial allocation of Adopters by *"modifying initial beliefs"* (McGrayne 2011).

Figure 3 is a simple conceptual visualization of the Bass-Bayes Spatial Extension process.

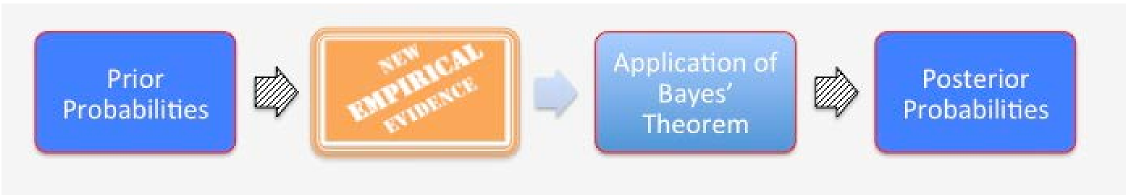

Figure 3. Conceptual Model of the Bass-Bayes Spatial Extension

Bayes' theorem can be written mathematically in several forms (Anderson, Sweeney, Williams 1984):

$$\Pr(A_i | B) = \frac{\Pr(A_i)\ \Pr(B|A_i)}{\Pr(B)} \qquad \textbf{(2)}$$

This is the alternate Bayes' theorem generalized form (Ibid.):

$$\Pr(A_i | B) = \frac{\Pr(A_i)\ \Pr(B|A_i)}{\Pr(B|A_1)\Pr(A_1) + \Pr(B|A_2)\Pr(A_2) + \ldots + \Pr(B|A_n)\Pr(A_n)} \quad (3)$$

for $i = 1,2,\ldots,n$.

"Pr" refers to the probabilities of the events A_i and B where Pr (B) ≠ 0. Pr (A_i) is the prior probability distribution *"...where the intermediate stage permits k different alternatives (whose occurrence is denoted by A_1 , $A_{, \ldots,}$ A_k)."* (Freund 2004).

This generalization *"requires the following theorem, sometimes called the **rule of total probability** or the **rule of elimination**. If the events A_1 , $A_{2, \ldots}$ and A_k constitute a partition of the sample space S and Pr (A_i) ≠ 0 for i =1,2,...,k, then for any event A in S ...the A's constitute a partition of the sample space if they are pairwise mutually exclusive and if their union equals S*." (Freund 2004).

The Pr (A_1) and Pr (B) are independent probabilities of observing event "A" and event "B", respectively. Pr (B | A_i) is a conditional probability distribution of the likelihood of event B occurring if A_i occurs; and Pr (A_i| B) is the posterior probability distribution for the likelihood of A_i given B (Ibid.). Table 2 shows the calculations for a factory with three stand-alone machines (A_1, A_2, A_3), all of which produce some small percentage of defective items (Kalbfleisch 2012). Total output for all three machines is 1000 items distributed as follows:

- Machine 1. Pr (A_1) = 200/1000 or 20% or 0.2
- Machine 2. Pr (A_2) = 300/1000 or 30% or 0.3
- Machine 3. Pr (A_3) = 500/1000 or 50% or 0.5

Table 2. Classic Bayesian 3-Machine/Defect Example

1	2	3	4	5	6	7	8
		PRIOR	Event A i			Event B	**POSTERIOR**
Producing	Items	%	Marginal	Adopter/	Conditional	Joint	Marginal
Units	Prod	Distribution	Pr (Ai)	QCB	Pr (B | Ai)	Pr (B | Ai) x Pr (Ai)	Pr (Ai | B)
Machine-1	200	0.2	0.2	10	0.05	0.010	0.417
Machine-2	300	0.3	0.3	9	0.03	0.009	0.375
Machine-3	500	0.5	0.5	5	0.01	0.005	0.208
	1000	1	1	24		**0.024**	1
						Pr (B)	

Consider the following question: If one item is selected randomly from the total items produced (i.e., 1000 items) and is found to be defective, what is the likelihood that Machine 3 produced the defective item?

The inverse probability Bayesian mechanism (i.e., P $(A_1|B)$, allows this question to be answered i.e., "What is the probability A_1 is correct, given that B has occurred?" In this example substituting the values, the likelihood was 50% (i.e., the Prior) that Machine 3 produced the defect, but with the empirical evidence (from column 6) of defect rates for Machine of 1%, the likelihood Machine 3 produced the defect has dropped to 20.8%; making Machine 1 (i.e., at 41.7% the most likely candidate for making a defect item).

This is a non-intuitive answer. It is sometimes referred to as "inverse probability". However the ability to ask "and" answer this question correctly is both a necessary and sufficient condition to allocate a spatial location to a Bass Adopters (or for "any" temporal only aspatial forecasting model output).

In other words it can be seen in Table 2 that (before any new evidence), the most likely machine to produce a defect is A_3 (Why? Because it produces 50% of all items produced). However after updating the model with new "defect rate" evidence (see Table 2, Column 6) the most likely machine to produce a defect is not " A_3 " but rather " A_1 " at 41.7% (Table 2, Column 8).

The evidence that leads to these posterior probabilities is derived from quality control procedures that track the actual dynamic defect rates as equipment ages (e.g., cutter tool wear, blade breaks and worn-out drill bits), to establish the conditional probability Pr (B| A_i). This enables computation of joint probabilities [Pr (B|A_i)) Pr (A_i)] for each machine and finally to calculate the overall defect probability (i.e., Pr (B)), which is also the normalizing constant.

BASS-BAYES SPATIAL EXTENSION (BBSE)

The Bass diffusion model specifically lacks any explicit spatial context. However the Bass model can be spatially invoked in the same way that the Three-Machine/Defect problem (Kalbfleisch 2012) can be structured spatially, if Euclidean distance were recorded for each machine in terms of its distance from the factory center point.

Utilizing this simple analog overcomes the "aspatial" Bass delimitation, and allows Bass to function within the context of the increasing availability of spatial-temporal data and information components.

The Bass-Bayes Spatial Extension (BBSE) approach operates parsimoniously based on the following assumptions.

- Assumption 1: The Bass Diffusion Model is both a necessary and sufficient condition for forecasting and predictive formulation, correctly estimating the timing and quantity of first-time Adopters (Imitators and/or Innovators) of an innovative product or service, throughout a store level trade area (SLTA) as defined by the Bass diffusion model.
- Assumption 2: There exists ongoing, geo-coded empirical, sales transaction data or "evidence" which provides the likelihood of the hypotheses being true. These initial random empirical adoptions of the innovation thus "qualify" certain census blocks in the store level trade area as "QCBs" for receipt of Bass predicted Adopters, after the innovation is released to begin diffusion.
- Assumption 3: We assume all Bass adopters are initially located in the store's census block (Note: this is an initializing placeholder condition only) until there is intervening evidence (i.e., empirical innovator sales transactions), in a Bass period to allow additional new QCB(s) into the innovation diffusion trade area shape. Gelman (2008b) points out that utilizing a "placeholder" for initializing Bayesian iterations is valid and has little or no distinguishing effects on evolving equilibria.

This is because a placeholder operates in a similar fashion to a statistical weighted in which each term of the average becomes relatively less important as new additional terms are added and the "weight" is spread across all terms.

WORKED EXAMPLE

The following explains procedures to spatially allocate the quantity of Bass forecasted Adopters per Bass time period, using the Bayesian process. The main purpose in allocating Bass Adopters to specific "qualified" census blocks is to support the microanalytic study of the spatial-temporal impacts during (a) each Bass period and (b) across all Bass time periods longitudinally (to eventual market equilibrium) within the "shape" of the innovation diffusion trade area.

For ease of explanation, Table 3 assumes the same values as the Three-Machine/Defect problem example, (i.e., Table 2). However in this example, assume a hypothetical home improvement/home-center retail store location in Southern California. Table 3 presents the tabular calculations necessary for Bayesian posteriors to be calculated for Bass time period 4. Within the store level trade area (SLTA) of this hypothetical store, a trade area "shape" has emerged based on empirical sales of the innovation. The trade area "shape" is made up of the following Qualified Census Blocks (QCBs) or "producing units" at Bass Period 4. The three (3) QCBs are (A_1, A_2, A_3).

Table 3. Simplified Bass-Bayes Spatial Extension Example Franklin 2018

1	2	3	4	5	6	7	8
Producing Units	Census Blk. POP	**PRIOR** % Distribution	Event A i Marginal Pr (Ai)	Adopter/ QCB	Conditional Pr (B | Ai)	Event B Joint Pr (B | Ai) x Pr (Ai)	**POSTERIOR** Marginal Pr (Ai | B)
QCB-A1	200	0.2	0.2	10	0.05	0.010	0.417
QCB-A2	300	0.3	0.3	9	0.03	0.009	0.375
QCB-A3	500	0.5	0.5	5	0.01	0.005	0.208
	1000	1	1	24		**0.024** Pr (B)	1

Constructing a Bass-Bayes Spatial Extension for achieving spatial allocations is accomplished again utilizing the "inverse probability" feature of Bayes theorem. First consider a simple but classic three-machine factory defect example (Kalbfleisch 2012). The objective is to answer the following critical question: "Given a randomly selected defective item (from all items produced by three machines), what is the probability a defective item, was produced by Machine #3".

Translating the Three-Machine/Defect problem example into the Bass-Bayes spatial extension case:

1. "Qualified census blocks" (QCBs) equate to Machines
2. Adopters who have made empirical purchases of the innovation are considered the "defective items" within the QCB's population.

The analogy may lack elegance, but it serves from a practical perspective, to demonstrate the functional equivalence of census blocks and machines in a classic machine-age example.

Recall a QCB (i.e., qualified census block) is defined as a census block, which has at least one empirical first-time Adopter (of the innovation) who has adopted the innovation and is located within its qualified census block with a polygonally defined areal geo-boundaries (i.e., a census block).

It is also important to assume that the home improvement store sells, delivers and installs special order fenestration products (e.g., custom exterior doors and windows) mostly to DIFM (i.e., Do-It-For-Me) type customers; who can afford to purchase both the product and the installation contract to have the innovation installed. Thus again the geo-coded location of the Adopter must be identifiable.

Each empirical DIFM (do-it-for-me) Adopters, by definition is from a QCB, has a specific and known geocoded residence address (required for installation of the innovation). Thus the spatial location of the Bass empirical Adopter is known.

Empirical Adopters (or as defined in the classic 3-Machine/Defect problem example as "defects") make purchases of the innovation for a specific QCB. Thus

1. QCBs = Machines
2. Bass Adopters = Defect

Census blocks also have important geospatial statistical attributes.

Once "Adopters" are linked to specific "qualified census blocks", the Adopter instantly becomes "spatially allocated" with a probability associated with their location.

With these facts, the following critical Bayesian question can be posed to establish the probability an Adopter is in a specific Qualified Census Block.

> *"If a population member is selected at random from the total population of all three QCBs, and is found to be an Adopter; what is the probability the Adopter resides in A_3 ?"*

Looking at just the "prior" population distribution, it would be logical to expect the Adopter allocations to be in the same proportion or ratio as the population distribution in each QCB; i.e., Pr (A_i) see column 3 of Table 3, i.e., 20%. However, with new evidence (e.g., a new empirical Adopter sale in one of the existing QCBs or in a new QCB for example), Table 3 is updated with new evidence and the posteriors calculated. For Pr (A_i| B) see column 8 of Table 3, i.e., 41.73%.

ADOPTER LOCATIONS

The BBSE's novel method for allocating Bass adopters to specific QCBs relies upon the core assumption that the highest probability location for "Innovator presence" is based on empirical evidence of "Adopters presence". Again a less than elegant example but if there is an oil slick floating on the water; that is a better place to start looking for something that has sunk (according to Coast Guard best practices and Bayesian search theory principles) than somewhere else in the middle of the ocean.

In other words, empirical sales of innovations to Adopters are a "necessary condition" for spatially positioning Bass forecast Adopters. This spatial assumption is widely supported by (1) Tobler's First Law of Geography (Tobler 1970) i.e., *"Everything is related to everything else, but near things are more related than distant things"*, (2) the concept of spatial autocorrelation and (3) Bayesian search theory (McGrayne 2011), used extensively and successively by the Coast Guard to locate sunken vessels at sea. Bayesian search theory creates geographical event scenarios as priors. Sales of the innovation to Adopters, while the diffusion is underway, act to direct and guide us to qualified census blocks (QCBs) where there are likely more Bass "M" ultimate Adopters.

For example, in Table 3 (i.e., column 3), QCB3 or " $\mathbf{A_3}$ " represents 50% of the total of the three QCB populations (i.e., 500/1000). However after updating the table with new evidence (i.e., Table 3, column 6), the new posteriors (i.e., column 9) suggest the most likely QCB to contain an Adopter is now representing 41.7% of the total three new posterior population distributions. The QCB "$\mathbf{A_1}$" represented only 20% of the total population in the earlier prior distribution.

In other words, the updated information informs the retail practitioner that Adopters are now more spatially likely to be allocated as shown in Table 3, Column 9 where now 41.7% of the all adopters are expected to be in QCB-1, 37.5% in QCB-2 and 20.8% in QCB-3.

This then allows the calculation of an exact Adopter rate (i.e., cumulative Adopters over Bass time periods for all QCB populations) in each QCB to establish the likelihood ratios or Pr $(B \mid A_i)$, a conditional probabilities. This then allows the next step, the computation of joint probabilities to be undertaken i.e., Pr (B | A_i) Pr (A_i) for each QCB (i.e., $A_{i...}A_n$).

The overall Adopter probability distribution for Pr (B) (i.e., the normalizing constant) is calculated and then the Pr (A_i| B) for each QCB within the SLTA. The "posterior" refers to the probability of an Adopter being located in a specific QCB (which are the smallest units-of-analysis in this study). The BBSE model does not include detailed spatial interaction modeling or analysis in the SLTA for the diffusion of an innovation, however both are addressed in chapter three (Temporalizing Huff: Pathways to Equilibrium).

ADOPTER ALLOCATIONS

To answer questions such as "Are Innovators uniformly distributed across the SLTA or within the QCBs (qualified census blocks)?", the process of allocation is detailed below for two Bass time periods.

Although the algorithm is relatively trivial (i.e., light on logic) and based on simple arithmetic and proportions, it is computationally intensive, typical of iterating Bayesian calculations (Gelman 2008) but ideal for computerization in a proprietary software suite that has been developed from this chapter. Also there are a few unique initialization steps in the first few iterations that will be identified.

To begin, there are three possibilities with respect to the "random empirical adoptions" (REA) or store level sales of the innovation that result from the diffusion of the innovation:

1. There are no REA,
2. There are REA in existing QCBs (i.e., "qualified census blocks") and

3. There are REA in new QCBs (informing the evolving "shape" of the innovation diffusion).

The following is a step-by-step explanation of the process (see Table 4 for the resulting calculations for Bass periods " t_0 and t_1 "). Formulas are shown in Table 5 for further analysis of " t_2 to t_4 ").

Step (A). At " t_0 " (Excel reference cell A6 in Table 4), the Store census block "A" (i.e., Pr (A_1)) is established as a "placeholder" (Gelman 2008), to initialize the BBSE (i.e., Bass-Bayes spatial extension). The Y-intercept of the Bass model from Figure 1 and Table 1 indicate a forecast of "80" Innovator (Adopters) (see L6) and zero Imitators. Additionally, with no REA yet established, we must initialize the BBSE process creating a QCB placeholder. This is done by assuming all 80 Innovators are "allocated" initially, to the Store census block "A". Like a weighted average calculation this initial assume with its obvious errors will diminish in importance to insignificance during the Bass interactions.

At this point the experienced retail practitioner may ask, "How is the Bass period by period forecast output being allocated?"

Specifically, (M6) shows the allocation to Store census block "A" (see M4 heading) of the total cumulative Bass output. This is accomplished by taking the posterior probability of 100% (see E6) and multiplying by "80" Innovator (Adopters) (see L6). The result is "80". (The procedure for more than one QCB follows after explanation "(B).")

Step (B). At " t_1 " (A11), the Store census block "A" i.e., Pr (A_1) and Census block "B" i.e., Pr (A_2) are available as QCBs to hold Bass Adopter outputs (see cell M11 and N12). Again, "80" Innovator

(Adopters) (see L11) and "74" (see L12) additional Innovator (Adopters) must be "allocated" to the two existing QCB i.e., Store census block "A" and Census block "B" respectively.

However in addition this time, there are also "0" Imitator (Adopters) ignored as a zero value in Step (A) (see Q11) and "37" (see Q12) new Imitator (Adopters) forecasted and needing "allocation" also to the Store census block "A" and Census block "B" respectively. Specifically (M11) is the allocation of "2" Innovators to Store census block "A" i.e., Pr (A_1) (see M4) and "152" Innovators to Census block "B" (see N12) i.e., Pr (A_2) of the total Innovator forecast to date of "154".

Thus the allocation algorithm operates by (a) establishing Priors and then (b) calculating Posterior probability in the traditional Bayesian method for Store census block "A" and Census block "B". The Bayes theorem calculations are detailed in columns "F through I".

Note, the priors have a two-stage construction from the REAs (see B11, B12) or "1" and "10". The Bass "M" ultimate adopters or Adopter Rate (i.e., defect rate in the 3 Machine/Defect problem) established based on the actual census block potential adopters (see D11, D12) or "1" and "200". For simplicity we assume the census block population is the same as Bass "M" ultimate adopters in this example. (In reality this would be parameterized to some percentage of the census block population subjectively or objectively with experience/knowledge or survey data).

Finally the posterior probability for " t_1 " is determined as follows:

(1 REA x 1 POP) / [(1 REA x 1 POP) + (10 REA x 200 POP)] or in Excel cell formulation [=(B11*D11)/((B11*D11)+(B12*D12))] or "0.0005". Then "0.0005" as the Store census block "A" PRIOR is multiplied by the Adopter rate (e.g., "defect rate" in the classic 3 Machine/Defect problem), calculated as [=B11/D11]. Now with a PRIOR and a CONDITIONAL (i.e., Adoption

rate for that QCB), the POSTERIOR is calculated as 0.00990 (based on the normalizing constant of Pr(B)=0.050).

Finally the POSTERIOR calculations allow "2" (see M11) of the "154" (see L13) Bass output Innovator Adopters to be allocated to the Store census block "A" and "152" are allocated algorithmically to Census block "B". In additional "0" Imitator Adopters are allocated to the Store census block "A" and "37" are allocated algorithmically to Census block "B".

As can be seen, the explanation is tedious and complicated (i.e., computationally complex) but simple in terms of logic, and as in most practical applications of Bayesian inference (Gelman 2008), is easily handled by a computerized software application.

Adopters are allocated spatially to QCBs. In a database (i.e., attribute table) all Innovators and Imitator allocations to specific spatial locations or QCBs is recorded. Using ratio analysis (e.g., mixture problem) the probability a randomly selected Adopter from a specific QCB is an "Innovator" can be proportionally calculated.

The rate of change (i.e., slope) in the Innovator and Imitator ratio proportions (per Bass time period), provides some insight into the impact and available "Innovator energy force" available to propel the diffusion in its early stages. Table 6 helps visualize these concepts for microanalytic purposes.

Firstly, the rate of change (i.e., slope) in the blue "CUM-IN" (i.e., CUMMULATIVE-INNOVATORS) and the red "CUM-im" (i.e., CUMULATIVE-imitators) curves (over all previous Bass time periods), indicates the 50-50 ratio proportion (of Innovators to Imitators) is reached more slowly than the actual "per Bass time period".

Table 4. Allocating Bass Forecasted

	A	B	C	D	E	F	G	H	I	J	K	L	M	N	O	P	Q	R	S	T	U
1	TIME	DEFECT						BAYES THEOREM CALCULATIONS													
2		ADOPTER		BASS	%																
3		EMPIRICAL	EVENT	"M"	Potential	PRIOR	CONDITIONAL	JOINT	POSTERIOR	POSTERIOR	EVENT										
4		SALES		Adopters	CONSUMERS	P(Census Tract i)	P(Customer/Census Tract i)	P(Customer/Census Tract i) P(Census Tract i)	P(Census Tract i/ Customer)	%			S	A	B	C		S	A	B	C
5		COUNT		POP	Pr (A)							Innovators					Imitators				
6	t_0	1	Store BLOCK A	1	100.0%	1.0000	1.0000	1.000	1.00000	100.0%	Store BLOCK A	80	80				0				
7		1		1				P(B)	1.00000			80					0				
8								1.000													
9																					
10																					
11	t_1	1	Store BLOCK A	1	0.0%	0.0002	1.0000	0.000	0.00111	0.1%	Store BLOCK A	80	0				0	0			
12		30	Census BLOCK A	200	100.0%	0.9998	0.1500	0.150	0.99889	99.9%	Census BLOCK A	74		154			37		37		
13		31		201	100.0%			P(B)	1.00000	100.0%	Total	154					37				
14								0.150													
15																					
16																					
17	t_2	1	Store BLOCK A	1	0.0%	0.0001	1.0000	0.000	0.00089	0.1%	Store BLOCK A	80	0				0	0			
18		15	Census BLOCK A	200	25.0%	0.2500	0.0750	0.019	0.19982	20.0%	Census BLOCK A	74		44			37		23		
19		30	Census BLOCK B	300	75.0%	0.7499	0.1000	0.075	0.79929	79.9%	Census BLOCK B	65			175		77			91	
20		46		501	100.0%			P(B)	1.00000	100.0%	Total	219					114				
21								0.094													
22																					
23																					
24	t_3	1	Store BLOCK A	1	0.0%	0.0000	1.0000	0.000	0.00038	0.0%	Store BLOCK A	80	0				0	0			
25		40	Census BLOCK A	200	36.4%	0.3636	0.2000	0.073	0.61515	61.5%	Census BLOCK A	74		167			37		##		
26		30	Census BLOCK B	300	40.9%	0.4091	0.1000	0.041	0.34602	34.6%	Census BLOCK B	65			94		77			78	
27		10	Census BLOCK C	500	22.7%	0.2273	0.0200	0.005	0.03845	3.8%	Census BLOCK C	53				10	111				9
28		81		1001	100.0%			P(B)	1.00000	100.0%	Total	272					225				
29								0.118													
30																					
31																					
32	t_4	1	Store BLOCK A	1	0.0%	0.0000	1.0000	0.000	0.00013	0.0%	Store BLOCK A	80	0				0	0			
33		60	Census BLOCK A	200	25.5%	0.2553	0.3000	0.077	0.46747	46.7%	Census BLOCK A	74		146			37		##		
34		50	Census BLOCK B	300	31.9%	0.3191	0.1667	0.053	0.32463	32.5%	Census BLOCK B	65			101		77			##	
35		40	Census BLOCK C	500	42.6%	0.4255	0.0800	0.034	0.20777	20.8%	Census BLOCK C	53				65	111				47
36												40					125				
37		151		1001	100.0%			P(B)	1.00000	100.0%	Total	312					350				
38								0.164													

Table 5. Excel Formulas for Table 4

	J	K	L	M	N	O	P	Q	R	S	T	U
1												
2												
3	POSTERIOR	EVENT										
4	%			S	A	B	C		S	A	B	C
5			Innovators					Imitators				
6	=I6	Store BLOCK A	43	=$J$6*$L$7				0				
7			=SUM(L6)					0				
8												
9												
10												
11	=I11	Store BLOCK A	43	=$J$11*$L$13				0	=$J$11*$Q$13			
12	=I12	Census BLOCK A	41		=$J$12*$L$13			10		=ROUNDUP($J$12*$Q$13,0)		
13	=SUM(J11:J12)	Total	=SUM(L11:L12)					=SUM(Q11:Q12)				
14												
15												
16												
17	=I17	Store BLOCK A	43	=$J$17*$L$20				0	=$J$17*$Q$20			
18	=I18	Census BLOCK A	41		=$J$18*$L$20			10		=$J$18*$Q$20		
19	=I19	Census BLOCK B	38			=$J$19*$L$20		21			=$J$19*$Q$20	
20	=SUM(J17:J19)	Total	=SUM(L17:L19)					=SUM(Q17:Q19)				
21												
22												
23												
24	=I24	Store BLOCK A	43	=$J$24*$L$28				0	=$J$24*$Q$28			
25	=I25	Census BLOCK A	41		=$J$25*$L$28			10		=$J$25*$Q$28		
26	=I26	Census BLOCK B	38			=$J$26*$L$28		21			=$J$26*$Q$28	
27	=I27	Census BLOCK C	35				=$J$27*$L$28	31				=ROUNDUP($J$27*$Q$28,0
28	=SUM(J24:J27)	Total	=SUM(L24:L27)					=SUM(Q24:Q27)				
29												
30												
31												
32	=I32	Store BLOCK A	43	=$J$32*$L$37				0	=$J$32*$Q$37			
33	=I33	Census BLOCK A	41		=$J$33*$L$37			10		=$J$33*$Q$37		
34	=I34	Census BLOCK B	38			=$J$34*$L$37		21			=$J$34*$Q$37	
35	=I35	Census BLOCK C	35				=$J$35*$L$37	31				=ROUNDUP($J$35*$Q$28,0
36			32					41				
37	=SUM(J32:J35)	Total	=SUM(L32:L36)					=SUM(Q32:Q36)				
38												

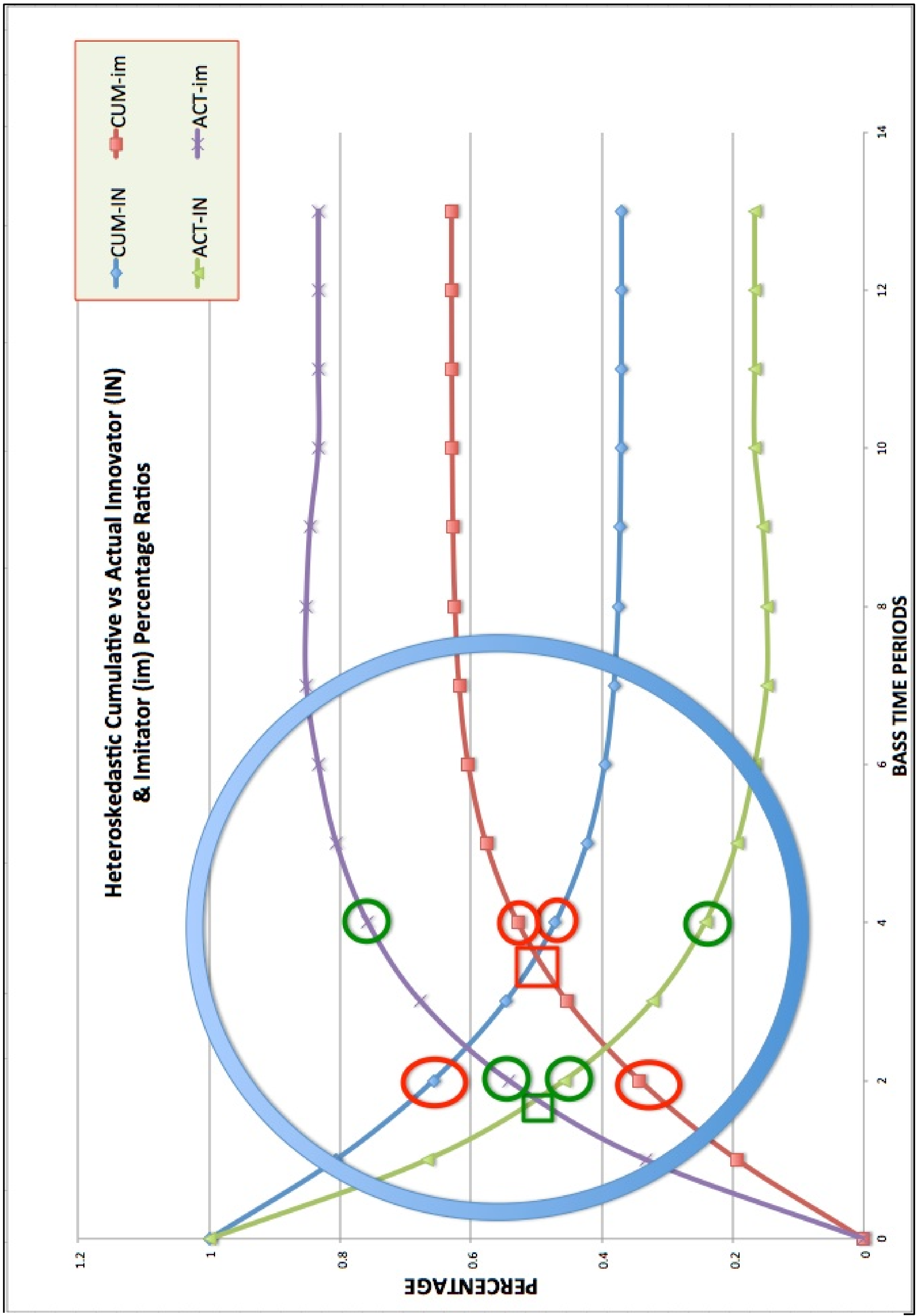

Figure 4. Heteroskedastic Ratio Variability for Innovators and Imitators

This can be seen in the green ACT-IN (i.e., ACTUAL-INNOVATORS) and the purple ACT-im (i.e., ACTUAL-imitators) curves. Reading from the Table 6 graph, the 50-50 ratio proportion (of Innovators to Imitators) for "CUM's" is reached approximately just before the 4th Bass period, while the "ACT's" 50-50 equivalence is reached just before Bass period 2 as can be approximated from studying the Table 6 graph.

This provides the retail practitioner with "advanced" warning. The "advanced" warning suggests that current forecast customers coming to the store to adopt the innovation have lost a lot of the Innovator force of urgency and price and risk insensitivity.

The REA (random empirical adopters) probably will exhibit more of the "imitator" "lack of urgency with high price and risk sensitivity" after Bass time period two (2). But this also suggests that the store's innovation campaign will still have two (2) more Bass periods (i.e., to approximately Bass time period four (4), in the trade area "pool", to enjoy a higher than 50-50 percent Innovator type adopter in which intervention and various targeted merchandising and marketing mechanisms could enjoy the higher proportion Innovator presence.

Secondly, the cumulative heteroskedastic ratio for Innovators and for Imitators can be treated as the probability that an adopter selected at random from the "mixture pool" will be an Innovator or Imitator. Therefore by definition, in the early stages of an innovation diffusion the likelihood will be high that a random selection of an adopter from the "mixture pool" will be an Innovator and as the diffusion progresses past Bass time period four (4) the likelihood will shift rapidly to being an Imitator, based simply on the population of each type of adopter in the trade area "mixture pool".

The heteroskedastic (i.e., dynamic variability), and decreasing ratio between Innovators and Imitators per Bass period is a powerful effect of the Bass diffusion model forecast. Firstly it signals the necessity of retail practitioners "locating" Innovators quickly in the first or second Bass period of a diffusion scenario. If left to later in a reactive "wait-and-see" management style, the Innovator advantage is wasted and their ability to positively impact the rate, extent and velocity of the diffusion process diminishes to insignificance to little effect by Bass time period 4 or 5.

Knowing the location of REA could assist in an innovation diffusion "steerage and throttling effect" (Allaway et al. 1994). In other words the Bass model period by period forecasting is a guide - if strategic action can engage the retail practitioner with the full number of Bass "M" ultimate adopters earlier than forecast by the Bass diffusion model, this is a desirable goal for any innovation diffusion "marketing intervention" operation.

Finally, the ratio of Innovators to Imitators being forecast by the Bass diffusion model both (a) per Bass period and (b) cumulatively, offers opportunities to utilize the ratio of Innovators to measure the likelihood of randomly selecting Adopters who are "Innovators" (i.e., as opposed to "Imitators) at all Bass time periods.

A Type-I, Type-II analysis utilizing these ratios is presented based on Figure 4. The dashboards represent original research utilizing the principles of Bayesian Machine Learning and Binary Classification.

VALIDITY, TYPE-I, TYPE-II ERRORS AND METRICS

Experienced practitioners will ask, "What is the accuracy of the "Bass-Bayes Spatial Extension" (BBSE), when attempting to locate "Innovators" from among the Adopters in specific qualified census blocks for marketing intervention actions?"

The following procedures have been developed to report Type-I and Type-II errors. In addition, a binary classification of Innovators and Imitators in dashboard form (i.e., Figure 4) has been constructed.

Metrics include:

1. Sensitivity (i.e., Market Share)
2. Specificity
3. True Positive (TP)
4. False Negative (FN)
5. False Positive (FP)
6. True Negative (TN)
7. Accuracy [(TP+TN)/(TP+TN+FP+FN)]
8. Prevalence or [Pr $\mathbf{(A_1)}$]

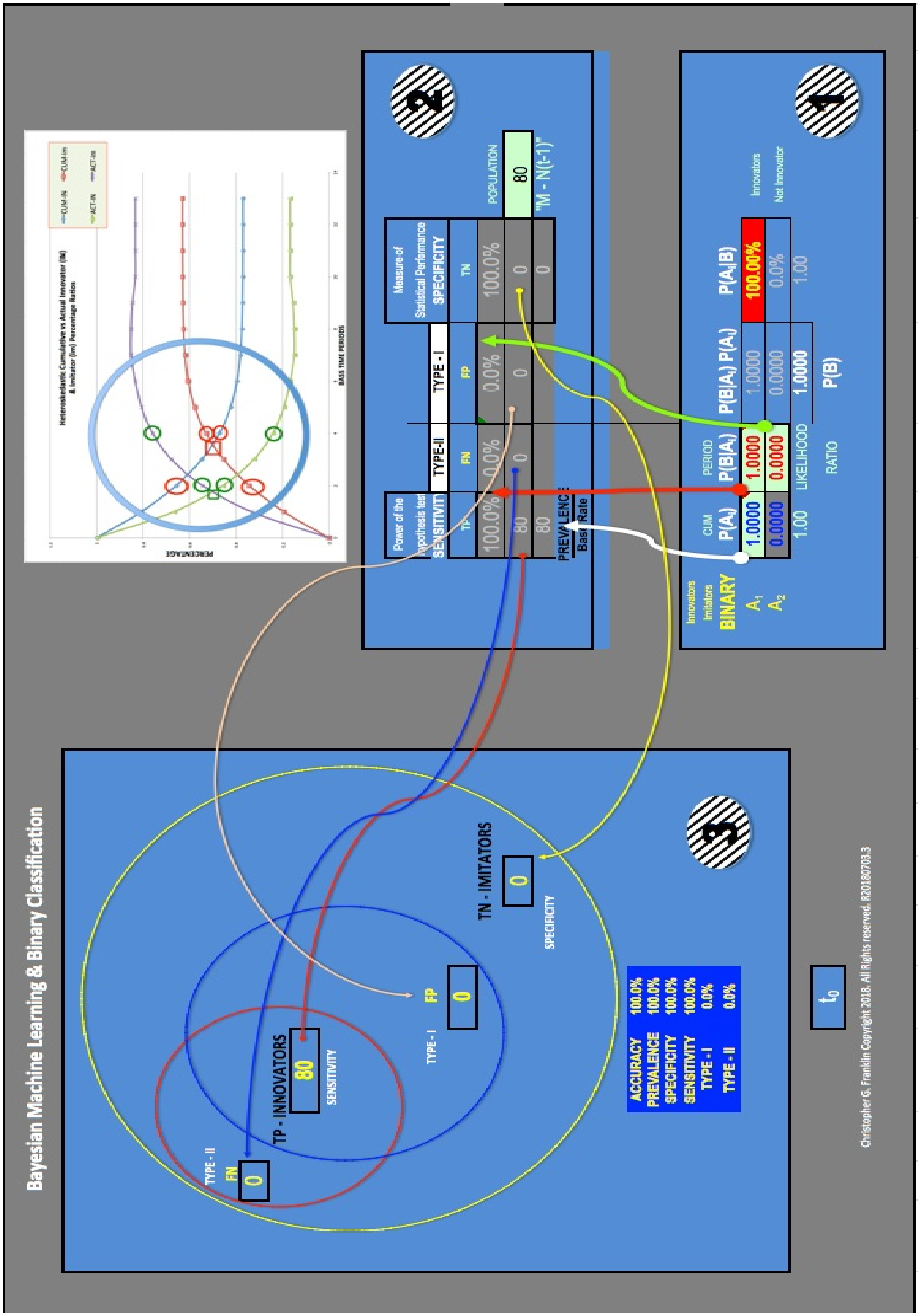

Figure 5. Dashboard for Bass time period = 0 with Y intercept = 80

Firstly, looking at Bass period zero there are no errors, since all Adopters at time=zero are Innovators in the Bass diffusion model. Looking in Appendix A at Bass period = one, the Type-I is 14%. There is a 14% chance of a Type I error (i.e., rejecting the null hypothesis when it is true). Specifically in this example that means the Type-I (Anderson, Sweeney and Williams 1984) error causes the "test" for Innovators to over-count Imitators as Innovators in the amount of 5 Innovators. This results in a False Positive value of "5" Imitators being classified as Innovators in " t_1 ".

Secondly, there is only a 8.1% chance of a Type II error (i.e., accepting the null hypothesis when it is false). Specifically in this example that means the Type-II error causes the "test" for Innovators to under-count Innovators as Imitators; resulting in a False Negative value of "12" Innovators being classified as Imitators in " t_1 ". For further parameters see the Blue Dashboard (i.e., Figure 4).

BINARY CLASSIFICATION

The Bass diffusion model was selected for study in this chapter based on its well-known accuracy in forecasting the quantity and timing of Innovator and Imitator innovation diffusion "adoptions". The thrust of this chapter so far has been to spatialize the Bass model Adopters by allocating them to "qualified" census blocks. The challenge remains to identify an Adopter as either an Innovator or Imitator and determine the accuracy of that process.

Although Bass (1969) designated only two categories of Adopters, "Innovators" as early adopters and "Imitators" as late adopters or followers; some "Imitators" begin right away adopting in Bass period 2 while some "Innovators" don't adopt until Bass period 4 or later, albeit very few according to the Bass diffusion model.

Thus when establishing a test for the likelihood of an Adopter being an Innovator or Imitator; there will be errors and variations.

Detecting the presence of a disease condition (e.g., Innovators among Adopters) has been widely studied and utilized in medical diagnosis tests using binary classification statistical measures. For example the Prostate-specific antigen screening test (PSA) medical test.

In medical testing "Sensitivity" (i.e., true positive rate) is the ability to correctly identify the presence of a particular disease. "Specificity" (true negative rate), another attribute of a medical test, is the ability to correctly detect when there is no disease present.

In terms of the diffusion of an innovation both Rogers (1958) and Bass (1969) developed a "relative test" for determining if an Adopter is an Innovator. They used the idea of "innovativeness", defined as the willingness and extent to which a person accepts new ideas (Rogers 1958) affected by the speed and number of word-of-mouth referrals generated by Innovators for the benefit of Imitators.

Early adopters exhibit high "innovativeness" while late adopters, followers and imitators displayed low "innovativeness".

A second aspect of "innovativeness" is "risk aversion". Rogers (2004) and Allaway et al. (1994) describe a psycho-demographic behavioral profile of Innovators as self-reliant, high socioeconomic means, leaders and very low in risk aversion traits.

The utilization of statistical measures in a binary classification is a first step toward identifying the probability of an Innovator being among Adopters in a specific QCB.

This procedure has made use of the heteroskedastic ratios operating during the diffusion process.

Retail practitioners will need to know where Bass Innovators are located. In every SLTA retail practitioners must take into account the active spatial and locational generating processes operating i.e., spatial autocorrelation and Tobler's Law for example.

To control marketing interventions and the "throttling" and "steerage" of diffusionary effects, retail practitioners will also need to know in what proportions Innovators occupy certain SLTA locations.

The binary classification methodology from Bayesian machine learning appears promising for establishing the reliability of the Bass-Bayes Spatial Extension approach.

The "PSI" (Period Specific Innovativeness) screening test (like the PSA screening test for prostate cancer) attempt to correlate likelihoods of one variable with the presence of another to discover "innovativeness" using the heteroskedastic ratios of "Innovators" to "Imitators" for each Bass period. Innovativeness " at t_0 " is 100%, by definition, and quantified based on the fact "Innovators" start with a Y-intercept at Bass time zero.

Of the total Bass "M" parameter ultimate Adopters (referring to Table 1) in the example 370 are "Innovator" Adopters (i.e., 370+630=1000). Assume there are some number of other potential consumers (e.g., hypothetically "7900") in the three Store Level Trade Area QCBs (A_1, A_2 and A_3). Then approximately 12.7% (1000/7900) of the potential consumers in the SLTA would be Bass "M" parameter ultimate Adopters (Note: "M" or potential market refers to "total adoptions". These values are estimated through surveys, subjective evaluation or a combination of techniques (BBRI 2010). The point is not all potential consumers in a QCB are counted by the Bass model parameter "M" (BBS 2010).

Based on Table 7 (Column "AD", Row "8") for the binary classification parameter "Prevalence" for Bass period (t_1), the probability of selecting a random Adopter who is an Innovator is 0.8063 (cell AD8) or 80.63% (i.e., 154/191) (cell AJ8/AN8).

The input information (Table 7) for this example, is loaded into the Dashboard algorithms (see Figure 5) for a "PSI" (Period Specific Innovativeness) screening test, for Bass time period " t_1 ", is as follows:

- Prevalence = 0.8063 (i.e., 154/191)
- Sensitivity = 0.9189 i.e., = (1 - [((37-(0.1937x111))/191])
- Specificity = 0.8604 i.e., = (1-[(154 - (0.6667x191))/191])

TYPE -1 (Rejection of the null hypothesis, when it is true)

The false positive (type I) error rate for the Bass period (t_1) is reported as 0.1396 or 13.96% or as high as 37.04% for the "PSI" (Period Specific Innovativeness) screening test.

TYPE-II (Acceptance of the null hypothesis when it is false).

The false negative (type II) error rate for Bass period (t_1) is reported as 0.0811 or 8.11% or as high as 8.52% resulting in 12 Innovators (see Table 8 in Appendix A) being wrongly classified as Imitators using the "PSI" (Period Specific Innovativeness) screening test.

If a randomly selected Adopter (from a Qualified Census block in the Store Level Trade Area), where the prevalence of Innovators, for Bass period (t_1), is 80.63%, receives a positive "PSI" (Period Specific Innovativeness) screening test, and that test has a false positive rate (i.e., Type I) of 13.96% and a false negative rate (i.e., Type II) of 8.11%, what is the chance that this Adopter actually is an Innovator?

Utilizing Bayes' theorem, the following analysis is guided by Benjamin Gregory Carlisle (2015) presentation on utilizing Bayes' theorem for medical diagnosis.

The Bayesian solution to the above question is organized as follows:

- Pr (A): The prior probability (Prevalence), an Adopter is an Innovator =0.8063
- Pr (B|-A): The (false positive - FP) Type I error rate = 0.1396 or 13.96%
- Pr (-B|A): The (false negative - FN) Type II error rate = 8.11%
- Pr (-A) = (1 - 0.8063) = 0.1937

There are only three inputs required to the Dashboard (see Figure 3):

1. Pr (A) = 0.8063 The prior probability (Prevalence) an Adopter is an Innovator
2. Pr (B|A) = 0.9189, [1 - 0.0811] Sensitivity = (1 - [((37-(0.1937x111))/191])
3. Pr (B|-A): The Type I error rate (false positive - FP) = 0.1396 or 13.96%

Given the defined parameters, the quantity we are interested in calculating is Pr(A|B) i.e., the probability that the Adopter is an Innovator given that they return a positive "PSI" (Period Specific Innovativeness) screening test.

$$\text{Solve: } Pr\ (A|B) = Pr\ (A)\ /\ [Pr\ (A) + [Pr\ (B|-A)\ Pr\ (-A)]/\ Pr\ (B|A)$$

Pr (A|B) is calculated using Bayes' theorem: There is a 96.48% chance that the randomly selected Adopter in question is an Innovator, given a positive "PSI" (Period Specific Innovativeness) screening test, to the end of the Bass period " t_1 ".

The heteroskedastic variability in the ratios between Innovators and Imitators both by time period (Actual) and in the Aggregate longitudinally (Cumulatively) provides evidence of the likelihood at different Bass times of the presence of Innovators.

Based on this outcome it is still very important to realize both the accuracy and probability measures demonstrated here (i.e., Pr (A_1|B), using this "PSI" (Period Specific Innovativeness) screening test all deteriorate quickly after Bass period " t_2 ".

The diffusion of an innovation not only happens quickly (and likely perhaps at non-uniform dispersion velocities across a trade area), as the falling prevalence numbers demonstrate (see Table 7, cell AD7).

This generating process is likely irreversible, despite the best efforts of retail practitioners, if the retailing efforts are ad hoc and unplanned. Efforts at intervention in a diffusion innovation require preplanning and preparedness. Like emergency management organizations, retail practitioners need a methodology to track and analyze diffusing innovations and they also need alternate plans depending on the speed of the diffusion after it is launched.

There are ample models and theories to support intervention strategies based on social and spatial science. For example the work of Berry (1967); Hägerstrand (1968); Rogers (1976); Haynes (1977); Allaway et al. (1994) and Shinohara (2012) are just a few in the literature focused on the power of innovation diffusion.

This investigation suggests a diffusion of an innovation can be intervened and monitored but retail practitioner's marketing controls, tools and options must be pre-organized and ready to deploy at a moments notice. The analogy is similar to emergency management and professional first responders.

Table 6. Binary Classification Parameters

	V	W	X	Y	Z	AA	AB	AC	AD	AE	AF	AG	AH	AI	AJ	AK	AL	AM	AN
4				4		3	2		1										
5	TIME	P(A|B)	Accuracy	CUM	Specificity	TYPE-I	Sensitivity	TYPE-II	Prevalence					ACT	CUM	ACT	CUM	ACT	CUM
6				TOTAL	TN	FP	TP	FN	CUM-IN	CUM-im	ACT-IN	ACT-im	TIME	IN	IN	im	im	TOTAL	TOTAL
7	0	100.00%	100.0%	80	1.0000	0.0000	1.0000	0.0000	1.0000	0	1	0	0	80	80	0	0	80	80
8	1	96.48%	89.0%	191	0.8604	0.1396	0.9189	0.0811	0.8063	19.4%	66.7%	33.3%	1	74	154	37	37	111	191
9	2	89.79%	85.7%	333	0.8001	0.1999	0.9148	0.0852	0.6577	34.2%	45.8%	54.2%	2	65	219	77	114	142	333
10	3	83.32%	85.1%	497	0.7759	0.2241	0.9260	0.0740	0.5473	45.3%	32.3%	67.7%	3	53	272	111	225	164	497
11	4	78.60%	85.7%	662	0.7711	0.2289	0.9430	0.0570	0.4713	52.9%	24.2%	75.8%	4	40	312	125	350	165	662
12	5	75.47%	86.6%	801	0.7710	0.2290	0.9603	0.0397	0.4232	57.7%	19.4%	80.6%	5	27	339	112	462	139	801
13	6	73.61%	87.3%	897	0.7709	0.2291	0.9755	0.0245	0.3958	60.4%	16.7%	83.3%	6	16	355	80	542	96	897
14	7	72.29%	87.7%	951	0.7664	0.2336	0.9867	0.0133	0.3817	61.8%	14.8%	85.2%	7	8	363	46	588	54	951
15	8	72.44%	88.3%	978	0.7729	0.2271	0.9937	0.0063	0.3753	62.5%	14.8%	85.2%	8	4	367	23	611	27	978
16	9	73.03%	88.9%	991	0.7815	0.2185	0.9971	0.0029	0.3724	62.8%	15.4%	84.6%	9	2	369	11	622	13	991
17	10	74.25%	89.7%	997	0.7956	0.2044	0.9988	0.0012	0.3711	62.9%	16.7%	83.3%	10	1	370	5	627	6	997
18	11	61.35%	81.4%	999	0.6296	0.3704	0.9993	0.0007	0.3704	63.0%	0.0%	100.0%	11	0	370	2	629	2	999
19	12	61.34%	81.5%	1000	0.6300	0.3700	0.9996	0.0004	0.3700	63.0%	0.0%	100.0%	12	0	370	1	630	1	1000
20	13			1000			1.0000	0.0000	0.3700	63.0%			13	0	370	0	630	0	1000
21														370		630			

Retail practitioners can improve the diffusion of retail innovations in their SLTA's. Utilizing the principles in this chapter and the available proprietary software that has been developed from this chapter, the retail professional will likely be ready to quickly engage Early Adopters, once an innovation is launched, and steer it to a successful and rewarding conclusion.

SUMMARY

The Bass diffusion model's Adopter output is spatialized from the Bass time period " t_0 " i.e., introduction of the "launch" of the innovation, to its eventual market equilibrium. The Bass model was selected because of its wide acceptance as a forecasting model in marketing science, albeit temporal-only and aspatial. The process developed in this chapter is referred to as the "Bass-Bayes Spatial Extension" or "BBSE".

The Bass diffusion model's Adopters are successfully allocated proportionally to specific spatial statistical demographic units (i.e., the areal units-of-analysis for this study and referred to as "qualified census blocks" or QCBs) across a store level trade area (SLTA).

The methodologies and procedures developed here are novel and innovative and make a contribution through rigor and practicality in benefiting the retail practitioner in implementations across retail management workflows.

To assist retail management, an additional proprietary software package has been developed offering the procedures and methodologies discussed in the chapter in one convenient and easy to use Microsoft Excel format. The software also presents unique and easy to interpret visualizations of the procedures and methods for the busy retail practitioner focused on operationalizing theoretical advances.

In summary, the BBSE (Bass-Bayes Spatial Extension) enhances and optimizes retail location intelligence across an SLTA for the retail practitioner and extends the microanalytic potential of "temporal-only legacy forecasting models" as new "hybrid spatial=temporal models" capable of evolving into new and emerging ecological spatial-temporal retail trade area environments. The BBSE is the first in a number of small steps intended to be taken in pursuit of better non-intuitive problem-solving methodologies and paradigms for studying "high spatial-temporal resolution" processes operating during the diffusion of an innovation, across a retail store level trade area (SLTA).

CONCLUSION

The integration of the Bass diffusion model, Bayes' theorem with applied business demography and census bureau geography (at the store level trade area, SLTA) was explained and demonstrated. The chapter provides ample evidence of successful "non-intuitive, problem solving" as a consequence of the Bass-Bayes Spatial Extension (BBSE) approach. Dynamic, non-intuitive problem solving strategies and positive Gestalt effects appear to emanate from the BBSE thus making the spatial-temporal ecological trade area environmental concept meriting further exploration and extensions.

The rewards for those retail practitioners able to see value in this approach may be a whole new set of competitive tools to increase market share and store-over-store returns over and above those dictated by the firm.

Finding imaginative, novel and beneficially innovative interconnections between various transdisciplinary approaches, as evidenced in this chapter, for retail practitioners suggests a valuable contribution. Utilizing non-intuitive problem solving techniques in the important retail economic sector of the economy make such intellectual pursuits vital for the economic health of corporations; especially in an area of increased information collection like high spatial and temporal resolution, microanalytic modeling and analysis of ecological trade area environments.

In conclusion, the greatest scientist of all time, often was quoted as saying the importance of non-intuitive problem-solving approaches and "imagination" are the keys to solving the worlds difficult problems. Einstein often used novel gedanken experiments and apprentice mathematicians to further his life long belief that *"Imagination is more important than knowledge. Knowledge is limited. Imagination encircles the world." Albert Einstein* (Harris 1995).

Since the concepts presented here are of little value if they cannot be utilized and visualized by retail practitioners, Excel is utilized to capture how easily the multi-equilibria visuals of "Bowling Alley" and "Sand Box" visualize the microanalytic view of cross sectional generating process occurring within each Bass time period.

With little time to acquire advanced training knowledge of complex statistics and mathematics, this type of analytic knowledge is both simple to understand and easy to acquire through software. Because these types of simple, yet, computationally intensive and repetitive, Bayesian arithmetic calculations, lend themselves well to algorithmic implementation, the modeling presented in this chapter was incorporated into a proprietary and licensable software application, operating in Microsoft's Excel environment.

The "*iST-Maap™"* (i.e., *integrated spatial-temporal modeling and analysis platform*) is a "cloud-based" implementation that includes the BBSE and other key modules. The BBSE components support Supply Chain Management forecasting and Merchandise Distribution in one decision management system. This suite of non-intuitive problem solving applications and dashboards is available to help the interested retail practitioner better manage innovation diffusion opportunities and facilitate strategic competitive differentiation, to achieve proactive intervention, throttling and steerage effects on the direction and speed of the diffusion of an innovation. Contact CFRANKLI@UCI.EDU for additional information (August 2018).

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CHAPTER 3

TEMPORALIZING HUFF: PATHWAYS TO EQUILIBRIUM

Analysis of high spatial-temporal resolution retail trade area (RTA) adoption behavior, under the generating forces of innovation diffusion, requires the use of spatial interaction and temporal equilibria modeling. But few, if any such parsimonious models exist to support retail practitioners interested in studying the innovation diffusion process and its effects, much less intervening and controlling its outcome within the store level trade area (SLTA) while working at the microanalytic level. It is hypothesized that the necessary and sufficient conditions exist to achieve such spatial interaction and temporal equilibria modeling utilizing existing models (i.e., the Huff gravity model, Bass diffusion model and Bayes' theorem). The objective is to hybridize these traditional, widely accepted, uni-dimensional models to function as multi-dimensional ensemble models composited across an ecological view of a RTA environment. Results are presented and future research suggested.

INTRODUCTION

In this chapter multiple micro-equilibrium retail trade area pathways are explored and their generating process studied as the result of the diffusion of an innovation. A temporalized and modified Huff gravity model is developed to capture the microanalytic structural changes as a result in the innovation trade area. A version of the traditional Huff gravity model is developed as a temporalized and modified Huff spatial interaction model, temporalized to the Bass diffusion model forecasting time periods.

The "Bass-Bayes Spatial Extension", developed in a companion chapter, i.e., the "BBSE" sets up an ecological environment for modeling and analysis in which to study the spatial interactions with the trade area. The choice agents are Bass model "Adopters" divided into early adopters, i.e., "Innovators" and later adopters, i.e., "Imitators".

The temporal units-of-analysis are the Bass model forecast time periods while "qualified" census blocks (i.e., QCBs) are the spatial areal units-of-analysis within which the altering distance decay structural composition of adopters takes place.

This capability is important to pursue to enable a better high spatial and temporal resolution understanding of the generating processes that take place within a trade area during the diffusion of an innovation.

There are many micro-pathways of generating processes and many states among equilibria in an evolving retail trade area for an innovation. During the presence of an innovation diffusion process in a store level trade area, well defined phases of distance/time related consumer behavior activities occur that can be studied and micro analyzed.

The retail industry however tends to project the results of firm wide analysis, forecasting and market area revenue and profit projections, from the general firm level perspective. *"We must ask ourselves the purpose we want predictive mechanisms to serve. If our needs are satisfied by gross decision parameters at the level of the firm, then perhaps our models of shape are satisfactory and our structural requirements are not overly monumental"* (Mason 1975). "*If, however, we are seeking the basics for a theory of consumer demand at the micro-analytic level, then our structural models particularly are woefully inadequate." (Ibid)*

Seeking the basics for a microscale (store level perspective versus the firm level) understanding of innovation diffusion generating-processes; we hypothesize the "affinity" behavior between buyers and sellers has a strong influence on the buying behavior of Adopters of innovation diffusion. This is based on widely know information that for example Apple products have an affinity with certain adopters that only can be explained by affinity marketing principles.

To study this question we adopt a logical approach in the search for factored scalars (vectors or variables) like time and distance to understanding the marketing microscale processes and/or high spatial and temporal resolution interactions occurring in the SLTA.

THE HUFF GRAVITY MODEL

Huff's traditional formulation enables the proportion of neighborhood trade directed to competitive alternatives to be estimated as the prime output. This approach requires coefficients of estimation from historical data and does not help in trade estimation in "dynamic" situations (at high spatial-temporal resolution).

The generalized Huff model states, *"the probability of demand at location **i** being satisfied by a retail outlet at location **j** equals the relative attractiveness of this **j**th retail outlet, where attractiveness is defined as*

$$\frac{size\ of\ retail\ outlet\ j}{(distance\ separating\ locations\ i\ and\ j)^{\lambda}} \tag{4}$$

where λ *is a distance decay parameter. Total demand is then allocated by multiplying expression (1) by the demand at each location **i** and then summing these demands*." (Griffith 1982).

The formula and definitions is from Huff (1963):

$$P_{ij} = \frac{A_j^{\propto} D_{ij}^{-\beta}}{\sum_{j=1}^{n} A_j^{\propto} D_{ij}^{-\beta}} \quad (5)$$

Where:

- "A" - refers to a specific retail outlet's (i.e., store) consumer "attraction" where the stores square footage (sq. ft.) is used as a proxy
- "D" - is the Euclidean distance from the geocoded customer address to the store's geo-coded address.
- " $\boldsymbol{i}$ " - the location of the consumer relative to the multiple shopping centers "j's"
- " j " - the location of the retail shopping center
- "β" is a parameter for distance decay - empirically estimated
- "α" parameter for adjusting attraction - empirically estimated
- "*n*" total number of units of a variable

Note: Definitions and wording taken from (Griffith 1982)

HUFF DECONSTRUCTED FOR THE QCB

With respect to selection of the census block as the qualified spatial unit of analysis; *"The level of geography...will depend on data availability as well as the degree to which you want to use modeling to overcome limitations due to statistical data quality and/or impute values for geographic units smaller than the level at which data were published. "* (Ratcliffe 2016).

For the purpose of this analysis it is assumed that the diffusion of an innovation takes place (ceteris paribus) within a monopolistic, single store level trade area. that being said the Huff model must be modified to perform "instant equilibria" for each Bass time period.

> *"The Huff model was introduced by David Huff in 1963 (Huff 1963). Its popularity and longevity can be attributed to its conceptual appeal, relative ease of use, and applicability to a wide range of problems, of which predicting consumer spatial behavior is the most commonly known."* (Dramowicz 2005)

In the classic Huff spatial interaction configuration the orientation is "consumer-to-stores" or "i" to "j's". This is necessary because in the classic (i.e., consumer-to-stores) Huff model, stores have an "attraction" attribute (based on their size for example) that is directed toward consumers while consumers have a "demand" attempting to be satisfied by the store.

In the Huff inspired "Retail Affinity Model" (i.e., RAM), QCBs have an "affinity" attribute (based on their demographic attributes). This requires "reversing" the orientation so that the focus is on one store to multiple QCBs" or one " J " to multiple " i's ".

In the "modified" Huff (i.e., "one store-to-multiple QCBs), the "Retail Affinity Model" stores have a "supply" of the innovation.

"Reversing" the polarity simply means - changing the focus from one consumer demand/attraction perspective " i " and competing alternatives; to one store's affinity/supply perspective of store offering affinity with multiple QCB Adopters. The classic Huff formulation is repeated below for comparison to the Affinity version of the Huff formulation.

$$P_{ij} = \frac{A_j^{\propto} D_{ij}^{-\beta}}{\sum_{j=1}^{n} A_j^{\propto} D_{ij}^{-\beta}} \tag{6}$$

$$a = \frac{[(Median\ Household\ Income\ QCB\boldsymbol{i})\ (\#\ Occupied\ housing\ units\ QCB\boldsymbol{i})]/QCB\boldsymbol{i}\ \boldsymbol{Census\ Population}}{(distance\ separating\ locations\ i\ and\ J)^{\lambda}} \tag{7}$$

$$P_{iJ} = \frac{a_i^{\propto} D_{iJ}^{-\beta}}{\sum_{i=1}^{n} a_i^{\propto} D_{iJ}^{-\beta}} \tag{8}$$

Where:

- " a_i " is a measure of affinity of Adopter "*i's*" in QCB_i, such as Medium Household Income
- " D_{iJ} " is the distance from *J* to *i*
- " α "is an affinitive parameter estimated from empirical observations
- " β " is the distance decay parameter estimated from empirical observations
- " *n* " is the total number of QCBs including QCB_i.

3.1 The quotient received from dividing $\boldsymbol{a_i^{\propto}}$ by $\boldsymbol{D_{iJ}^{-\beta}}$ is known as the perceived "affinity" of $\mathbf{QCB}_i$ *by* the store located at *J*. The α parameter is an exponent to which a QCB's affinity value is raised, and enables the user to account for nonlinear behavior of the affinity variable. The β parameter models the rate of decay in the affinitive strength of Adopters, as QCBs are acquired further away from the store. Increasing the exponent would decrease the relative influence of more distant QCBs on a store. *(The preceding paragraph reworded by substitution from a summary by Dramowicz (2005).*

AFFINITY AND THE DECONSTRUCTED HUFF MODEL

From a retail practitioner's perspective, taking an "affinity/supply" marketing approach to a SLTA means "quietly" (i.e., from a competitive posture) facilitating the affinity groups interest in supply of a diffusing innovation,

> *"Definition of affinity marketing; marketing directed specifically toward a group of people who have a common interest, support a common cause, etc.,...marketing directed toward a*

particular affinity-group: 'A Dallas-based company's new twist on *affinity marketing*—selling electricity to the fans of the Texas Longhorns and Texas Aggies—appears to be a first for electricity providers and may represent a new way to reach customers in Texas … —Vicki Vaughan, *Houston Chronicle*, 31 Aug. 2010 " (Merriam-Webster 2014a)

The chapter posits that under innovation diffusion conditions (Hägerstrand 1952,1968; Allaway et al. 1994) the "affinity/supply" retail marketing strategy has certain advantages over the "loud" marketing stance of "attraction/demand".

Affinity is defined by Merriam-Webster as "a feeling of closeness and understanding that someone has for another person because of their similar qualities, ideas, or interests" (Merriam-Webster 2018). Creating "affinity" through recognizing and appealing to certain social groups and/or individuals is not new. It is typically practiced and implemented by marketers who use only demographic variable profiling by census division (Sweitzer 1975).

A Census block becomes "qualified" based on one or more ongoing geo-coded, empirically derived sales of the innovation (i.e., Adoptions) occurring in that particular census block. In the classic Bayesian three machine/defect problem (Kalbfleisch 2012); the census blocks can be crudely thought of as the "machines"; the census block population being analogous to the items produced by the "machines" and finally the Adopters allocated to the census block based on the proportion of ongoing geo-coded, empirically derived sales of the innovation in that census block, as the "defects".

The Adopters are allocated based on the proportional probability ratio of ongoing geo-coded, empirically derived sales of the innovation (i.e., the numerator) in that specific Qualified Census Block (QCB) and divided by the "census population" in the denominator. Posteriors are updated using Bayes theorem. In addition the census block "population" value may be weighted with demographic attributes for hierarchical ranking purposes with other census blocks.

An alternative ecological modeling and analysis environment appears to provide dynamic capability. By simulating the developing trade area formation, under an assumption of monopolistic conditions; high spatial-temporal resolution delimits the eventual market equilibrium to Bass time periods. These aggregating equilibria, forming longitudinally over all Bass time periods, result in the ultimate eventual retail market equilibrium. This approach provides a means of analysis of high-resolution, spatial-temporal innovation diffusion adoption behavior across a retail trade area (RTA). There have been several attempts (e.g., Haynes 1977, Shinohara 2012) but few, if any parsimonious, to support retail practitioners interested in studying microanalytic innovation diffusion effects at the store level trade area.

The Bass-Bayes Spatial Extension (BBSE) is a method by which a temporal-only Bass innovation diffusion forecast of Adopters can be located spatially within a SLTA and then be allocated to specific areal units (i.e., census blocks, the units-of-areal/spatial analysis). Only census blocks that contain at least one geo-coded REA (random empirical adoption) are included in the "shape" (Mason 1975) of the Innovation trade area. These "qualified" census blocks are referred to as "qualified census blocks" or QCBs and can accept the Bass forecasted Adopters according to a proportional formulations.

Linking the aspatial Bass temporal-only diffusion model to qualified spatial-only census blocks is interesting, novel and has many potential retail practitioner uses (e.g., proactive intervention with ongoing innovation diffusion processes). In addition utilizing the census blocks as the units-of-spatial-analysis provides demographic statistics and attributes that can assist the retail practitioner. Thus a store level trade area (SLTA) could be disaggregated into its smallest units-of-temporal-analysis (i.e., Bass time periods) for study and analysis.

Bayes' theorem and the Law of Total probability provide the probabilistic values for each census block based on the proportions of adoptions made to each census block. REAs can signal potential spatial auto-correlation and Tobler's first law of Geography, among Adopters and adoptions, indicating potential new neighborhood market potential and opportunities for the retailer.

To initialize the Bayesian inference engine, placeholders (Gelman 2008a, 2008b) are used to accept the Bass forecast Adopter output until actual store sales (i.e., presence data, REA or random empirical adoptions) of the innovation begin and can be used to disperse the Bass forecast adoptions. "Presence" data or geo-coded evidence (i.e., REAs) creates empirical priors with the QCBs representing different Bayesian " A_i " events in Bayesian search theory (McGrayne 2001).

Thus the BBSE provides the retail practitioner a simple, yet comprehension spatial-temporal ecological environment in which to work, test, experiment, monitor and conduct microanalytic studies on the generating processes of innovation diffusion at high spatial and temporal resolutions.

One major benefit to retail practitioner derived from the BBSE environment is the ability to develop strategies on "where" and "when" to best intervene in a store level trade area and the likelihood of "who" will be affected by such interventions. Table 7 provides the Bass specifications and parameters used for the example in this chapter.

Table 7. Bass Parametric Coefficients

M / p / q	1000 / 0.080 / 0.500						
		p	q	S(t)	M		
Period	Time	Innovators	Imitators	Adoptions	Total	N(t-1)	S(t)
t_0	0	80	0	80	920	0	80
t_1	1	74	37	110	810	80	110
t_2	2	65	77	142	668	190	142
t_3	3	53	111	164	503	332	164
t_4	4	40	125	165	338	497	165
t_5	5	27	112	139	199	662	139
t_6	6	16	80	96	103	801	96
t_7	7	8	46	55	49	897	55
t_8	8	4	23	27	22	951	27
t_9	9	2	11	12	9	978	12
t_{10}	10	1	5	5	4	991	5
t_{11}	11	0	2	2	2	996	2
t_{12}	12	0	1	1	1	998	1
t_{13}	13	0	0	0	1	1000	0
	Total	370	630	1000			

TEMPORALIZING THE HUFF MODEL

The chapter attempts to leverage the *"considerable literature...for the gravity model of spatial interaction"* and in particular develop a nonintuitive approach where *"Part of this literature deals with the use of a gravity model perspective to analyze market areas for retail outlets."* (Griffith 1982).

"Temporalizing" the modified Huff model, allows the "temporalized modified Huff" to report on the dynamic structure of the market area during the evolving diffusion of the innovation at each Bass time period. The "Retail Affinity Model" or RAM is utilized to refer to the "temporalized modified Huff model" so as to clearly distinguish it from the classic Huff model.

Essentially "delimiting" the RAM to thirteen (13) "instant equilibria" for each Bass time period (see Table 7) increases the temporal resolution from 1 end state equilibrium to 13 instant equilibria across the same spatial and temporal (i.e., chronological) timeframe. In geospatial science the M.A.U.P. is used to describe the problems in referring to spatial or areal units that change in shape and extent over time (ESRI 2018). Therefore there will likely be issues with the M.T.U.P. (modifiable temporal unit problem) as spatial-temporal data analysis evolves, but that topic is outside the scope of the current chapter.

Changing the resolution also changes the analysis from "static" (i.e., all Bass time periods) "end state market equilibrium" of the classic Huff approach to 13 periods of different "dynamic" instant equilibria. Modifying the Huff gravity model to analyze "affinity" versus "attraction" of the QCBs is required to capture the strong "silent" word-of-mouth effects.

"The keys to understanding the shape of market areas are found in central place literature. Our concepts of market area structure are anchored in the refinements of the traditional gravity model." (Mason 1975)

The Affinity modeled approach to merchandising and marketing of the diffusion of an innovation is the polar opposite of the techniques assumed in the creation of "demand" and "attraction" of consumers to stores used in the classic Huff model.

In other words the classic Huff model assumes "demand" from an "*i*" (i.e., consumer) being "attracted" to multiple shopping centers "*j's*" based on a store square footage size attribute. Most consumers are familiar with the noisy, expensive advertising and promotion required to create demand.

Two Figures, the "Sandbox" (Figure 5) and the "Bowling Alley" (Figure 6) may assist in visualizing the ecological high-resolution environment. The BBSE creates a "Bowling Alley" like image shown in Figure 7. It takes the major step of spatializing the Bass forecast Adopters. In (Figure 3) the rectangular "Sand Box" takes the next major step of temporalizing the Huffian spatial interaction to each Bass period.

Thus the BBSE is a necessary condition to "temporalize" the modified Huff gravity model or the Huff-inspired "RAM" (i.e., Retail Affinity Model). The RAM shows the "instantaneous market area (QCBs) equilibria" within each Bass time period and the evolving incremental and cumulative "structure" of the QCBs. A second necessary condition is the Sandbox concept.

The sandbox creates an "instant equilibrium" for the (Huff-like) Retail Affinity Model (i.e., RAM) where the small side of the rectangular Sand Box (the width is one Bass period and the length is the maximum distance from the store to the QCB where the sale are recorded.

This Huff inspired retail affinity model with spatial interaction as it core process depicts the evolving "structure" (Mason 1975) of the SLTA at the microanalytic level at various stages of the Bass longitudinal time series.

In the spatial-temporal ecological trade area environment paradigm, created by the Bass-Bayes Spatial Extension (BBSE), "supply/affinity" replaces "demand/attraction" subject to Retail Affinity Model or RAM.

In order to operate in a high spatial-temporal resolution environment the traditional Huff gravity model is "temporalize" (i.e., delimited to "instant equilibrium" with a Bass period as opposed to "eventual equilibrium across a market area"). In this environment the individual Bass time periods delimit the typical longitudinal equilibrium of the traditional Huff model to what is referred to as "instant equilibria" equivalent to each Bass time period.

Figure 7 and 12 display the "shape" and "structure" (Mason 1975) of an actual SLTA in Southern California from a private sourced retail SLTA database.

Spatial interactions of Adopters (i.e., Innovators and Imitators), facilitated by the BBSE are analyzed per Bass time period using a conceptual model called a "Sandbox" (see Figure 6).

The "spatial" length of the "sand box" is simply the range of the Euclidean distance of the furthest qualified census block (QCBs) centroid to the store census block centroid.

Conceptually, for 13 Bass periods, the length of the "Bowling Alley" (see Figure 7) would consist of 13 rectangular Sandboxes placed next to each other. Thus micro-analytic generating processes occur (1) spatially (i.e., cross-sectionally on the distance axis) and (2) temporally (i.e., along the Bass time line).

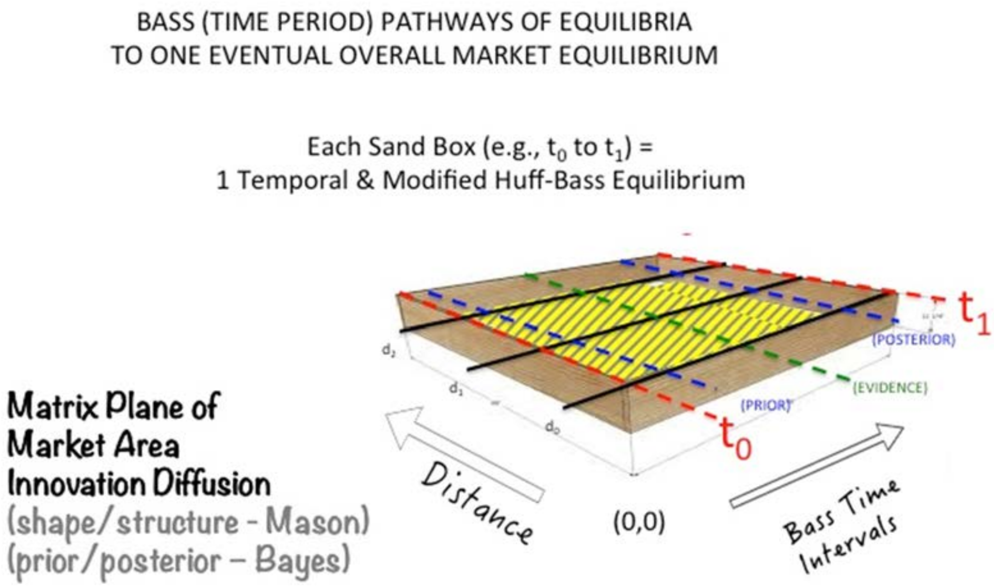

Figure 6. The "Sandbox" - One Bass period of time

However projecting logical inferences about structural or shape "cross sectional processes" subjects the prediction to Exception fallacy. If on the other hand the more traditional Huff gravity model spatial interaction analysis is relied upon, the longitudinal or "eventual market equilibrium" idea of identifying distance decay structure exposes the paradigm's logical deductions to Ecological fallacy.

The economic health of the retail sector and the profitability of retail firms can be positively impacted by a better understanding of how "affinity" not "attraction" better aligns retailers with potential innovation diffusion adopters.

From this perspective, "early adopter" interactions, both spatial and temporal, operate and aggregate to form successive "eventual mini-equilibriums" within the virtual temporal width of "prior Bass time period to posterior". These mini-equilibriums (i.e., multiple equilibria) allow a "temporalized modified Huff" or a supply/affinity "Retail Affinity Model" (RAM) to measure the dynamic structural effects for each Bass time period based on temporal (i.e., transaction time/date) and spatial (i.e., qualified census block centroid Euclidean distance to the store's census block centroid) while characterizing those dynamic effects with the additional magnitudes of each hierarchical transactional value (i.e., sales order value).

In other words time, space, and value serve as dynamic critical three-way factors from a SLTA perspective under the forcings of the diffusion of innovation. With this knowledge the strategy of intervening to improve the speed and depth of early adoptions becomes viable.

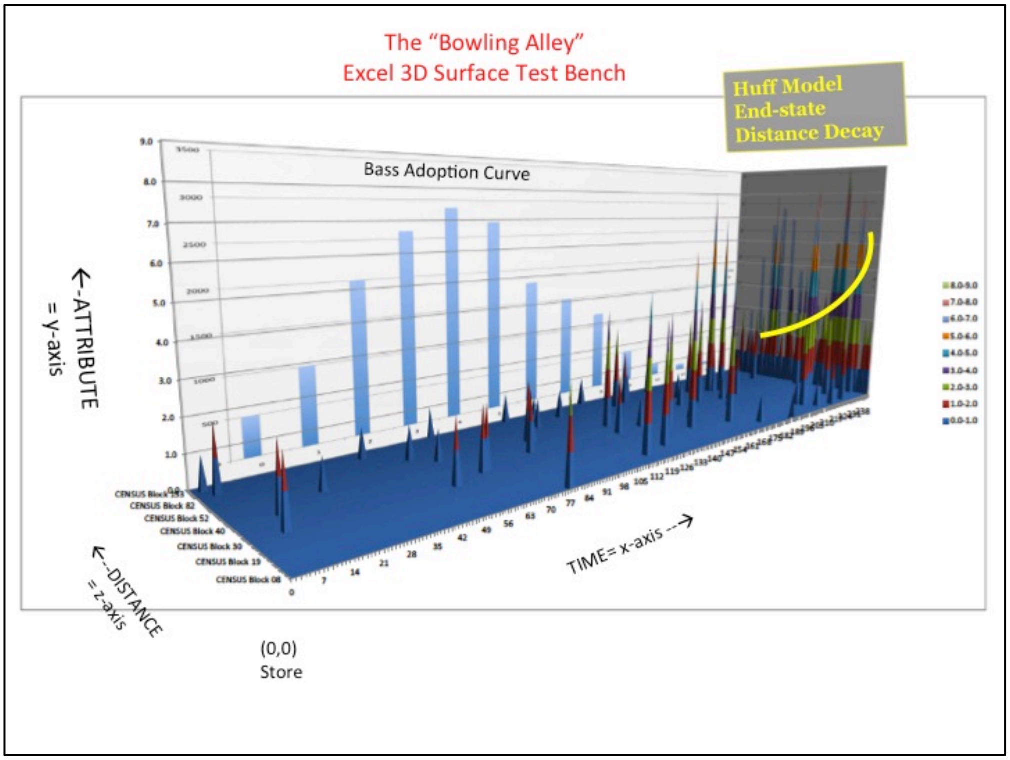

Figure 7. The "Bowling Alley" - Random Empirical Adoptions (REA)

Thus in order to visualize the impact on the dependent variable with time and Euclidean space (i.e., distance in miles) during the innovation diffusion process and any possible evolving structural changes happening in an SLTA (at each Bass time period); the traditional Huff gravity model must be "modified".

The modifications to accommodate a reversed Huff focus from (1) a single "consumer's" (i) "demand and attraction" directed to competitive alternatives i.e., "stores" (j) and alternatives to (2) a single "store's" (J) "supply and affinity" associated with like-mined "consumers" (i) with either homogeneous socioeconomic and demographic attributes and/or those with similar determinant attributes (Sweitzer 1975).

This approach maximizes the contribution the Huff model's spatial interaction analysis and other theoretical mechanisms (i.e., distance decay and forcing factors of innovation diffusion, word-of-mouth and early or late adoption stages) make within a new and innovative spatial-temporal ecological SLTA environment.

Each Bass time period is a "equilibria" unit-of-temporal-analysis. We call each unit a "Sandbox" The ecological trade area environment necessary for utilizing the "temporalized" and "modified" Huff gravity Model or RAM (for spatial interaction analysis of the effects of evolving generating processes on the structure of a SLTA under the conditions of innovation diffusion), is conceptually visualized in (Figure 11).

Hypothesis 1:

It is hypothesized that all necessary and sufficient conditions exist to temporalize and modify the traditional Huff gravity model and incorporate it into a spatial-temporal ecological modeling and analysis environment. A Temporalized "Modified Huff gravity model or Retail Affinity Model is proposed to test this hypothesis.

The following steps produce dynamic instant equilibrium in each Bass time period (i.e., multiple equilibria before reaching an end "static" state). Across multiple Bass time periods these microcosms of equilibria, are captured and aggregated each generation leading up to the traditional and eventual total trade area end state equilibrium.

First the Huff model's typical "instant equilibrium" (Allaway et al 1994) assumption in an eventual market saturation end state has to be "delimited" to each of the Bass periods, consecutively, one at a time. Secondly, the "orientation" of the traditional Huff model had to be "reversed". Instead of one consumer "i" representing "demand" and being "attracted" to multiple potential shopping centers jn; the methodology required one store representing "supply" and seeking "affinity" with multiple potential adopters.

Thus the "Temporal Modified Huff" (T&MH) or RAM emerges. Maximum entropy of the diffusion energy (i.e., the spreading out of Innovator and Imitator referrals through positive word-of-mouth and social contacts) is achieved when the multiple Bass time period microcosms of specific spatial temporal generating processes reach their end state.

Figure 6 visually presents how the conceptual model of the "reversed" Huff traditional gravity model was developed and devised. For example the Qualified Census Block (QCB) or areal unit "AU_3" has a population of 500 and one empirical Customer "i_3" (i.e., an Adopter). In addition, Euclidean distance is shown to calculate attenuating distance decay and determine the Huffian "structure" incrementally from posterior values from each Bass time period. When combined with demographic information, hierarchical ranking of the QCBs (i.e., qualified census blocks) can be achieved. The "shape" of each Bass time period market area is determined by REA (random empirical adoptions). This "evidence" is used to update the next time period priors whenever there are new QCBs and REA information.

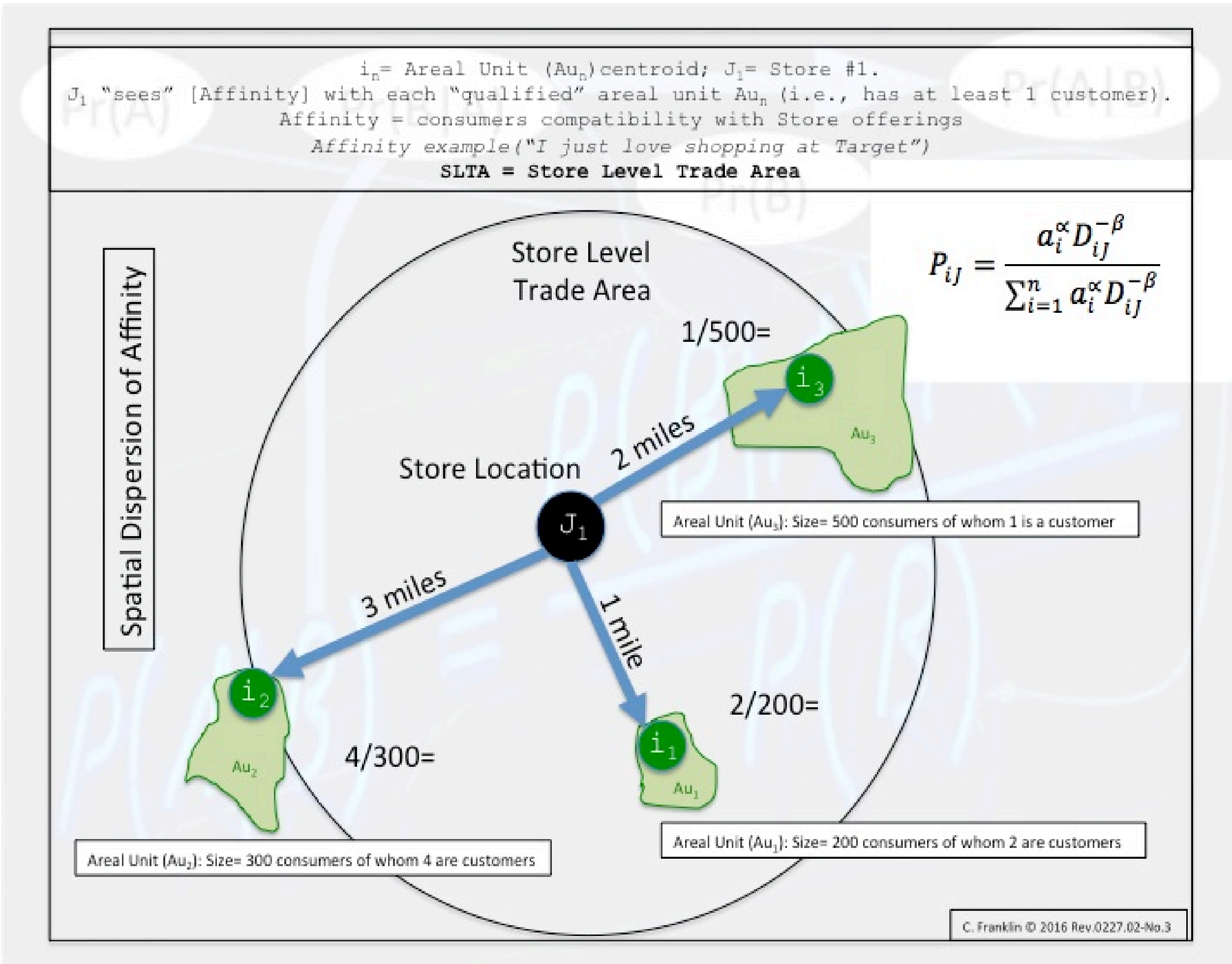

Figure 8. Spatial Dispersion of Affinity in Market Area with QCBs
Based on Bayesian 3 Machine Factory Defect Example
Franklin 2018

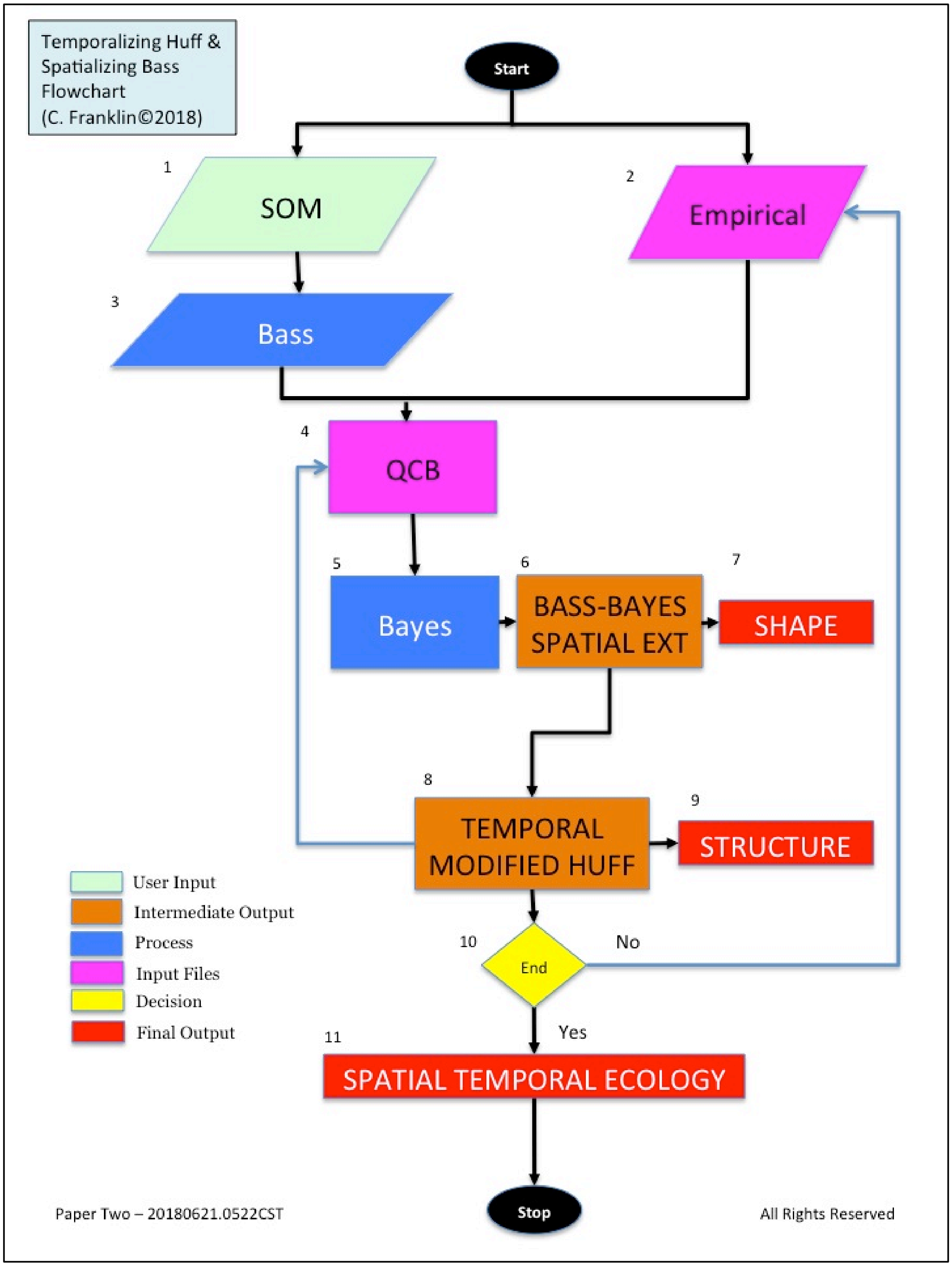

Figure 9. Flow Chart for Integrated Bass, Bayes Huff Market Area Ecology Franklin 2018

DEFINITION OF TERMS

1. SOM - **Determines** the marketing area of interest and assign values to the Bass parameters "M" total market, coefficient of innovation, "p" and coefficient of imitation, "q"
2. EMPIRICAL - **Collects** ongoing empirical geo-coded addresses of REA (random empirical adoptions), and selects the associated QCBs
3. BASS - **Calculates** a prediction table of Adoptions by quantity and timings
4. QCBs - **Aggregates** all "qualified census blocks" i.e., $Pr(A_i)$ events to identify the shape of an innovation diffusion
5. BAYES - **Calculates** Bayes theorem for each $Pr(A_i)$ event (QCB)
6. BASS-BAYES SPATIAL EXT - **Establish** likelihood of Adopter locations based on the REA (random empirical adoptions) as evidence
7. SHAPE - **Identifies** all QCB's for later visualization (e.g., GIS mapping)
8. MODIFIED TEMPORALIZED HUFF - **Integrates** the various data components to estimate "Affinity" [(Median Household Income x Occupied HU) / D^2]
9. STRUCTURE - **Calculates** hierarchical ranking of QCB demographics
10. END - **If** last Bass period reached then STOP
11. SPATIAL TEMPORAL ECOLOGY - **Ensembles** of spatial-temporal generating processes modeled and analyzed
12. SOM - **Determines** the marketing area of interest and assign values to the Bass parameters "M" total market, coefficient of innovation, "p" and coefficient of imitation, "q"
13. EMPIRICAL - **Collects** ongoing empirical geo-coded addresses of REA (random empirical adoptions), and selects the associated QCBs

14. BASS - **Calculates** a prediction table of Adoptions by quantity and timings
15. QCBs - **Aggregates** all "qualified census blocks" i.e., Pr(A_i) events to identify the shape of an innovation diffusion
16. BAYES - **Calculates** Bayes theorem for each Pr (A_i) event (QCB)
17. BASS-BAYES SPATIAL EXT - **Establish** likelihood of Adopter locations based on the REA (random empirical adoptions) as evidence
18. SHAPE - **Identifies** all QCB's for later visualization (e.g., GIS mapping)
19. MODIFIED TEMPORALIZED HUFF - **Integrates** the various data components to estimate "Affinity" [(Median Household Income x Occupied HU) / D^2]
20. STRUCTURE - **Calculates** hierarchical ranking of QCB demographics
21. END - **If** last Bass period reached then STOP
22. SPATIAL TEMPORAL ECOLOGY - **Ensembles** of spatial-temporal generating processes modeled and analyzed

When there is a Random Empirical Adoption (i.e., REA) the empirical sales event is treated as a "defect" by analogy to the three-machine example

The necessary steps are outlined in the above flow chart (Figure 7) with detail below:

1) Generate the Bass diffusion model forecast (See Table 2).
   a) "coefficient of innovation" p=0.08
   b) "coefficient of imitation" q=0.5
   c) M=1000. (Bass 1969).
2) Cycle through each Bass time period (utilizing the Excel calculations) and adding empirical sales data to determine the QCB (qualified census blocks).

3) The information is now put into an attribute table for calculation. For Bass periods "t_n" see column A8, A13, A19, A26 AND A34 in Table 1.
4) Next the census data is loaded into the table for QCB (a) Row 8; Columns J, K, and L; (b)
5) The store's trade area for a specific new innovative product consists of adopters in one census block initially. A simplistic analogy is the classic factory/three-machine defect problem (Kalbfleisch, 2012) is shown in Figure 8. Let us take a very simple example of how the ensemble modeling process works. Stage one in a two-stage chrysalis that reverses the Huff model from [one "i" and many "j's"] to [one "j" (the store location) and many "i's", in other words the centroid of the Census block containing the Bass adopter (since we do not have a geocoded address as we would in a sales transaction for delivery and install of an innovative product). The concept of store centric Affinity replaces consumer centric Attraction.
6) Assume the example is a home improvement, hardware or home center type store outlet operating in the densely populated limited and delimited Southern California geography. Examples of "real" stores like the example would include Ace Hardware, Lowes, Home Depot and Dixieline.
7) The Innovations are special order, delivered and installed products/services for a DIFM (i.e., DIFM - do-it-for-me) Adopter type customer. Thus so the geo-coded address is known and from that the census block number can be determined, its centroid (e.g., for store centroid to QCB centroid Euclidean distance measures) and its demographic attributes and statistical variables can be obtain or inferred.
8) The QCBs (i.e., qualified census blocks) become "qualified" simply by having one or more empirical innovation adoptions recorded within their areal unit during the Bass time periods.

9) The innovative product in the example can be visualized as a special-order, energy-efficient, architecturally high-end fenestrations (i.e., exterior window system).

WORKED EXAMPLE

The "modified Huff model" or to distinguish from the classic Huff model in this chapter; the Huff-inspired "Retail Affinity Model" (i.e., RAM), reverses the " *i* " and the " *j's* " focus of the classic Huff.

In addition, Huff measured relative "attractiveness" of alternative "store shopping locations" and "consumer demand" assumed to be continuous across the trade area geography (Huff 1963).

The RAM (or Huff-inspired Retail Affinity Model) measures relative "affinity" of alternative "QCB locations containing Adopters" assumed to be discrete areal units (i.e., census blocks, the spatial units-of-analysis), across an "innovation's market area" (possibly spatially "within" a store level trade area and probably noncontiguous) and "store supply" with alternative qualified census blocks (QCBs).

Affinity is defined as:

$$a = \frac{[(Median\ Household\ Income\ QCB\boldsymbol{i})\ (\#\ Occupied\ housing\ units\ QCB\boldsymbol{i})]/QCBi\ \boldsymbol{Census\ Population}}{(distance\ separating\ locations\ i\ and\ J)^{\lambda}} \quad (6)$$

with the general form being:

$$P_{iJ} = \frac{a_i^{\propto} D_{iJ}^{-\beta}}{\sum_{i=1}^{n} a_i^{\propto} D_{iJ}^{-\beta}} \qquad (9)$$

For simplicity, assume four qualified census blocks (QCB1, QCB2, QCB3 and QCB4). From the statistical demographic census block data, the number of "Occupied housing units" are respectively is 297, 611, 358 and 439; "Median Household Income" is $52,688, $33354, $53,292 and $43, 450 respectively; Population is 713,1230, 839 and 943 for the QCBs respectively. Finally Euclidean mileage distances between the QCB centroids and the centroid of the store location are: 1.0, 2.9, 4.3 and 6.7 respectively.

Not all statistics are available at the census block level. But for instance, medium household income, median value of housing, number of occupied housing units could be combined in a weighted index and then summed over all QCBs using the Law of Total Probability. *"That is, just because demographic characteristics are not available at the block level doesn't mean you couldn't "parse" block group or tract level data to derive a modeled set of block level statistics."* (Ratcliffe 2016).

In marketing terminology a trade area is hierarchically grouped as TAM or total available market (i.e., the size of the market in total) which might be global, SOM (service obtainable market) usually the same as the Store level trade area and SAM (i.e., Serviceable Available market), the market the store might expand into to service.

Since not all of the QCB populations are Bass "M" ultimate adopters, a reducing factor is applied i.e., Pi /100 or .0314 (Budak 2016). The new population figures are then 22, 39, 26 and 30 i.e., potential adopters within each identified QCB.

Finally calculating "Affinity" per the formula, the Affinity for QCB1 = (($52,688 x 297) / 713)(Prior) where Prior = (SAM Adopters per QCB/ normalizing factor of all SAM Adopters). Although this can become tedious to explain, software can easily manipulate the repetitions to beneficial effect.

The parameters, values, calculations and formulas can be seen as summarized in the Excel Tables 8 and 9.

Table 8. Affinity Bayesian Priors

Time	Demographic QCB Rank	Miles	AFFINITY "a"	Huff Bayes	Affinity Supply	TMH	Housing Units	MedHouseholdIncome	Population	PI	e
	Raw		Perceived						TAM	SAM	SOM
	Affinity		Affinity								
			of QCB							3%	
t_4	4,205	1.0	4,205	0.9%	10.5%	80.2%	297	$52,688	713	22	19
	5,470	2.9	650	5.8%	65.2%	12.4%	611	$33,354	1230	39	33
	5,119	4.3	277	1.7%	19.4%	5.3%	358	$53,292	839	26	23
	5,117	6.7	114	0.4%	4.9%	2.2%	439	$43,450	943	30	26
			5,246	8.8%	100.0%	100.0%					

Table 9. Excel Formulas

Time	Demographic QCB Rank	Miles	AFFINITY "a"	Huff Bayes	Affinity Supply	TMH	Housing Units	MedHouseholdIncome	Population	PI	e
	Raw		Perceived						TAM	SAM	SOM
	Affinity		Affinity								
			of QCB							0.03	
t_4	=(((I34*H34)/J34)*Q34)	1	=POWER(B34,1)/(PO	=G34*V34	=E34/E$38	=D34/D$38	=X113	=Y113	=T113	=(PI()/100)*	=(EXP(1)/10
	=(((I35*H35)/J35)*Q35)	2.9	=POWER(B35,1)/(PO	=G35*V35	=E35/E$38	=D35/D$38	=X116	=Y116	=T116	=(PI()/100)*	=(EXP(1)/10
	=(((I36*H36)/J36)*Q36)	4.3	=POWER(B36,1)/(PO	=G36*V36	=E36/E$38	=D36/D$38	=X114	=Y114	=T114	=(PI()/100)*	=(EXP(1)/10
	=(((I37*H37)/J37)*Q37)	6.7	=POWER(B37,1)/(PO	=G37*V37	=E37/E$38	=D37/D$38	=X115	=Y115	=T115	=(PI()/100)*	=(EXP(1)/10
			=SUM(D34:D37)	=SUM(E34:E37)	=SUM(F34:F37)	=SUM(G34:G37)					

Table 10. Bayes' Theorem Calculations

	N	O	P	Q	R	S	T	U	V
1									
2	"Defects"	BASS	QCB			"Defects"			POSTERIOR
3	Adopter			Prior%	PRIORS	CONDITIONAL	JOINT	POSTERIOR	
4	Empiricial	TOTAL		%	Pr (A_i)				
5	Sales	ADOPTERS	EVENTS	Potential	PRIOR	CONDITIONAL	JOINT	POSTERIOR	POSTERIOR
6	#	"M"	A_i	ADOPTERS	P(Census Tract i)	P(Customer/Census Tract i)	P(Customer/Census Tract i) P(Census Tract i)	P(Census Tract i/ Customer)	%
7									
34	1	22	QCB1	19.1%	0.1915	0.0446	0.009	0.01163	1.2%
35	40	39	QCB4	33.0%	0.3303	1.0348	0.342	0.46512	46.5%
36	28	26	QCB2	22.5%	0.2252	1.0624	0.239	0.32558	32.6%
37	17	30	QCB3	25.3%	0.2530	0.5740	0.145	0.19767	19.8%
38									
39	86	117		100.0%			P(B)	1.00000	100.0%
40							0.735		

Table 11. Excel Formulas

	N	O	P	Q	R	S	T	U	V
1									
2	"Defects"	BASS	QCB			"Defects"			POSTERIOR
3	Adopter			Prior%	PRIORS	CONDITIONAL	JOINT	POSTERIOR	
4	Empiricial	TOTAL		%	Pr (A_i)				
5	Sales	ADOPTERS	EVENTS	Potential	PRIOR	CONDITIONAL	JOINT	POSTERIOR	POSTERIOR
6	#	"M"	A_i	ADOPTERS	P(Census Tract i)	P(Customer/Census Tract i)	P(Customer/Census Tract i) P(Census Tract i)	P(Census Tract i/ Customer)	%
7									
34	1	=K34	QCB1	=O34/O$39	=Q34	=N34/O34	=S34*R34	=S34*R34/$T$40	=U34
35	40	=K35	QCB4	=O35/O$39	=Q35	=N35/O35	=S35*R35	=S35*R35/$T$40	=U35
36	28	=K36	QCB2	=O36/O$39	=Q36	=N36/O36	=S36*R36	=S36*R36/$T$40	=U36
37	17	=K37	QCB3	=O37/O$39	=Q37	=N37/O37	=S37*R37	=S37*R37/$T$40	=U37
38									
39	=SUM(N34:N37)	=SUM(O34:O38)		=SUM(Q34:Q37)			P(B)	=SUM(U34:U37)	=SUM(V34:V37)
40							=SUM(T34:T37)		

Table 12. Huffian Temporalized Data

Time	Numerator	AFFINITY "a"	Huff Bayes	TMH	Affinity Supply	Miles	Housing Units	MedHouseholdIncome	Popularion	PI	e
									TAM	SAM	SOM
t_0	21,959	21,959	1.0000	100.0%	100.0%	1.0	297	$52,688	713	22	19
		21,959	1.0000	100.0%					713		
t_1	10,092	10,092	0.0853	93.8%	60.3%	1.0	297	$52,688	713	22	19
	12,285	664	0.0562	6.2%	39.7%	4.3	358	$53,292	839	26	23
		10,756	0.1414	100.0%	100.0%				1552		
t_2	6,279	6,279	0.0223	91.5%	40.5%	1.0	297	$52,688	713	22	19
	7,643	413	0.0147	6.0%	26.6%	4.3	358	$53,292	839	26	23
	7,640	170	0.0181	2.5%	32.9%	6.7	439	$43,450	943	30	26
		6,862	0.0552	100.0%	100.0%				2495		
t_3	4,205	4,205	0.0164	80.2%	24.3%	1.0	297	$52,688	713	22	19
	5,470	650	0.0253	12.4%	37.6%	2.9	611	$33,354	1230	39	33
	5,119	277	0.0151	5.3%	22.4%	4.3	358	$53,292	839	26	23
	5,117	114	0.0106	2.2%	15.8%	6.7	439	$43,450	943	30	26
		5,246	0.0674	100.0%	100.0%				3726	117	101
t_4	4,205	4,205	0.0093	80.2%	10.5%	1.0	297	$52,688	713	22	19
	5,470	650	0.0577	12.4%	65.2%	2.9	611	$33,354	1230	39	33
	5,119	277	0.0172	5.3%	19.4%	4.3	358	$53,292	839	26	23
	5,117	114	0.0043	2.2%	4.9%	6.7	439	$43,450	943	30	26
		5,246	0.0885	100.0%	100.0%						

Table 13. Excel Formulas

	A	B	C	D	E	F	G	H	I	J	K	L
1												
2	Time	Numerator	AFFINITY	Huff	TMH	Affinity	Miles	Housing	MedHouseholdIncome	Popularion	PI	e
3			"a"	Bayes		Supply		Units				
4										TAM	SAM	SOM
5												
6												
7												
8	t_0	=(((I8*H8)/J8)*Q8)	=POWER(B8,1)/(POWER(G8,2))	=E8*V8	=C8/C$9	=D8/D$9	1	=X113	=Y113	=T113	=(PI()/100)*J8	=(EXP(1)/100)*J8
9			=SUM(C8)	=SUM(D8)	=SUM(E8)					=SUM(J8)		
10												
11												
12												
13	t_1	=(((I13*H13)/J13)*Q13)	=POWER(B13,1)/(POWER(G13,2))	=E13*V13	=C13/C$15	=D13/D$15	1	=X113	=Y113	=T113	=(PI()/100)*J13	=(EXP(1)/100)*J13
14		=(((I14*H14)/J14)*Q14)	=POWER(B14,1)/(POWER(G14,2))	=E14*V14	=C14/C$15	=D14/D$15	4.3	=X114	=Y114	=T114	=(PI()/100)*J14	=(EXP(1)/100)*J14
15			=SUM(C13:C14)	=SUM(D13:D14)	=SUM(E13:E14)	=SUM(F13:F14)				=SUM(J13:J14)		
16												
17												
18												
19	t_2	=(((I19*H19)/J19)*Q19)	=POWER(B19,1)/(POWER(G19,2))	=E19*V19	=C19/C$22	=D19/D$22	1	=X113	=Y113	=T113	=(PI()/100)*J19	=(EXP(1)/100)*J19
20		=(((I20*H20)/J20)*Q20)	=POWER(B20,1)/(POWER(G20,2))	=E20*V20	=C20/C$22	=D20/D$22	4.3	=X114	=Y114	=T114	=(PI()/100)*J20	=(EXP(1)/100)*J20
21		=(((I21*H21)/J21)*Q21)	=POWER(B21,1)/(POWER(G21,2))	=E21*V21	=C21/C$22	=D21/D$22	6.7	=X115	=Y115	=T115	=(PI()/100)*J21	=(EXP(1)/100)*J21
22			=SUM(C19:C21)	=SUM(D19:D21)	=SUM(E19:E21)	=SUM(F19:F21)				=SUM(J19:J21)		
23												
24												
25												
26	t_3	=(((I26*H26)/J26)*Q26)	=POWER(B26,1)/(POWER(G26,2))	=E26*V26	=C26/C$30	=D26/D$30	1	=X113	=Y113	=T113	=(PI()/100)*J26	=(EXP(1)/100)*J26
27		=(((I27*H27)/J27)*Q27)	=POWER(B27,1)/(POWER(G27,2))	=E27*V27	=C27/C$30	=D27/D$30	2.9	=X116	=Y116	=T116	=(PI()/100)*J27	=(EXP(1)/100)*J27
28		=(((I28*H28)/J28)*Q28)	=POWER(B28,1)/(POWER(G28,2))	=E28*V28	=C28/C$30	=D28/D$30	4.3	=X114	=Y114	=T114	=(PI()/100)*J28	=(EXP(1)/100)*J28
29		=(((I29*H29)/J29)*Q29)	=POWER(B29,1)/(POWER(G29,2))	=E29*V29	=C29/C$30	=D29/D$30	6.7	=X115	=Y115	=T115	=(PI()/100)*J29	=(EXP(1)/100)*J29
30			=SUM(C26:C29)	=SUM(D26:D29)	=SUM(E26:E29)	=SUM(F26:F29)				=SUM(J26:J29)	=(PI()/100)*J30	=(EXP(1)/100)*J30
31												
32												
33												
34	t_4	=(((I34*H34)/J34)*Q34)	=POWER(B34,1)/(POWER(G34,2))	=E34*V34	=C34/C$38	=D34/D$38	1	=X113	=Y113	=T113	=(PI()/100)*J34	=(EXP(1)/100)*J34
35		=(((I35*H35)/J35)*Q35)	=POWER(B35,1)/(POWER(G35,2))	=E35*V35	=C35/C$38	=D35/D$38	2.9	=X116	=Y116	=T116	=(PI()/100)*J35	=(EXP(1)/100)*J35
36		=(((I36*H36)/J36)*Q36)	=POWER(B36,1)/(POWER(G36,2))	=E36*V36	=C36/C$38	=D36/D$38	4.3	=X114	=Y114	=T114	=(PI()/100)*J36	=(EXP(1)/100)*J36
37		=(((I37*H37)/J37)*Q37)	=POWER(B37,1)/(POWER(G37,2))	=E37*V37	=C37/C$38	=D37/D$38	6.7	=X115	=Y115	=T115	=(PI()/100)*J37	=(EXP(1)/100)*J37
38			=SUM(C34:C37)	=SUM(D34:D37)	=SUM(E34:E37)	=SUM(F34:F37)						
39												
40												
41												

In Figure 10 the Huffian distance decay structure (i.e., the blue line RAM) can be clearly seen as a traditional negative exponential or power curve. Key insights can be obtained by the retail practitioner, on the spatial disposition and structural features of QCBs, with respect to adoptions, throughout the innovation market area and its proportion of the store level trade area (i.e., SLTA) Table 14 shows an analysis summary of metrics developed in this chapter. Figure 10 presents the graphical visualization of those summary metrics (i.e., from Table 14) of the Retail Affinity Model (RAM) in relation to the "a" Infinity measure and the Posterior distribution, all ranked by Euclidean distance.

Table 14. Summary of QCB Values for RAM, Affinity and Posterior

EVENT	DIST	RAM	Affinity	POSTERIOR
QCB1	1.0	80.2%	10.5%	1.2%
QCB4	2.9	12.4%	65.2%	46.5%
QCB2	4.3	5.3%	19.4%	32.6%
QCB3	6.7	2.2%	4.9%	19.8%

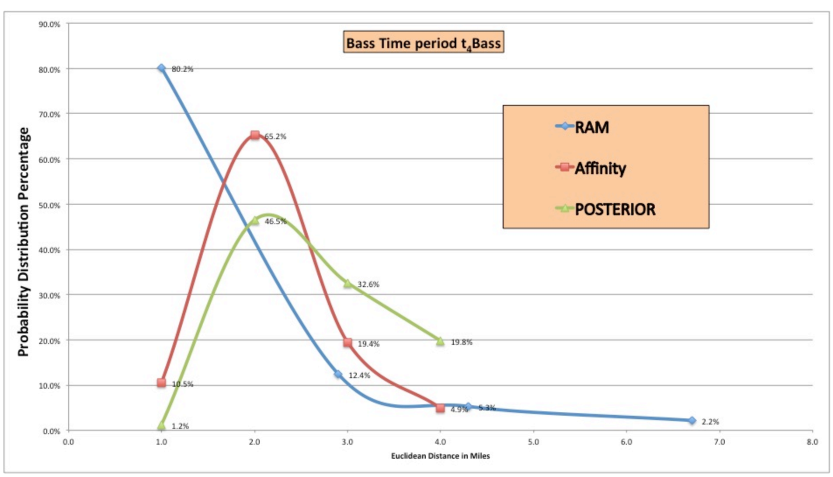

Figure 10. RAM + Affinity + Posterior Spatial Distributions

NEW TRADE AREA ECOSYSTEM

Ecology is defined as *"pattern of relations between organisms and their environment"* (Merriam-Webster 2018). Market area shape and structure (Mason 1975) play a major role in the store level trade area (i.e., SLTA), which is the geographic interest of this chapter. Again the two key features of a market area for a product or service (i.e., within an SLTA) are

- "Shape" (measured as contiguous or non-contiguous "areal units of spatial analysis" and
- (b) "Structure" (measured by customer density per Euclidean distance with gravity models).

> *"Marketers frequently use various types of spatial analyses in market area planning without understanding the underlying bases of the implicit spatial and temporal relationships in the paradigms used. Most students of consumer behavior are familiar with the contributions of Reilly, Converse, and perhaps one or two other individuals (Schwartz, 1963; Don and Ruth Mulvihill, 1970; Revzan, 1968) such as Huff but their knowledge for the most part does not go beyond this. Perhaps a reason for relative lack of interest is that many researchers do not believe that these paradigms offer the field of marketing explicit means for explaining or predicting the spatial behavior of consumers (Moore and Mayer, 1966). Thus, marketers have turned to a variety of other disciplines in developing useful spatial and temporal constructs for market analysis. Primary contributions have been made by geographers (Berry and Horton, 1970), economic historians (Fite and Reese, 1965), land economists (Hoover, 1948), regional scientists (Isard, 1956), and others." (Mason 1975)*

One of the objectives of this chapter is to explore how traditional, widely accepted and commonly used, uni-dimensional models (e.g., the spatial Huff Gravity model) might be modified for multi-dimensional use (i.e., the "temporalized" and "modified" spatial Huff Affinity model) and then composited into a new retail trade area (RTA) ecological modeling and analysis environment.

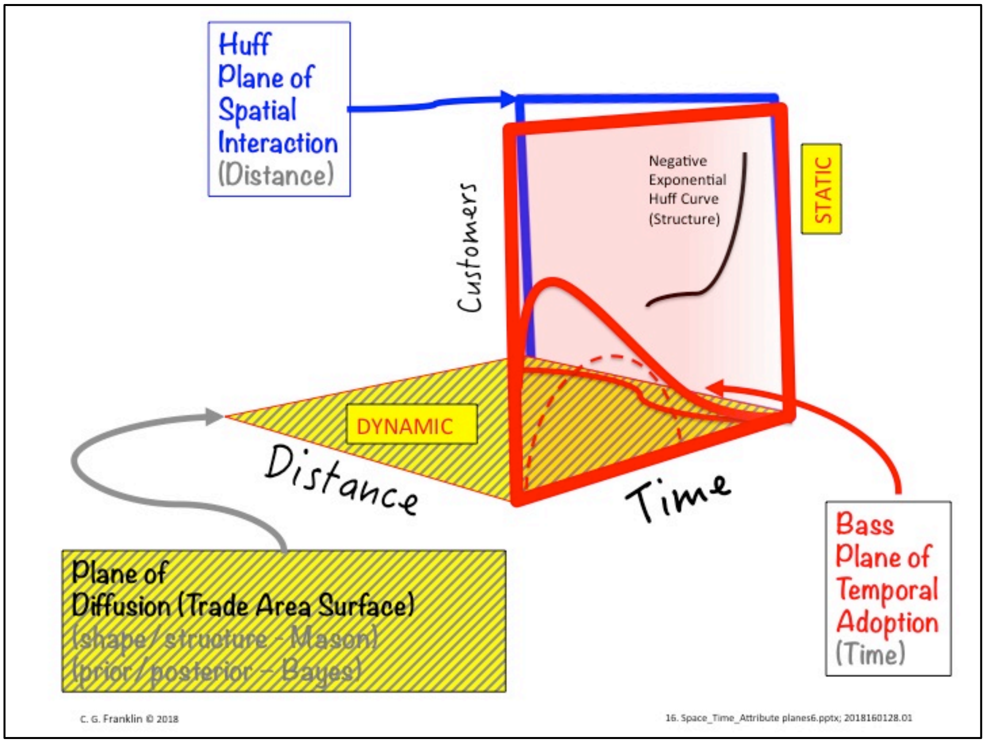

Figure 11. Conceptual "Spatial Temporal Ecological Pathway" (STEP) Franklin 2018

Figure 11 is a generalized, 3D conceptual model of the SLTA spatial-temporal, ecological environment envisaged in this chapter. The red plane is the temporal Bass; the blue plane is the spatial Huff and the yellow plane is the SLTA geography. For orientation, the intersection of the Time and Distance axis (i.e., in foreground) is equivalent to the Cartesian 3D coordinate position (i.e., x, y, z) "0,0,0", the store's census bloc*k centroid* location.

The three planar surfaces and the x, y, z space they describe can be defined as follows:

*1. **Plane of Diffusion (Trade Area Surface):*** This is the "Distance-Time", 2D "dynamic" plane (i.e., the yellow "floor" of the 3D box) that captures the evolving "shape" of a retail trade area by aggregating QCBs (qualified census blocks that contain at least one geocoded, empirical, innovation purchase transaction). The QCB centroid is utilized for Euclidean "distance" calculations from the store and "time" of transaction (i.e., space-time or spatial-temporal); where the concept of retail trade area "shape" is derivative of Central Place theory (Mason 1975).

2. ***Huff Plane of Spatial Interaction (Distance):*** This is the "Distance-Customers", 2D "static" plane (i.e., the blue "end-wall") that captures the eventual market equilibrium and distance decay effects of customer spatial locations. Those customer spatial locations are generalized in this method to QCB (i.e., qualified census block) centroids. The QCB contains at least one geocoded empirical innovation purchase transaction. The concept of a retail trade area "structure" (typically depicted as a negative exponential curve) by gravity models emerges as a subset of "shape" (Mason 1975). Generally, "structure" is defined as the organization of consumers on a vector of their Euclidean distance relationship to the store location (i.e., where the X-axis is Euclidean distance and the Y-axis is the number of customers). This typical negative exponential structure of a trade area indicates a drop in consumer attraction to the store location as the customer distance increases from the store. There are anomalous situations however when the exponential structure of a trade area form a positive exponential curve. For example, some types of innovations e.g., high-end, all-wood, energy efficient, architectural fenestrations (such as exterior window and door systems) being installed on gated community golf course multi-million dollar homes may increase the number of customers with distance in some circumstances.

3. ***Bass Plane of Temporal Adoption (Time):*** This is the "Time-Customers", 2D "static" plane (i.e., the red "side-wall") that captures the timing and quantity of Bass forecast innovation Adopters from innovation launch to market saturation (i.e., it tracks the quantity of Innovators and Imitators arising for each consecutive Bass time period), where the concept of predictive forecasting of the timing of Adoption regarding the spread of word-of-mouth among early to late adopters of a new product or services is supported by the work of Hägerstrand (1952, 1965, 19668), Rogers (1958, 1962, 1976) and Bass (1969, 1980, 1994).

BEYOND THE DELIMITED SLTA

Another feature of the RAM i.e., the Huff-inspired "Retail Affinity Model" is the two tier "spatial pricing" rationale it offers the retail practitioner an objective method for establishing the value of a sale "inside" and "outside" the "delimited" (by whatever means) SLTA.

Establishing affinity with customers beyond the SLTA "delimited" boundary is often possible. In marketing terminology this area is referred to as the SOM (i.e., serviceable obtainable market). A "supply/affinity" (versus demand/attraction) merchandising and marketing strategy, develops patronage "quietly" in terms of promotion.

No "loud screaming ads" attempting to attract attention, instead a low-key information al approach that relies on diffusion through the spread in target consumers social networks.

It can also utilize determinant attribute and demographic similarities (e.g., spatial autocorrelation and Tobler's law) through word-of-mouth (e.g., innovation diffusion) gives a retail practitioner what is referred to in economic as leverage. Leverage means that the revenue obtained is marginal, attracting only variable costs. Such revenue is spatially outside the expected trade area required to satisfy contribution to fixed costs.

As such, through affinity, capturing revenue and profits outside the spatially "delimited" (by whatever means) SLTA while servicing the SAM (i.e., Serviceable available market) introduces the concept of "spatial discounting" to bring the value in line with the pricing outside the Innovation Market area, SLTA and in the SAM.

Defining the situation is the first step in addressing such market potential and opportunities when the situation arises outside an SLTA. Thus taking advantage of breaches in the perceived trade area catchment is part of a trade area analysis and can lead to permanent expansion of the trade area. The experienced retail practitioner has learned to treat the outer boundaries of trade areas as fluid, shifting zones (Huff 1961), due to a multitude of dynamic exogenous and endogenous factors. Here briefly is how the FRPM operates.

Figure 6 depicts a retailer's multiple store locations in Southern California, symbolized as small yellow circles with black center dots. The "boundary conditions" are complex due competition and revenue cannibalization when new "sister" stores are opened because of the close proximity of existing stores. The vertical green bars are geocoded, residential home improvement customer transactions. The corresponding blue bars are a "re-valuation" of the green transaction value bars based on being "outside" the perceived catchment of the SLTA.

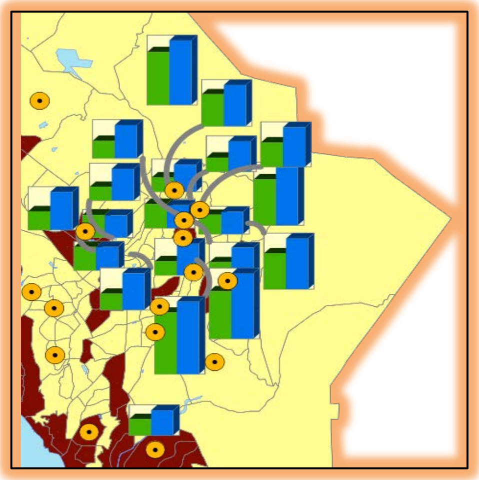

Figure 12. Typical Stores and their SLTA Franklin 2018

ADJUSTED STORE SALES FOR TRADE AREA BOUNDARY CATCHMENTS

The blue bars reflect a higher value to the store of such a sale and thus compensate for a more competitive pricing strategy in those outlier areas i.e., sales coming from outside the perceived store level trade area (SLTA).

The formula postulates as distance increases, the "apparent" value of a sale increases in areas at or beyond the perceived trading area catchment, zones and boundaries.

Retail Pull Model:

$$RPM_{iJ} = \sqrt{\frac{E*V_i}{\frac{T_i}{DiJ}}} *100 \qquad (10)$$

where:

RPM - Valuing distant transactions as discounted marginal revenue

E - In-store Experiential display factor = (1.4)

V - Actual transaction value ("Green" bar) = ($10,000)

T - Time in minutes from customer to store = (20 minutes)

D - Euclidean distance store and customer locations = (21 miles)

J - Store

i - Geo-coded customer residential address

[Source: Franklin 2018, FRPM calculations.xlsx]

In this overly simplified example, a standard priced "Green bar" store special order sales transaction of $10,000, converted to a "Blue bar" (FRPM) marginal revenue priced valuation, at a distance of 21 miles, actually has an equivalent value of $12,124 supporting a more competitive pricing strategy to bring the "Blue bar" valuation to equal the "Green bar valuation of $10,000. The argument being a discounted pricing strategy to customers outside the "reach of a trade area (i.e., customers in a SAM versus closer customers in a SOM) is justified as a management decision on the marginal growth in revenue expected the expansion geographically of a growing the trading area.

SUMMARY

Based on (1) the worked examples presented in this chapter and (2) the enabling Bass-Bayes Spatial Extension (BBSE) (developed in a previous chapter) the following summary can be offered: Bayesian methodology is a relatively simple procedure to apply to uni-dimensional models (i.e., either spatial or temporal) to create a "hybrid" multi-dimensional (i.e., spatial-temporal) model. Such hybrid models, in turn support a new, innovative and high resolution, spatial-temporal RTA ecological environment.

These are new findings in innovation diffusion practice, retail trade area theory, spatial interaction theory quantitative geography, and in the marketing science literature. The findings not only confirm the viability of re-purposing uni-dimensional models to "hybrid" multidimensional models, in new ways, but also validate the need for further nonintuitive and exploratory research in this new high resolution, hybrid-retail trade area environment.

Such further research would likely involve CyberGIS, Geospatial Information Science, Geographical Systems, spatial statistical modeling as well as development of best practices and the establishment of additional extensions and improvements to this approach on behalf of retail practitioners.

This powerful differentiating strategy is built upon a marketing approach of "affinity" with homogeneous target markets. "Affinity" is a force that simply means an emotional bond formed by individuals for a particular brand or business. This strategy can be supported by marketing information. It also relies heavily on word-of-mouth within the social networks of a store level trade area (SLTA). Word-of-mouth in social networks is also the primary mechanism of innovation diffusion between Innovators (early adopters) and Imitators (later adopters). Affinity can be determined through determinant attribute segmentation and utilization of census based demographics.

LIMITATIONS AND DELIMITATIONS

Bayesian search procedures typically begin with prior probabilities for initial hypotheses that contain large statistical errors. For example, a missing aircraft in the Atlantic Ocean has the initial prior hypotheses that it is "Somewhere in the Atlantic ocean". The strength of Bayes search theory is that with further evidence and iterations over time, the variance and errors are narrowed and reduced as new evidence and information is received in an iterative cycle. The human brain operates as a Bayesian inference engine eliminating the influence of the earlier defective values. With new evidence such as "triangulated signals showing a small area" the aircraft was last communicated the hypothesis improve. However without additional new evidence or long lead times on new evidence and information attainment, Bayesian procedures tend to stall. This also applies to the Bass-Bayes Spatial Extension. Because of the fast decay rate of innovators as first time buyers it is critical to locate them immediately, once a innovative product is launched, if ongoing geo-coded empirical sales of the innovation are slow, the ability to locate Innovators takes longer and may negatively impact the ability of the retail practitioner to maximize market potential and opportunities.

CONCLUSION

In conclusion it appears that rich new methods can be developed to present spatial-temporal data about trade area environments. In this chapter it has been a relatively simple matter to convert uni-dimensional models (e.g., spatial-Huff, temporal-Bass) to "hybrid" multi-dimensional spatial-temporal models through the use of Bayesian procedures. Therefore the overall conclusion is that Hybrid models are possible, potentially unique and valuable and more importantly, necessary conditions for the emergence of novel, innovative, high spatial-temporal resolution ecological modeling and analysis of trade area environments. Based on a review of the literature across a transdisciplinary spectrum, including diffusion of innovation theory, retail trade area theory, spatial interaction theory, quantitative geography and marketing science; this appears to be a new finding.

More specifically, spatial-temporal (i.e., multi-dimensional) analysis of the individual innovation diffusion generating processes e.g., (a) spatial-temporal consuming behavior of Adopters (i.e., Innovators and Imitators) (b) the aggregating "shape" of the evolving innovation market area in QCBs and (c) the distance-decayed, hierarchically demographic structure of QCBs as measured by the "temporalized modified Huff" model (TMH) or to avoid confusion with the classic Huff model, the "RAM" (i.e., a Huff-inspired "Retail Affinity Model"), that produces a rich detailed analysis.

As a final conclusion the chapter has developed its first rounds of spatial-temporal microanalytic results with the following 27 SLTA business scenarios in mind facing the retail practitioner. The chapter provides nonintuitive problem solving techniques with Bayesian logic such as the Retail Affinity model and an Ecological Spatial-Temporal Environment to provide innovative approaches to assist retailers with the launch of successful innovation-diffusion programs.

> *"The keys to understanding the shape of market areas is found in central place literature. Our concepts of market area structure are anchored in the refinements of the traditional gravity model. Central place theory provides the key assumptions underlying the structure of market areas. Thus, structure emerges as a subset of shape."* (Mason 1975)

Pre-identifying appropriate innovation intervention strategies and spatial entry points for QCB high density Innovator groupings is another important contribution of this chapter for intervention programs, after launch, which can greatly improve the chances of success. However once an innovation diffusion process gains momentum and begins rapidly moving randomly toward an end state market equilibrium, any intervention becomes less and less effective and more and more difficult for the retail practitioner.

Table 15. Store Business Outcome Scenarios

SCENARIOS	Trade Area Shrinking	Trade Area Unchanged	Trade Area Expanding
Sales Up	Profit Up	Profit Up	Profit Up
Sales Up	Profit Flat	Profit Flat	Profit Flat
Sales Up	Profit Down	Profit Down	Profit Down
Sales Flat	Profit Up	Profit Up	Profit Up
Sales Flat	Profit Flat	Profit Flat	Profit Flat
Sales Flat	Profit Down	Profit Down	Profit Down
Sales Down	Profit Up	Profit Up	Profit Up
Sales Down	Profit Flat	Profit Flat	Profit Flat
Sales Down	Profit Down	Profit Down	Profit Down

This chapter not only confirms the viability of re-purposing uni-dimensional models for multi-dimensional uses, but also informs the strong need to conduct further and immediate nonintuitive/exploratory research focusing on novel and innovative hybrid modeling and the ecological SLTA modeling and analysis environment.

Models, frameworks and methodologies from CyberGIS, Geospatial Information Science, Geographical Information Systems, spatial statistics and marketing science likely will support the ongoing development of the best practices needed to further establish this new ecological trade area paradigm with additional extensions and improvements on behalf of retail practitioners.

Viewing "location optimization" of SLTAs in an innovative and novel spatial-temporal ecological paradigm (STEP) may also shorten the time retail practitioners must spend learning new methods and tools. By simply utilizing existing tools, albeit reconfigured through new software algorithms; dashboards will be the graphical user interface of the future.

The approaches outlined in this chapter are designed for innovative products/services but may also prove beneficial in the marketing of low-innovation products and services to increase effectiveness and desired contributions to retail operations (see "pizza" store analysis Allaway et al 1994).

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CHAPTER 4

SPACE-TIME DIFFUSION: VISUALIZATION USING BAYESIAN INFERENCE

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Retail marketing geography has traditionally employed static gravity models for location analytics based on probabilistic locational consumer demand. However, such retail trade area models provide little insight into the dynamic space-time hierarchical diffusionary processes that aggregate to an eventual market structure equilibrium (Mason et. al., 1994), which gravity models attempt to predict for retail trade areas. In addition, most attempts to display the aggregating dynamic space-time hierarchical diffusionary processes of space, time and attributes of interest, in a geographical information system (GIS), produce visualizations that are overly complex and typically displayed utilizing unfamiliar paradigms. Further, these attempts fail to take into account the extensive body of literature in psychology and brain science that stress the importance of perceptual elements and design in achieving optimum visualization comprehension. In other words, simplicity (e.g., three-way factor analysis) and visual familiarity (cognitive fit theory, Vessey, 2006; mere-exposure effect in psychology, Dajonc, 1968) are key to better understanding. This will provide faster perception and better visuospatial and temporal understanding of objects and trends. In this study we incorporate these elements in our visualization object that we refer to as “Avatar”. A Huff inspired, Bayesian framework of inference for spatial allocation and hypothesis testing allows the Avatar object to display the spatial allocation of the Bass model’s innovators and imitators for sales forecasts of new product diffusion (e.g. a mathematical version of Everett Roger’s adoption concept), thus enabling and supporting faster and improved visuospatial understanding of very large data repositories of unbounded and/or “countably infinite” sized geo-big-data (referred to throughout the rest of this paper as GBD). We then introduce the three steps necessary to create an Avatar object (i.e., a 3-D semaphoric, space-time diffusion visualization object). The Avatar object is designed specifically to visualize determinant attributes (e.g. demographics) for the Bass, Bayes, Berry and Huff integrated ensemble model forming part of an ancillary paper to this study. In this way we display the timed hierarchical diffusion of new innovative products throughout store trade areas and across the ensuing and evolving store networks. In addition, by calculating Bayesian conjugate priors and posterior spatial allocation probabilities for the "smallest units of human settlement" (Christaller, 1966) or in our case statistical demographic units (i.e., Census Blocks), we establish customer (innovator and imitator) spatial distributions for the Bass temporal-only model for the case of the aggregating store level trade area (SLTA) scenario. Our approach is empirically supported by five years of new product diffusion geocoded panel data from the Southern California market. We conclude that our cognitive fit theory validated Avatar space-time diffusion visualization strengthens "location analytics" and "location intelligence" and provides a simple and familiar tool for displaying GBD across a growing domain of varying applications and end-user knowledge and needs.

INTRODUCTION

"As the mass of data generated to understand business problems increases both in terms of variety as well as complexity, better methods are needed to display, communicate and analyze such information" (Huff, 1981).

When Huff published this statement in the fall of 1981, the first fledging IBM-PC had just been introduced and yet Huff's statement today is timely. Today increasing quantities of digital information, in various forms and complexity, are accumulating at increasing orders of magnitude, over shorter and shorter periods of time. If this information can be presented in familiar and simple visualizations, the literature of psychology and brain science provide ample theories and models to connect our powerful cognitive and visual recognition strengths with certain visuodesign principles (Gestalt), resulting in faster perceptual transfers of both meaning, understanding and knowledge from analysis of geo-big-data (i.e., GBD).

In retail marketing geography, going past 2D/3D symbolized map visualizations, may hold potential for greater understanding of complex consumer behavior, primarily through "theories of consumer demand at the micro-analytic store level" (Mason, 1975) and cognitive theories from the psychological and brain science literature. Detecting the micro, as well as the macro space-time adoption effects over increments of time, for new innovative product diffusion and life cycles (Bass, 1969), allows a chain retailer the ability to investigate (a) the effects of individual store level market potential and opportunities and (b) the effects at the firm level as a whole (Lusser's Law) with respect to maximizing market share, revenues and profits across growing (i.e., aggregating) trade area shapes and structures (Mason, 1975). Also within the context of store level trade area diffusion of adoption (Rogers, 1976), visually identifying processes from positive statistical areal unit autocorrelations of "radial customer zones" (Huff, 1963), to better understanding the impacts on internal sales related processes such as supply chain management, may also contribute to strengthening the timely management decision making processes for new innovative product market penetrations across an entire, evolving store network (Berry, 1971).

To this end our study presents a novel approach to visualizing and exploring the salient space-time dimensions of GBD, while adhering to validating perceptual principles from psychology and brain science. Such visualizations may offer further insights into the importance of Euclidean distance as a standardizing factor for analysis (ceteris paribus) in the search for interesting federated trends and directions (Bunge, 1962) in the GBD from a mathematical geographic perspective (Goodchild, 2008), at which point further detailed and traditional GIS methods and techniques may be applied.

We shall make use of the polar azimuthal equidistant projection with the meridians modified to represent time. Polar coordinate systems are not new to geography or geometry. The polymath and Muslim scholar “al Biruni”, over 1000 years ago, discovered the polar azimuthal equidistant projection (King, 1996). In terms of today’s use, one familiar example of a polar azimuthal equidistant projection is the United Nations emblem (Figure 13).

There are few, if any, examples in the geographical information science (GISc) literature, of experimentation with polar coordinate visualizations by modifying the meridians to represent temporal frames of reference for space-time analysis. Further by assuming that Time, Distance, and Variables are the important factors to consider (in a three-way factor analysis approach) as Brian J.L. Berry and his students had done in geography during the early 1960’s at the University of Chicago, we can construct a virtual and physical visualization model in Excel software and as a 3D printed semaphore object, which we refer to as the Avatar model.

Figure 13. Example of a familiar polar azimuthal equidistant projection

The Avatar was developed to be a novel and innovative visualization in support of a novel and innovative theoretical attempt to link two well-known models Frank Bass’s temporal diffusion model and David Huff’s spatial interaction model. The methodology, through which this linkage is achieved, consists of modifications to the focus of the Huff model with an additional temporal iteration mechanism developed using Bayes theorem. Finally, to this conceptual visualization we add Berry's hierarchical innovation diffusion concepts of an evolving network of retail store locations, creating the robust Avatar ensemble model.

The robustness of the ensemble model is based on the fact it *"introduces a Bayesian framework for inference, visualization and hypothesis testing of…" (Lemey, 2009)* innovative product diffusion and life cycles (e.g. Bass Model) for inter and intra store level trade areas (i.e., Berry's concepts).

We begin with a brief review of the history of store level trade area visualization and analysis, which provides the contextual background for what follows. We then outline the steps needed to construct our Avatar ensemble space-time trade area visualization model, proceeding sequentially through these steps and ending with the final result, a 3-D printed Avatar model.

DISCUSSION

Store level trade area analysis and visualization, using gravity models (e.g. the Huff Model) have been popular in geographic information systems (GIS) applications and continue to inform and dominate retail trade area analysis, location analytics and location intelligence activities in response to increasing quantities and types of GBD. A fundamental "…prerequisite in determining the potential market demand for the products or services of a prospective retail firm or of agglomerations of prospective retail firms, within an urban area, is a geographical delineation of the region containing the probable customers for such goods. Such a region is called a retail trade area." (Huff, 1963).

Gravity models as mathematical and statistical tools are a class of discrete choice spatial interaction models (Haynes, 1984) that calculate non-subjective, empirically based, probability estimates of static retail trade area demand distance limits (Huff, 1963).

A typical trade area analysis visualization consists of contour lines. However, it provides little, if any, information about the incremental spatial-temporal interactions of new product diffusion, or information about the intervening spatial-temporal hierarchical relationships developing throughout the store or evolving network trade areas. In other words there is no dynamic spatial temporal resolution, much less any visualization of spatial temporal events and scenarios.

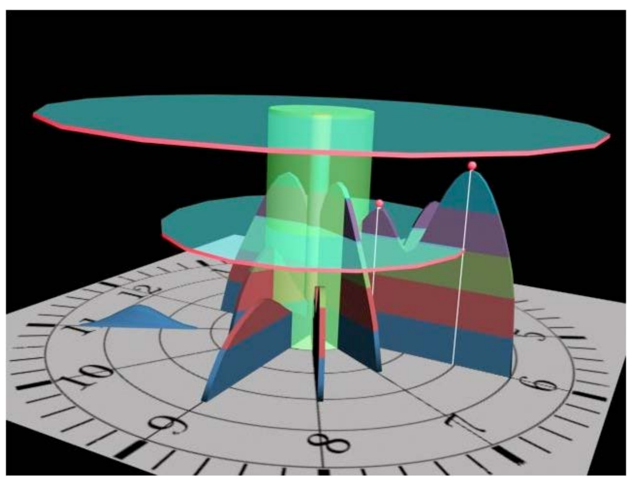

Figure 14. The Avatar Concept – a time-modified polar azimuthal equidistant projection

GOAL

> *"When a new phenomenon (bus stop, emergency center, retail store or even advertising campaign) is introduced into an existing spatial market, discrete choice models estimate the eventual market structure equilibrium rather than the process of attaining it." (Allaway, Black, Richard, Mason, 1994)***.**

The key theoretical ambition of this novel research is to design a familiar visualization capable of (a) displaying three simple factors (Time, Space and Variables) and (b) the interim processes necessary for "attaining market structure equilibrium" versus a visualization capable of only displaying the eventual end-state market structure equilibrium such as the Huff Gravity Model.

There have been space-time visualizations attempted before in GIS (e.g. Kwan, 2004) resulting in various degrees of visual intuitiveness and clarity; typically dependent on the complexity of the data and the comprehensibility and experience of the observer. For example "aquariums" visualizations combine a digital map (in the X-Z plane), joined to a vertical dimension (a Y axis perpendicular to the X-Z plane) that represents increasing increments of time. In other words time becomes physically "vertical".

Some geodatabases (e.g. New York Taxi pick-up and drop off geocoded database) if displayed as an "Aquarium" or a "Rubik cube" paradigm, would generate such large point densities, that visually comprehending the individual points would be difficult or even impossible at any practical scale. Additionally it would be perceptually difficult or simply impossible to sort out and keep track of the intricate temporal increments. Eventually the brain's visual perceptual system overloads and the comprehensibility factor for this type of paradigm reduces significantly, even for the trained and educated observer.

COGNITIVE FIT THEORY APPLIED TO GBD VISUALIZATIONS

Information systems research conducted by Iris Vessey produced Cognitive Fit Theory in 1991. The theory attempts to relate task performance variations to different types of data visualizations such as pictures, graphs and tables (Vessey, 2006; Umanath & Vessey, 1994). In GIS specifically, researchers have used cognitive fit theory to explore user performance variations when presented with spatial relationships tasks (Dennis and Carte, 1998; Smelcer and Carmel, 1997). In other words, Cognitive fit theory provides a theoretical structure to explore and explain variations in task performance due to varying comprehensibility of different types visual presentations. In other words, notwithstanding different levels of training, knowledge, age, health and experience; the brain's understanding of spatial relationships, for a controlled set of geodata, will vary depending on the type of visualization used to present that geodata. Continuing research into the effects of visualization on task performance for spatial relationships may also be enhanced to include simultaneous aggregating temporal effects using our semaphore object (i.e., Avatar) visualization approach.

The brain's perceptual system thus appreciates, anticipates and expects familiar and simple objects and shapes when viewing complex GBD. There may also be a perceptual and inferred frame of reference the brain sets up to anticipate and interpret visual images within the context of a particular situation or scenario (e.g. across time the brain may seek a familiar clock face in the visualization).

Familiarity with objects we posit is learned behavior and socialized from childhood through norms, values and mores in a sociological sense. Generally, shared belief systems inform comfortable cultural familiarity and context for individuals. Simplicity and familiarity likely will reduce the permutations and combinations of possible outcomes of anxiety, inherent when the brain is faced with interpreting and understanding unfamiliar, complex GBD visualizations. When the brain's visual anticipation or expectation is not in synch with the visual "reality" being perceived, dissonance and anxiety ensue. Abstract visual concepts, like "vertical time", become confounding abstractions creating potential distraction in the brain's focus and attempt to understand and interpret GBD. Finally, the risk of error, and unintentional consequences such as misreading, misunderstanding or misinterpreting unnecessarily complex visualizations of GBD increases in the same way a poorly or incorrectly symbolized traditional GIS maps would lead the observer to potentially fallacious conclusions.

Researchers connecting spatial and temporal dimensions, without regard to psychological literature, run the risk of developing myopic scholarship. Cognitive fit theory appears to confirm the brain's ability to perform object recognition and understanding, with relatively high error free comprehension, went the visualization is designed and executed with simplicity, familiarity and proportionality, independent of the complexity of the GBD.

Therefore in this study our key strategy is to reduce, whenever possible, what we consider noise or confusing and unnecessary details, and create simple, familiar, proportional, efficient and effective methods for presenting only the necessary and sufficient dimensions of space, time, and attributes or variables required initially for an observer to easily decipher global perspectives from the information generated from GBD and thus reach the goal of understanding and having the option of applying the knowledge gained from such information.

RESEARCH QUESTIONS

There are two primary research questions that arise. Is there a perceptual-neutral, space-time visualization device, that anyone can easily comprehend (regardless of analytic skill), which at the same time is capable of reducing large volumes of GBD into a simple and familiar visualization object?

Secondly, as retail “consumers” convert to new “customers” and existing customers repeat their purchasing behavior (through the process of adoption during product life cycles of new innovative products); How can diffusing customer location distribution patterns be best visualized in space and time across a Store Level Trade Area (SLTA) and across evolving hierarchical networks of store level trade areas?

METHODOLOGY

Factor analysis is generally described as a statistical method for reducing clusters of variables down to several representative key variables or “factors”. Noted geographer Brian Berry, beginning in the 1960’s, made significant contributions to the use of factor analysis in geography. However it has not been fully explored by other scholars in geography (Tucker, 1964).

Figure 15. First Conceptualization Sketch

In this chapter, three-way factor analysis reduced the GBD for a store level trade area and an evolving store network down to three principle components: space (distance/location), time (daily special orders) and attribute (number of customers, i.e., innovators and imitators per Bass). Our initial sketch of a preliminary three-factor approach is presented in (Figure 15). After many refinements and discussions, it leads to development of our primary visual hypothesis: Avatar. The concept, although simple, exposed a number of incompatibilities. One major incompatibility between the Huff (space) and Bass (time) models is the lack of a mechanism to spatially distribute the Bass innovators and imitators or a mechanism to iterate the Huff model through each time interval. Early attempts to study and analyze this incompatibility lead to a developmental visualization and conceptual model (Figure 16).

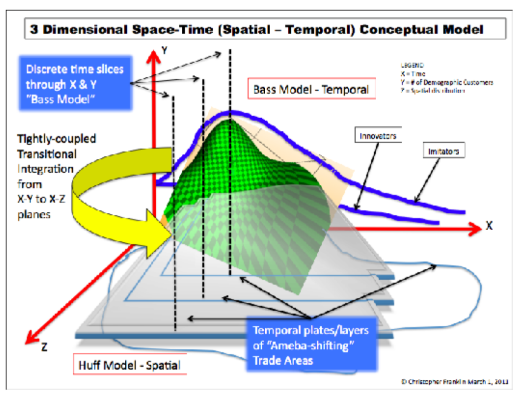

Figure 16. First Huff-Bass Conceptualization

Reducing this conceptual model to a simple and familiar object was the next task. Ultimately we formed an approach that included transdisciplinary collaborations with the schools of computer science, natural science and mathematics, arts and technology, brain science and business. With many serendipitous inputs from various colleagues and conversational "*Gedankenexperiment*" along the way, a test version of Avatar was developed in a standard Excel 3D graph before attempting a polar azimuthal equidistant projection version for experimentation and testing purposes.

This perceptual approach is grounded and validated in the psychological and brain science literature (e.g. Gestalt theory, Cognitive fit theory, mere exposure effect). The three-factors of space, time and attribute can be seen as the distance "Z-axis", time "X-axis" and attribute or variable "Y-axis" respectively (Figure 17). Note the store is located at the X-Z intersection in the foreground of the graph, i.e., position (0, 0). To visualize the transformation of this image to the Avatar, imagine the graph is wrapped clockwise in a circle. All of the components would then be aligned with the Avatar visualization.

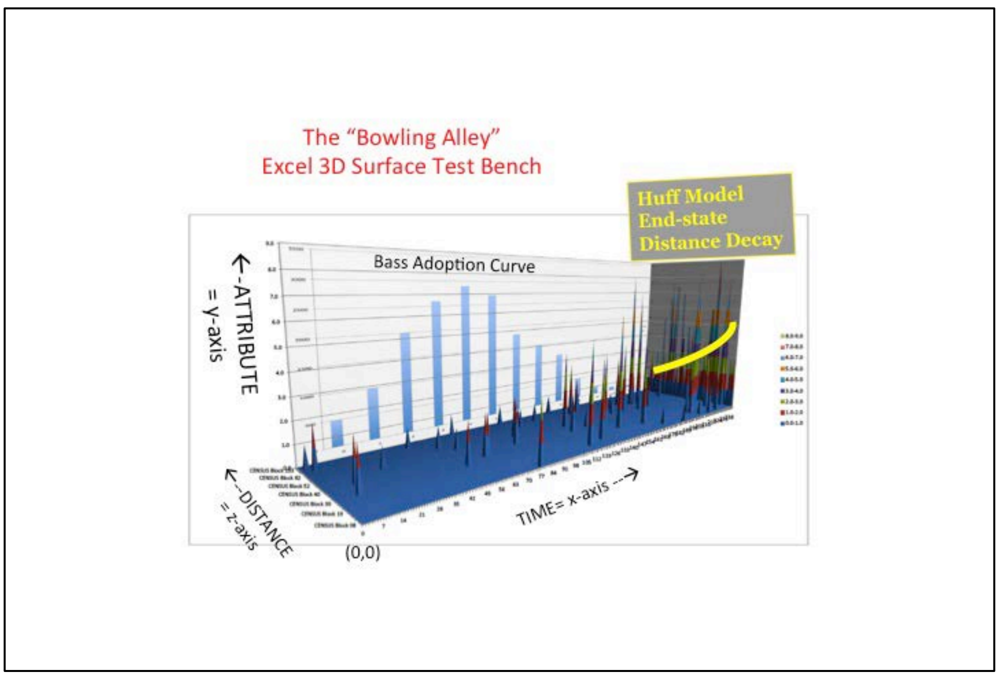

Figure 17. Early Excel 3D Surface Test bench for Prototyping the Space-Time Visualization

CONSTRUCTING AN ENSEMBLE SPACE-TIME DIFFUSION VISUALIZATION MODELESEARCH QUESTIONS

STEP ONE

The first step is to obtain geocoded addresses for a store and its customers (Figure 18). We begin with a simple example of five (5) customers arrayed around a store in a standard GIS coordinate system with various bearings and Euclidean distances from the store located at the center.

STEP TWO

Next we perform a rotational transformation of the vectors (i.e., Store to customer distances) to one angle, 90 degrees in the horizontal (Figure 19). This new radius or summing vector has all of the unique customer Euclidean distances indicated on its line. Thus this radius becomes the x horizontal axis in the avatar.

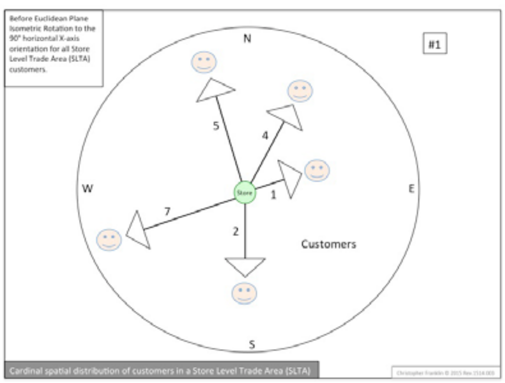

Figure 18. Begin with Geo-coded Data

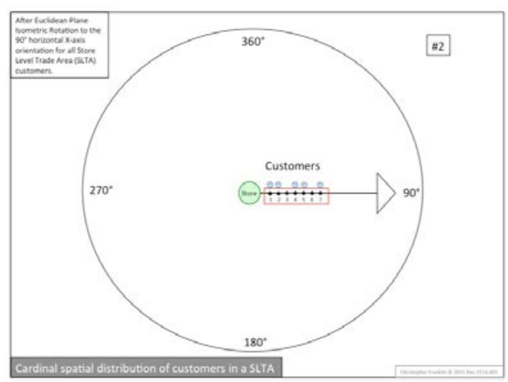

Figure 19. Rotate all distances to the X-axis

Next we transpose from a compass rose to a clock face. The final step is to add the multi-attribute vertical Y-axis attributes (e.g., # of Customers) to the clock face plane see Figure 20. We utilize a proportional measurement scale for the horizontal and vertical axis. Thus the magnitude of the GBD is represented proportionally (i.e., one Avatar could represent 1 store or 2500 stores). The concentric isometric circles emitting from the center are the standardized Euclidean distance on the X-axis an depict 1 mile, 2 miles, 3 miles from the store to the customer.

The circumferential axis represents whatever time denominations deemed appropriate (e.g. years, months, days, hours and so on).

Finally, we create two versions of the Avatar. The first is a physical 3-D printed object (Figure 21) and the second a virtual 3-D software object operating in a special version of Microsoft's EXCEL.

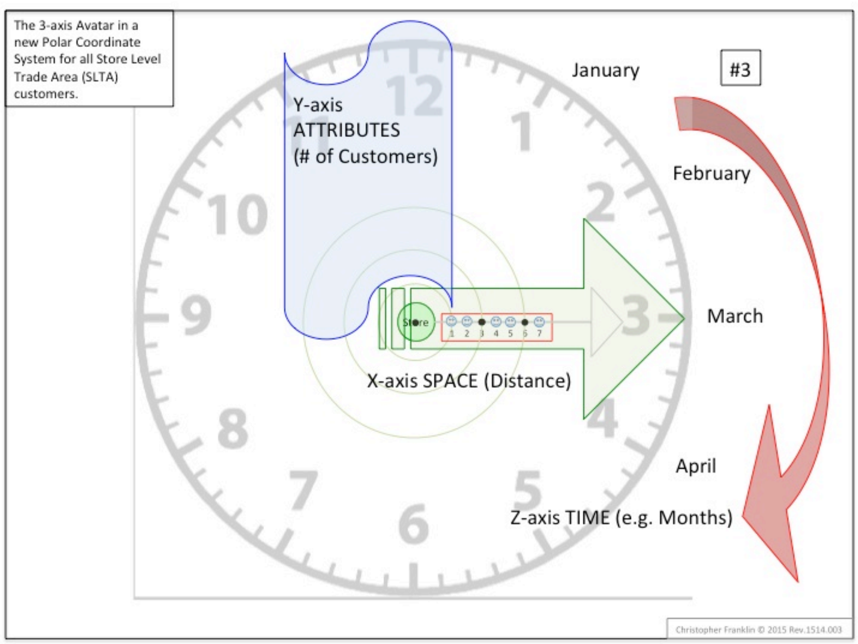

Figure 20. Core Space-Time Model

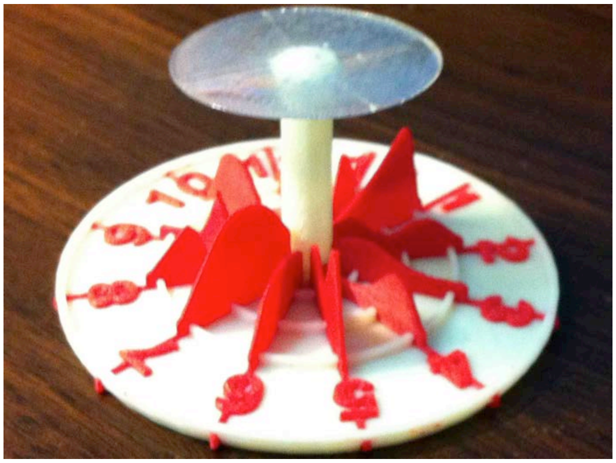

Figure 21. 3D printed Avatar model

FINDINGS

Based on informal discussions to date with typical categories of respondents in retail, academia and social networks is indicating initial positive perceptions of the Avatar 3D printed model. It is being enthusiastically received as a valuable visual-kinetic tool and semaphore object. Typical phenomenological comments are: *"After a brief explanation, the Avatar is very easy to extract information from. It allows the visual learner to understand complicated data quickly."*[1] And *"It materializes data to be analyzed or perceived with more than just the visual sense."* [2]

After only moments of viewing and handling the 3D printed Avatar semaphore device, respondents are eager to test their understanding through role-playing. Assuming the role of "store manager" they easily develop realistic and likely scenarios (explanations) and rationales for the temporal spatial distributions and displacements they observe in the object, resulting from new product

[1] Christyna Marguerite

[2] Jose Manuel

spatial temporal hierarchical innovation diffusion processes, throughout a store level trading area or an evolving network of stores.

Reactions appear to comply with the psychological literature on cognitive fit theory, Gestalt theory and mere-exposure effect. This suggests that visualizing GBD as an object, across a space-time continuum, is a viable, effective, efficient approach and of potential benefit to a wide audience of variously skilled practitioners and scholars.

Additionally, the “proportional” effect of displaying data in this way allows different scales of GBD to be easily comprehended (e.g. the 3D printed Avatar could represent a 1 store or 2500 stores nationally in a similarly sized Avatar).

The normalizing constant benefit of displaying proportionality allows the advantage of easily co-mingling data in the same object, similar to the technique used in geosciences and referred to as “stratigraphy” (see the mocked-up visual of this below in Figure 22).

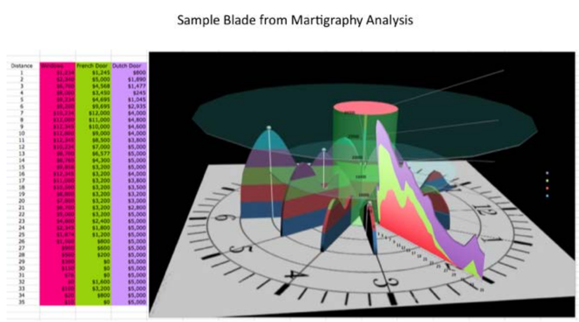

Figure 22. Mock-up of "Stratigraphy" among Avatar variables

SUMMARY

One of geography’s relative weaknesses is the difficulty in displaying complex and large quantities of GBD. This weakness exposes a reliance on traditional maps and symbology for all stages of GBD exploration and visualization. This weakness further signals the need for ancillary and complimentary visualization methodologies over standard GIS mapping symbologies. Exploratory data analysis would be enhanced with a simpler three-factor approach (time, space, attribute) using a semaphore object technique. Perhaps moving past 2D/3D only GIS symbolized maps, toward space-time visual objects, may hold potential for increasing the quantity of GBD that can be instantly viewed as a semaphore, while also increasing for retail trade area analyses, the degree of comprehension, insight and understanding gained.

The Avatar object used for federated searches of GBD three-way factor trends in space and time could direct and guide the focus later for more detailed and traditional GIS analysis. The Avatar is effective in both micro-analytic store level visualization and analysis (Mason, 1975) and at the macro-analytic firm level (Robert Lusser's Law)

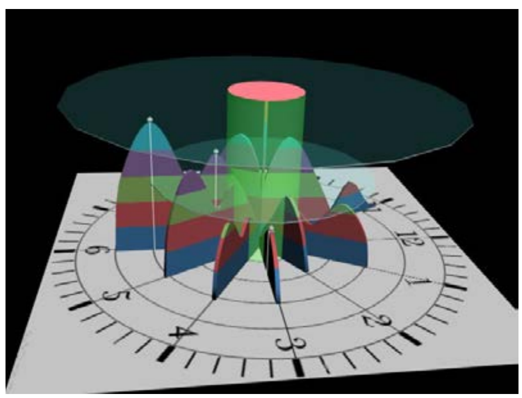

Figure 23. Future Avatar Semaphore Object

Finally, the 3-D printed version of the Avatar is also interesting for its "tactile" or physical representation of data in terms of human reference frames (i.e., tactile and haptic perception). By examining the Avatar physically, while viewing, discussing and hypothesizing; the brain collects a plethora of information about the object, much more than a 2D/3D map visualization and similar to the process an archaeologist employs when finding artifacts in the filed (e.g. a skull). It is common knowledge that Archaeologist even use tongue tests to determine the origin of certain object fragments.

CONCLUSION

We believe research initiatives like this are needed, across the GISc discipline, to cultivate innovative thinking of extensions and new theories to advance the frontiers of integrative applications benefiting from a GISc space-time perspective. Because GISc is inherently transdisciplinary in nature, adapting visualizations of GBD to 3-D semaphore objects will not initially be comfortable for some Geographers.

However resistance to geographic quantitative methods during the 1950's and 1960's faced similar challenges – initially. This chapter purposefully moves from the traditional symbolic geographic visualizations of GIS digital mapping, to investigating the possibilities of other complementary and potentially novel forms of GBD semaphore object visualization within their own coordinate systems. New insights, such as Avatar may lead to a strengthening of the traditional and contemporary tools available to support new directions of inquiry in the endless pursuit of improving location analytics and intelligence.

Human perception has a rich history in the psychological and brain science literature and the perception of objects is well documented. Transdisciplinary theories will and should play an important validating role in any innovative and potentially novel GISc data visualization system. The Avatar, grounded in transdisciplinary visual-kinetic concepts, promises the potential of such a standardized space-time modeling approach, to meet the almost insurmountable challenges facing users of GBD and the need to simplify and familiarize the geovisualization experience. The Avatar will identify spatial temporal consumer buying strength, leverage socio-demographic geosegmentation targeting knowledge and facilitate fast and easy dynamic trade area analysis, especially as an exploratory tool and process while guiding further targeted traditional GIS based map investigations.

The Avatar 3-D object is designed to leverage and enhance the synergy of existing GIS knowledge by maintaining links to existing GIS data structures (i.e., attribute table and geodatabase files).

Developing innovative GBD visualizations may open knowledge pathways to ever-widening and expanding non-GIS audiences, from children to adults, from neophytes to experts, in many walks of life, all eager to easily consume and quickly understand growing volumes of GBD of interest. The analogy comes to mind of those born in the mid to late 1800's, who eventually could speak with their grandchildren, after the invention of the telephone, without any knowledge of engineering, electricity, Morse code or the other components necessary to operate a telephone system. Wide consumerism of GBD will require approaches like Avatar.

Finally, our approach leverages the multidisciplinary knowledge that can be applied to GBD (e.g. Human perception) and in so doing, breaks the bonds of maps and compasses.

LIMITS AND DELIMITS

The Avatar is currently limited to analysis of transactions where the customer geocoded residence addresses are available. Only Euclidean distance is used to calculate mileage from residence to store. Avatar is limited to three factor analysis. These necessary and sufficient dimensional components are used in an initial exploratory fashion to become acquainted with the spatial temporal skeletal framework of shape (time and distance) and structure (i.e., time and variable or attribute). Avatar visualizations are limited to the current alpha versions of 3D printed objects.

Avatar visualizations are limited to the current software running in the Excel engine and game software engine renderings in UDK and 3DMAX.

There is no current linkage of the models to GIS "attribute tables" but is planned using Python during further development.

The unit of analysis consists of the “smallest unit of human settlement” (Mason, 1975) and is limited to the decennial Census Block. Only those Census Blocks at each time step “t” where the number of customers in a Census Block is greater than zero are included in the store level trade area shape.

The existing Census Block population conjugate priors are delimited to non-children (defined by census as 18+) population statistics. Visualizations are currently limited to inelastic demand for each store location in a monopolistic competitive market.

Space or distance is fixed as Euclidean distance only for this study and assumes (ceteris paribus). In other words we specifically delimit distance to be defined as Euclidean (straight line) distance only. There is an important rationale here for using Euclidean as opposed to shortest path or drive time. Unlike shortest path or drive time measurements, Euclidean distance measurements remain unchanged in growing urban landscapes and environments (e.g. changing road networks).

More research will extend the addition of drive time and shortest path for possible additional perspectives.

FUTURE VISUALIZATION DEVELOPMENT

The Avatar visualizations are being developed further, with game software quality imagery, environments and interactions, to strengthen the dynamic modeling depictions for Location Analytics and to improve high-level location intelligence.

The game software quality of the Avatar visualization below in Figure 23 was rendered in 3D-MAX, Unity and UDK rendering engines.

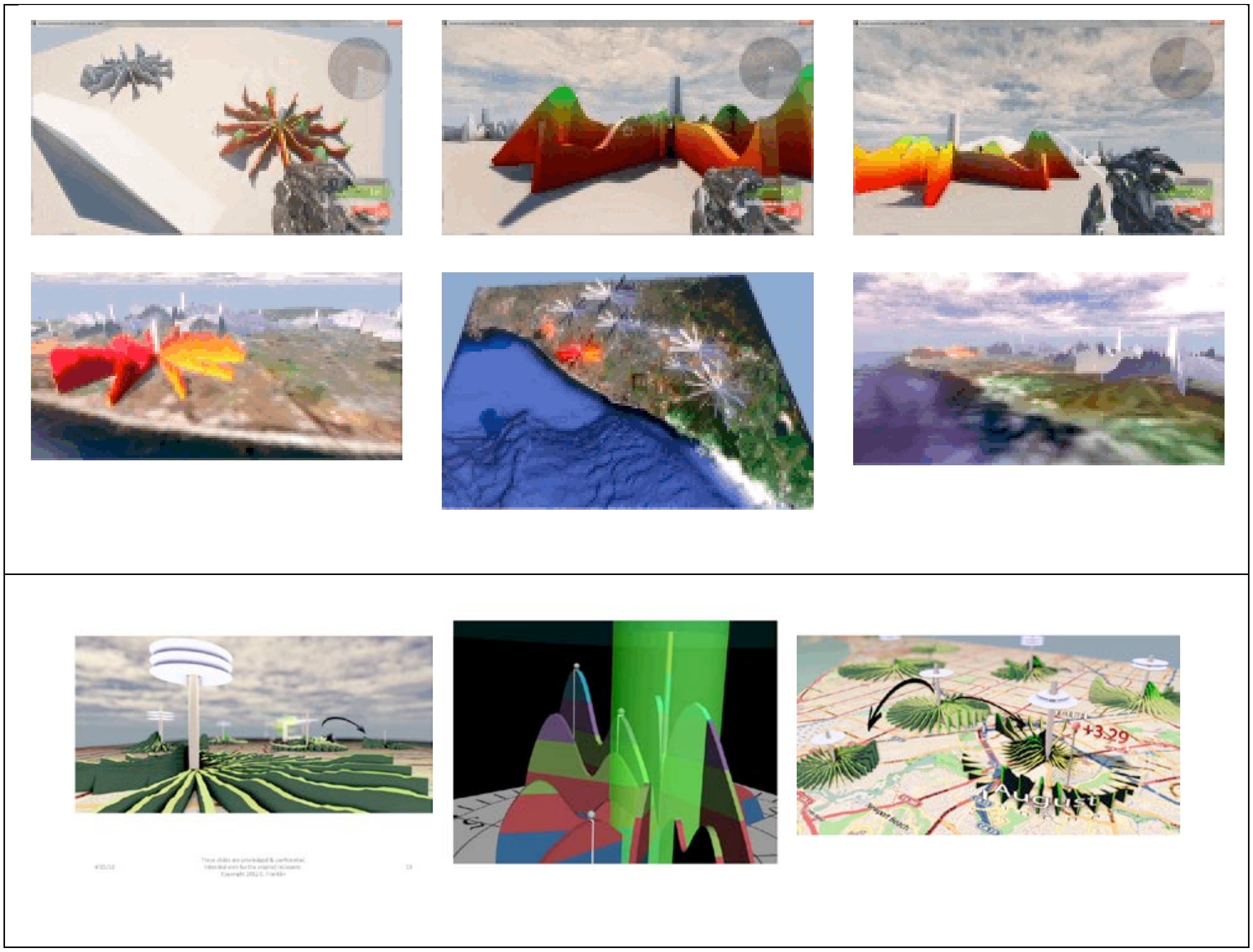

Figure 24. Game Software renderings of Store locations in Orange County, CA

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Zammit-Mangion, Andrew, 2012. “Point process modeling of the Afghan War Diary”, July, Proceedings of the National Academy of Sciences, Vol. 109, No. 3

BIOGRAPHICAL SKETCH

Christopher Franklin born in Chatham, New Brunswick. After finishing at Moncton High School in 1970, Christopher entered the University of New Brunswick in Fredericton, New Brunswick. He received a Bachelor of Arts degree with a major in sociology from Loyola University in Montreal in June 1977. In October 1979 he received a bachelor of commerce degree from Dalhousie University in Halifax, Nova Scotia. He returned to graduate school at the University of California Irvine, earning a Master of Arts in Social Science in demographic and social analysis in 2009. Subsequently he entered the geospatial information sciences PhD program at The University of Texas at Dallas and graduates in 2018.

CURRICULUM VITAE

CHRISTOPHER G. FRANKLIN

1. EDUCATION

University of Texas at Dallas, Richardson, Texas, USA
Doctor of Philosophy, Geospatial Information Science
Manuscript dissertation: Three essays for the retail planner: Spatializing Bass, Temporalizing Huff and Visualizing the Ensemble
Committee: Profs. Brian JL Berry (Chair), James Block Pick (U. Redlands), Daniel Alva Griffith, Wayne Gearey, Stuart Murchison (In Memoriam).

University of California Irvine, California, USA
Masters of Arts in Social Science, Demographic & Social Analysis (DASA)
Thesis: *Demographic & Geo-Spatial Factors Affecting Purchasing Behavior: Among Home Improvement Customers in Orange County*
Advisor: Prof. George Tita.

Dalhousie University, Halifax, Nova Scotia, Canada
Baccalaureatus in Commerrio, Marketing Research,

Concordia University (Loyola), Montreal, Quebec, Canada
Bachelor of Arts, Sociology

2. PROFESSIONAL EXPERIENCE

- Home Depot - Adjunct Corporate Trainer, Regional Headquarters, Anaheim, CA
- Fine Finish/Home Depot - Territory Sales Manager, Home Depot Division, Orange Co., CA
- Home Depot - Consumer Retail Specialist, Mission Viejo, CA
- Channel Partner Consultant - Professional Services Contractor to Data General, Mississauga, ON
- Data General Corporation - Large Account Development Executive, Toronto, ON
- International Marketing Consultant - European Manufacturers entering N.A. Markets, Irvine, CA
- Entrepreneur - Minicomputer Software Development & Hardware manufacturing, Ottawa, ON
- Air Canada - Market Research Analyst to Office of Sr. VP Sales & Marketing, Halifax, NS
- Air Canada - Station Attendant/Ramp Operations, Montreal, QB
- Air Canada - Station Attendant/Ramp Operations, Fredericton, NB

3. TEACHING/RESEARCH ASSISTANT EXPERIENCE

Experience to date includes:

- Research Assistant for three years
- Teaching Assistant for undergraduate GIS courses for two years at UT Dallas
- Teaching Mentor to the Geospatial Information Science Student Organization (Ex-Officio President)
- Teaching coordinator for collaborative problem solving at the UT Dallas, Center for GIS Innovation in Industry and Institutions (CG3i).
- Corporate Manufacture Ad hoc Trainer, Home Depot University, California Regional headquarters based on the teaching philosophy of Dr. Malcolm Knowles and utilizing Adult Learning Theory and Andragogical (versus Pedagogical) teaching styles and methods.
- Under/Graduate Advisor, CG3i (Center for GIS Innovations in Industry and Institutions)
- Group Leader, CG3i Collaborative Transdisciplinary STEM Flood Inundation Project, UT Dallas & Department of Homeland Security Fusion Center contract
- Industrial Research Assistant - Advising Commercial Real Estate Industry firms on Location Analytics
- Graduate Research Assistant – Research on remote sensing & land use (Professor Brian Berry)
- Regional Store-Trainer – The Home Depot

4. RESEARCH INTERESTS

My research interests center on non-intuitive problem solving, at high spatial and temporal resolutions, and visualizing dynamic and complex data interactions in focal areas such as:

Integrated Simplified Inundation Mapping **-** GIS integrated simplified inundation mapping and hydrologic modeling, Demographically Aware Emergency Response and Community Resilience**,** Emergency Planning & Tabletop Simulations
Spatial Temporal Visualization
Interpretation & Analysis of 3D Space-Time Semaphore Objects (Avatars), Visualization & Pattern Recognition Methodologies, Hierarchical Innovation Diffusion Modeling, Bayesian Influenced Visualizations, Advanced Spatial Temporal Analysis of Geo-Big-Data
Information Data Analytics
Geo-big-data Analysis, Location Intelligence & Decision Management, Applied Business Demographic and Social Analysis

5. PUBLICATIONS

- Franklin Christopher, W. Henson, R. Garth, 2016, *Translator TTX - Bridging the Communication Gap between Researchers and Emergency Responders*, Chpt. 14, pp. 105, Eds., Maidment, D.R., Rajib, A., Lin, P., Clark, E. P., National Water Center Innovators

Program Summer Institute Report. Consortium of Universities for the Advancement of Hydrologic Science, Inc. (CUAHSI), Technical Report No. 13, 122 p. DOI: 10.4211/technical.20161019

- Franklin Christopher, 2015. *Space-Time Diffusion Visualization using Bayesian Inference: Research-in-Progress*, Location Analytics, ICIS 2015 Pre-Conference Workshop: Thirty Sixth International Conference on Information Systems, https://www.redlands.edu/globalassets/depts/school-of-business/gisab/workshops-conferences/christopher-franklin_space-time-visualization_pre-icis-workshop-2015_12-3-15.pdf, Fort Worth, TX

Submitted For Publication

- *Facilitating Effective Utilization of Water Science Research: Among Emergency Flood Responders*, (with Whitney Henson, Richard Garth), Journal of Open Water, Dan Ames, Editor-in-Chief, 2016
- *Bridging the Communication Gap: Researchers and Emergency Responders*, (with Whitney Henson, Richard Garth), Environmental Modeling & Software, Dan Ames, Editor-in-Chief, 2016

Posters

- PhD Category- *Improving Emergency Response and Community Resilience*, Emergency Management, University of Texas at Dallas, GIS Day, Juried award winner, November 2015

- Graduate Fellow Category - *Improving Emergency Response and Community Resilience*, University of Alabama, NOAA National Water Center/CUASHI National Interoperability Flood Experiment (NFIE) Capstone Conference, Emergency Management, Juried special mention, July 2015

6. CONFERENCES, WORKSHOPS AND PRESENTATIONS

- **University of California Irvine**, invited Demographic and Social Analysis (DASA) alumnus panel speaker *The Value of a GISc Demographic and Social Analysis Perspective*, Irvine, CA, January 2017
- **University of California Irvine**, invited book chapter reviewer, *The Global Digital Divides: Explaining Change,* Pick & Sarkar - UCI Libraries Series, Irvine, CA, January 2017
- **US Army Corp of Engineers, US Army Engineer District** invited presenter with national security clearance required**,** *Integrated Simplified Inundation Mapping of Flood Scenarios*, Vicksburg, MS, June 2016
- **University of Alabama, NOAA National Water Center** invited assistant theme

coordinator with national security clearance**,** CUASHI Summer Research Institute, *Translator TTX - Bridging the Communication Gap between Researchers and Emergency Responders*, Tuscaloosa, AL, June/July 2016

- **Association for Information Systems (AIS), International Conference on Information Systems** invited presenter, *Space-Time Diffusion Visualization using Bayesian Inference (Introducing Avatar - a geo-big-data, exploratory research, three-way factor analysis, shape changing 3D printed or SW visualization semaphoric object tool),* Fort Worth, TX, December 2015
- **Pioneer Natural Resources** (Oil&Gas), invited presenter of sponsored corporate research, Irving, TX, December 2015
- **Lockheed Martin Aeronautics Security and Emergency Services**, invited co-presenter with national security clearance, Regional Emergency Managers Conference, GIS, Hydrology, *Integrated Simplified Inundation Mapping of Flood Scenarios*, Fort Worth, TX September 2015
- **ESRI User Conference, Lightning Talks** invited co-presenter, *Integrated Simplified Inundation Mapping of Flood Scenarios*, San Diego, CA, July 2015
- **University of Alabama, NOAA National Water Center** invited graduate fellow with national security clearance, CUASHI National Interoperability Flood Experiment (NFIE), *Improving Emergency Response and Community Resilience*, Tuscaloosa, AL, June/July 2015
- **ESRI DEV (Developer) Summit Conference** invited conference attendee and discussant, Palm Springs, CA, April 2015
- **University of Texas at Austin, Center for Research in Water Resources**, Civil Engineering Department, invited colloquium co-presenter, GIS and *Integrated Simplified Inundation Mapping of Flood Scenarios* Austin, TX, Feb 2015
- **North Central Texas Council Of Governments (NCTCOG)** invited co-presenter *Integrated Simplified Inundation Mapping of Flood Scenarios*, iSIM Software as a Service Solution, Arlington, TX, June 2014
- **US Department of Homeland Security, Collin County FUSION Center** - invited in-camera co-presentation to USDHS-Critical Infrastructure, USDA, Collin County Engineering & Emergency management, Texas Commission on Environmental Quality, Texas Department of Public Safety Critical Infrastructure, NCTCOG), Collin, TX, May 2014
- **University of California Santa Barbara, Center for Spatially Integrated Social Science,** invited Graduate Fellow/Presenter for NIH sponsored Advanced Spatial Analysis one week Summer Research Workshop for "...early-career population scientists (i.e., graduate students, post-docs, and junior faculty/researchers in demography-related disciplines)", Santa Barbara, CA, July 2010
- **University of California Irvine**, invited Demographic and Social Analysis (DASA) alumnus panel speaker, *Transdisciplinarity in Demographic and Social Analysis Approaches*, Irvine, CA, May 2010
- **ESRI Space-Time Modeling and Analysis,** invited three day workshop attendee and discussant, ESRI World Headquarters, Redlands, CA, Feb 2010

7. GRANT SUBMISSIONS

- NSF $3.5M 2015

8. TECHNICAL SKILLS

- ArcGIS
- Python, ArcGIS,
- Applied Business Demography
- Market Research
- Microsoft Suite

9. AWARDS AND HONORS

- **Pioneer Natural Resources -** Graduate Geospatial Fellowship, Irving, TX 2016
- **Pioneer Natural Resources -** Graduate Geospatial Fellowship, Irving, TX, 2015
- **Association for Information Systems (AIS) -** Best Paper Award, International Pre-Conference of Information Systems (ICIS), Fort Worth, TX 2015
- **UT Dallas Office of VP of Technology** - Technology & Commercialization Innovators honor for "Avatar" 3D Space-Time Semaphoric Object, 2014

10. LANGUAGES

- English - Highly developed and strong interpersonal communication skills.

11. PROFESSIONAL MEMBERSHIPS

- AAG
- GISSO