SHAPING CONSUMER EXPECTATIONS

THROUGH INTEGRATED MARKETING

by

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In dedication to my husband Duo and my parents.

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In my dissertation, I study the managerial implications when firms can use different strategies to influence consumers' learning process. For example, in the first chapter, I focus on consumers' learning about product fit, which is relevant to the frequently discussed phenomenon of showrooming. Showrooming is the phenomenon where consumers visit a brick-and-mortar (B&M) store to examine the products but then buy online to obtain lower prices. Though it is usually considered a major threat to the B&M retailers, the popular arguments ignore the strategic role of the manufacturer in the distribution channel. After all, the manufacturer's need for retail informational services has always been one of the essential reasons for retailers to exist and is a means for retailers to achieve profitability. This chapter analytically shows that when the manufacturer's decisions are considered (i.e., when the manufacturer-retailer contract is endogenous), consumers' ability to engage in showrooming may lead to increased, rather than decreased, profitability for B&M retailer(s). Thus, retail efforts to restrict showrooming behavior may be misguided. This result holds even if the manufacturer is restricted to wholesale-only contracts and is not allowed to price discriminate between channels.

In the second chapter, I focus on consumers' learning about product quality. It is common for consumers to rely on opinion leaders, who presumably have a higher expertise in the product category, to form beliefs about the product quality. At the same time, the evaluations and

product adoptions of opinion leaders are influenced both by the product quality and by their idiosyncratic preferences (fit). When opinion leaders do not provide a very detailed review, their followers need to form expectations of how much the opinion leader's recommendation is driven by product quality and how much it is driven by an idiosyncratic fit of the product to the opinion leader. This chapter considers how the firm should adjust its optimal choice of the product variety in the presence of word of mouth, given that the opinion leader is likely to have more expertise and therefore, be better able to choose the version of the product that fits her best. It shows that while the opinion leader's presence may be a force toward either an increased or decreased number of variants, generally speaking, the distortion is upward if it is more difficult to satisfy the opinion leaders, which could be either due to the higher importance of fit or due to their higher standards. I further show that the firm's knowledge of the true quality may increase the distortion of the number of product variants it offers even when the equilibrium number of variants is pooling across the product qualities.

The third chapter of my dissertation looks into the effect of scalping on consumers' expected market structure and in turn the firms profit. Scalpers purchase the products with limited supply for reselling them later at inflated prices. Though firms often impose restrictions on scalping, we rarely observe actions that completely eliminate scalpers. This chapter explains why and how an intermediate level of restrictions on scalping can be optimal for the firm purely from the perspective of the firm's profitability. I consider a firm with limited capacity. Consumers decide between purchasing the product before resolving the uncertainty, and waiting at the risk of the price increasing and the product selling out. I find that the firm's profitability can indeed be maximized at an intermediate level of scalping. This result is an outcome of two opposing effects of scalping. On the one hand, the scalpers' higher flexibility in setting the price decreases consumers' expected payoff of waiting, making them more eager to pay a high price right away. On the other hand, the competition between the scalpers and the firm can decrease the firm's equilibrium price. This result provides an explanation for the firms' seemingly contradictory practices: they do impose some but not complete restrictions on scalping.

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CHAPTER 1

WHEN SHOWROOMING INCREASES RETAILER PROFIT

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1.1 Introduction

Showrooming is the phenomenon of consumers visiting a B&M store to examine the product(s) but then buying online to obtain lower prices. It has recently become a common practice with the emergence and spread of the Internet and mobile technology. According to an Accenture (2013) report, 73% of survey respondents indicated that they have participated in the practice of showrooming. A more recent report by SecureNet (2014) indicates that 55% of consumers have used a mobile device to research a product while in a B&M store, and 21% of consumers have purchased a product with their smartphone while browsing for the same item in a B&M store. This behavior is recognized as a major threat facing B&M retailers. It is easy to see why: B&M retailers have higher selling costs than e-tailers, and it is therefore questionable whether they can withstand price competition with e-tailers if they cannot use their ability to provide better service to their advantage.[1] Some leading retailers, such as Best Buy, have had a difficult time finding ways to combat showrooming in their B&M stores.

The extant academic literature on showrooming (e.g., Balakrishnan et al. 2014; Jing 2016; Jing 2018; Mehra et al. 2018) also begins with the premise that showrooming hurts B&M retailers. A consideration missing in both the popular press and academic discussions is the role of the manufacturer in the channel with showrooming. It has long been recognized that manufacturers use retailers not only for logistics but also for informational services. In some categories, a direct experience with the product is important.[2] Consumers engage in showrooming because they need information about a product. It may be that they are uncertain about the product's features or their individual fit with the product; they may not

[1] While marginal (product) costs may be the same for the B&M and online retailers, the fixed costs (e.g., retail space, sales force) are lower online. Therefore, as competition erodes margins, online retailers may survive, whereas the B&M ones may not.

[2] For example, Bell et al. (2018) empirically show that allowing consumers to view and try the products at B&M stores can increase the overall demand as well as the online demand.

even be aware of the product's existence. For example, it is difficult to evaluate softness, size, or fit of clothes; luster of a necktie; firmness and feel of mattresses; or workmanship of furniture when evaluating products online. Without the in-store evaluation, a "random" product in the category may have a low expected value to the consumer. In this case, the consumer may need to form his or her valuation at the physical store but may then be indifferent between buying there or online and end up buying online due to a lower price. In turn, retailers design the level of informational services they provide, which affects the level of consumer demand. These services include product display in an accessible/highly visible position, allocation of the shelf space to display many product variations (or to ensure that the product is not out of stock on the shelf), salesperson assistance in finding the right product and explaining the product benefits and features, product demonstrations, free trials, and so on. Consumer unwillingness to purchase a product in some category without physically examining it implies that the B&M retailer's service is especially valuable for the manufacturer in this product category. One could expect that if the informational service of a B&M retailer becomes more important to generate overall demand (e.g., because online retailers are unable to provide it), the manufacturer(s) will provide more incentives for the B&M retailer to provide service and, as a result, the B&M retailer's profit may increase.[3]

When demand-generating services are costly to the retailer, the manufacturers design compensation structures to incentivize retail provision of the level of service optimal for the manufacturer. Examples of such incentives are lower wholesale prices (higher retail margin incentivizes the retailer to put more effort into selling), exclusive distribution, exclusive territories, and display and slotting allowances. For example, auto manufacturers sometimes compensate car dealerships for consumer test drives. A case in point is General Motors offering dealers $5,000 for each Cadillac ELR the dealer assigns to the test fleet (up to

[3]Note that in this article, we consider only the inspection aspect of showrooming and not the phenomenon of consumers comparison-shopping between brands; therefore, we do not consider how showrooming affects the manufacturer competition.

two) in 2004. The retailers may abuse such incentives, and thus, manufacturers also try to implement controls. In this instance, to qualify for the $5,000 incentive, a dealer must log at least 750 test drive miles on each ELR assigned to the test fleet (arguably, this is an imperfect control of the potential dealers' abuse of the incentive; see Naughton 2014).

If a retailer does nothing to prevent showrooming, it may suffer from consumers using its informational services but buying online. However, if this regularly happens and the manufacturer's sales increase, the manufacturer may value this retailer's service more and consequently may find a way to compensate the retailer more for the valuable informational services it provides (to ensure that it continues to provide them). The question then arises whether the net effect could be that the retailer is better off allowing (or even facilitating) showrooming, rather than preventing it.[4]

Of course, competition from low-cost e-tailers is an important issue facing B&M stores. However, to develop the optimal competitive strategy, a manager needs to differentiate combating competition and combating showrooming. To combat showrooming, a retailer could, for example, try to make the product information consumers obtain at its B&M store not useful for the purchase decision at a different e-tailer by using brand variants (Bergen et al. 1996). A case in point is mattress manufacturers labeling their mattresses with a model number specific to the retailer to ameliorate the retailers' concern about service free riding. With branded variants, consumers who want to experience the feel of a mattress at a B&M store cannot then ensure that they are buying an identical mattress from the online competitor. However, combating showrooming in this way would not help if the underlying problem is due to competition from low-cost e-tailers for consumers who do not need a showroom in the first place.

[4]Whether a retailer would be able to prohibit showrooming effectively is a different question. The point is that before thinking about how to prevent showrooming, the retailer should first determine whether showrooming has a negative or positive impact on its bottom line.

To formally consider how the manufacturer's role in the channel modifies the implications of showrooming for B&M retailers, we construct a model with one manufacturer and two competing retailers: one is a B&M retailer able to provide informational demand-increasing service and the other is a purely online retailer (e-tailer) unable to provide such a service. Within this setup, we consider several variations of the demand structure and the manufacturerretailer contract possibilities.

The main result is that the consumer ability to engage in showrooming may benefit the B&M retailer when the manufacturer is able to provide direct compensation for the retail service but the level of service is not perfectly observable by the manufacturer. In a model variation where the online and B&M retailers are differentiated, we also show that the B&M retailer could be better off due to showrooming even when the manufacturer is restricted to use only wholesale prices to incentivize service (under imperfect competition, a lower wholesale price incentivizes higher demand-expanding retail efforts because retailers have a higher margin on each sale) and when the manufacturer is able to price discriminate between retailers by offering them different wholesale prices. The intuition for this result is that when service is important enough, due to the optimal manufacturer decision of how much to reduce the wholesale price to incentivize the service level optimal for the manufacturer, the B&M retailer's profits are directly linked to its ability to provide service and, in equilibrium, do not depend much on the sales. Showrooming then leads to higher retail profits because it results in a higher benefit of retail service to the manufacturer.

The rest of the article is organized as follows. After discussing the related literature, we formally define and analyze the main model, in which the manufacturer imperfectly observes the level of retail service and can offer the B&M retailer compensation for the service conditional on not having observed the retailer's shirking on the service level. In the following section, we analyze several extensions and modifications of the main model, including the endogenous detection rate of retail shirking, different functional forms of the cost of service for the B&M retailer, retail differentiation, wholesale price discrimination, and wholesale-price-only contracts. We conclude with a further discussion of the implications and potential variations of the model.

1.2 Related Literature

This research belongs to a small but growing literature stream investigating the implications of showrooming for retailers and the strategies of coping with it. For example, Balakrishnan et al. (2014) analyze a model where, in equilibrium, some consumers engage in showrooming behavior; Jing (2018) shows that showrooming need not benefit the online retailer, as showrooming may increase competition; and Mehra et al. (2018) consider several strategies to combat showrooming, including price matching[5] as the short-term strategy and product exclusivity as the long-term strategy. The models of each of these cited works share the underlying premise that showrooming hurts the B&M retailer and do not consider the manufacturer's response to the consumer ability to showroom.

This article is also related to the extant research on retail service and information provision and retail service free riding. Starting with Jeuland and Shugan (1983), marketing literature has considered the issue of retail service underprovision and the manufacturer's contractual ability to incentivize retail demand-enhancing service. Service underprovision could be either a result of horizontal free riding or due to the retailer not facing the full benefit of making the sale (when the marginal wholesale price is above marginal production cost).[6] Correspondingly, manufacturers can use lower wholesale prices (Jeuland and Shugan 1983; Mathewson and Winter 1984), slotting allowances (Desai 1997; Lal 1990; Shaffer 1991),

[5]Chen et al. (2001) caution that price-matching guarantees may lead to intensified competition and reduced profits.

[6]Retail competition may also result in overprovision of service (see, e.g., Iyer 1998; Iyer and Kuksov 2012).

retail price maintenance (Mathewson and Winter 1984; Miklos-Thal and Shaffer 2015; Shaffer 1991), or exclusive territories (Iyer 1998; Mathewson and Winter 1984) to incentivize retail service.[7] In the context of showrooming, Miklos-Thal and Shaffer (2015) examine how retail price maintenance, in addition to a customized-to-retailer two-part-tariff contract, is necessary to fully coordinate the channel and induce the efficient service level at a B&M retailer. That article focuses on how the first-best outcome can be achieved from the perspective of the manufacturer. In contrast, we consider situations in which the channel is not fully coordinated and study whether and when the phenomenon of showrooming may benefit the B&M retailer.[8] With respect to consumers' uncertainty about product fit, extant studies have shown how retailers could provide a flexible return policy to facilitate consumer learning (e.g., Shulman et al. (2010) consider optimal return policies for multiproduct retailers when consumers need to experience the products to understand their value).[9] One of the effects of the informational service provision identified in the literature is that given differentiated products, information about fit may increase differentiation and lessen competition (Gu and Xie 2013; Kuksov and Lin 2010; Shin 2007). Wu et al. (2004) show that providing service may result in a competitive advantage and higher profits (relative to the free-riding retailers) even when retailers can free ride on service. Shin (2007) shows that the ability to free ride on service may benefit both the free-riding and the service-providing retailers by softening price competition (i.e., by enabling the competitor to profitably increase its price). This result

[7] Although much of the extant research on service provision abstracts from the specifics of why service increases consumer valuation, Wernerfelt (1994) formalizes the role of sales assistance as helping the consumer find the best match, and there is considerable recent literature specifically on the informational provision/disclosure (e.g., Gu and Xie 2013; Guo and Zhao 2009; Jing 2016; Kuksov and Lin 2010; Sun 2011).

[8] If the manufacturer achieves its first-best outcome, it appropriates all the channel surplus, resulting in zero retailer profit; thus, the retailer is, by definition, indifferent between different options.

[9] If returns are inefficient (e.g., have transaction costs), the flexible return policy could also signal product quality (Moorthy and Srinivasan 1995). An interest observation in this context is that signaling quality through higher wholesale price is less costly when manufacturers need to incentivize retail service through lower wholesale price (Desai and Srinivasan 1995).

relies on uninformed consumers having sufficiently large and heterogeneous shopping costs for shopping at the retailer that provides no service. The idea is that the service provider can use service to attract consumers to the store and effectively "lock them in" if they have a high shopping cost of going to another store. If the service provider were to lock in all customers who visit its store, the retailers would engage in intense price competition up front. Allowing free riding can soften price competition because the retailer that does not provide service can receive those consumers who visited the service-providing retailer and have low shopping cost. In contrast, the current article highlights the role of the manufacturer and shows that consumer free riding can make the service-providing retailer better off even if consumers have zero costs of shopping online and when the retailer faces undifferentiated competition with an online retailer or with a competitive online market. This captures the showrooming context because the idea of showrooming is that consumers may have virtually zero shopping costs of buying online once they know the exact product they need, and that the online prices may be lower as a result of competition in the online marketplace (as opposed to the online prices driven by the competition with B&M retailers). In other words, this article shows how the service-providing retailer may be better off as a result of consumer free riding even when consumer free riding intensifies competition.

Inasmuch as this article is related to the increasing importance of the online marketplace, it is related to the research on the effects of the Internet as well as the the multimarket (online vs. offline) and multichannel competition. For example, Lal and Sarvary (1999) provide an early analysis of the conditions under which the Internet is likely to decrease competition (and highlight the potential importance of nondigital attributes), Balasubramanian (1998) analyzes how the entry of a direct-mail retailer affects competition between conventional retailers, Biyalogorsky and Naik (2003) study the effect of online sales on offline business, Liu et al. (2006) consider how expanding online could facilitate e-tailer entry and hurt a B&M retailer's profitability (unless the B&M retailer has different prices online), Ofek et al. (2011)

study how multichannel strategy should depend on product returns, Gu and Tayi (2017) consider how retailer choice of product assortment online and offline should be affected by consumer search for information, and Kireyev et al. (2017) consider how a retailer's price matching to its own online store could allow it to price discriminate between consumers who prefer to buy offline and those who are indifferent.

Our article is also related to recent studies on when the entry of a competitor into the market increases the profits of the incumbents (e.g., Chen and Riordan 2007; Ishibashi and Matsushima 2009; Pazgal et al. 2016). In these studies, the entry of a newcomer changes the marginal consumers the incumbents are competing for and may increase equilibrium prices in the market when each firm expects the competitors to raise prices. When a manufacturer introduces a direct online channel, it may also decide to change the wholesale price to the B&M retailer(s) because the presence of the online channel may change the customer base of the B&M channel (see, e.g., Kumar and Ruan 2006). In contrast, in our article, we maintain the intuitive effect of showrooming increasing competition and show that the B&M retailer may still benefit from showrooming as a result of the manufacturer's response.

1.3 Main Model

One manufacturer sells a product with many variants to consumers through two retailers: a B&M Retailer 1 and an online Retailer 2.[10] Each variant of the product has constant and equal marginal cost normalized to zero (i.e., consumer valuation and prices should be understood as relative to the marginal cost). There is a unit mass of consumers with valuation V for the best-fitting product, where V is uniformly distributed on $[0, 1]$ across consumers. The product has many variants (e.g., different tailoring of a shirt), only one

[10]In this model, it is inconsequential whether the online market by itself is competitive or has only one e-tailer. For ease of presentation, we therefore assume one retailer, but the same analysis applies if the online market is perfectly competitive.

of which "fits" an individual consumer and generates the above utility V, while the other variants' value is below cost, such that the expected utility of a random choice will be too low for the consumer to justify a purchase even if the price is equal to cost.[11] Retail service is the only way for consumers to determine whether a product fits them, and only the B&M retailer is able to provide this service (through, e.g., a product showcased in a desirable position, a salesperson's help and recommendations about the product variant choice and/or explanations of the product's benefits, a fitting room). This is a simplification of what we observe in real life, in which people visit B&M stores to test products and find the one with a good match. In the main model, we assume that the consumer does not have preference toward either retailer, and his or her surplus from purchasing the product at either retailer is $V - p$, where p is the price paid. Furthermore, we assume that consumers have zero cost of shopping.

Both retailers decide on their respective prices p_1 (B&M retailer's price) and p_2 (e-tailer's price).[12] In addition, the offline retailer sets the level of service(s) it provides to consumers visiting its store. We allow the service level to be variable. The B&M retailer may be able to change the space allocated to the product line under consideration on the shelves, shift its display location, and/or alter the number of sales assistants or their incentives to promote the particular product. For parsimony, we model service outcome (with respect to an individual customer) as a binary variable: given service level s, an individual consumer visiting the store finds his or her fit with products with probability s and gains no information about it with probability $1 - s$. Thus, the feasible range of s is $s \in [0, 1]$, and (assuming that all consumers in the market visit the B&M store) s becomes the mass of consumers who

[11] An alternative interpretation is that there is one product, $1/\delta$ mass of consumers, and a consumer's valuation of the product is ξV, where $\xi = \begin{cases} 0, & \text{probability } 1-\delta, \\ 1, & \text{probability } \delta, \end{cases}$ with the value of ξ (fit) observed by the consumer on inspection only and the value of V (the value of the ideal product) known a priori.

[12] It is clear from the symmetric setup that the same price is optimal for all variants. Chen and Cui (2013) provide an additional rational for uniform pricing across variants through consumer fairness concern.

end up knowing the right product variant and their valuation for it. As standard in the literature, assume that the cost of service to the offline retailer is an increasing and convex function $c(s) = ks^2$ of service (we consider some variations in Section 1.5.2). Furthermore, we normalize the product cost to zero.

We consider several possibilities of the contract structure the manufacturer is able to offer the retailers. First, we assume in the main model that the manufacturer is restricted to offering the same contract to both retailers (this assumption simplifies technical analysis; in Section 1.5.3, we relax this assumption and confirm the main implications) and the contract consists of a wholesale price w, the desired retail service level s*, and service compensation R, which a retailer receives if the manufacturer did not detect that the retailer provided a lower service level.[13] Naturally, because it is common knowledge that the online retailer cannot provide service, the online retailer will not receive R. However, detecting whether the B&M retailer is shirking may be difficult.[14] Let us denote the probability with which the manufacturer detects the B&M retailer's service by r. One can think of the probabilistic detection as the manufacturer's having an inspector to check the retail service level. Limited by his or her time and energy, the inspector can only make inspections at a rate r (relative to the frequency with which the retailer may change the service level). For now, this rate is exogenous, but in Section 1.5.1, we endogenize this detection rate by assuming a cost per inspection (as in the classic model of the inspection game) and show that the conceptual

[13]One may consider a more general service compensation contract $R = R(s)$ for $s \in [0, 1]$, but in our setting, the restriction to one-point incentive is without loss of generality. If, however, there would be some uncertainty (in, e.g., the manufacturer's observation of the service level, consumer demand, or retail service cost), a more general compensation scheme would be useful.

[14]Given no uncertainty of demand, which we assumed to simplify the model, the manufacturer could technically deduce the retailer's service from total sales. However, more realistically, the realized demand is uncertain, and the demand we specify in the model reflects the expected demand (conditional on the retail price and service), while the actual demand could be not very informative of a given retailer's service. Furthermore, conditioning retailer's compensation on total sales could introduce a moral hazard problem for the manufacturer's demand-enhancing activities that we do not explicitly consider (e.g., advertising). Thus, in the model, we do not allow the manufacturer to condition service compensation on sales.

results remain unchanged. Technically, $r \in [0, 1]$. In this article, we put more emphasis on $r < 1$ so that, consistent with practice, the channel does not end up being fully coordinated.

Finally, the timing of the game is as follows. First, the manufacturer offers a contract to the retailers. Then, the retailers simultaneously set their respective prices p_i, and the B&M retailer sets its service level s (conceptual implications remain unchanged if any of these decisions are sequential). Then, consumers observe prices and visit the B&M store (technically, they decide whether to visit, but because the cost of visiting is zero, it is a dominant strategy for them to do so), find which product fits (with probability s), and decide whether and where to buy the product. Finally, with probability r, the manufacturer detects the actual service level set by the B&M retailer. When consumers are not able to showroom, those who are in need of service can only purchase offline. When showrooming is possible, they can purchase either online or offline.

Note that our objective is to analyze whether and when a B&M retailer may be better off due to showrooming and not to determine what type of contract would coordinate the channel. Therefore, we consider contracts that capture the usual elements of the manufacturers' practices to induce the retail service in industry and that do not achieve the first-best outcome for the manufacturer. For an example of how the question of whether showrooming increases or decreases retail profits is relevant to the retailer, note that showrooming can be prevented by changing the names of the product variants between online and offline versions, and just reading a description or looking at the picture online is not enough to figure out which product is which.

1.4 Model Analysis

Because we aim to study the effect of showrooming on B&M retailer's profit, we will compare the equilibrium strategies and profits when showrooming is possible and when it is not (i.e., when consumers cannot apply information obtained at the B&M store to resolve their uncertainty about a product bought online). When showrooming is not possible, given our simplifying assumption of homogeneous and extreme consumer need for information, the e-tailer can generate no sales, and the B&M retailer is the de facto monopoly in the market. This means that providing service yields benefits such as (1) increased sales at the monopoly margins and (2) compensation from the manufacturer, if offered. When showrooming exists, according to our simplifying assumption, all consumers are indifferent between buying from the e-tailer and buying from the B&M store. Because the two retailers face the same product costs (wholesale prices), the ensuing perfect Bertrand competition results in zero profitability of sales regardless of where consumers end up buying from. This means that compensation for service is the only incentive the retailer has. The following three subsections analyze the two market conditions (consumers allowed and not allowed to showroom); compare the outcomes under these two conditions; and derive the conditions under which the retailer, the manufacturer, or both are better off allowing or prohibiting showrooming.

1.4.1 Benchmark: Showrooming Is Not Possible

When showrooming is not possible, given the assumption that consumer valuation for a random product variant is below its cost and that consumers have no information about which variant fits them better without retail service, all consumers who end up buying the product, buy it from the B&M retailer. No sales can be generated online, and the B&M retailer is a de facto monopoly in the market. If the manufacturer does not offer compensation for service (i.e., sets $R = 0$), the B&M retailer's profit maximization problem is

$$\max_{p_1 \geq w,\ 0 \leq s \leq 1} s(1 - p_1)(p_1 - w) - ks^2. \tag{1.1}$$

The optimal decision is to set the retail price at $p_1 = \frac{1+w}{2}$, and the service level at $s = \min\{1, \frac{(1-w)^2}{8k}\}$.

As a result, the manufacturer profit is $\pi_m = \frac{1-w}{2}w$ if $\frac{(1-w)^2}{8k} \geq 1$ and $\pi_m = \frac{(1-w)^3 w}{16k}$ if $\frac{(1-w)^2}{8k} < 1$. In the former case, the retail margin for the B&M retailer is large enough relative to the service cost, so that the retailer is willing to provide the highest level of service (i.e., $s = 1$). As long as the cost parameter k is such that the former condition holds, the level of service is unaffected by small changes of the wholesale price. The wholesale price still influences the B&M retailer's price and, in turn, influences the realized demand but (locally) has no effect on the retail service. If k is such that the optimal w without consideration of its effect on retail service is already in the former range (i.e., when k is small), any extra incentive for service is not necessary. Therefore, for low k, in equilibrium, the B&M retailer bears all the cost of service provision.

When k is larger, however, the problem becomes more complex. If k is such that the retailer sets $s < 1$ when the manufacturer has set the wholesale price while ignoring service incentives, the manufacturer may consider inducing a higher service level by (1) decreasing the wholesale price and increasing the retailer's margin and the incentive to increase demand or (2) offering compensation R for some target service level s^*, or (3) both. If the manufacturer offers $R > 0$ for some $s = s^*$ the retailer would not otherwise choose, the retailer trades off between setting the target service level s^* and shirking to the optimal service driven by the sales margins alone (but at the risk of being caught). The first choice leads to the retailer's profit of $s^* \frac{(1-w)^2}{4} - ks^{*2} + R$, and the second choice leads to its expected profit of $\frac{(1-w)^4}{64k} + (1-r)R$. Therefore, the minimum value of R to ensure the suggested service level is $\frac{k}{r}(s^* - \frac{(1-w)^2}{8k})^2$. Not surprisingly, the optimal compensation is increasing in the difference between the suggested service level and the retailer's optimal service without compensation. Moreover, the optimal compensation is decreasing in the detection rate, which reflects the information rent.

Thus, the manufacturer's profit maximization problem is equivalent to

$$\max_{w \leq 1,\ 0 \leq s^* \leq 1} s^* \frac{1-w}{2} w - \frac{k}{r} \left[s^* - \frac{(1-w)^2}{8k} \right]^2, \tag{1.2}$$

where the first term reflects the manufacturer's sales revenue and the second equals minimal service-compensation level resulting in the retailer's choice to set service level $s = s^*$.

Note that by setting the wholesale price w such that $s^* = \frac{(1-w)^2}{8k}$, the manufacturer is able to induce the desired retail service level s^* purely through lowering the wholesale price. However, intuitively, because the manufacturer has two instruments at its disposal to affect retail service (the wholesale price and the direct compensation), it will find it optimal to use a combination of both. The exact solution for the optimal w, s^*, and R appears in the Appendix.

Note that it is intuitive that the manufacturer is willing to make at least some costly effort to increase the retailer's service level (if it is below $s = 1$) because, without the manufacturer's incentive, the retailer's service level is a solution to the first-order condition equating the retailer's benefit to the retailer's cost, and therefore a small extra incentive results in a disproportionally larger increase in the service level. The result is that for larger k, by incentivizing a service level higher than what the retailer would choose under $w = \frac{1}{2}$ and $R = 0$, the manufacturer starts to effectively share the cost of service with the retailer (for a formal derivation of how parameters affect the manufacturer and retailer profits without showrooming, see the Appendix). If the direct compensation is more difficult to implement (i.e., when r is small), the manufacturer decreases the direct compensation and instead decreases the wholesale price more. Note, however, that if the optimal retail price is higher than the one under the vertically integrated channel, the manufacturer's incentive to subsidize the retail service cost is lower than it would be under the vertically integrated channel. This leads to the idea that if competition from the online marketplace reduces the retail price toward the optimal retail price of the vertically integrated channel (the manufacturer can always keep it at or above optimal by increasing the wholesale price), the manufacturer may be willing to incentivize the retail service more, which could benefit the B&M retailer and offset the profits lost from the lower margin.

1.4.2 Model Solution When Consumers are Allowed to Showroom

When consumers are able to engage in showrooming, it is optimal for them to compare the online and the B&M retailer's prices after discovering which variant fits them at the B&M retailer. Then, those who decide to buy the product do so at the retailer with the lower price. This undifferentiated Bertrand competition results in both retail prices equal to the wholesale price w (in Section 1.5.3, we analyze a model where the e-tailer is differentiated from the B&M retailer, such that the B&M retailer does not lose all the sales if it has a higher price). Facing zero retail margin, the B&M retailer has no incentive to provide the costly service to consumers unless the manufacturer offers a direct compensation for it. Consequently, without the manufacturer's compensation, the market would shrink to zero, resulting in zero payoff for all players.[15] Therefore, the manufacturer is forced to offer at least some direct compensation no matter how low the cost of service is.

Again, we solve for the optimal w, s^*, and R for the manufacturer given that the B&M retailer will choose the level of service to maximize its own profit. Given the target service level s^* and the associated compensation R, the B&M retailer trades off between setting service level s^* and obtaining profit $R - ks^{*2}$ and providing no service ($s = 0$) and receiving an expected shirking payoff of $(1 - r)R$. Thus, to induce the retailer's choice of $s = s^*$, the minimal compensation – and therefore, the optimal manufacturer's choice of R – is $R^* = \frac{ks^{*2}}{r}$. Note that because of the imperfect detection, the manufacturer is paying an amount more than what the retailer needs to cover its service costs. Given this choice of R as a function of s^*, the manufacturer needs to decide on w and s^*, and its objective function becomes

$$\max_{w \le 1,\ 0 \le s^* \le 1} \left[s^*(1 - w)w - \frac{ks^{*2}}{r} \right]. \tag{1.3}$$

[15]Note that in this case, effectively, consumer ability to showroom does not result in actual showrooming, as consumers are unable to determine fit. The effect of consumers' ability to showroom on sales is indirect: it causes the B&M retailer choose to provide no service, which in turn leads to zero demand regardless of the price as far as it is at or above cost.

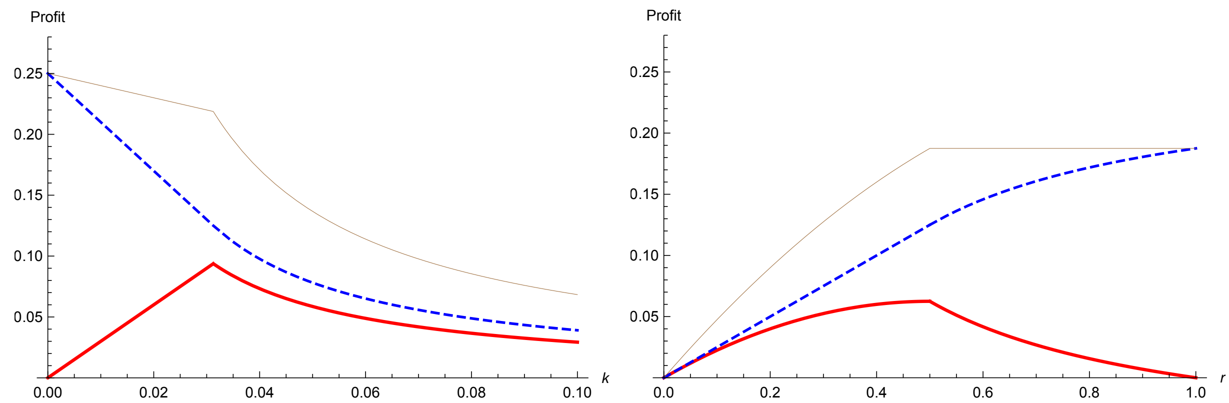

Figure 1.1: Profits with Showrooming (Main Model)
Notes. Profits as functions of k fixing $r = \frac{1}{4}$ (Left) and r fixing $k = \frac{1}{16}$ (Right). Thick red line = B&M retailer; dashed blue line = manufacturer; thin brown line = industry.

As the Appendix shows, the optimal wholesale price under showrooming is always $w = 1/2$, and the manufacturer incentivizes the retail service of $s = 1$ for small enough k ($k \leq r/8$, to be precise) and targets a lower level of return service for higher k. The following proposition summarizes how profits depend on the service cost and the service detection rate.

Proposition 1.1. *When showrooming is allowed, (a) the B&M retailer's profit has an inverted U-shape in both k and r, and (b) both the manufacturer's profits and the total industry profits strictly decrease in the service cost k and increase in the service-level detection rate r.*

Figure 1.1 illustrates how the B&M retailer's, the manufacturer's and the total industry profits depend on k and r when consumers are able to showroom.

The B&M retailer's profit is increasing in r and k when they are small enough and increasing in r and k when they are large. The intuition for such dependence on r is that the manufacturer's ability to detect the actual service level has two effects on retailer profit. On the one hand, it increases the manufacturer's incentive to induce a higher service level, which provides an opportunity for the retailer to achieve some profit from compensation due to the imperfect detection. On the other hand, as the detection improves, the manufacturer can induce a higher service level more efficiently, reducing the retailer's information rent. Thus, from the retailer's point of view, the optimal detection level is in the intermediate range.

The intuition for the effect of k on the retailer's profits is that, up to a point, the manufacturer chooses to provide sufficient incentive for the retailer to set the full service level. This incentive must be larger when k is higher.

1.4.3 Effect of Consumers' Ability to Showroom on Profits

Not surprisingly, when consumers engage in showrooming, the B&M retailer might reduce service level when it is costly, and that could mean lower profits for both the manufacturer and the retailer. Therefore, the manufacturer may want to increase compensation for service. As we have discussed, when showrooming is not possible (and the B&M retailer is a de facto monopoly), the manufacturer has two instruments in incentivizing service: wholesale price and compensation for service. With showrooming and no discrimination in wholesale prices, only one instrument remains. This lowered efficiency may be good or bad for the B&M retailer: conceptually, the manufacturer could either give up or increase its efforts to incentivize the retailer. Which outcome holds depends on the specific values of the service cost and the manufacturer's detection rate of the retail service. With less efficiency in incentivizing service, one may wonder if it is possible for the manufacturer and the retailer to be better off simultaneously with than without showrooming. After all, within our model's demand structure, the manufacturer could prevent showrooming by simply not allowing online sales. However, the reason the manufacturer could be better off is that showrooming, through increasing competition, reduces double marginalization problem: although allowing retail margin makes it easier to incentivize service, higher retail price hurts the manufacturer's profits through reducing consumer demand.

By comparing the results of the previous two subsections, one can derive how showrooming affects the B&M retailer's and the manufacturer's profits. Table 1.1 and Proposition 1.2 summarize how the profit contribution of showrooming depends on k and r.

Proposition 1.2.

(a) The contribution of showrooming to the equilibrium B&M retailer profit is increasing in r if r is small and decreasing in r if r is large. It is increasing in k for large r.

(b) The contribution of showrooming to the equilibrium manufacturer profit is increasing in r. Furthermore, it is decreasing in k for large r but is U-shaped in k for small r.

Figure 1.2 illustrates how the equilibrium contribution of showrooming to the B&M retailer's, the manufacturer's, and the industry profits depend on k and r for some parameter values. The intuition of the effect of showrooming on the B&M retailer's profit is that the consumer ability to showroom implies that the B&M retailer cannot achieve profitability through margins. This becomes less important when service is costly, as in that case, the B&M retailer's profitability is derived from the manufacturer's compensation for service. Furthermore, forcing the manufacturer to rely on the direct compensation is more beneficial to the B&M retailer when the detection is less perfect (i.e., when r is small) but of course only if the manufacturer does not "give up" on inducing service (i.e., not when r is too small or k is too large).

The intuition for the effect of showrooming on the manufacturer's profit is that an increased r makes it easier for the manufacturer to induce a higher service level through the direct compensation for service and thus, the ability to use wholesale price as a service-inducing instrument is less important. Showrooming can then benefit the manufacturer as it solves the double-marginalization problem and expands the market. To see the intuition for the effect of k on the difference in the manufacturer's profits with and without showrooming,

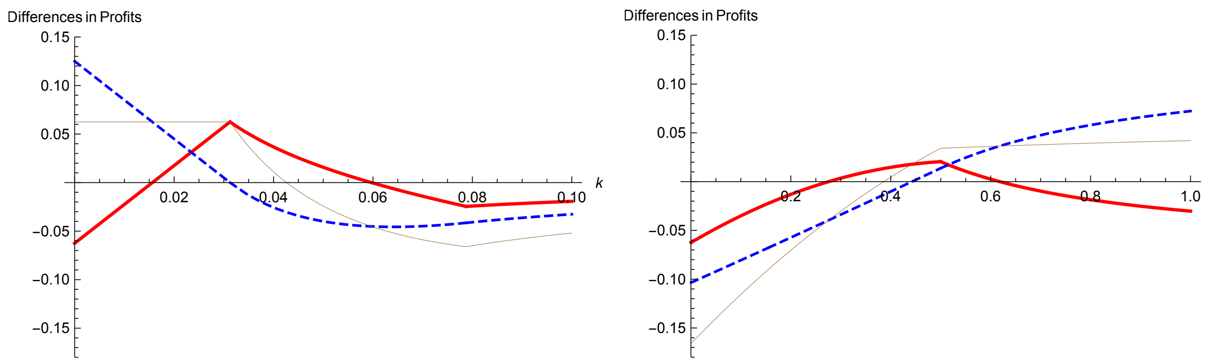

Figure 1.2: Effect of Showrooming on Profits (Main Model)
Notes. Differences in profits as functions of k fixing $r = \frac{1}{4}$ (Left) and r fixing $k = \frac{1}{16}$ (Right). Thick red line = B&M retailer; dashed blue line = manufacturer; thin brown line = industry.

note the following. First, when k is small, inducing service is not a big issue with or without showrooming, but the reduction of the double-marginalization problem is a plus. Therefore, the manufacturer is better off with showrooming when the service cost (k) is small. As service cost increases, the lower ability to induce service with showrooming hurts the manufacturer and, thus, the effect of showrooming becomes less positive and – if r is small (and thus direct compensation is not a very effective mechanism to induce service) – could become negative. Finally, as $k \to \infty$, all profits tend to zero and, thus, the difference between the manufacturer's profits with and without showrooming must tend to zero as well. Whether this difference is increasing or decreasing depends on whether it is negative for intermediate k (and it is negative when r is small).

To return to our research question – Can consumer ability to showroom benefit the B&M retailer? – the following proposition summarizes when showrooming benefits the manufacturer, the retailer, or both. Note that although our original question was whether showrooming could benefit the retailer, our result is strengthened by the observation that the positive effect of showrooming on the B&M retailer's profit does not necessarily come with the cost of decreased manufacturer's profit (otherwise, the manufacturer could try to prevent showrooming as well, and then showrooming may not be observed in equilibrium).

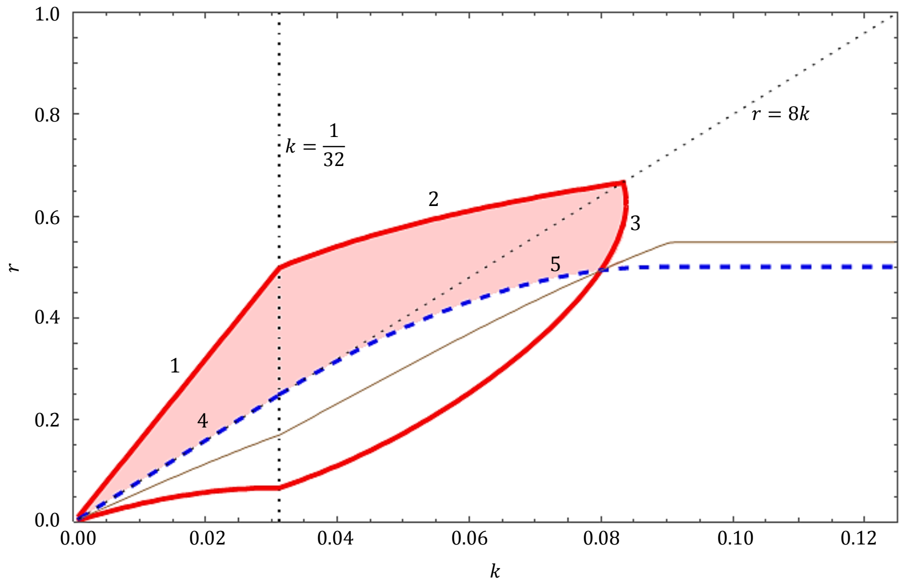

Figure 1.3: When Showrooming Increases Profits (Main Model)

Notes. Showrooming increases profits of the following: within the thick red curve = B&M retailer; above the dashed blue curve = manufacturer; shaded area = both; above the thin brown curve = industry. Equations defining boundaries are specified in the Appendix.

Proposition 1.3. *The consumer ability to showroom increases the B&M retailer's equilibrium profit when k is not too large and r is in an intermediate range. It increases the manufacturer's equilibrium profits when r is relatively large (a sufficient condition is $r > 8k$). Furthermore, when k is not too large, the consumer ability to showroom increases both the B&M retailer's and the manufacturer's equilibrium profits for intermediate r. These regions of parameters are illustrated in Figure 1.3 and the equations defining the boundary curves are reported in the Appendix.*

Thus, contrary to the popular wisdom that showrooming is always a threat to the B&M retailers, our analysis shows that when the manufacturer's decisions are added to the consideration (i.e., when the manufacturerretailer contract is endogenous), the ability of consumers to engage in showrooming can make the B&M retailers better off. Moreover, this does not necessarily happen at the cost of reduced manufacturer profit. Clearly, if showrooming benefits consumers, a well-coordinated industry should be able to take advantage of that and achieve overall higher profitability. The problem lies in the allocation of the increased profit so that both the coordination is achieved and all the individual channel members are better off. As we have shown, depending on the market conditions, rather than combating showrooming, the B&M retailer may be better off taking advantage of its role as the service provider.

Note that in the main model we analyzed, if the manufacturer can perfectly detect retailer's service level (i.e., if $r = 1$), the retailer cannot be better off due to showrooming. This is because with showrooming, retailers are undifferentiated, and therefore without manufacturer's direct compensation, the B&M retailer has zero profits. Thus, if $r = 1$, the manufacturer can provide direct compensation just above the service cost, and the B&M retailer's profit is zero. However, if retailers are differentiated, this intuition no longer applies, and as we show subsequently, the B&M retailer can be better off due to showrooming even when $r = 1$. In the following section, we consider several extensions to better understand the robustness of the results and the intuitions discussed previously.

1.5 Extensions

There are several assumptions we have made either for parsimony or analytical tractability. In this section, we consider several model variations and extensions to determine how the results are robust and to note some alternative forces that lead to similar results.

In the following subsection (Section 1.5.1), we extend the main model by endogenizing the service level detection rate (i.e., by making the costly inspection the manufacturer's decision unobserved by the retailer). In Section 1.5.2 we consider a variation of the service cost function to show that the same results can be obtained when the cost function is smooth (so that the interesting effects are obtained through internal solutions as opposed to at the kink in the service cost function) as well as a variation to consider just two discrete service levels, which leads to similar results and is analytically simpler than the main model. In Section 1.5.3, we extend the main model to allow for differentiated consumer valuations for the online and B&M retailers. In addition to con firming robustness of the main results, the differentiated demand enables us to show how the B&M retailer may benefit from showrooming even if the manufacturer is restricted to wholesale-price-only contracts and when the manufacturer is able to price discriminate between the B&M and online retail channels (i.e., set different wholesale prices for them). In Section 1.5.4, we formulate a model extension where the online retailer is able to offer a free return policy to adjust for its inability to offer in-store service and where (some) consumers face a shopping cost of visiting the B&M store. Finally, we discuss several other extensions. The formal analysis of all extensions is relegated to the Appendix.

1.5.1 Endogenous Detection Rate

One characteristic of the main model we analyzed is that, in equilibrium, the B&M retailer never deviates from the service level suggested by the manufacturer. This may raise the question of why then the manufacturer would inspect the service given that the inspections are presumably costly to the manufacturer. Here, we relax the exogenous detection rate assumption and let the manufacturer decide whether to conduct the inspection at a cost, which we denote by d. Thus, if the manufacturer in equilibrium inspects with probability r, it will incur the expected equilibrium cost of $d \times r$.

The timing is as follows. First, as in the main model, the manufacturer offers a contract to the retailers. Then, simultaneously, both retailers set their respective prices, the B&M retailer sets its service level s, and the manufacturer decides whether to inspect the actual service level. The rest of the model setup is as in the main model.

In this model modification, when the manufacturer incentivizes a higher service level through the direct compensation, there is no equilibria with the B&M retailer never shirking.

The reason is that if the B&M retailer never shirks, the manufacturer would decide never to inspect, which in turn makes shirking strictly optimal for the B&M retailer. Nevertheless, the main results about the equilibrium effect of showrooming on profits remain. To show this, it is sufficient to consider the case of $k \leq \frac{1}{32}$.

Proposition 1.4. *When the manufacturer's detection is endogenous and $k \leq \frac{1}{32}$, the consumer ability to showroom increases the B&M retailer's profit when d is in an intermediate range and increases the manufacturer's profit when d is small.*

Intuitively, when d is small, the manufacturer has good control over the retailer's service decision without paying much compensation. In this case, the B&M retailer is hurt by showrooming, but the manufacturer benefits from the elimination of the double-marginalization problem. When d is larger, the manufacturer chooses to detect the retail service with lower probability and therefore needs to provide a larger compensation to ensure that the suggested service level is often offered. As a result, the B&M retailer benefits from the higher compensation, but the manufacturer is hurt by the costlier detection.

In the current extension, it turns out that because of the cost of detection, whenever the retailer is better off due to showrooming, the manufacturer is worse off. However, the lower bound on the detection cost required to make the retailer better off is exactly the upper bound when the manufacturer becomes better off. Therefore, considering a convex detection cost as a function of detection probability could allow both the manufacturer and the retailer to be better off.[16] A convex (as opposed to linear) cost of detection is more realistic because, in practice, it is easier to inspect at some times of the day than others and because some complications (e.g., traffic jams, the inspector's sickness, the retailer's ability to mislead the inspector) may be more difficult to prevent.

[16]For instance, when the cost of detection is $dr + dr^2$, showrooming can increase both the manufacturer's and the B&M retailer's profit (e.g., when $k = \frac{1}{32}$ and $d = \frac{1}{128}$).

1.5.2 Variations of the Retailer's Cost of Service Function

In the main model, the service level is assumed to be quadratic at first and then bounded by 1, which results in a kink in the retail response to the service incentive. Moreover, the noteworthy results held when the manufacturer finds it optimal to provide just enough incentive for the B&M retailer to keep the service level at 1. In this subsection, we first show that this is not an important assumption, but a smooth service cost function also works. We then return to the simpler idea of the "high" versus "low" service level and illustrate how a simpler model with just two possible service levels works as well.

Smooth cost of service. Instead of the piece-wise cost function of the main model, assume that the cost of service is $c(s) = k_1 s + k_2 s^2$ for any $s \geq 0$ (note that this means the retailer may provide service $s > 1$, which technically voids the interpretation of service as a probability of the fit discovery by an average customer; yet as we see next, the parameters can be assumed to be high enough that it would never be practical for the B&M retailer to set $s > 1$, which allows us to keep that interpretation).

The formal analysis of this model variation is very similar to that of the main model and is relegated to the Appendix (of course, as the equilibrium service level is now a nontrivial function of the wholesale price and the direct compensation, the analytical expressions become much more complicated). Figure 1.4 illustrates the region of parameters under which the B&M retailer, the manufacturer, and the industry profits increase under showrooming.

As in the main model, the consumer's ability to showroom increases the B&M retailer's profit when both the service costs and the detection rate are intermediate and increases the manufacturer's profit when the detection rate is relatively large. Moreover, when k_1 is neither too large nor too small and r is intermediate, showrooming increases both the B&M retailer's and the manufacturer's equilibrium profits (note that while k_2 affects decision variables and profits, it ends up not affecting profit comparisons with vs. without showrooming and thus does not enter Figure 1.4).

Table 1.1: Effect of Showrooming on Profits (Main Model)

Region		$\frac{\partial \Delta\pi_m}{\partial k}$	$\frac{\partial \Delta\pi_m}{\partial r}$	$\frac{\partial \Delta\pi_{B\&M}}{\partial k}$	$\frac{\partial \Delta\pi_{B\&M}}{\partial r}$
$\mathbf{R_I}$	$k \leq \frac{1}{32}$ and $0 \leq r < 8k$	$-$	$+$	$+$ for small r $-$ for large r	$+$
$\mathbf{R_{II}}$	$k \leq \frac{1}{32}$ and $8k \leq r \leq 1$	$-$	$+$	$+$	$-$
$\mathbf{R_{III}}$	$\frac{1}{32} < k < \frac{1}{9}$ and $\max\{0, r^*\} \leq r < 8k$	$+$ for small r $-$ for large r	$+$	$-$	$+$ for small r $-$ for large r
$\mathbf{R_{IV}}$	$\frac{1}{32} < k < \frac{1}{9}$ and $\max\{8k, r^*\} \leq r < 1$	$-$	$+$	$+$	$-$
$\mathbf{R_V}$	$k \geq \frac{9}{128}$ and $0 \leq r < \min\{8k, r^*, 1\}$	$+$ for small r $-$ for large r	$+$	$+$	$+$ for small r $-$ for large r
$\mathbf{R_{VI}}$	$k \geq \frac{9}{128}$ and $8k \leq r \leq \min\{r^*, 1\}$	$-$	$+$	$+$	$-$

Notes. See Lemma A.1 in the Appendix for the definition of r^*.

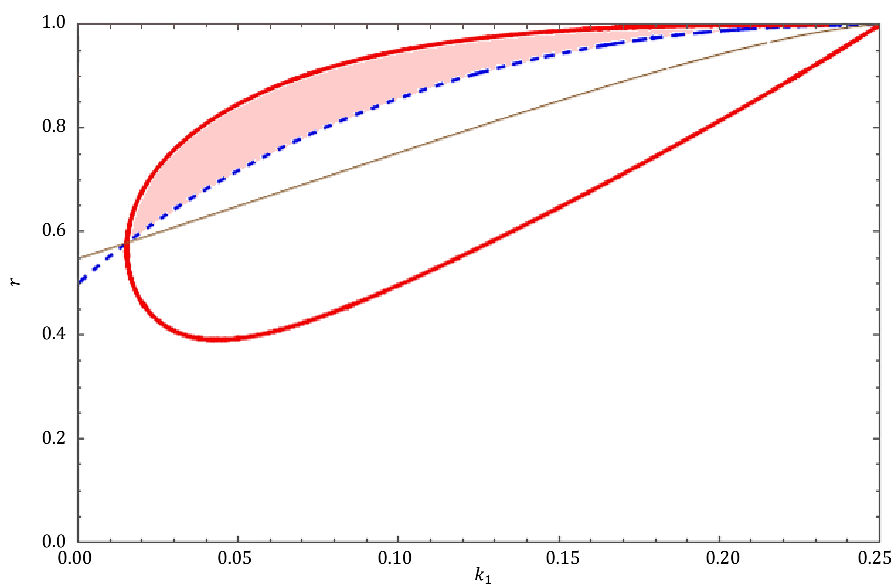

Figure 1.4: When Showrooming Increases Profits (Extension: Smooth Cost of Service)
Notes. Showrooming increases profits of the following: within the thick red curve = B&M retailer; above the dashed blue curve = manufacturer; shaded area = both; above the thin brown curve = industry.

Discrete service levels. An alternative way of thinking about service levels is that it is, conceptually, either "high" or "low," and the manufacturer may prefer the B&M retailer to set it at the "high" level. This leads to the same conceptual results with much simpler analytical expressions (albeit with more numerous cases to consider).

Specifically, assume that the B&M retailer is choosing between two service levels only: the "low" level of s_l and the "high" level of s_h (as our analysis shows, s_h can be normalized to 1; the results work with $c(s_l) = 0$, but that is not a normalization). Let the B&M retailer's cost of providing these two service levels be, respectively, $c(s_l) = 0$ and $c(s_h) = C > 0$. Again, the B&M retailer may have the incentive to choose a lower service level than the one desired by the manufacturer due to the cost, but the manufacturer can incentivize a higher service level (i.e., s_h) with both wholesale price w and, possibly, the direct compensation R.

The formal analysis of this model has multiple cases but is technically straightforward and relegated to the Appendix. To illustrate how the main result works in this model variation, let us note that when $s_l = \frac{2}{5}$, $s_h = 1$, $r = \frac{1}{2}$ and $C = \frac{3}{80}$, in the no-showrooming case, $\pi_m = \frac{1}{8}$ and $\pi_{B\&M} = \frac{1}{40}$; in the showrooming case, $\pi_m = \frac{7}{40}$ and $\pi_{B\&M} = \frac{3}{80}$.

1.5.3 Differentiated Retailers

The main model assumed that the B&M and online retailer are not differentiated to simplify the analytical analysis and to show that showrooming may benefit the B&M retailer (1) even if the online marketplace does not expand the market and (2) when potential competition is the most severe. In the main model, with consumer ability to showroom, Bertrand competition forces the retail prices both online and offline to the wholesale price, fully eliminating the retailers' profits from product sales. In this subsection, we relax this assumption and show that our result is robust when the retailers are differentiated and both retailers can make positive profits from the product sales. Furthermore, we show how retail differentiation can strengthen the main result, as the result is possible even when the manufacturer is unable to offer direct compensation for service (i.e., when $r = 0$) and when the manufacturer can set different wholesale prices for online and B&M retailers (Luchs et al. 2010 provide evidence that the legal restriction on wholesale price discrimination has become weaker over the years).

With regard to retail differentiation, one may speculate that the valuation of buying online may be smaller due to the necessity of waiting for the product to arrive, the possibility of the product incurring damage in transit, or some trust issues related to providing a credit card or bank account number online or trusting that the online merchant will send the product (Forman et al. 2009 find empirical support for lower valuations online). We consider such a model in the following subsection. Alternatively, some consumers may prefer to shop offline and some online. We consider such a model in the "Hotelling-Like Retailer Differentiation" subsection. A conceptual advantage of the former model variation relative to the latter one is that the former preserves the idea in the main model that the results could be obtained without the online marketplace expanding the market.

Consumers have lower valuations online. As in the main model, assume that when a consumer finds the best-fitting product after visiting the B&M retailer, his or her utility of purchasing a product that fits at the B&M retailer is $u_1 = V - p_1$, where p_1 is the offline retail price and V is uniformly distributed on $[0, 1]$ across consumers. In contrast to the main model, assume that the consumer valuation of purchasing a product online that fits is $u_2 = \delta V - p_2$, where p_2 is the online retail price. The discount factor $\delta < 1$ captures consumers' discounted utility of getting a product online (because of, e.g., having to wait for delivery, the hassle of placing the order, the possibility of receiving a damaged package). Thus, if showrooming is possible, after determining which products fits at the B&M retailer, the consumer will purchase at the B&M retailer if $V \geq \max\{p_1, \frac{p_1 - p_2}{1-\delta}\}$, and will purchase online if $\frac{p_1-p_2}{1-\delta} > V \geq \frac{p_2}{\delta}$.

The full analysis of this model appears in the Appendix. It turns out that in this model variation, not only does the main result that showrooming possibly benefits the B&M retailer continue to hold, but the manufacturer's ability to directly compensate for the retail service is unnecessary. Note that this implies that retail differentiation may expand the parameter range under which showrooming benefits the B&M retailer. Furthermore, if the manufacturer is able to set different wholesale prices (restricted not by law but by no-arbitrage condition), showrooming may still benefit the B&M retailer (for parameter values establishing these results and for the clarification of the no-arbitrage restriction on the manufacturer's ability to price discriminate, see the Appendix). The model in the next subsection strengthens this statement even more by removing the no-arbitrage restriction.

Hotelling-like retailer differentiation. Alternatively, retail differentiation may be modeled as a horizontal one (as in Hotelling 1929), so that the consumer valuation at the B&M retailer is $u_1 = V - xt - p_1$, and at the e-tailer, it is $u_2 = V - (1-x)t - p_2$, where t is a parameter, $x \in [0,1]$ is the uniformly-distributed-across-consumers preference for shopping online vs. offline, V is the consumer value of the product (as before, distributed uniformly on $[0,1]$ and independently of x), and p_i are the retail prices.

The Appendix provides details of this model analysis and an example of parameter values for which showrooming increases the B&M retailer's and the manufacturer's profits when the manufacturer is able to exercise unrestricted wholesale price discrimination (i.e., when retailers are not allowed to engage in arbitrage) even when it is not allowed to offer direct compensation for service (e.g., when $r = 0$). The following proposition summarizes the main implications of the analysis in Section 1.5.3.

Proposition 1.5. *When the B&M and online retailers are differentiated, consumers' ability to showroom may increase the B&M retailer profits even if the manufacturer is restricted to wholesale-price-only contracts (i.e., not allowed to compensate for service directly). Furthermore, this result is robust to allowing the manufacturer to set different wholesale prices to the B&M and online retailers.*

Next, we relax the assumption of zero consumer search costs and the inability of online retailer(s) to provide some uncertainty-resolving service.

1.5.4 The Possibility of Online Returns and Offline Shopping Costs

A couple of other simplifications we have made in the main model is that the online store(s) have no way of offering fit-revealing services to consumers and that consumers have no shopping costs of visiting either B&M or online stores. In practice, online retailers have some service instruments, such as allowing easy returns, and one could argue that many consumers face shopping costs at B&M stores. The following model considers these possibilities.

In addition to the decisions in the main model, the online retailer can choose to offer a return service. If it does so, consumers unsure about product fit can buy multiple items online and return all except the one that fits. To avoid the possibility that the cost of consumer returns drives results, let us assume that returns are costless for the consumer, but we impose a tie-breaking rule that if a consumer is indifferent between using versus not using the return service (e.g., if the maximal possible valuation is below price, if it is costless for the consumer to use the B&M service), (s)he will choose not to use it. This essentially means that showrooming can reduce the online return costs because consumers who engage in showrooming reduce the online retailer's marginal costs (coming from the returns).

Furthermore, assume that a proportion $\lambda \leq 1$ of consumers are "shoppers" (i.e., like to shop) and face no shopping costs to go to the B&M store, while the rest $1 - \lambda$ are "not shoppers" (i.e., do not like to shop) and face a cost of $t > 0$ to visit the B&M store. To avoid the phenomenon of all consumers with positive shopping costs strictly preferring the online retailer, consider the product valuations as in the "Consumers Have Lower Valuations Online" subsection. However, to simplify the analysis, assume that the B&M retailer's service decision is discrete: $s = 0$ or 1 with $c(0) = 0$ and $c(1) = C$ (i.e., the simplification introduced in the "Discrete Service Levels" subsection). The rest of the setup is the same as in the main model. Although the formal analysis of this model is complicated, the Appendix presents the equilibrium conditions and shows that showrooming can benefit the B&M retailer in this model variation as well.

1.5.5 Other Extensions

There are many possible variations one may consider beyond those we have explored in this section. The Appendix discusses some model variations where some consumers do not need a showroom (which is an alternative way of considering how the online marketplace may exist without showrooming), a two-segment model of differentiation between the online and offline marketplace (which is an alternative to the model variation in Section 1.5.3), and a model variation with discrete set of consumer valuations, which also leads to similar results to those in the main model.

1.6 Discussion and Conclusion

The popular press has depicted online retailers as "eating the lunch" of B&M stores. The latter have dwindling market share and decreasing margins; even worse, online retailers free ride on them as showrooms. After considerable expenditures on shop associate salaries, store organization, and product demonstrations to generate consumer demand, B&M retailers may discover that although they are welcoming high consumer traffic, at the end of the day, a considerable portion of consumers switch to the online competitors.

However, in this article, we argue that lumping together the issue of new competitors with the issue of showrooming is not necessarily justified. Consumer need for a showroom is an important reason for retailers to exist in the first place, and it can be used as a driver of retail profitability. If online competitors are not able to provide an essential service, B&M retailers should be able to leverage their advantage. By definition, showrooming refers to consumers buying online the product they have identified as the best offline, and therefore, it is about the allocation of sales of a particular manufacturer. Therefore, the manufacturer is both the beneficiary of the increased demand due to online sales and at the threat of collapsing demand if the B&M retailers do not provide sufficient service. The manufacturer is also in a position to offer contracts to coordinate the competition versus service trade-off. This article argues that the effect of the technological advance that led to showrooming could result in increased B&M retailer profitability. Of course, this is only true if showrooming is important, but not if consumers switch to online purchases as they become better educated about the online marketplace and do not need a showroom. In other words, although the competition from the e-tailers may be bad for the B&M retailers, showrooming itself may be goodin the sense that preventing showrooming would further decrease B&M retailers' profits.

Incidentally, the intuition formalized in our model justifies the advice in one of the popular-press essays detailing various ways to deal with showrooming: perhaps, in response to customers wanting to showroom, the B&M retailer should provide a free wireless service (i.e., to facilitate showrooming; see point 6 of Charlton 2013; this is also in line with the traditional advice to "do what the customer wants"). Indeed, instead of covering the ceilings with wire nets to block cell-phone reception, many retailers (e.g., Barnes & Noble, Best Buy, Macy's, Nordstrom) now provide a complimentary wireless Internet access. Of course, the models presented in this article also show the possibility that showrooming could be detrimental to the B&M retailer; in that case, it may want to consider strategies to prevent showrooming, such as requesting exclusive products from the manufacturer, restricting the consumer use of cell phones in its store, and bundling the common products with unique products or services. Note that whether showrooming is desirable or not, B&M retailers may also want to use strategies that do not address showrooming directly but are designed to reduce competition, such as price obfuscation (e.g., bundle, discounts, loyalty points) or price matching.

Although a particular model requires many assumptions for tractability, we have shown that the basic idea is robust to several variations. One possibility we did not allow for is an e-tailer opening a physical location to offer a showrooming service. For example, Amazon is opening some B&M stores. This possibility could be worth considering in future research. At first glance, such a strategy appears suboptimal: Why open a showroom when you can free ride on another retailer? However, if manufacturers encourage service, an e-tailer may also want a piece of that pie. The results would likely depend on how services from two retailers interact in affecting consumer valuation.

Another possibility is that a manufacturer may want to open showrooms itself and avoid channel inefficiencies altogether. For example, Tesla and Apple are famous for opening retail showrooms. In such situations, the considerations and results of this article do not apply. In many categories, however, the advantage of retail experience in providing service (and perhaps, credibility of sales assistance, as a retailer's sales force likely does not seem as potentially biased as a manufacturer's) and the benefit of one-stop shopping could outweigh the benefits of vertical integration. We empirically observe that distribution channels are often both decentralized and without perfectly aligned incentives.

In conclusion, we offer suggestions for strategies retailers could use to take advantage of showrooming – namely, charging customers for the use of the showroom (e.g., an entrance fee) and taking advantage of the increased store traffic by selling more unique products (or, at least, products that are inconvenient to buy online). The former strategy sounds appealing, especially in the framework of the models we considered, and indeed, it may work in some cases, but in many situations, consumers may face uncertainty about the value of the best match they can find. In this case, if the retailer has some control over the best match (e.g., it chooses the number of products or their quality) or has superior information about the possible matches, consumers faced with an entrance fee may contemplate the possibility that the retailer, having already collected the entrance fee, no longer has an incentive to display a good assortment or provide good service in the store. In other words, in some situations, an entrance fee could signal low value of items in the store. On the other hand, retailers have traditionally used loss-leader pricing to attract customer traffic. Showrooming could be perceived as exactly that opportunity: a store with a nice display and good wireless/Internet connectivity can attract customers without even taking a loss on the sale of a loss-leader product (e.g., Barnes and Nobles bookstores also receive revenue from their cafe sales).

CHAPTER 2

OPINION LEADERS AND PRODUCT VARIETY

2.1 Introduction

The importance of word of mouth and opinion leader recommendations for consumer purchase decisions has been recognized by marketing professionals for a long time (see e.g., Katz and Lazarsfeld 1955 or Dichter 1966). The idea is that some consumers have higher expertise and/or willingness to spend effort to understand products, and then, they are willing to propagate their recommendations to other consumers. The other consumers (which eventually result in most of the product sales), while may learn about products from mass media, make purchase decisions in large part on the basis of the opinion leader recommendations. Bass (1969) model captures these types of consumers mathematically through the terms of the "innovators" and "imitators," respectively. According to Whitler (2014), a majority (64%) of marketing managers believe that word of mouth is the most effective marketing, yet only 6% claim to have mastered it.

The effectiveness of word of mouth lies perhaps in its relative trustworthiness. Recent technological advances in social media make posting and receiving opinions even easier. Did the consumer trust change? According to a 2012 Nielsen study, 92% of global consumers trust word of mouth and recommendations from friends and family, and 70% trust online reviews, both numbers an increase of at least 15% over the previous 5 years (Grimes 2012). Pertaining to the opinion leader influence through social media, Karp (2016) reports a joint study by Tweeter and Annalect indicating that 49% of respondents rely on digital influencers for product recommendations, and nearly 40% of Tweeter users have made a purchase because of an influencer's tweet. On the opposite side, consumer trust in advertising declined by at least 20% across the advertising type spectrum, and a majority of consumers no longer trust advertising, regardless of whether it is on TV, in newspapers, or online (Grimes 2012). It appears that the importance of word of mouth is increasing even further in the digital age.

When opinion leaders are not professional judges of quality but rather more-or-less "usual" consumers from the point of view of their preferences, their evaluation and recommendation tend to be not very detailed and to be influenced both by the product quality and by their personal preferences for features, colors, styles, etc. In other words, an opinion leader often provides a feedback indicating whether she likes the product overall in a way that may be difficult for the followers (if not for the opinion leader herself) to see how much of this judgement is based on the quality (i.e., that which is positively valued by all) and how much it is based on the idiosyncratic preference of the opinion leader (i.e., fit) which may not be relevant for other consumers. For example, in addition to the product quality, mangakas choose to adopt drawing tools that match their drawing habits, make-up artists choose to demonstrate cosmetics suitable for their skin types, and tennis players choose to use racquets satisfying their needs for size and length.

In terms of how information flows online vs. offline, one can make the following observation. Offline, when a recommendation comes from a family member or a friend, the recipient may have a natural chance to follow up with questions pertaining to the basis of the recommendation. On the other hand, online, a recommendation may come in a form of a selfie on Instagram, a short note on Tweeter, a "like" on Facebook, or the fact that the opinion leader uses certain products. For example, consumers see the drawing tools in the painting tutorials (e.g., Steve Mitchell's pins on Pinterest), the cooking utensils in the cookery courses (e.g., Gordon Ramsay's videos on YouTube), the sport equipments in the sport-related blogs (e.g., Janae Jacobs's running blog *Hungry Runner Girl*) and the beauty products in the make-up demonstration videos (e.g., Michelle Phan's tutorials on YouTube). These situations do not present easy follow-up opportunities.

An implication of this observation is that when consumers are trying to infer their utility of a product from an opinion leader's recommendation, especially in the case of digital influencers, they may not have an easy time figuring out whether the recommendation is due to an idiosyncratic fit of the product version the opinion leader uses, or due to the quality of the product or brand that applies to every later adopter.[1]

Knowing that consumers infer the quality of products and brands from the positive or negative evaluations of opinion leaders, firms are trying to win the opinion leaders' support. Paying them for good reviews directly could be a risky strategy since if the opinion leader reveals to the general public that the brand offered a bribe to write a favorable review for a bad product, there could be either a considerable damage to the brand's image or consumers may no longer trust opinion leader product recommendations of this brand.

As a result, to earn an opinion leader's endorsement, the firm not only needs to have high-quality products, but also needs to provide a variant that satisfies the opinion leader's personal preferences.

This strategy dates back to a long time before the rise of online opinion leaders, with some famous recent examples including the Birkin bag produced for the actress and singer Jane Birkin by high fashion company Hermès, and the Air Jordan shoes produced for the basketball player Michael Jordan by sportswear company Nike. The product adaptation solution is straightforward when the company is very familiar with the opinion leader and understand her personal preference well. However, in the case of a multitude of less influential opinion leaders, providing a product variant customized for each of them becomes infeasible, and firms can only encourage the product adoption and recommendation through careful product line design, including the decision on product variety or customizability.

Let us consider a firm's optimal choice of product variety, i.e. the number of product variants to provide. An opinion leader is likely to have a higher expertise and/or desire to educate herself about the product category, which is how she obtains popularity and trust of her followers in the first place, and thus, is likely to be better able to understand

[1] Another difference between recommendations from family and friends and those from the online influencers is that online influencers may be "sponsored" by the firm, an issue we will return to in Section 2.5.6

and choose which variant fits her the best. On the other hand, the uninformed consumers, not being familiar with the product category or not wishing to spend the time and effort, may not be able to choose the best-fitting alternative and may benefit less from having many alternatives in the market. Moreover, while a larger number of alternatives is likely to increase an expert's satisfaction and therefore, the likelihood of the (positive) product recommendation, the uninformed consumers' rational inference of quality *conditional* on the (positive or negative) evaluation will likely decrease due to the anticipation of a better fit found by the opinion leader. Because of this decreased quality expectation, increasing the product variety may have a negative effect on the firm's profit.

In this paper, we study how the optimal product variety (assortment) should be adjusted when uninformed consumers rely on the product evaluations of opinion leaders (or, experts) to make purchase decisions. Henceforth, we will use the terms "opinion leader" and "expert," as well as "opinion leader's product evaluation" and "expert opinion," interchangeably. We default to the latter for brevity. We focus on the informational aspect of this context (i.e., on the flow of information) and seek to answer the following questions. Should the product variety be always adjusted in one direction (up or down) due to the presence of the expert opinion or is there an intermediate optimal number of product variants from the point of view of affecting consumer inference in the most beneficial-to-the-firm way? If the latter, what does the optimal variety depend on? And how are decisions affected if the firm knows the true quality vs. the quality being uncertain both to the firm and to the (non-expert) consumers?

To analyze these questions, we first consider a model in which in the absence of the expert opinion, the number of product variants neither affects the firm's cost nor the consumer utilities, and the firm has no more information about the quality than the consumers. Furthermore, we assume that the experts constitute a negligible part of the consumer demand. These assumptions allow us to isolate the effect of the consumer inference from the expert's product opinion. We then see how the presence of the expert's opinion affects the optimal decisions when increasing the number of product variants is costly to the firm (so that in the absence of the expert opinion, a single product would be strictly optimal to offer) and when consumers can partially observe their individual fit to each product variant (so that more variants would be preferable to the consumers, and in the absence of a per-variety cost, an infinite number of them spanning the range of consumer preferences would be optimal). We also consider how different the equilibrium number of variants would be when the firm knows the product quality before choosing the number of product variants. In the main model, we assume the expert's evaluation is only driven by the product itself, but in an extension, we also consider the possibility that the expert opinion depends on the product price.

The main results are as follows. First, we show that the informational forces discussed above may lead to an intermediate optimal number of product variants. In other words, there can be a strictly optimal number of variants (more than one and less then infinity) even if the firm's costs do not increase in the product variety, and providing more alternatives does not increase the expected fit for an uninformed consumer. The intuition for this result is that increasing the number of product variants has two opposing effects on the firm's profit: (i) it increases the probability of getting a positive expert opinion and therefore, the product being recognized by consumers as of high quality, and (2) it decreases the consumer certainty that the quality is high conditional on the realized (positive or negative) expert opinion. Though the first effect is positive, the second one is negative. It turns out that for a small number of variants, the first effect dominates the second, and for a large number of variants, the second effect dominates the first. We further find that the optimal number of product variants increases if the importance of fit for the expert or the expert's selectiveness (unwillingness to provide a positive recommendation) is higher.[2] These results are robust

[2]Note that this result is not due to the uninformed consumers valuing fit since in the main model, they are unable to choose a product which fits them better.

to the firm not being able to adjust the price based on the outcome of the expert's opinion, and to the expert opinion depending on the product's price.

Unsurprisingly, when the firm knows its quality, the low-quality firm may want to hide its quality by limiting the information transmitted through the expert opinion and therefore, mimicking the high-quality firm's product variety decision. It turns out that when the consumer inference from the expert opinion is the only factor affecting the length of the product line, the optimal number of variants when the firm is unsure of its quality remains the equilibrium choice when the firm knows its quality. However, when (a) increasing the number of product variants is costly or (b) having more alternatives helps uninformed consumers improve fit, the high quality firm is able to signal its quality through the product variety decision. Moreover, in (a), the high-quality firm may further decrease the variety as compared to the case when the firm is uncertain about its quality, whereas the opposite may be true in (b). Using the undefeated equilibrium concept to derive a unique equilibrium prediction, we show that in the above two extensions, the unique equilibrium may be pooling and may exhibit a larger distortion due to the consumer inference from the expert opinion when the quality is known to the firm than when it is not.

To relate our considerations to business practice, one can observe that firms often introduce a product line with a small number products (or one product) promoted to opinion leaders. The idea is that the promotion of a smaller number of products makes communication easier and clearer. For example, Griner (2015) describes a promotional campaign by Lord & Tailor to introduce Design Lab collection, with success attributed to its choice to promote exactly the same version of a dress to many influencers on Instagram. If it were to offer a large product choice, the influencers could each choose a different dress from the collection, and the followers might have attributed the choice to each influencer's idiosyncratic preference for a specific color pattern, which may or may not look well on a different person. But the fact that all agreed to post the same version perhaps indicates that the reason is not the fit to the individual complexion, but some good underlying quality of the dress. Note that in this case, the firm (Lord & Tailor) explicitly connected with the influencers and must have compensated them for the post, but the informational value of this act could be similar to the organic word of mouth because presumably, the value comes from the understanding that a firm would find it more difficult to convince the influencers to post if the dress was not good (most influencers disclosed the relationship by the hashtag #ad, and comments left on the feeds of others also made the sponsored nature of the posts clear).

The rest of the paper is organized as follows. Section 2.2 discusses the literature related to this paper. The following section (Section 2.3) formally defines the main model. Section 2.4 analyzes the main model where the firm faces no variety costs, the uninformed consumers have no information about their individual fit, and the firm does not know its true product quality. Section 2.5 examines several variations of the main model to both confirm that our main results are robust and to establish additional insights. It also presents an empirical validation of the intermediary result important for our main insight that consumer inference from a positive expert recommendation is hindered by an increased product variety. Section 2.6 concludes the paper. Detailed proofs are relegated to the Appendix.

2.2 Related Literature

There is a rich and growing literature on firms' strategies to provide or affect information available to consumers, e.g., through informative advertising (e.g., Butters 1977, Lal and Matutes 1994, Soberman 2004, Villas-Boas 2004, Iyer et al. 2005, etc.), sales assistance (e.g., Wernerfelt 1994), or product returns (e.g., Shulman et al. 2010). A number of papers also examined the underlying question of whether and under what conditions firms benefit from providing information (e.g., Guo 2009, Guo and Zhao 2009, Kuksov and Lin 2010, Gu and Xie 2013, Branco et al. 2016, etc.), and what makes firm-supplied information credible (e.g., Milgrom and Roberts 1986, Balachander and Srinivasan 1994, Moorthy and Srinivasan 1995, Simester 1995, Anderson and Simester 1998, Zhao 2000, Shin 2005, Miklos-Thal and Zhang 2013, etc.).

Even closer to our paper is research on how product assortment affects consumer information and purchase decisions. For example, Villas-Boas (2004) considers how firms should choose the product line length given that communicating to consumers is more costly if the number of products is larger. Iyengar and Lepper (2000) and Boatwright and Nunes (2001) show that increased assortment may result in lower sales in an experimental and empirical contexts, respectively. Kuksov and Villas-Boas (2010) explains this effect through an analytical analysis of how the expected consumer evaluation (search) costs depend on the number of products, and Kamenica (2008) considers how the number of products affects consumer inference of the likelihood of fit. This stream of research points to the importance of considering consumer information and communication when deciding on the product variety.

As related to how product variety affects word of mouth, we know that word of mouth depends on customer satisfaction, in particular, that highly satisfied consumers are more important for promoting products to other consumers than less highly satisfied consumers (see e.g., Anderson 1998 or Reichheld 2003), and Diehl and Poynor (2010) experimentally show that even conditional on consumers choosing a product, customer satisfaction could be lower if they have chosen from a larger assortment. Their rationale for this finding is that when consumers face a larger assortment, they expect to end up with a higher utility, and if satisfaction is driven not by the consumer's realized utility of the chosen product but by how much that utility exceeds expectations (a finding in Anderson and Sullivan 1993), then the higher expectations imply a lower satisfaction. Combining this result with the effect of satisfaction on word of mouth implies that larger assortment leads to a less beneficial word of mouth, which is an effect similar to one we have in this paper, although justified through a somewhat different mechanism. One may view understanding the implications of product variety on word of mouth as especially useful given that most managers do not have a very good grasp of how to manage word of mouth (Whitler 2014).

Extant research also examined how third-party reviews impact consumers' beliefs and in turn, consumer decisions. For example, Reddy et al. (1998) find that critics have a significant impact on the choice of artistic goods. There is also analytical research on how reviews should impact consumer beliefs (e.g., Sun 2012), how firms should adapt their strategies in response to reviews (Chen and Xie 2005) and how to affect them (e.g., Kuksov and Xie 2010). As in most of the previous literature, we assume that the expert always truthfully reveals the private information she has (although Durbin and Iyer 2009 consider the seller being able to make not fully truthful recommendations) and uninformed consumers make a rational inference from it. Extending the previous literature, we consider how the firm should choose the product variety to affect consumer inference from the opinion leader recommendations, given that the assortment size is observed by all consumers and used in the inference process.

2.3 Main Model

Consider a market with one firm, one expert consumer, and a unit mass of uninformed consumers each with a single-unit demand. The firm can produce a product in any number of variants at zero cost (we later consider positive costs).

We denote a consumer's valuation for the product category by v and assume it is heterogeneous across consumers and distributed uniformly on $[0, V]$.[3] Further, we assume that there are some characteristics of the product that are common across all versions of the firm's products in this category (which could be either some objective quality or just a preference shared by all consumers), and some characteristics that are designed to match various idiosyncratic preferences. We call the former characteristics *quality* and represent it by the

[3] We assume consumer heterogeneity over v to generate a continuous downward sloping demand: consumer heterogeneity over fit we define later would not lead to downward sloping demand in the main model because we assume consumers do not observe this value prior to purchase. Alternatively, one can consider consumer heterogeneity over the value of quality. This alternative assumption leads to conceptually similar results but more complex analytical expressions.

parameter q. We call an individual consumer i's utility from the latter characteristics *fit* and model it following Salop (1979) as the importance of fit parameter t times the distance between consumer i's ideal point x_i and product variant j's location l_j on a circumference of a unit length, with x_i distributed uniformly on the circumference across consumers.[4] To summarize, consumer i's utility from version j of the product is

$$U_{i,j}(p) = v + q - d(x_i, l_j) \cdot t - p,$$

where p is the product's price.

Turning to the information structure, we assume that neither the firm nor the consumers know the exact value of q, but only know that it can be either low or high with equal probability, which reflects a situation when a firm introduces a new brand or a conceptually new product (the alternative assumption that the firm knows the exact value of q is considered in Section 2.5.1).[5] For example, one could argue that when Microsoft introduced and highly promoted its Zune MP3 player, it did so because it did not know how cool the consumers will perceive it to be. The same can be said about Apple when it introduced iPod MP3 player. These two products could have had similar costs, but one was a flop and the other was a huge success. Similar uncertainty probably applied to the original iPhone and applies to many apparel brands. We normalize the high quality value to 1 and denote the low value by $q_0 \in (0, 1)$. The firm decides on the price p of the product and on the number n of the product variants to offer to the consumers (all variants are of the same quality). One can think of the product as the brand, and product variants as the different versions within the brand (i.e, customizable attributes of technology products, or different styles and colors of

[4]Using Salop's circular preference space instead of Hotelling's unit interval allows us to avoid the technical complications associated with considering consumers located closer to the center vs. closer to the end of the preference space.

[5]According to Ogawa and Piller (2006), the new product failure rates often reach 50% or greater, suggesting that manufacturers of new products have limited knowledge about the average consumer valuation.

apparel), or the product can be more specific than a brand and the number of variants could represent how customizable it is (i.e., a TV could have plenty or a few picture and color adjustment options).

We further start with the assumption that consumers are uncertain about their own personal preferences (this assumption is relaxed in Section 2.5.3). An alternative interpretation is that the uninformed consumers do not observe the location of any product variant. Either way, uninformed consumers are uncertain about the product fit, and thus they are indifferent between different variants. This allows us to isolate the effect of the number of variants as coming only through its effect on the expert product opinion and the consequent consumer inference.

The expert, on the other hand, knows her personal preference and the exact characteristics of the variants. Therefore, she is able to correctly identify the product variant that suits her best, and always chooses to obtain it. This last assumption is only to make sure that expert always posts her product opinion (the fact of purchase itself is inconsequential since the mass of uninformed consumers is assumed to be infinitely larger). In other words, the expert observes the precise value of $q - \min_j d(x_e, l_j) \cdot t$, where $\min_j d(x_e, l_j)$ is the distance between the expert's ideal variant, x_e, and the product variant providing her with the best match. She then broadcasts her opinion about the product, which is either positive or negative. Specifically, we model the opinion as being positive if and only if $q - \min_j d(x_e, l_j) \cdot t$ exceeds a certain threshold parameter u_0. One interpretation of u_0 is that it represents the utility of the outside option the expert considers, which means u_0 would be higher in a well-developed product category with a lot of good brands. Alternatively, it could be a consumer behavior or a social norm parameter indicating how critical opinion leaders tend to be. Note that if $u_0 \geq 1$, the expert opinion about the product will always be negative, and thus not informative. To focus on the interesting case where the expert product opinion can help resolve consumer uncertainty, we assume that $u_0 < 1$. Moreover, we assume that the expert's personal preference (x_e) is her private information. In other words, the location of the expert's ideal variant on the circumference is unknown to both the firm and the other consumers.

To simplify the model and reduce the number of cases to consider, we assume that $V \geq \max\{1 - \frac{t}{4}, \frac{t}{4}\}$, which will imply that in equilibrium, the market is always partially covered.

The timing of the game is as follows. First, the firm makes the product line decision, i.e., how many product variants to offer and where on the circle to locate each variant. Then, the expert chooses the variant she likes best and posts the product opinion based on her overall experience. After observing the expert's opinion, the firm sets the price of the product to uninformed consumers, which amounts to the assumption that the firm can adjust the price after observing word of mouth (we later consider a model variation where the firm has to commit to price before knowing the expert opinion (Section 2.5.4 and 2.5.5). This assumption can be viewed as realistic because most of the demand from opinion leaders (innovators) comes earlier than most of the demand from followers (imitators), and firms can increase prices if products turn out to be popular and offer discounts when demand is slack. Finally, uninformed consumers decide whether to purchase the product based on the price and their expectation of the product quality given their prior beliefs, the expert opinion, and the number of variants the firm offers.

2.4 Main Model Analysis

We use Perfect Bayesian Equilibrium as a solution concept. Consequently, we first derive the uninformed consumers' beliefs about product quality as a function of the expert opinion and the number of variants n. Since we have so far assumed the firm does not know the quality beyond what it can infer from the expert opinion, the price is not informative of the true quality. Note that while the firm's *decision* on the number of variants is likewise not informative about the quality, it may enter the consumer evaluation because it affects the outcome of the expert's opinion. Then, we derive the optimal price given the number of product variants and conditional on the expert opinion. Finally, we derive the optimal number of product variants for the firm and how the profit depends on the existence of the expert opinion and the parameters of the model.

To analyze the game, it is useful to observe from the onset that since the uninformed consumers do not know the location of each variant and the positive expert opinion is preferred by the firm, the firm's objective in deciding the variant locations is to maximize the probability of the positive expert opinion given n. The expert opinion is an additive function of the product quality and the disutility of the mismatch with the closest variant. Having no power over the value of q, the firm can only try to minimize the expert's mismatch between her ideal variant and the best available variant. If the location of the expert's ideal product would be known, the firm could simply provide the ideal variant to the expert. In this case, the expectation of the uninformed consumers would be that the expert's disutility from mismatch is zero, and for the purpose of the product opinion the expert is only comparing the value of q and u_0. However, if the firm cannot customize the product to the expert's preference, to minimize the expert's expected mismatch, the optimal locations of the variants are equidistant (i.e., $\frac{1}{n}$ from the adjacent ones along the circle). The probability of the expert opinion being positive can then be stated as in the following lemma.

Lemma 2.1. *If $q_0 > u_0$, the probability of a positive expert opinion is*

$$Prob(positive) = \begin{cases} \frac{n(1+q_0-2u_0)}{t}, & \text{if } n \leq \frac{t}{2(1-u_0)} \\ \frac{t+2n(q_0-u_0)}{2t}, & \text{if } \frac{t}{2(1-u_0)} < n \leq \frac{t}{2(q_0-u_0)} \\ 1, & \text{if } n > \frac{t}{2(q_0-u_0)} \end{cases}. \tag{2.1}$$

If $q_0 \le u_0$, on the other hand, it is

$$Prob(positive) = \begin{cases} \frac{n(1-u_0)}{t}, & \text{if } n \le \frac{t}{2(1-u_0)} \\ \frac{1}{2}, & \text{if } n > \frac{t}{2(1-u_0)} \end{cases}. \tag{2.2}$$

Thus, when $q_0 > u_0$ and $n \le \frac{t}{2(q_0-u_0)}$, or $q_0 \le u_0$ and $n \le \frac{t}{2(1-u_0)}$, the probability of the positive opinion is decreasing in the importance of fit parameter t and the threshold of the positive opinion u_0. Moreover, the probability is increasing in the number of variants n. This result is intuitive since as n increases, the expert's evaluation of the best-suited-for-her variant increases. To see where the above thresholds of n come from, note that $n \le \frac{t}{2(1-u_0)}$ corresponds to $1 - \frac{t}{2n} \le u_0$, i.e., the expert opinion may be negative even for a high-quality product; in contrast, $q_0 > u_0$ and $n > \frac{t}{2(q_0-u_0)}$ correspond to $q_0 - \frac{t}{2n} > u_0$, i.e., the expert opinion will always be positive even for a low-quality product.

We now consider how the uninformed consumers form their expectations of product quality rationally expecting that the expert's evaluation follows the rule derived above.

2.4.1 Consumer Expectation of Product Quality

When uninformed consumers observe a positive expert opinion, they do not know whether it is driven by a high quality or a low disutility from mismatch. Bayes rule then leads to the following expected product quality conditional on the observed expert opinion.

Lemma 2.2. *If $q_0 > u_0$, the expected quality conditional on a positive expert opinion is*

$$\hat{q}_p = \begin{cases} \frac{1+q_0^2-u_0(1+q_0)}{1+q_0-2u_0}, & \text{if } n \le \frac{t}{2(1-u_0)} \\ \frac{t+2nq_0(q_0-u_0)}{t+2n(q_0-u_0)}, & \text{if } \frac{t}{2(1-u_0)} < n \le \frac{t}{2(q_0-u_0)} \\ \frac{1+q_0}{2}, & \text{if } n > \frac{t}{2(q_0-u_0)} \end{cases}, \tag{2.3}$$

while conditional on a negative opinion, it is

$$\hat{q}_n = \begin{cases} \frac{t(1+q_0)-2n(1+q_0^2-u_0(1+q_0))}{2(t-n(1+q_0-2u_0))}, & \text{if } n \le \frac{t}{2(1-u_0)} \\ q_0, & \text{if } n > \frac{t}{2(1-u_0)} \end{cases}. \tag{2.4}$$

If on the other hand $q_0 \leq u_0$, the expected quality conditional on a positive opinion is $\hat{q}_p = 1$, and conditional on a negative opinion, it is

$$\hat{q}_n = \begin{cases} \frac{(1+q_0)t - 2n(1-u_0)}{2(t-n(1-u_0))}, & \textit{if } n \leq \frac{t}{2(1-u_0)} \\ q_0, & \textit{if } n > \frac{t}{2(1-u_0)} \end{cases}. \tag{2.5}$$

The expected quality of the product conditional on a positive expert opinion is larger than $\frac{1+q_0}{2}$. It is decreasing in n if $\frac{t}{2(1-u_0)} < n \leq \frac{t}{2(q_0-u_0)}$, and constant in n otherwise. It is increasing in t and u_0 if $n \leq \max\{\frac{t}{2(1-u_0)}, \frac{t}{2(q_0-u_0)}\}$, and constant in t and u_0 otherwise. Moreover, it is first constant, then decreasing, and then increasing in q_0. The expected quality of the product conditional on a negative expert opinion, on the other hand, is smaller than $\frac{1+q_0}{2}$. It is decreasing in n and increasing in t and u_0 if $n \leq \frac{t}{2(1-u_0)}$ and constant in n, t and u_0 if $n > \frac{t}{2(1-u_0)}$. It is always increasing in q_0. Figure 2.1 illustrates how the expectation of product quality changes as a function of the parameters.

Intuitively, as long as $q_0 \leq u_0$ or $n \leq \frac{t}{2(q_0-u_0)}$, both positive and negative expert opinions about the product are informative, in the sense that they imply an expected quality of the product other than the mean of the prior distribution $\frac{1+q_0}{2}$. Furthermore, and also intuitively, the informativeness (indicated by the difference between the prior mean and posterior mean) of a negative expert opinion weakly increases in n, and the informativeness of a positive expert opinion weakly decreases in n. As the number of product variants provided in the market increases, the satisfaction of the expert implied by a positive opinion should be attributed more to a low mismatch of personal preference instead of a high product quality.

The positive monotonicity of the conditional expected qualities in t and u_0 is also intuitive. A larger t corresponds to a higher expected disutility of mismatch, and a larger u_0 represents a higher baseline that the utility of the focal product is compared with.

Perhaps the most surprising part is that the uninformed consumers' expectation of product quality given a positive expert opinion may decrease in q_0, the value of the low quality

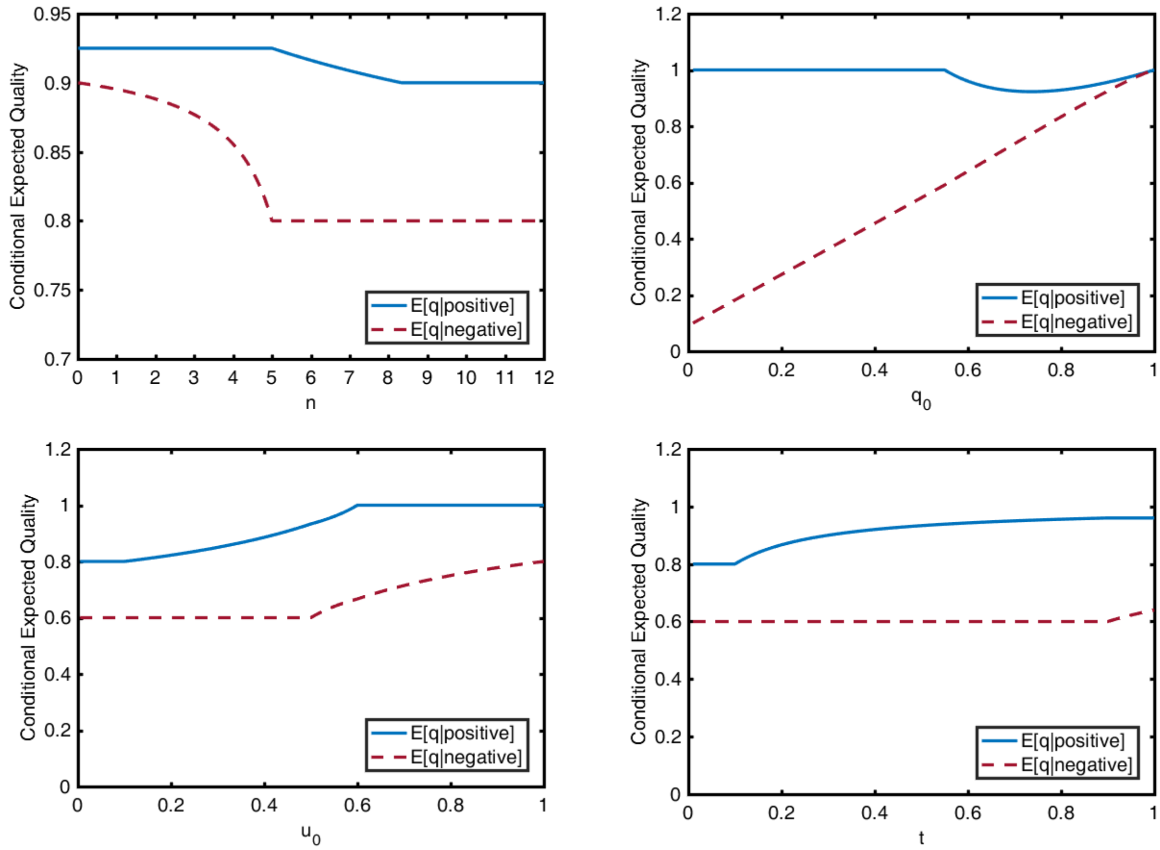

Figure 2.1: The Conditional Expected Product Quality

Notes. Upper-left: $t = 5$, $q_0 = 0.8$, and $u_0 = 0.5$. Upper-right: $t = 1$, $u_0 = 0.55$, and $n = 1$. Lower-left: $t = 1$, $q_0 = 0.6$, and $n = 1$. Lower-right: $q_0 = 0.6$, $u_0 = 0.55$, and $n = 1$.

level. In fact, when $q_0 \leq u_0$, a positive opinion indicates a high-quality product for sure. When $q_0 > u_0$ and q_0 is very small, a positive opinion indicates a very large probability of high-type product. As q_0 increases a little bit, the probability of the product being low-quality increases, and therefore there should be a higher weight on the low quality possibility, which decreases the expected quality. When the value of q_0 further increases, even though the weight on q_0 increases, the mean of the posterior distribution increases as the difference between q_0 and 1 becomes smaller.

2.4.2 The Pricing Decision

Denote the uninformed consumers' expectation about the product quality based on the expert opinion derived in the previous subsection by $\hat{q}$. Recall that since an uninformed consumer is uncertain about the fit of the product variant, she essentially chooses a variant at random. Thus, the expected mismatch between the uninformed consumer's ideal variant and her chosen one is

$$E(d(x_i, l_j)) = \frac{1}{4}. \tag{2.6}$$

As a result, an uninformed consumer's expected utility from purchasing the product is $v + \hat{q} - \frac{t}{4} - p$, where v is her valuation for the product category, $\hat{q}$ is the expected product quality, t is the importance of fit, and p is the product price. An uninformed consumer will purchase the product if $v + \hat{q} - \frac{t}{4} - p \geq 0$, i.e., if $v \geq p - \hat{q} + \frac{t}{4}$.

Since the distribution of v across consumers is $U(0, V)$, the expected demand for the product is 0 if $p > V + \hat{q} - \frac{t}{4}$, $\frac{V+\hat{q}-\frac{t}{4}-p}{V}$ if $\hat{q} - \frac{t}{4} < p \leq V + \hat{q} - \frac{t}{4}$, and 1 if $p \leq \hat{q} - \frac{t}{4}$, meaning that the expected revenue is 0 if $p > V + \hat{q} - \frac{t}{4}$, $\frac{(V+\hat{q}-\frac{t}{4}-p)p}{V}$ if $\hat{q} - \frac{t}{4} < p \leq V + \hat{q} - \frac{t}{4}$, and p if $p \leq \hat{q} - \frac{t}{4}$. Therefore, given our assumption that $V \geq \max\{1 - \frac{t}{4}, \frac{t}{4}\} \geq \max\{\hat{q} - \frac{t}{4}, \frac{t}{4} - \hat{q}\}$, the optimal price is

$$p^*(\hat{q}) = \frac{1}{2}(V + \hat{q} - \frac{t}{4}), \tag{2.7}$$

and the profit at the optimal price is

$$\pi^*(\hat{q}) = \frac{1}{4V}(V + \hat{q} - \frac{t}{4})^2. \tag{2.8}$$

Intuitively, the firm can obtain a higher profit if consumers have higher valuation (parameter V) for the product category, higher expectation $\hat{q}$ of the product quality, or lower weight (parameter t) they place on the fit.

The above derivation implies that in the benchmark case of no expert (i.e., consumers cannot observe expert's opinion), $\hat{q} = (1 + q_0)/2$ and therefore, the optimal price is $\frac{1}{2}(V +$

$\frac{1+q_0}{2} - \frac{t}{4})$ and the expected firm profit is $\frac{1}{4V}(V + \frac{1+q_0}{2} - \frac{t}{4})^2$. Since consumers have the same expected disutility of mismatch with any product variant regardless of the firm's variety decision, the firm is indifferent between any n. If we would assume that the profit from sales to expert consumers is at least slightly positive (instead of zero), we would have that the optimal number of variants is infinite. We next consider the optimal number of variants in the presence of the expert.

2.4.3 The Optimal Number of Variants

To simplify the expressions, let us ignore the integer constraint on n and allow it to be continuous.[6] Since the firm does not know the realization of q prior to seeing the expert's post, in deciding the number of variants to provide, the firm maximizes the expected profit:

$$\max_n E[\pi(n)] = E[R(n)] = E[R(n)|\text{pos.}] \cdot Prob(\text{pos.}|n) + E[R(n)|\text{neg.}] \cdot Prob(\text{neg.}|n), \quad (2.9)$$

where $R(n)$ is the firm's revenue from sales, "pos." stands for the expert opinion being positive, and "neg." stands for it being negative. Substituting the optimal price obtained in the previous subsection and the consumer expectation about quality based on the number of product variants and the expert opinion, we obtain that if $q_0 > u_0$, the expected profit is

$$E[\pi(n)] = \begin{cases} \frac{1}{4Vt}\Big[n(1+q_0-2u_0)(\frac{1+q_0^2-(1+q_0)u_0}{1+q_0-2u_0} + V - \frac{t}{4})^2 + \\ \qquad (t - n(1+q_0-2u_0))(\frac{t(1+q_0)-2n(1+q_0^2-u_0(1+q_0))}{2(t-n(1+q_0-2u_0))} + V - \frac{t}{4})^2\Big], \\ \qquad\qquad\qquad\qquad\qquad\qquad \text{if } n \leq \frac{t}{2(1-u_0)} \\ \frac{1}{4V}[(V-\frac{t}{4})^2 + (V-\frac{t}{4})(1+q_0) + \frac{t(1+q_0^2)+4nq_0(q_0-u_0)}{2(t+2n(q_0-u_0))}], \quad \text{if } \frac{t}{2(1-u_0)} < n \leq \frac{t}{2(q_0-u_0)} \\ \frac{1}{4V}(\frac{1+q_0}{2} + V - \frac{t}{4})^2, \qquad\qquad \text{if } n > \frac{t}{2(q_0-u_0)} \end{cases} \quad (2.10)$$

[6]The integer-constrained choice of n will be the nearest integer above or below the unconstrained one. Careful consideration of the integer constraints and the results that depend on when the value of n jumps from below to above or vice versa do not appear to be insightful.

while if $q_0 \leq u_0$, the expected profit is

$$E[\pi(n)] = \begin{cases} \frac{1}{4V}[(V - \frac{t}{4})^2 + (V - \frac{t}{4})(1 + q_0) + q_0 + \frac{(1-q_0)^2 t}{4(t-n(1-u_0))}], & \text{if } n \leq \frac{t}{2(1-u_0)} \\ \frac{1}{8V}((1 + V - \frac{t}{4})^2 + (q_0 + V - \frac{t}{4})^2), & \text{if } n > \frac{t}{2(1-u_0)} \end{cases}. \quad (2.11)$$

Examining how the above expression depends on the number of variants n, we obtain the following proposition.

Proposition 2.1. *The expected firm profit is increasing in n if $n \leq \frac{t}{2(1-u_0)}$, decreasing in n if $\frac{t}{2(1-u_0)} < n \leq \frac{t}{2(q_0-u_0)}$, and constant in n otherwise. Thus, if $q_0 > u_0$, the strictly optimal number of product variants is $\frac{t}{2(1-u_0)}$, while if $q_0 \leq u_0$, any number at or above $\frac{t}{2(1-u_0)}$ is optimal.*

For an intuitive interpretation of the optimal product variant number, note that $n \leq \frac{t}{2(1-u_0)}$ is equivalent to $1 - \frac{t}{2n} \leq u_0$, meaning that the expert opinion may be negative even for a high-quality product. In this range, expanding the assortment size increases the probability of positive opinion for both low- and high-type firms. Although uninformed consumers' expected product quality conditional on the expert's opinion is weakly decreasing in n, as indicated by Lemma 2.2, the loss is outweighed by the gain from having more positive opinion, and the firm still benefits from having more product variants. When $q_0 \leq u_0$ and $n > \frac{t}{2(1-u_0)}$, or $q_0 > u_0$ and $n > \frac{t}{2(q_0-u_0)}$, neither the probability of positive expert opinion nor the conditional expected product quality changes in the value of n, and the firm is neutral over different values of n in this range. As for the case with $q_0 > u_0$ and $\frac{t}{2(1-u_0)} < n \leq \frac{t}{2(q_0-u_0)}$, only the low-quality firm obtains higher probability of positive expert opinion by increasing n. However, the expected product quality conditional on a positive opinion is decreasing in n. In this range, a firm uncertain about its own type does not find it profitable to offer a large number of product variants. Figure 2.2 illustrates how the expected profit changes in the number of product variants n.

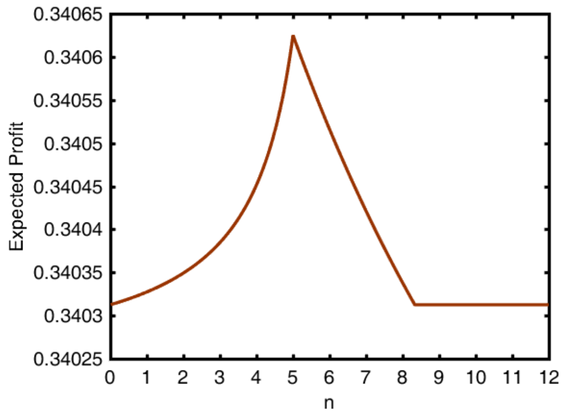

Figure 2.2: The Expected Profit as a Function of n

Notes. Left: $V = 2$, $t = 5$, $q_0 = 0.8$, and $u_0 = 0.5$. Right: $V = 2$, $t = 5$, $q_0 = 0.4$, and $u_0 = 0.5$.

According to Proposition 2.1, the firm may find it strictly optimal to provide more than one product variant, even if consumers do not directly benefit from more alternatives. The reason is that although increasing n does not change the uninformed consumers' expected mismatch cost, it changes their inference of the product quality. The firm has the incentive to be perceived as of high quality, but it also understands that consumers adjust their inference from the expert product opinion as they recognize that the number of variants affects it. The firm can increase the probability of the positive opinion by providing more alternatives to the expert to choose from. This strategy is anticipated by the uninformed consumers, and therefore, while increasing the number of variants increases the probability of the positive opinion, it decreases the expected quality conditional on a given (positive or negative) opinion. One could speculate that since a rational consumer is able to solve back the firm's strategy, the firm would not be able to be better off due to a distortion of the number of product variants away from the one optimal to satisfy consumer preferences. However, according to Proposition 2.1, the firm may benefit from increasing the number of variants even if consumers are able to solve back the firm's strategy. Moreover, when $q_0 > u_0$, the firm may find it strictly optimal to limit the number of product variants provided to the market, even if it is able to add new variants at no cost. In this case, the expert opinion may be positive for a low-quality product, as long as the expert is able to choose a variant with low enough disutility of mismatch.

The expression for the optimal number of variants in Proposition 2.1 immediately implies the following corollary.

Corollary 2.1. *The optimal number of the product variants is higher if the expert has a stronger personal preference (high t) and/or a higher threshold for the positive opinion (high u_0).*

By substituting the optimal number of variants for n in the expression for $E[\pi(n)]$, we derive the expected firm profit given the optimal number of variants and price. Specifically, if $q_0 > u_0$, the expected profit is

$$\pi^* = \frac{1}{4V}\left[\left(V - \frac{t}{4}\right)^2 + \left(V - \frac{t}{4}\right)(1+q_0) + \frac{1+3q_0^2-(1+q_0)^2u_0}{2(1+q_0-2u_0)}\right], \tag{2.12}$$

while if $q_0 \le u_0$, the expected profit is

$$\pi^* = \frac{1}{8V}\left(\left(1+V-\frac{t}{4}\right)^2 + \left(q_0+V-\frac{t}{4}\right)^2\right). \tag{2.13}$$

Since uninformed consumers' expectation of product quality is weakly increasing in u_0, the equilibrium firm profit is weakly increasing in u_0. When $u_0 < q_0$, both high- and low-quality firms can get a positive expert opinion, and thus the high-quality firm is not perfectly distinguishable from its low-quality counterpart. In this range, the firm benefits from a higher u_0, i.e., a more demanding expert. Moreover, this benefit also increases in u_0. In fact, when $u_0 < q_0$, the profit is convex in u_0. When $q_0 \le u_0 < 1$, on the other hand, the separation between the high- and low-quality firm can be perfect, and the firm is thus indifferent over u_0. Figure 2.3 illustrates how the equilibrium profit changes in u_0. Unsurprisingly, the equilibrium firm profit is always decreasing in t (consumers' weight on mismatch), and increasing in V (consumers' valuation for the product category).

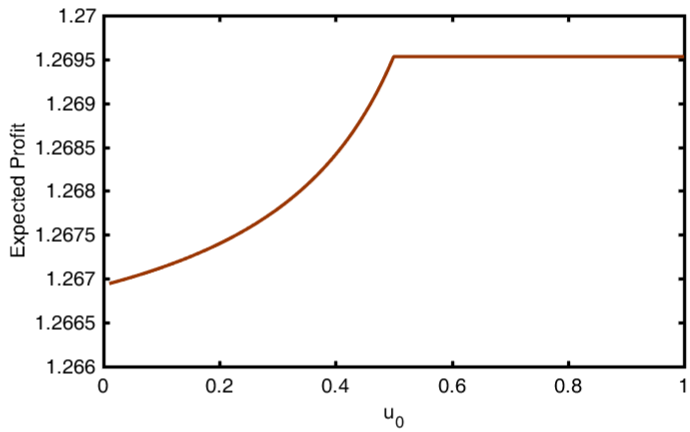

Figure 2.3: Equilibrium Profit as a Function of u_0

Note also that an implication of the above profit Equations (2.12) and (2.13) is that if the firm is able to affect u_0, the firm would prefer it to be not too low (i.e., not below q_0), but even if the firm can choose u_0, the choice of product variety is still relevant: without the expert, $n = 1$ is optimal, while with the expert, it is strictly optimal to increase n to $\frac{t}{2(1-u_0)}$, i.e., at least to $\frac{t}{2(1-q_0)}$. Assuming the firm weakly prefers not to have too many product variants due to the costs of variety (see e.g., Section 2.5.2), the optimal choice of u_0 would be q_0 and the optimal variety would be $n = \frac{t}{2(1-u_0)}$.

The equilibrium profit with the existence of the expert opinion (see Equations (2.12) and (2.13)) is always higher than it is without the expert opinion ($\frac{1}{4V}(\frac{1+q_0}{1} + V - \frac{t}{4})^2$). In a nutshell, this is because the firm can choose a (price) response to the changed beliefs and therefore, it benefits from resolving consumer uncertainty. A more detailed intuition is as follows. When the market is partially covered, the demand is a linear function of the consumer belief about the quality. Therefore, the average demand is the demand at the average quality. If there would be no response of the firm in reaction to the expert opinion, then the expected (average) profit would be the profit at the average quality level. By optimally changing the price in response to the expert opinion, the firm strictly increases its expected profit relative to the alternative case where it would keep the price constant in the absence of the expert opinion.

2.5 Model Variations and Extensions

In the main model, we made some simplifying assumptions to both zero in on the effect of the expert opinion and to simplify the derivations of the firm's optimal decisions. In this section, we relax some of our assumptions and consider several extensions to see how our main results are robust to model variations and what additional insights may be gained. We first consider the possibility that the firm knows the exact quality level before choosing the number of variants (Section 2.5.1) and show that the equilibrium decisions remain exactly the same (in particular, the equilibrium is pooling across the quality levels). We then consider two model variations where either the firm prefers to have a lower number of variants due to the positive per-variant fixed cost (Section 2.5.2), or a larger number of product variants has a positive direct effect on the profits because uninformed consumers can partially observe fit before purchase (Section 2.5.3). Within each of these extensions, we show that the main results remain qualitatively valid, but now the firm knowing its quality may change the equilibrium outcomes. Specifically, it turns out that in each of the two cases, the equilibrium choice of the number of variants could be distorted further in the direction of the original distortion due to the presence of the expert opinion. We then consider an extension where the firm has positive marginal costs but cannot change the price based on the expert's opinion (Section 2.5.4). It turns out that the main implications are unchanged when these two features are added. We then also change the expert recommendation rule to consider the value net of price rather than gross of price in forming her recommendation (Section 2.5.5) and conceptually discuss the industry practice of paying influencers for recommendations (Section 2.5.6).

2.5.1 Quality Known to the Firm

In the main model, we assumed that the true value of q is unknown to the firm when the product first comes out. Although justifiable due to firms' frequent miss-predicting of the consumer reception and aggregate demand, this assumption was mainly done for ease of the analytical analysis. The consequence of this assumption is that the decision on the number of variants n does not depend on the firm's type. We now consider the alternative assumption that the firm knows the exact value of q and makes the product line decisions based on that.

In this case, the optimality of having equidistant variants and the analysis of Section 2.4.2 (pricing as a function of the consumer expectations of quality) still apply. However, since the firm knows q before making the assortment decision, the choice of n may depend on the value of q, and therefore consumers may infer q not only from the expert opinion but also from the firm's decision on n. Nevertheless, it is easy to see that if the low- and the high-quality firm choose different n in equilibrium (i.e., if equilibrium is separating), the uninformed consumers would perfectly infer quality from n, and therefore the low-quality firm will do no worse by imitating the high-quality firm (since the cost of offering any number of alternatives is the same and the consumers do not care about the number provided beyond what it implies about the quality). Therefore, the equilibrium has to be pooling on n and price.[7] In a pooling equilibrium, the previous statement about the uninformed consumers' expectations of product quality, as listed in Lemma 2.2, also applies here.

It remains to consider the firms' optimization problem over n, since now the preferred n may depend on the quality. Clearly, there are multiple pooling equilibria sustained by the consumer beliefs that any deviation implies a low-quality product. To select a unique equilibrium, we will apply the concept of undefeated equilibrium (see, e.g., Mailath et al.

[7]When $q_0 \leq u_0$ and $n_h^* \geq \frac{t}{2(1-u_0)}$, we may also have separating equilibria where the low-quality firm is always identified as the low type and thus does not have strictly positive incentive for mimicry. However, as we discuss later, the equilibrium profits are the same.

1993, Gill and Sgroi 2012, Miklos-Thal and Zhang 2013, Subramanian and Rao 2016), which focuses on the equilibrium outcome where the high-quality firm achieves its highest profit possible across all equilibria.

To derive the undefeated equilibrium, we need to consider the firm's profits assuming that consumers believe the choice of n is an equilibrium one and therefore, as discussed above, a pooling one. In other words, consumer inference is as if they do not take n into account when forming their expectations of quality. Under this rule, the probability that the expert product opinion is positive conditional on the product quality q is

$$Prob(\text{pos.}|n,q) = \begin{cases} 0, & \text{if } q \leq u_0 \\ \frac{2n(q-u_0)}{t}, & \text{if } q > u_0 \text{ and } n \leq \frac{t}{2(q-u_0)} \\ 1, & \text{otherwise} \end{cases} . \tag{2.14}$$

Here, the threshold of n depends on the value of q. When $q > u_0$ and $n > \frac{t}{2(q-u_0)}$, we have $q - \frac{t}{2n} > u_0$, and the expert will always post a positive opinion about the product given the variants provided.

As in the main model, the expected profit conditional on q is

$$E[\pi(n)|q] = E[R(n)|q] = E[R(n)|\text{pos.}] \cdot Prob(\text{pos.}|n,q) + E[R(n)|\text{neg.}] \cdot Prob(\text{neg.}|n,q), \tag{2.15}$$

where $R(n)$ is the revenue when both firms choose to provide n variants to the market (and consumers expect this).

Therefore, if $q_0 > u_0$, the conditional expected profits are

$$E[\pi(n)|q=1] = \begin{cases} \frac{1}{4Vt}(2n(1-u_0)(\frac{1+q_0^2-(1+q_0)u_0}{1+q_0-2u_0} + V - \frac{t}{4})^2 + \\ \qquad (t-2n(1-u_0))(\frac{t(1+q_0)-2n(1+q_0^2-u_0(1+q_0))}{2(t-n(1+q_0-2u_0))} + V - \frac{t}{4})^2), \\ & \text{if } n \leq \frac{t}{2(1-u_0)} \\ \frac{1}{4V}(\frac{t+2nq_0(q_0-u_0)}{t+2n(q_0-u_0)} + V - \frac{t}{4})^2, & \text{if } \frac{t}{2(1-u_0)} < n \leq \frac{t}{2(q_0-u_0)} \\ \frac{1}{4V}(\frac{1+q_0}{2} + V - \frac{t}{4})^2, & \text{if } n > \frac{t}{2(q_0-u_0)} \end{cases} \tag{2.16}$$

and

$$E[\pi(n)|q=q_0] = \begin{cases} \frac{1}{4Vt}(2n(q_0-u_0)(\frac{1+q_0^2-(1+q_0)u_0}{1+q_0-2u_0} + V - \frac{t}{4})^2 + \\ \qquad (t-2n(q_0-u_0))(\frac{t(1+q_0)-2n(1+q_0^2-u_0(1+q_0))}{2(t-n(1+q_0-2u_0))} + V - \frac{t}{4})^2), \\ & \text{if } n \leq \frac{t}{2(1-u_0)} \\ \frac{1}{4Vt}(2n(q_0-u_0)(\frac{t+2nq_0(q_0-u_0)}{t+2n(q_0-u_0)} + V - \frac{t}{4})^2 + \\ \qquad (t-2n(q_0-u_0))(q_0 + V - \frac{t}{4})^2), & \text{if } \frac{t}{2(1-u_0)} < n \leq \frac{t}{2(q_0-u_0)} \\ \frac{1}{4V}(\frac{1+q_0}{2} + V - \frac{t}{4})^2, & \text{if } n > \frac{t}{2(q_0-u_0)} \end{cases}, \tag{2.17}$$

and if $q_0 \leq u_0$, the conditional expected profits are

$$E[\pi(n)|q=1] = \begin{cases} \frac{1}{4Vt}(2n(1-u_0)(1 + V - \frac{t}{4})^2 + \\ \qquad (t-2n(1-u_0))(\frac{(1+q_0)t-2n(1-u_0)}{2(t-n(1-u_0))} + V - \frac{t}{4})^2), & \text{if } n \leq \frac{t}{2(1-u_0)} \\ \frac{1}{4V}(1 + V - \frac{t}{4})^2, & \text{if } n > \frac{t}{2(1-u_0)} \end{cases} \tag{2.18}$$

and

$$E[\pi(n)|q=q_0] = \begin{cases} \frac{1}{4V}(\frac{(1+q_0)t-2n(1-u_0)}{2(t-n(1-u_0))} + V - \frac{t}{4})^2, & \text{if } n \leq \frac{t}{2(1-u_0)} \\ \frac{1}{4V}(q_0 + V - \frac{t}{4})^2, & \text{if } n > \frac{t}{2(1-u_0)} \end{cases}. \tag{2.19}$$

Moreover, examining when $\frac{dE[\pi(n)|q]}{dn}$ is positive, we obtain that when consumers do not infer the product quality from the number of variants provided by the firm (as in the case of pooling equilibrium), $E[\pi(n)|q=1]$ is increasing in n if $n \leq \frac{t}{2(1-u_0)}$, decreasing in n if $\frac{t}{2(1-u_0)} < n \leq \frac{t}{2(q_0-u_0)}$, and constant in n otherwise. On the other hand, $E[\pi(n)|q=q_0]$ is decreasing in n if $n \leq \frac{t}{2(1-u_0)}$, increasing in n if $\frac{t}{2(1-u_0)} < n \leq \frac{t}{2(q_0-u_0)}$, and constant in n otherwise. In other words, when uninformed consumers purely rely on the expert opinion for their quality inference, whereas the high-quality firm favors an assortment size such that the firm type can be identified,[8] the low-quality firm favors the one such that the firm type is indistinguishable.[9] Comparing the preferred choice of the high-quality firm with the equilibrium in the main model, we obtain the following proposition.

Proposition 2.2. *The equilibrium product line decisions when the quality is uncertain to the firm are also equilibrium ones when the quality is known to the firm.*

According to the above proposition, firm's knowledge of the true quality level does not change the equilibrium product line decision. The intuition is that the high-quality firm favors the number of product variants that facilitates the uninformed consumers to identify the high type, which is also the optimal choice when the firm is uncertain about its product quality. By choosing the value of n optimal for the high-quality firm, the low-quality firm may still be identified as the low type through the expert product opinion. However, if it chooses another value of n, uninformed consumers expect it to be of the low type even

[8]If $q_0 \leq u_0$, $E[\pi(n)|q=1]$ is maximized when $n \geq \frac{t}{2(1-u_0)}$, where the expert opinion will always be positive for the high-quality firm and negative for the low-quality firm. If $q_0 > u_0$, on the other hand, $E[\pi(n)|q=1]$ is maximized at $n = \frac{t}{2(1-u_0)}$, where the expert opinion will always be positive for the high-quality firm and sometimes be negative for the low-quality firm.

[9]If $q_0 \leq u_0$, $E[\pi(n)|q=q_0]$ is maximized at $n=0$ ($n=1$ under the integer constraint). If $q_0 > u_0$, on the other hand, $E[\pi(n)|q=q_0]$ is maximized at $n=0$ or $n \geq \frac{t}{2(q_0-u_0)}$ ($n \geq \frac{t}{2(q_0-u_0)}$ under the integer constraint). We can think of the $n=0$ case as the one where n is too small to satisfy the expert even if $q=1$, and the $n \geq \frac{t}{2(q_0-u_0)}$ case as the one where n is large enough so that the expert is satisfied even if $q=q_0>u_0$.

before seeing the expert's post. As a result, the low-quality firm has an incentive to mimic the decision of its high-quality counterpart if it is costless to do so. We will see in the following sections that this result may not hold when changes in the number of product variants are costly either due to the per-variant fixed cost or due to consumers being able to find a better fitting product if more alternatives are provided.

Moreover, if $q_0 > u_0$, the equilibrium profit is $\frac{1}{4V}(\frac{1+q_0^2-(1+q_0)u_0}{1+q_0-2u_0} + V - \frac{t}{4})^2$ for the high-quality firm, and $\frac{1}{4V(1-u_0)}[(q_0 - u_0)(\frac{1+q_0^2-(1+q_0)u_0}{1+q_0-2u_0} + V - \frac{t}{4})^2 + (1 - q_0)(q_0 + V - \frac{t}{4})^2]$ for the low-quality firm. If $q_0 \leq u_0$, on the other hand, the equilibrium profit is $\frac{1}{4V}(1 + V - \frac{t}{4})^2$ for the high-quality firm, and $\frac{1}{4V}(q_0 + V - \frac{t}{4})^2$ for the low quality firm.

As expected, the equilibrium profit is decreasing in t, increasing in V, and weakly increasing (decreasing) in u_0 for the high-quality (respectively, low-quality) firm. Note that the value of u_0 (compared with q_0) determines how easy it is for uninformed consumers to identify the firm type from the expert product opinion. Although a high u_0 benefits the high-quality firm, it is detrimental to the low-quality firm. Moreover, the equilibrium profit is first constant, then decreasing, and then increasing in q_0 for the high-quality firm, and increasing in q_0 for the low-quality firm.

Note that when $q_0 \leq u_0$, the low-quality product can be identified perfectly in equilibrium. In other words, the "mimicking" is not fully successful. In this case, the low-quality firm can do as well by not choosing the same assortment size as the high-quality one. However, even if the separating equilibrium is chosen, the resulting expected payoff of each player is equivalent to the one in the pooling equilibrium.

In the main model, since the firm did not know q, we had to assume the marginal production cost of the high- and the low-quality products is the same (otherwise, the firm could infer quality from cost). This is the case when quality represents the average consumer valuation and is not driven by an observable quality of the input materials (in the case of apparel) or whether the components are high or low end (in the case of technology products).

It can also represent the case when a new technology is unproven, and it is not clear under what conditions it works well.

On the other hand, when quality is driven by the known quality of input materials, the firm may know the quality and high quality is likely to have higher cost. It makes sense then to consider whether the cost increasing in quality would affect the results within the model variation of this section (i.e., when quality is known to the firm). It turns out that the results remain conceptually the same. The reason is that (1) the high quality firm still wants to choose the number of variants as to best separate from the low quality firm, (2) production costs do not enter the expert recommendation and the consumer inference, and (3) the low quality firm still wants to pool. Of course, per-variant fixed costs (or marginal costs increasing in the number of variants) would affect the decision on product variety. We consider these costs in the following subsection.

2.5.2 Positive Fixed Per-Variety Cost

As an extension of the main model, let us now assume that the firm faces a positive cost C per product variant, so that the firm incurs a cost to increase the number of variants. Another alternative reason for why the firm may face cost of variety is that larger variety results in higher marginal production costs due to the economies of scale. The implications of this alternative reason for variety being costly in our setting turn out to be the same, and to be specific, we consider fixed costs of variety. In the following two subsections, we consider in turn the case when the firm does not know the product's quality and the case when it does.

The firm is uncertain about quality

In this case, the firm chooses n to maximize the expected revenue net of the accumulated per-variant cost:

$$\max_n E[\pi(n)] = E[R(n) - C \cdot n] = E[R(n)] - C \cdot n. \tag{2.20}$$

Extending the analysis of Section 2.4, we obtain that the firm may provide many variants even though it faces costs of doing so and a higher number of variants does not increase the consumer utility directly. Interestingly, when the optimal number of variants is higher than 1, the optimal choice is exactly the same as when $C = 0$. This is because the profit function is piecewise convex in the number of product variants (the derivative is first positive and increasing and then negative and increasing). To point up this curious result, we summarize the optimal choice of variety under fixed costs as the following proposition.

Proposition 2.3. *Although with $C > 0$, $n = 1$ is optimal for the firm in the absence of expert opinion, the firm uncertain about its true quality level may choose a higher value of n when C is small in anticipation of consumers' quality inference. Moreover, this optimal choice n_u^* is the same as in the case of $C = 0$ for small and positive C (i.e., $n_u^* = \frac{t}{2(1-u_0)}$), whereas $n_u^* = 1$ otherwise.*

Again, the firm may provide more than one variant to the market, even if it is costly and it does not help average consumers to find a better match, to affect the expert's opinion about the product. Moreover, with positive per-variant fixed cost, there is a jump in the value of n_u^*. In fact, as we have shown in the main model, even if $C = 0$, the firm does not earn a higher profit by providing more than $\frac{t}{2(1-u_0)}$ product variants. Here, $n = \frac{t}{2(1-u_0)}$ is the number of product variants that is just enough to guarantee a positive expert opinion for the high-quality product. Thus, when $C > 0$, the optimal n may be further decreased. Moreover, when we look at the expected revenue $E[R(n)]$ in the range of $n \leq \frac{t}{2(1-u_0)}$, it is both increasing and convex in n. In other words, the marginal benefit of adding one more variant increases in n. The marginal cost of doing so (i.e., C), on the other hand, is constant. Consequently, as long as $\frac{t}{2(1-u_0)} > 1$ and it is profitable to provide more than one product variants, it is optimal to provide $\frac{t}{2(1-u_0)}$ product variants.

Quality is known to the firm

First, let us consider the case where the equilibrium decision of n is pooling (in which case the decision on price is also pooling). Here, the low-quality firm mimics the high-quality firm on the number of product variants provided to the market, and the uninformed consumers cannot infer the value of q from n. A similar analysis to the one in Section 2.5.1, leads to the following result.

Lemma 2.3. *When both high- and low-quality firms choose the same number of product variants and C is small, the value of n favored by the low-quality firm is smaller (larger) than the high-quality firm if $q_0 \leq u_0$ ($q_0 > u_0$).*

Again, the high- and low-quality firms may favor different values of n in the pooling equilibrium. When $q_0 \leq u_0$, the low-quality firm never gets a positive expert opinion, and it favors a smaller equilibrium number of variants so that the high-quality firm often gets negative expert opinion, too. When $q_0 > u_0$, on the other hand, the low-quality firm favors a larger equilibrium number of variants so that oftentimes the expert opinion is positive for both types of the firm and thus consumers cannot identify the firm type. Again, many values of n may be sustained as a pooling equilibrium, as long as uninformed consumers attribute any deviation to the low-quality firm, unless the expert's post identifies the quality for sure. We again turn to the undefeated equilibrium concept to focus on the unique equilibrium where the high-quality firm achieves its optimal profit.

Unlike in the main model, it is now costly for the low quality firm to mimic the high-quality firm (recall that in the case under consideration, if the quality is known to consumers, due to the fixed costs and no consumer ability to find the product variant that fits them better, one product variant is optimal). While the cost of choosing a higher number of variants is the same for the high- and the low-quality firms, the benefit of doing so is not the same. This is because the probability of a positive expert opinion depends on the actual quality. It turns out that the high-quality firm has a higher benefit of setting a high n. Therefore, the high-quality firm is able to signal high quality by increasing n sufficiently. However, when C is too large, the cost of separation may outweigh the benefit, and the high-quality firm may instead choose to pool with the low-quality firm.

If the equilibrium is pooling, the process of equilibrium selection is similar to the one in Section 2.5.1. However, the fixed costs can result in the optimal n preferred by the high-quality firm being higher (instead of the same) than the optimal n for the firm who does not know the quality. Formal analysis of the considerations above lead to the following proposition.

Proposition 2.4. *With positive per-variant fixed cost, when quality is known to the firm,*

i. If C is large enough or if $q_0 > u_0$ and C is small enough, the equilibrium is pooling with $n_h^* = n_l^* = n_p^* = \frac{t}{2(1-u_0)}$ or 1. Moreover, n_p^* may be strictly higher than n_u^*.

ii. If $q_0 > u_0$ and C is in an intermediate range, or $q_0 < u_0$ and C is small enough, the equilibrium is separating with $n_h^* > n_l^* = 1$. Moreover, it could be that $n_l^* < n_u^* < n_h^*$, in which case n_h^* is strictly higher than $\max\{\frac{t}{2(1-u_0)}, 1\}$.

Comparing Propositions 2.3 and 2.4, we obtain that similar to the case of zero per-variant fixed cost, the knowledge of the quality may not change the firm's equilibrium decision (when C is large, or $q_0 > u_0$ and C is small). In addition, by comparing the results here with the equilibrium discussed in Section 2.5.1, we obtain that compared with the case of $C = 0$, having positive C may not change the equilibrium decision of the firm knowing its own quality level if C is small or $\frac{t}{2(1-u_0)} \leq 1$.

Unlike in the case with $C = 0$, when the per-variety fixed cost is positive, the separating equilibrium may be the unique equilibrium when quality is known to the firm. Moreover, in a separating equilibrium, it is the high-quality firm that chooses the higher n to signal its high type, although this signal comes with a cost. Especially, when $q_0 \leq u_0$, the expert product opinion is always negative for the low-quality firm. If C is small, the high-quality firm finds it profitable to signal its high type through setting a high n, though it is costly, whereas the low-quality firm chooses to save on the fixed cost knowing that it would be identified as the low type by a negative expert opinion even if it sets a high n. When $q_0 > u_0$, on the other hand, the expert's opinion can be positive for the low-quality firm too. Thus, when C is positive but small, the low quality firm may find it optimal to pool with the high-quality firm. In this case, the equilibrium n is the same as before. However, as C increases to an intermediate range, the high-quality firm may want to prevent the low-quality firm from mimicking by further increasing n. The intuition is that for the low-quality firm, the benefit of pooling at high n is the profit difference between the imperfectly- and perfectly-identified low-quality firm, whereas the cost is proportional to the difference between the high n and 1 (i.e., $n_h - 1$). When C is not too small, the low-quality firm may give up mimicking, and the signaling is feasible. On the other hand, the high-quality firm benefits from successful signaling by increasing n due to the profit difference between the perfectly- and imperfectly-identified high-quality firm, whereas the additional cost is proportional to the increase in n (i.e., $n_h - n_p < n_h - 1$). When C is not too large, the high-quality firm can earn a higher profit by signaling as compared with pooling.

Note that when the equilibrium choices are different for the two types of firms, uninformed consumers can perfectly identify the firm type before seeing the expert's post. This may suggest that the expert opinion is not informative. However, the validity of this separation relies on the anticipation of a credible expert product opinion.

Considering the equilibrium choice of n given in Proposition 2.4 as a function of C, one can observe that since for some positive C, the equilibrium choice n is higher than the equilibrium choice $\frac{t}{2(1-u_0)}$ when $C = 0$, we obtain the following result.

Corollary 2.2. *The equilibrium number of product variants chosen by the high-quality firm may increase as the per-variety fixed cost increases.*

The intuition for this result is that positive fixed costs allow the high-quality firm to engage in credible signaling, and it is done through increasing the number of variants.

2.5.3 Expected Consumer Fit Increases in n

Let us now consider the opposite case where having more alternatives is beneficial. To achieve that, we assume that the firm has no cost of increasing the number of variants, but the uninformed consumers can benefit from having more alternatives (i.e., not only the expert consumer can see the horizontal differences between the product versions, but so can other consumers).

Specifically, let us assume that although uninformed consumers do not know which product location is the best (i.e., consumers do not know their ideal product), each uninformed consumer receives a signal about its personal preference $\hat{x}_i$. With probability λ, the signal is accurate, i.e., the observed value $d(\hat{x}_i, l_j)$ is indeed the mismatch between her ideal variant and variant j. In this case, by choosing the available variant that is the closest one to her observed personal preference along the circle, the mismatch is minimized to $\min_j d(x_i, l_j)$. However, with probability $1 - \lambda$, the signal is inaccurate (a mistake) and represents a random location. In the latter case, although the consumer chooses the available variant with the smallest observed mismatch (hoping that the signal is accurate), the actual expected mismatch is $\frac{1}{4}$ as in Equation (2.6).

Let us denote consumer i's smallest observed mismatch $\min_j d(\hat{x}_i, l_j)$ by $\Delta\hat{x}_i$. After choosing the variant with the smallest observed mismatch, consumer i's expected actual mismatch is $\lambda \times \Delta\hat{x}_i + (1-\lambda) \times \frac{1}{4}$. Thus, consumer i will make the purchase if $v \geq p - \hat{q} + (\lambda \cdot \Delta\hat{x}_i + \frac{1-\lambda}{4})t$, where $\hat{q}$ is the expected product quality. Since $\Delta\hat{x}_i \sim U(0, \frac{1}{2n})$, the expected consumer demand equals to

$$Prob(v \geq p - \hat{q} + (\lambda \cdot \Delta\hat{x}_i + \frac{1-\lambda}{4})t) = \frac{V + \hat{q} - (\frac{1}{4} - \frac{(n-1)\lambda}{4n})t - p}{V}$$

if $\hat{q} - \frac{1-\lambda}{4}t < p \leq V + \hat{q} - (\frac{1-\lambda}{4} + \frac{\lambda}{2n})t$. Similar to Section 2.4.2, to make the market not fully covered in equilibrium, we assume $0 \leq \lambda \leq \min\{1, \frac{4(V-1)+t}{2t}, \frac{4V-t}{2t}\}$. Thus, given the expected product quality $\hat{q}$, the optimal price is

$$p^*(\hat{q}) = \frac{1}{2}(V + \hat{q} - \frac{n-(n-1)\lambda}{4n}t), \tag{2.21}$$

and the profit at the optimal price is

$$\pi^*(\hat{q}) = \frac{1}{4V}(V + \hat{q} - \frac{n-(n-1)\lambda}{4n}t)^2. \tag{2.22}$$

As we can see, for a given $\hat{q}$, consumers' willingness to pay for the product increases both in n and λ. Since increasing the product variety improves consumers' fit, without the consideration of consumer quality inference, the firm should provide as many product variants as possible. However, as we will show below, because uninformed consumers infer the product quality from expert's positive or negative opinion, the firm interested in providing uninformed consumers with product information may limit the number of variants if $q_0 > u_0$ and λ is small. This can be true for the firm uncertain about its own type and the firm of the high quality. For clarity, we consider the cases of quality unknown and known to the firm separately in the following two subsections.

The firm is uncertain about quality

Recall that when $q_0 > u_0$ and $\lambda = 0$ (the main model), the expected profit is first strictly increasing, then strictly decreasing, and then constant in n. Since the profit function is piecewise polynomial in λ, this trend remains when λ is positive but close to 0. Thus, it directly follows that limiting n is optimal if $q_0 > u_0$ and λ is small. We therefore obtain the following proposition.

Proposition 2.5. *Even when $\lambda > 0$, when the firm is uncertain about the product quality, the optimal number of product variants is finite if $q_0 > u_0$ and λ is small.*

This result shows the robustness of the effect of the expert opinion on the optimal product variety when there is a direct benefit of a larger set of alternatives for consumers.

Quality is known to the firm

Here, a positive λ provides extra incentive for the firm to increase the number of product variants, which imposes an opposite effect to that of a positive per-variety cost C. It is therefore intuitive that it influences the equilibrium in the opposite way (again, we focus on the undefeated equilibrium) as indicated by the following proposition.

Proposition 2.6. *When $\lambda > 0$ and the firm knows its quality,*

i. The equilibrium is pooling with $n_h^* = n_l^* = n_p^*$ if either q_0 or λ is small, or both q_0 and λ are large. Moreover, it could be that $n_p^* < n_u^* < +\infty$.

ii. The equilibrium is separating with $n_h^* < n_l^* = +\infty$ if both q_0 and λ are intermediate. Moreover, it could be that $n_h^* < n_u^* < n_l^* = +\infty$.

Again, the equilibrium results underline the importance of the consideration of affecting the consumer quality inference from the expert's positive/negative product opinion. Even though having more alternatives is welcomed by both the expert and the uninformed consumers, the equilibrium number of variants provided to the market may be finite, no matter what the firm's type is. Since we expect the firm of the high quality to have a higher incentive to provide quality information than the firm uncertain about its own type, it is not surprising that the equilibrium n of the former one can be smaller than the latter one. As the above proposition shows, this may happen both when equilibrium is pooling or separating.

In contrast to the results in Section 2.5.2, when $\lambda > 0$, the high-quality firm signals its high type through decreasing, rather than increasing the number of product variants provided to the market. The reason is that with $\lambda > 0$, the opportunity cost of reducing n is larger for the low-quality firm. When λ is not small, the low-quality firm does not find it optimal to mimic its high-quality counterpart if n_h is small enough. Even if it were to mimic, it may be identified as the low type by a negative expert opinion, and thus the mimicry is not as beneficial. On the other hand, it faces a cost of mimicking as it may need to lower the price since the consumers' disutility of mismatch increases as n decreases.

2.5.4 Constant Price and Positive Marginal Costs

In the main model, we allowed the firm to adjust the product price p after observing the expert opinion. This was to capture the idea that pricing decision is more flexible than the product variety decision and that the price can be changed based on any new information about the expected aggregate demand. Alternatively, and especially if the timeline of purchase decisions of opinion leaders is overlapping with the purchase decisions of consumers, one could argue that it would be reasonable to assume that the opinion leaders and regular consumers face the same price, i.e., that the firm should not have the opportunity to adjust the price after observing the recommendation of the opinion leader. Furthermore, it is also realistic to assume that the marginal (per product) cost is positive rather than zero. In this section, we consider a model variation with both of the above features to verify the robustness of the main results.

Namely, in a change from the main model, assume that the marginal cost is $c > 0$ per product sold, and the game sequence is: the firm decides on the product variety and the price first, then the expert chooses a product and evaluates it, and finally, the consumers make purchase decisions based on the price, variety, and the expert opinion. Following the main model, for the sake of the ease of analysis, we assume that the firm does not know the product's quality. We still assume that the expert opinion is based on whether the expert sees the value she derives from her chosen product as clearing the bar defined by the parameter u_0 and is not dependent on price (as we show in the following section, the results are similar, albeit more analytically complex, when the expert opinion is based on her utility net of price).

Since this model variation does not change the expert's situation, the probability of a positive expert opinion and the consumers' expectations of the product quality given the expert opinion are the same as in the main model (see Lemma 1 and Lemma 2, respectively). Therefore, so is the consumer demand as a function of price and variety. However, the firm's pricing decision is now affected both by not being able to condition on the expert opinion and by having to account for the marginal cost. The expected firm's profit is

$$E[\pi(n,p)] = \begin{cases} p-c, & \text{if } p \leq \hat{q}_n - \frac{t}{4} \\ 1 \times (p-c) \times Prob(pos.) & \\ \qquad + \frac{V+\hat{q}_n-\frac{t}{4}-p}{V} \times (p-c) \times Prob(neg.), & \text{if } \hat{q}_n - \frac{t}{4} < p \leq \hat{q}_p - \frac{t}{4} \\ \frac{V+\hat{q}_p-\frac{t}{4}-p}{V} \times (p-c) \times Prob(pos.) & \\ \qquad + \frac{V+\hat{q}_n-\frac{t}{4}-p}{V} \times (p-c) \times Prob(neg.), & \text{if } \hat{q}_p - \frac{t}{4} < p \leq V + \hat{q}_n - \frac{t}{4} \\ \frac{V+\hat{q}_p-\frac{t}{4}-p}{V} \times (p-c) \times Prob(pos.) + 0, & \text{if } V + \hat{q}_n - \frac{t}{4} < p \leq V + \hat{q}_p - \frac{t}{4} \\ 0, & \text{if } p > V + \hat{q}_p - \frac{t}{4} \end{cases}, \tag{2.23}$$

where $Prob(pos.)$ is defined in Lemma 2.1, $Prob(neg.) = 1 - Prob(pos.)$, and $\hat{q}_p$ and $\hat{q}_n$ are stated in Lemma 2.2.

The following proposition implies robustness of our main result to the model variation of this section.

Proposition 2.7. *When the product price cannot be conditioned on the expert opinion, the firm finds it optimal to use* $n = \frac{t}{2(1-u_0)}$ *product variants, and this choice is strictly optimal if* $q_0 > u_0$ *and the marginal cost* c *is neither too large nor too small.*

Note that while the expression for the optimal price would be necessarily different from that in the main model (both because it cannot be conditioned on the expert opinion and because it has to depend on the cost), the expression for the optimal product variety turns out to be exactly the same, i.e., the ability to adjust price may not matter for the product variety choice.

To understand the intuition for this result, recall that the intuition for the optimal variety choice in the main model was that the profit function was increasing in a mean-preserving spread of consumer beliefs (a belief updating has to be mean-preserving to be rational) due to the linear demand and the ability of the firm to adjust price based on the consumer beliefs. Therefore, the firm was benefiting from the expert opinion being the most informative (the more information, the more consumers adjust their beliefs). In the current model variation, this effect still holds for a range of marginal costs, although for a slightly different reason: a mean-preserving spread of consumer beliefs is profit increasing if the cost is high enough to sometimes result in zero sales (given the optimal price), since sales cannot decrease below zero (which may happen in the case of a negative recommendation) but are strictly increasing in consumer beliefs in case of a positive recommendation (unless costs are so high that there are no sales under any condition). Thus, the firm still strictly benefits from the product variety that results in the best information in the marketplace under the following two conditions: 1) the cost is not too large, so that zero profit is not guaranteed, and 2) the cost is large enough to sometimes (given a negative recommendation) result in zero profit.

Note that if either of the above two conditions does not hold then product variety does not matter: if the first condition is not satisfied, profits are always zero; if the second one is not satisfied, the expected profit does not depend on the expert opinion. The first condition above can be easily interpreted for managerial guidance: it states that although a manager may be uncertain about the recommendation of the opinion leaders, she is expecting positive sales at least if they are good. The second condition may then be interpreted as saying that the higher the marginal cost, the more important it is to adjust the product variety. Furthermore, just as in the main model, it should be adjusted downward if the opinion leaders are more likely to like the product (i.e., if they are expected to have a lower acceptance threshold u_0) and upward otherwise.

2.5.5 Expert Opinion Depends on the Price

So far, we considered expert evaluation to be only driven by the product itself. Let us now consider the possibility that the expert opinion depends on the price so that the lower the price, the higher the chance that the expert is positive. Namely, suppose the expert opinion is positive if and only if her product valuation net of price exceeds a certain threshold, i.e., if $q - \min_j d(x_e, l_j) \cdot t - p > u'_0$. We use u'_0 here instead of u_0 to indicate that due to the difference in the evaluation rules, there is no reason to expect this parameter to be the same across the two specifications. Assume the rest of the model setup is as in the previous section, i.e., the firm faces marginal cost $c > 0$ and cannot change the price after the expert posts her product opinion.[10]

[10] In this setup, since the total demand from the opinion leaders is assumed to be negligible, allowing full flexibility in varying price to the expert and other consumers would amount to the firm having complete control over expert evaluations, and is therefore not realistic. If the expert knows that consumers in her audience would face a different price, she should at least try to factor the price she paid out of her evaluation.

Similar to the main model, the probability of a positive expert opinion can be derived to be

$$Prob(positive) = \begin{cases} 1, & \text{if } p < q_0 - u_0' - \frac{t}{2n} \\ \frac{t+2n(q_0-u_0'-p)}{2t}, & \text{if } q_0 - u_0' - \frac{t}{2n} \le p < \min\{1 - u_0' - \frac{t}{2n}, q_0 - u_0'\} \\ \frac{n(1+q_0-2u_0'-2p)}{t}, & \text{if } 1 - u_0' - \frac{t}{2n} \le p < q_0 - u_0' \\ \frac{1}{2}, & \text{if } q_0 - u_0' \le p < 1 - u_0' - \frac{t}{2n} \\ \frac{n(1-u_0'-p)}{t}, & \text{if } \max\{1 - u_0' - \frac{t}{2n}, q_0 - u_0'\} \le p < 1 - u_0' \\ 0, & \text{if } p > 1 - u_0' \end{cases} . \tag{2.24}$$

As expected, the probability of a positive expert opinion decreases in the price. As a result, the consumers' expected product quality conditional on the positive expert opinion is

$$\hat{q}_p = \begin{cases} \frac{1+q_0}{2}, & \text{if } p < q_0 - u_0' - \frac{t}{2n} \\ \frac{t+2nq_0(q_0-u_0'-p)}{t+2n(q_0-u_0'-p)}, & \text{if } q_0 - u_0' - \frac{t}{2n} \le p < \min\{1 - u_0' - \frac{t}{2n}, q_0 - u_0'\} \\ \frac{1+q_0^2-(u_0'+p)(1+q_0)}{1+q_0-2u_0'-2p}, & \text{if } 1 - u_0' - \frac{t}{2n} \le p < q_0 - u_0' \\ 1, & \text{if } p \ge q_0 - u_0' \end{cases} , \tag{2.25}$$

and conditional on the negative opinion, it is

$$\hat{q}_n = \begin{cases} q_0, & \text{if } p < 1 - u_0' - \frac{t}{2n} \\ \frac{t(1+q_0)-2n(1+q_0^2-(u_0'+p)(1+q_0))}{2(t-n(1+q_0-2u_0'-2p))}, & \text{if } 1 - u_0' - \frac{t}{2n} \le p < q_0 - u_0' \\ \frac{(1+q_0)t-2n(1-u_0'-p)}{2(t-n(1-u_0'-p))}, & \text{if } \max\{1 - u_0' - \frac{t}{2n}, q_0 - u_0'\} \le p < 1 - u_0' \\ \frac{1+q_0}{2}, & \text{if } p > 1 - u_0'. \end{cases} \tag{2.26}$$

Both of these expectations increase in p. This is because consumers observe the price and expect the expert to have formed the opinion using the same price. Therefore, a higher quality should be needed to compensate for a higher price to result in a positive recommendation. This means that a positive opinion implies a higher expected quality if the price is higher. Similarly, a negative opinion does not decrease the probability of the high quality as much in the case of a higher price as in the case of a lower price.

Substituting Equations (2.24), (2.25), and (2.26) in Equation (2.23), we obtain the firm's expected profit as a function of n and p, and then solve for the firm's optimal pricing and product variety decisions. While the detailed analysis is relegated to the appendix, let us discuss here some intuition.

If the firm decreases the price, the probability of a positive expert review increases, but the consumers' expectation of quality conditional on the expert opinion (both positive and negative) decreases. This is a trade-off similar to the one coming from increasing the product variety. In addition to the informational effect of price, a price distortion from the one optimal without the informational effect results in a decreased profit due to the price's direct effect on the consumer demand. Hence, the firm would choose not to rely on the price distortion too much. We therefore have the following proposition.

Proposition 2.8. *Even when the expert opinion depends on price, the optimal variety satisfies* $n^* = \frac{t}{2(1-u_0'-p^*)}$, *where* p^* *is the optimal price, and this variety is strictly optimal for some range of marginal costs.*

Note that $u_0' + p^*$ now serves the function of u_0 in the expression for the optimal product variety. This is intuitive, since $u_0' + p^*$ is the bar the product value (gross of price) now must clear to result in a positive expert opinion. Thus, the main result is conceptually robust to the possibility that the expert opinion depends on price.

From the discussion above Proposition 2.8 one could be tempted to infer that the firm would not want to use the price as an instrument of affecting the expert opinion at all because changing price is costly (due to its direct effect on the demand), while changing variety is assumed not to be costly. But this is not exactly true because the effect of a price change on the probabilities of the positive and negative expert opinions are not exactly the same as the effect of a change in variety. In other words, the expert opinion is maximally informative when the firm uses both the price and the variety as instruments. Note also that if consumers have an increased direct utility when the assortment is larger (as in the model variation of Section 2.5.3) or the firm had additional per-variety cost (as in the model variation of Section 2.5.2), the variety adjustment to affect expert opinion would have a detrimental effect on profits, and some price adjustment is definitely optimal. This is because in the neighborhood of the optimal point, the cost of adjustment is quadratically small, and therefore, it would be optimal to adjust both variety and price.[11]

Of course, the above analysis assumed that the firm cannot secretly promote to the opinion leader (i.e., in a way unobserved to the rest of consumers), which is a possibility we discuss in the following section.

2.5.6 Independent Word of Mouth vs. Sponsored Influencer Marketing

Along with the traditional word of mouth now magnified (albeit at a cost of being less personal) by the ease of communication online, recent years have witnessed an increased popularity of another phenomenon called sponsored influencer marketing. For example, Schafer (2018) states that as opposed to word of mouth marketing, which stands for creating and managing marketing environment as to facilitate consumer-to-consumer information dissemination about the products of a given company, influencer marketing increasingly refers to directly and actively promoting products to the influencers, i.e., people who are identified as having a large following, usually, based on some social media metrics.[12]

[11] Another reason for optimality of price adjustment is the integer constraint on the product variety, which would make setting the exact optimal (non-integer) variety impossible, and therefore, the informational effect would need to be fine-tuned through price.

[12] Technically, this is also not really a new phenomenon, but could be thought of as a variation of a referral program (Godes 2012).

In another dichotomy, WOMMA (Word of Mouth Marketing Association, a trade group founded in 2004 and acquired by Association of National Advertisers in 2018) defines word of mouth marketing as all-inclusive, but splits it into organic and amplified (see WOMMA 2005). The first occurs when a customer is happy with a product and spreads the word without an active intervention from the firm, while the second is due to an active effort by the firm to affect word of mouth and ranges from developing consumer-to-consumer communication platforms to explicitly reaching out to influencers and possibly offering monetary compensation for positive reviews (a.k.a. sponsoring). While the latter may have recently increased in popularity, according to FTC, a clear disclosure of the financial connection is legally required in the influencer's posts.

A decrease of sponsored influencers' impact on consumer beliefs due to the relationship disclosure may have not been very pronounced so far for two reasons. First, for a while, the practice of non-disclosure went under-the-radar of FTC (Shin 2006). In fact, before around 2010, FTC attention was directed toward firms' rather than influencers' disclosures. However, recently, FTC started to take steps to more actively enforce influencer disclosures. In 2017, FTC sent letters to a number of influencers stating that "any 'material connection' between an endorser and an advertiser ... should be clearly and conspicuously disclosed" in all influencers' posts (including tweets), and it further mentioned "monetary payment or the gift of a free product" as examples of a material connection (see FTC 2017). Furthermore, not disclosing relationship may end up detrimental to both the influencer and the firm in the long run. As WOMMA puts it, "Attempting to fake word of mouth is unethical and creates a backlash, damages the brand, and tarnishes the corporate reputation" (WOMMA 2005).

Second, since sponsored influencer posts are a relatively new phenomenon, many consumers perhaps have not yet learned to identify such short-hand disclosures as #sp ("sponsored post") or #partner. However, if the practice of sponsoring continues, consumers are likely to learn those hashtags. When consumers are able to discern sponsored influencer marketing from independent (non-sponsored) word of mouth, it stands to reason that a firm would benefit from managing its marketing mix to best affect the second (independent) type of word of mouth, regardless of whether it engages in the first kind.

Moreover, the usefulness of sponsored influencer posts is based on the consumer belief that these posts at least to some extent reflect independent and truthful opinions. This could be because the digital influencers' popularity highly relies on their trustworthiness in the specific areas they cover, and they may not want to lose relevance.[13] One should also remember that many of these influencers are not professional marketers and thus their motivation in posting reports may not be driven purely by the monetization of their influence over their followers. Therefore, in the long term, either influencers (on average) will have enough incentives to be (partially) truthful, perhaps due to other reviewers policing their posts, or due to firms' restraint coming from the concern that unduely high encouragement or payment could result in an influencer's viral post about unethical marketing tactics of that firm, or consumers will completely lose trust in opinion leaders they don't personally know and the word of mouth will revert to the old-fashioned one. Either way, the ideas in our paper about influencing opinion leaders' opinions with a product selection also available to other consumers may apply. For example, the main model may be thought of as capturing the effect of an influencer's opinion when the influencer (consistently with her followers' expectations) have received the product for free. For example even if an influencer never posts a negative opinion, if she receives free products from various firms, a remark about or the use of a product would be interpreted as a positive recommendation, and an absence of those forms of communication could be interpreted as a negative recommendation. At

[13]For example, Shi and Wojnicki (2014) point out that while extrinsic (monetary) incentives increase opinion leaders' referrals, the reason opinion leaders are willing to respond to monetary incentives is that they have developed a reputation for intrinsically (based on their own opinion) motivated referrals. The argument is that while intrinsically motivated (organic) referrals increase the social capital of an opinion leader, externally incentivized ones (compensated by the firms) decrease it. It then follows that compensation to incentivize opinion leaders' positive recommendations cannot replace organic recommendations, but rather the latter ones are essential for the former ones to have an effect.

the same time, although outside of the scope of the current paper, sponsored influencer marketing strategies is an interesting topic which warrants further research.

2.6 Discussion and Conclusion

Nowadays, with the development of information technology and the growth of social media, it becomes increasingly easy for consumers to both post a comment or a picture of a product viewable by others and to notice whether somebody else uses or praises a product or a brand, making word of mouth and the effect of opinion leader recommendations on consumer purchase decisions more important than ever. However, in spite of the positive relationship between product choices and product quality, it can sometimes be difficult to interpret why a person likes a product. This is because in addition to the product quality, the attitude toward the product is also influenced by the person's idiosyncratic preference. Although valuation of quality may be shared by all consumers, the idiosyncratic consumer preferences are not. Moreover, due to the lack of detailed clarification in the product recommendations (such as in many online posts), consumers may not be able to perfectly tease out the relevant information.

In this paper, we consider how a firm should adjust its product line when consumers use opinion leader recommendations in their purchase decisions. An intuitive solution is to increase product variety. With more alternatives available, it is more likely that the expert can find a variant that fits her personal preference, and post a positive product opinion. Since consumers expect quality to be higher when expert opinion is positive instead of negative, they are willing to pay a higher price. From this perspective, an increased assortment size can benefit the firm. However, apart from the sentiment of the expert's post, the firm also need to consider how consumers' inference from an expert opinion is affected by the number of variants. When many product variants are available, consumers may expect an expert to find a better fit, and this consideration would then decrease consumer expectations of product quality given either a positive or negative expert opinion. As a result, consumers' valuation may decrease, which possibly leads to a lower profit.

One of the predictions of our model is that when opinion leaders are more positively inclined (i.e., more likely to be happy with the product regardless of the exact quality they see), a firm should prefer to offer a smaller product selection. For example, the fact that Apple does not provide as many customization opportunities in iOS for iPhones as Android systems usually do could be seen as consistent with the above prediction, since many opinion leaders like Apple products better and therefore, their positive recommendation threshold (u_0) derived from the value of the outside option (Android for iOS, and v.v.) is lower for iPhone.

Although we considered a number of variations of the main model and relaxed several assumptions, there are several other simplifying assumptions worth mentioning. One of them is that we have considered consumers uniformly distributed on a circumference. This assumption implies that there are no "central" and "extreme" preferences. In practice, the consumer preference space or the distribution of preferences is likely to be such that some consumers are more representative of the "average" consumer than others. Those that are more representative will find it easier to become opinion leaders. In other words, opinion leaders are likely to be more representative of consumer preferences that a randomly chosen consumer. An implication of this is that to satisfy opinion leaders better, the firm may want to reposition more products towards more central preference locations and towards areas where the consumer preference distribution is more dense than it would without the opinion leader effects. At the same time, the conceptual results of the main model about the adjustments to the number of products is likely to be robust. This is because the effects of the number of products on the expected fit and therefore, also on the consumer inference remains (as far as the preference of all opinion leaders is not the same and predictable precisely).

Another simplifying assumption we made is that the expert has the same valuation of quality as uninformed consumers. If this is not so, the expert's judgements would be affected by her preference for quality. For example, if the expert values quality more, this would act as reducing the importance of fit in the expert's recommendations. In addition, if consumers do not know the expert's valuation of quality, this would add an extra uncertainty to the consumer inference, but there is no reason to expect that our main conceptual results would not hold.

We have also assumed that the expert always posts the product evaluation. In practice, opinion leaders could be silent on many products. An absence of a recommendation may be interpreted as a negative opinion (the expert did not find it worthwhile to choose the product) or could be decreasing the informativeness of the recommendations. One interesting possibility for future research is to consider how the uncertainties facing the expert before and after the purchase affect the likelihood of purchase and the likelihood of posting a review.

While the effect of product variety on an expert's satisfaction with the chosen product may be intuitive, one may question whether consumers are sophisticated enough to discount a positive recommendation more when it is based on a choice from a larger variety. To provide some empirical validation to this prediction, we have conducted a survey-based experiment using 80 subjects recruited through Amazon.com's mturk and paid $2 each. We used a randomized two by two factorial between-subjects design to test whether a recommendation of a brand would indeed have a weaker effect on buying propensity when the brand has a larger number of product versions. Specifically, we first showed the subjects a situation description in which we asked them to imagine that they are looking for a cashmere sweater as a Christmas gift for a friend, and they either have found an XYZ store's webpage through a search for cashmere sweaters (Condition 1: no recommendation), or have heard a co-worker opinion that she liked an XYZ's cashmere sweater and search for the store online (Condition 2: recommendation). Following the situation description, a webpage displayed

Table 2.1: Summary of the Experimental Results

(a) Average Purchase Likelihood

	No Recommendation	Recommendation
Small Variety	5.3684	7.8571
Large Variety	7.3333	7.5263

(b) Estimation Results

	Estimate	Standard Error
Intercept	5.3684	0.4912
Recommendation Dummy	2.4887	0.6779
Large Variety Dummy	1.9649	0.6779
Recommendation Dummy × Large Variety Dummy	−2.2957	0.9587

Note. All parameter estimates are significant at 2% level.

the XYZ's "exclusive cashmere sweaters" with a short description of the product line, the price ($89.99), and a choice (pictures) of sweaters. In Condition I (small variety) there were only two sweaters, while in Condition II (large variety) there were 27 sweaters. We have then asked subjects to rate their likelihood to buy on a scale of 0 to 10. Table 2.1 summarizes the results. As expected, the main effect of the recommendation on the purchase likelihood is positive. The main effect of larger variety turned out to be positive as well, and, in line with our prediction, the interaction effect of larger variety and the recommendation is negative (all three effects are significant at 2% level). Thus, we believe this experiment provides some face validity to our assertion that consumers would discount a positive recommendation of a brand when the product variety within the brand is large.

CHAPTER 3

SCALPERS: WHEN "HOW MANY" IS THE QUESTION

3.1 Introduction

In 2007, with the rapidly increasing number of students considering studying abroad, demand in China for the TOEFL test was huge. Due to the scarcity of supply, the test slots filled up several months in advance. Apart from the students who registered early at the risk of future time conflicts, the slots were also booked by scalpers. Unlike the "real" test takers, scalpers did not plan to take the test. However, because "transference" between registrants was possible,[1] scalpers booked multiple slots for reselling them at a premium. Indeed, many students who registered late had to turn to the resale market and ended up paying scalpers a considerable amount of money.[2] Given this profitable arbitrage possibility, the emergence of scalping is not unexpected.

Scalping has long been an issue in many markets. As long as the supply is less than the demand and opportunities for arbitrage exist, scalpers hinder consumers from purchasing in the primary market and force them to pay a significant premium in the resale market. As early as the 19th century, scalpers purchased Broadway show tickets in advance to sell outside of the theaters. Nowadays, though street scalping has become old-fashioned, scalpers are still active in scalping various products with limited supply: Super Bowl tickets, railway tickets, visa appointments,[3] as well as Apple iPhone X at its launch. Unlike the firm that

[1]ETS, the organization hosting the test, immediately released the slots canceled by registrants for (new) booking. Thus, the scalper could cancel his slot at a pre-agreed upon (weird) time of the day (e.g., late at night) and allow his client to get the slot. Here, a simple way to eliminate scalping is to release the canceled slots to the public at a certain pre-announced time of day.

[2]Similar cases occurred in other countries with high demand for the TOEFL test. See, for example, Su-Hyun Lee (2007) *High demand causes 'Toefl crisis' in South Korea* available at https://www.nytimes.com/2007/05/14/world/asia/14iht-english.1.5699917.html.

[3]See *New Target for Chinese Scalpers: German Visa Interview Appointments* available at https://blogs.wsj.com/chinarealtime/2012/04/24/new-target-for-chinese-scalpers-german-visa-interview-appointments/

sells in the primary market, scalpers face very limited restrictions on pricing and have little reputation concern. They exploit the consumers as much as possible in order to maximize profit. Therefore, consumers who fail to compete with scalpers for the limited capacity often end up paying exorbitant premiums.

Firms take different actions to limit scalping. For instance, Apple once required shoppers to present a picture ID matching the online reservation made in advance.[4] Thus, purchasing in bulk became much more difficult. As another example, Ticketmaster sometimes allows verified fans (identified through the platform's sales record as well as social media data) priority access to popular concert tickets.[5] This practice makes getting in front of the crowds harder for scalpers. In a more extreme case, the Glastonbury music festival has begun featuring the holder's photograph on each ticket, making tickets non-transferable.

However, despite the different methods available to reduce scalping, firms still allow it to some extent. For example, Apple allowed each Apple ID to order more than one iPhone Xs at its launch. In addition, some event organizers verify the tickets sold in the resale market (e.g., through Ticketmaster's Verified Ticket System) so that consumers do not need to worry about the authenticity of scalpers' tickets.

These seemingly contradictory practices are puzzling at first. If firms benefit from restricting scalping, why don't they impose stricter restrictions (if they are not costly)? To understand why, let us start with how firms with limited capacity can make a profit. Consumers often have valuation uncertainties when products go on sale. For example, football fans do not know which teams will play in the Super Bowl until after conference championships, and gamers do not know whether Nintendo Switch is a suitable gaming console for them until they have read enough reviews. However, out of fear of not getting the Super

[4]See Chenda Ngak (2012) *Apple fights calpers with iPhone 4S lottery system in Hong Kong* available at https://www.cbsnews.com/news/apple-fights-scalpers-with-iphone-4s-lottery-system-in-hong-kong/.

[5]See Anne Steele (2017) *Ticketmaster asks: are you a big enough fan?* available at https://www.wsj.com/articles/ticketmaster-asks-are-you-a-big-enough-fan-1504636200.

Bowl tickets or Nintendo Switch in time (at a good price) due to limited supply, consumers may rush to purchase the products when they first come out. (The literature refers to this phenomenon as "buying frenzies." See, e.g., DeGraba 1995.) After all, getting a product with some risk of disliking it may be better than not getting it at all. In fact, inducing scarcity is a marketing tool many businesses apply to encourage making purchase decisions before resolving valuation uncertainty and increase the demand. For example, according to Miguel Diaz Miranda, a vice president at Zara, the culture they are creating with their customers is "you better get it today because you might not find it tomorrow" (Fraiman et al. 2008). So what role do scalpers play here? In this case, the presence of scalpers makes the scarcity even worse. Though the units obtained by the scalpers will be put back on the market later, they are unlikely to be good deals: scalpers never hesitate to exploit consumers in order to make more money. Given this fact, consumers may be more eager to acquire the products in the first place, leading to higher firm profitability. However, the presence of too many scalpers can also create a problem. For one thing, the scalpers selling in the resale market are competing with the firm for customers and may decrease the firm's margin. For another, some consumers driven out of the primary market by scalpers may never make the purchase, resulting in a decreased total demand.

This paper focuses on the effect of scalping on a firm's profit to explain why and how an intermediate level of restrictions on scalping can be optimal for the firm. In turn, it answers the following questions: How do the restrictions on scalping (i.e., the maximum number of units scalpers can acquire)[6] influence the equilibrium market prices? What is the optimal level of restrictions? How are the firm's profit and the optimal level of restriction affected by the rationing rule, the scalpers' market power, and the firm's ability to vary the price? How does scalping affect the firm's optimal decision on capacity? This paper considers a

[6] In practice, sometimes the restrictions on scalping are imposed on the reselling stage rather than the purchasing stage. Here, knowing that the risk of being punished is nonnegligible once they resell too many units, scalpers will limit the number of units they purchase in the first place.

market with a monopoly firm selling a product with limited quantity in a two-period model. Though consumers may arrive early (in period 1) or late (in period 2), they cannot learn their valuation until the beginning of period 2. Thus, early consumers need to compare their payoffs of purchasing immediately with the risk of disliking the product and making the purchase decision later with certainty. In addition to the early consumers, scalpers may also buy in period 1. Therefore, in the first period, the firm sells to consumers and scalpers, and in the second period, the firm and the scalpers jointly sell to the consumers.

The main results are as follows. First, a firm's profitability may first increase and then decrease in the severity of restriction on scalping, and thus an intermediate level of it can be optimal for the firm, even if altering the level of restriction is costless. It results from two opposing effects of increased scalping on a firm's profit. On the positive side, increased scalping decreases consumes' expected utility of delaying purchase and increases their willingness to pay right away, because they expect scalpers to exploit them more than the firm does. On the negative side, increased scalping displaces the demand to the earlier period and increases the supply in the later period, decreasing the firm's ability to raise prices. This result implies that reducing scalping does not always benefit the firm, even if doing so is costless. In some sense, this result emphasizes the conflict of interest between the firm and the consumers. Moreover, the firm does have the incentive to limit scalping to some extent. Going back to the examples discussed before, firms often allow scalping by permitting purchasing in bulk even if the supply is limited, and they may even make scalping more appealing by authenticating the products in the resale market (e.g., Ticketmaster's Verified Ticket System). At the same time, they make take costly actions to limit scalping, such as verifying the real consumers and giving them priority access to the products (e.g., through Ticketmaster's Verified Fan Program). Though alternative explanations (which rely on multiple drivers) are possible for these firms' seemingly contradictory practices[7] – they

[7] For example, firms impose some restrictions on scalping because they care about consumer surplus or their own reputation, and do not eliminate scalping because doing so would be infeasible or too costly.

impose some but not the most stringent restrictions on scalping – this paper abstracts from the additional considerations and explains this phenomenon purely from the perspective of the firm's profitability.

Second, a firm can benefit from an intermediate level of capacity, even if the unit cost of production is zero. It results from two opposing effects of increasing the capacity on the firm's profit. On the positive side, it increases the maximum number of consumers the firm can serve, leading to a potentially higher profit given a fixed price. On the negative side, it decreases the risk of the product being sold out, and increases consumers' expected utility of delaying the purchase. Thus, a lower price is needed to persuade consumers to purchase immediately. In fact, sometimes a larger supply may result in a smaller demand. This result supports the popular practices in businesses whereby firms (e.g., Zara) induce scarcity to increase the demand. Moreover, the existence of scalpers can result in an increase in a firm's optimal capacity by attenuating the negative effect of enlarging the capacity, increasing the social welfare.

By analyzing several model variations, I show the robustness of the main results. Moreover, I find that a strict restriction on scalping is less important with a higher level of coordination among scalpers, because the coordinated scalpers may themselves limit the number of units acquired and resold. On the other hand, a strict restriction on scalping is more important when a firm has more flexibility in setting the prices, because the firm's ability to exploit the (informed) consumers can increase the consumers' willingness to pay before they have enough information even without the "help" of scalpers, and the firm would rather take a larger slice of the pie by restricting the scalpers in the market.

The rest of the paper is organized as follows. After discussing the related literature (section 3.2), I formally define (section 3.3) and analyze (section 3.4) the main model in which the firm sets a fixed price over the two periods and scalpers compete with each other. In the following section (section 3.5), I discuss several extensions and modifications of the main model, including the alternative rationing rule, the coordinated scalpers, and the possibility of the firm varying the price over time. The extensions show the robustness of the main result and provide some additional insights. Section 3.6 concludes.

3.2 Related Literature

This paper is related to the topics on reselling, current and future sales conflict, consumer uncertainty, and firm-induced scarcity. In the following, I briefly discuss the literature on each of these topics and then conclude with a review of several papers specifically about scalping.

Inasmuch as scalping is one type of reselling, this paper is within the broad literature on marketing channels (Jeuland and Shugan 1983, McGuire and Staelin 1983, Moorthy 1988, Raju and Zhang 2005, Dukes et al. 2006, Geylani et al. 2007, etc). Apart from selling to consumers directly, firms often sell their products through intermediaries, such as retailers, who help them reach consumers or provide services (Lal 1990). Though conflicts can arise between the direct and indirect channels (Chiang et al. 2003, Vinhas and Anderson 2005, Kumar and Ruan 2006), firms often have some power to dictate the authorized resellers' behavior, for example, explicitly through contracting. The firm's control of the reselling process is diminished when the resellers are unauthorized. For instance, when gray marketers procure products in a lower-priced geographic market and sell them in a higher-priced market, there is unintended competition among the authorized and unauthorized resellers. However, it has been shown that gray marketers can benefit the manufacturer through providing a lower-quality product version (Ahmadi and Yang 2000) or reducing the double-marginalization problem (Xiao et al. 2011).[8] In comparison to the literature, this paper shows that scalping

[8] In the cases discussed above, (re)sellers deal with authentic products. Note that the extant literature also studies the phenomenon of product piracy (Givon et al. 1995, Vernik et al. 2011) and counterfeiting (Qian 2014, Qian et al. 2014, Gao et al. 2016), where the inauthentic products compete with the authentic ones. Nevertheless, the illegal copies and counterfeits may actually increase the sales of authentic products through network effect or advertising effect.

(as a special type of reselling) can benefit the firm even if the resellers do not help to reach the consumers, increase the value, or expand the demand.

This paper is also related to other literature studying the conflicts between current the future sales. For example, the firm's sales in the later period can be diminished due to the increased sales now because the old (used) goods behave as (imperfect) substitutes for the new goods (Levinthal and Purohit 1989, Purohit and Staelin 1994, Chen et al. 2013). Some strategies for reducing this type of conflict are: using leasing instead of selling (Desai and Purohit 1998), buying back old items (Levinthal and Purohit 1989), and offering trade-in programs (Rao et al. 2009).

In the literature on exploiting consumer uncertainty, researchers show how firms can induce consumers to purchase before they determine their idiosyncratic valuation. For example, firms can make the immediate purchase more appealing than waiting by offering discounted prices in the advance period (Gale et al. 1993, Shugan and Xie 2000, Xie and Shugan 2001, Fay and Xie 2010) or creating rationing risk (Gale et al. 1993, DeGraba 1995, Liu and Van Ryzin 2008). The main rationale is that the firm can have larger sales (at higher prices) when there is smaller variation in the consumer valuation. Although this paper and the closest literature focus on exploiting consumer uncertainty, there is also research on the firms' incentive to provide information (e.g., Shin 2007, Bhardwaj et al. 2008, Guo and Zhao 2009, Kuksov and Lin 2010, Gu and Xie 2013, Branco et al. 2016).

Given this paper's emphasis on products with limited supply, it is related to the literature on firms' scarcity strategy. Apart from the purpose of creating rationing risk discussed above, firms may also intentionally limit supply to credibly commit to a high price (Denicolo and Garella 1999, Xie and Shugan 2001), signal high quality (Stock and Balachander 2005, Balachander et al. 2009), or create the perception of a status good (Balachander and Stock 2009, Amaldoss and Jain 2010).

Though the extant literature specifically on scalping is rather sparse in marketing, there are a few papers in economics and operations management studying how scalping influences firms' profits. Despite the arguments against scalping, they show that scalpers can benefit the firm through extracting all consumer surplus and transferring the profit to the firm (Karp and Perloff 2005), or sharing the risk of demand uncertainty (Su 2010). In this paper, I consider the effect of scalping that can influence consumers' decisions on whether to wait. Cui et al. (2014) also consider this possibility, but they only allow two extreme levels of scalping. In comparison with Cui et al. (2014), this paper shows that an intermediate level of scalping can actually benefit the firm the most, thereby reconciling the theoretical findings with the observed firms' practices.

3.3 Model Setup

The market consists of three groups of agents: a monopoly firm, consumers, and scalpers. The firm has a capacity of K units, and it sells the products over two time periods, $t = 1, 2$. Consumers can arrive in both periods. Without loss of generality, I assume a mass 1 of consumers arrives in period 1 (early consumers) and a mass 1 of consumers arrives in period 2 (late consumers). A consumer's valuation v is independent and identically distributed uniformly over $[0, 1]$. Though consumers may arrive in both periods, they cannot learn their valuations until the beginning of period 2. This assumption captures the phenomenon that some uncertainties are associated with the product (e.g., how much the new gaming console will increase a consumer's enjoyment of playing the video game, or whether a consumer's favorite team will make the playoffs) when it first comes out. The scalpers do not value the product for themselves, but they arrive in period 1 and buy the product with the hope of reselling it in period 2 at a higher price. Because the scalpers have more expertise in purchasing the products, I assume scalpers have higher priority in getting the products than consumers when capacity is inefficient.[9] Due to the firm's anti-scalping efforts (e.g., the

[9] This assumption is made for simplification. The main results will hold as long as scalpers can sometimes get in front of the consumers in the line.

banning of scalper bots, or the requirement for identity verification), the scalpers can obtain at most β units of the products. In the main model, I assume the firm cannot vary the prices across time (e.g., the firm commits to the price in pre-announced advertisements). Thus, the firm charges a fixed price p_f over the two periods. (I consider the firm's ability to price dynamically in section 3.5.3.) The scalpers, who are more flexible in setting prices, can charge prices other than p_f in the second period. For simplification, I assume in the main model that scalpers maximize their individual utility. (In practice, scalpers may know each other and interact with each other, which allows them to coordinate on prices. I consider this possibility in section 3.5.2.)

The firm sets p_f to maximize the expected total revenue in both periods. Each individual scalper decides whether to purchase one unit of product (if the upper limit β is not reached) in period 1 and what price to set in period 2, in order to maximize the expected profit. Each consumer decides whether, when, and where to purchase the product to maximize her expected payoff.[10] If supply falls short of demand, consumers with the highest willingness to pay will get the products first. That is, I assume the efficient-rationing rule. Later, I consider alternative rationing rules, for example, the proportional-rationing rule, in section 3.5.1. Agents do not discount future payoffs.

The timing is as follows. In period 1, the firm sets the fixed price p_f. The scalpers and early consumers arrive. Each scalper and consumer decides whether to purchase at p_f. However, some scalpers may not obtain the product if the upper limit β is reached, and some scalpers as well as some consumers may not obtain the product if the supply is short of demand. In period 2, the late consumers arrive. Both the early and late consumers learn about their valuations, and the scalpers set price p_s. Given the valuation and prices, each

[10]To simplify the discussions, I make the following implicit assumptions on consumer decisions in the event of a tie: First, an early consumer prefers to purchase in period 1 if $u_1 = u_2$. Second, in period 2, a consumer prefers to purchase from the firm (scalpers) rather than purchase nothing if $u_{2,f} = 0$ ($u_{2,s} = 0$), and he prefers to purchase from the scalpers rather than the firm if $u_{2,f} = u_{2,s}$.

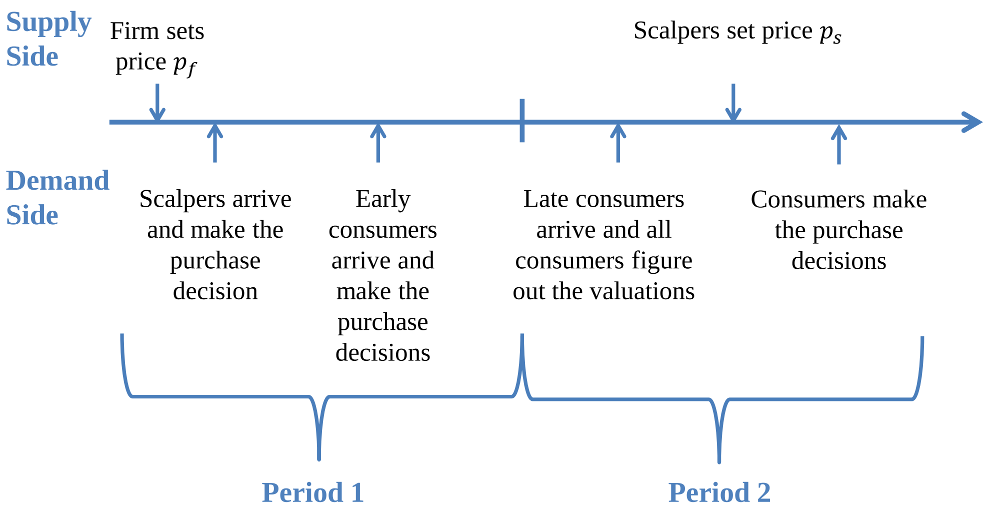

Figure 3.1: Sequence of Events

consumer who remains in the market chooses whether and where to purchase the product while supplies last. Figure 3.1 summarizes the sequence of events.

3.4 Analysis and Results

Because this paper aims to study how much scalping is optimal for the firm, I will first analyze the equilibrium firm's, scalpers', and consumers' decisions given an upper bound β of scalping. I solve for the Perfect Bayesian Equilibrium. I start with the decisions in period 2 and then analyze the decisions in period 1. Second, based on the equilibrium obtained in the first step, I study the effect of scalping on the firm's equilibrium profit and look for the optimal level of restrictions on scalping. Last, I discuss the firm's optimal capacity.

3.4.1 Decisions in Period 2

Denote the mass of early consumers purchasing in the first period by D_{1c}, where $D_{1c} \leq 1$. Thus, the mass of consumers in the second-period market is $M = (1 - D_{1c}) + 1 = 2 - D_{1c}$. A consumer with valuation v obtains $u_{2,f} = v - p_f$ if he purchases from the firm, $u_{2,s} = v - p_s$ if he purchases from the scalpers, and $u_{2,0} = 0$ if he does not purchase the product.

While supplies last, the consumer will make the decision corresponding to the highest utility. Because the supply is limited, some consumers may not be able to get the product even if $\max\{u_{2,f}, u_{2,s}\} \geq 0$. Given the efficient-rationing rule, consumers with the highest willingness to pay will be served first.

Denote the mass of scalpers purchasing in the first period by D_{1s}, where $D_{1s} \leq \beta$. Thus, the scalpers' and firm's capacity in period 2 are $Q_{2s} = D_{1s}$ and $Q_{2f} = K - D_{1s} - D_{1c} = M + K - 2 - Q_{2s}$, respectively. The competition among scalpers will drive the price in the resale market to a single price, which is denoted by p_s.[11] Here, if p_s is less than p_f, consumers with $v \geq p_s$ all choose to purchase from the scalpers first; if p_s is more than p_f, on the other hand, consumers with $v \geq p_s$ purchase from the scalpers only when the product is out of stock at the firm. Note that the former case (p_s is less than p_f) can never exist in equilibrium, because the scalpers get the products from the firm at price p_f, and they can only make positive profit by setting $p_s > p_f$. Moreover, under the efficient-rationing rule, only consumers with $v \leq 1 - \frac{Q_{2f}}{M}$ consider purchasing in the resale market. Thus, when scalpers' supply equals the demand, $Q_{2s} = M(1 - \frac{Q_{2f}}{M} - p_s)$. This equation leads to $p_s^* = \frac{2-K}{M} = \frac{2-K}{2-D_{1c}}$. Clearly, scalping can only be profitable if $K < 2$; that is, the firm's capacity is less than the number of consumers in the market.

3.4.2 Decisions in Period 1

Let us start with the scalpers' and early consumer' purchases decisions in period 1. Here, an individual scalper purchases one unit of product as long as $p_f \leq p_s^*$ and the upper bound β is not reached. As for the early consumers, they choose between purchasing immediately with an expected product valuation of $Ev = \frac{1}{2}$ and delaying the purchase until the exact

[11] In practice, the coordination among scalpers (or the market friction) and their information about consumers can lead to multiple prices in the resale market. I consider this possibility in section 3.5.3.

value of v is known. The expected utility of purchasing immediately is

$$u_1 = Ev - p_f = \frac{1}{2} - p_f. \tag{3.1}$$

On the other hand, if an early consumer expects that a total mass D_{1c} of early consumers will purchase immediately, her expected utility of waiting is

$$\begin{aligned} u_2 &= \int_{1-\frac{Q_{2f}}{M}}^{1} (v-p_f)^+ dv + \int_{0}^{1-\frac{Q_{2f}}{M}} (v-p_s^*)^+ dv \\ &= \int_{1-\frac{K-D_{1c}-D_{1s}}{2-D_{1c}}}^{1} (v-p_f) dv + \int_{\frac{2-K}{2-D_{1c}}}^{1-\frac{K-D_{1c}-D_{1s}}{2-D_{1c}}} (v - \frac{2-K}{2-D_{1c}}) dv \\ &= \frac{1}{2} - p_f - \frac{(2-K)^2 - 2(2-D_{1c})(2-K)p_f + 2(2-K-2p_f+D_{1c}\cdot p_f)D_{1s}}{2(2-D_{1c})^2}, \end{aligned} \tag{3.2}$$

where $p_s^* = \frac{2-K}{2-D_{1c}}$ is the expected scalpers' price in period 2. An early consumer compares u_1 and u_2 and may purchase immediately only if $u_1 \geq u_2$ (i.e., $p_f \leq \frac{(2-K)(2-K+2D_{1s})}{2(2-D_{1c})(2-K+D_{1s})}$). Note that $\frac{(2-K)(2-K+2D_{1s})}{2(2-D_{1c})(2-K+D_{1s})} \leq \frac{2-K}{2-D_{1c}}$. Therefore, as long as the scalpers expect some early consumers to purchase immediately, they acquire a total of $D_{1s}^* = \beta$ units.

In expectation of scalpers' and consumers' decisions, the firm chooses p_f to maximize its total payoff over the two periods. I relegate the detailed analysis to the Appendix and only discuss the intuition and key results in the main text. Note that early consumers' willingness to pay right away depends on the number of units acquired by the scalpers and the expected mass of early purchasers. Generally speaking, the more scalpers (i.e., D_{1s}) or (and) the larger expected mass of early purchasers (i.e., D_{1c}), the higher the price the firm can charge. Following the literature (e.g., Bagwell and Riordan 1991, Pesendorfer 1995), I assume the firm can coordinate the consumers (as a big player), and focus on the equilibrium corresponding to the highest firm's payoff. Note that the firm does not always find it optimal to sell in the first period. That is, when a very low p_f is required to persuade early consumers to purchase immediately, the firm would rather set a higher price and only serve consumers in the second period.

By comparing the different cases, I obtain the firm's optimal pricing decision in period 1, as well as scalpers' and consumers' response in equilibrium. Specifically, if

$$\begin{cases} 1 < K \leq \frac{1+\sqrt{5}}{2} \\ \frac{(2-K)(K-1)^2}{-1+4K-2K^2} \leq \beta \leq (2-K)(K-1+\sqrt{K(K-1)}) \end{cases},$$

the firm's price is $p_f^* = \min\{\frac{(2-K)(2-K+2\beta)}{2(2-K+\beta)}, \frac{(2-K)(2-K+2\beta)}{2(2-K+\beta)^2}\}$. The scalpers purchase a total of β units of products in period 1 and sell them at price $p_s^* = \min\{2-K, \frac{2-K}{2-K+\beta}\}$ in period 2. Early consumers purchase immediately as long as the firm's supply lasts, and those who cannot obtain the products in period 1 due to the limited supply, together with the late consumers, purchase in period 2 as long as their valuations are high enough. Otherwise, the firm does not find selling in the advance period profitable and sets the price $p_f^* = \max\{\frac{2-K}{2}, \frac{1}{2}\}$. Neither the scalpers nor early consumers purchase in period 1. A consumer purchases in period 2 as long as her valuation is high enough.

3.4.3 Optimal Restriction on Scalping

In this section, I examine the effect of scalping on the firm's decisions and profit, and discuss the optimal level of scalping.

Based on the equilibrium firm's decisions derived above, we can observe that the firm finds selling in the advance period more profitable (and is willing to set a lower price to ensure immediate purchase) when the restriction on scalping is intermediate. This result is summarized in the following proposition.

Proposition 3.1. *Restricted scalping increases the firm's incentive for selling in the advance period.*

To see why restricted scalping (i.e., an intermediate value of β) can increase the firm's incentive to sell in advance, let us take a closer look at the effect of scalping on early consumers' purchase decision. Note that scalping decreases consumers' expected likelihood of getting the product from the firm (that charges a lower price) and increases the likelihood of being exploited by the "evil" scalpers. As a result, early consumers will be more willing to purchase immediately from the firm, making selling in advance profitable. However, when scalping is too severe and scalpers obtain too many units because of their expertise in obtaining the products (e.g., efficient bots or knowledge about when to start standing in line), the scalpers' increased supply in the second-period market will lead to lower second-period prices and consumers' higher expected utility from purchasing in period 2. In this case, persuading early consumers to purchase immediately becomes costly, and the firm does not find selling in period 1 to be profitable. Note that although restricted scalping can increase the expected profit from advanced selling, whether scalping can indeed result in the firm selling in advance also depends on the firm's capacity (i.e., K). That is, when the firm's capacity is very limited, the firm would rather sell to "informed" consumers (in the spot period) who are willing to pay a higher price, whereas when the firm's capacity is very large, consumers do not expect a shortage, and selling in advance (by lowering the price) is not profitable.

Recall that this paper's primary interest is the effect of scalping on the firm's profitability. This result is described in Proposition 3.2.

Proposition 3.2. *The firm's equilibrium profit first (weakly) increases and then (weakly) decreases in β, the upper bound of units acquired by scalpers. Moreover, an intermediate value of β is strictly optimal for the firm when the firm's capacity K is intermediate.*

According to Proposition 3.2, a non-monotonic relationship exists between β, the restriction on the maximal units scalpers can acquire, and $\Pi_f(K, \beta)$, the firm's expected total payoff. I illustrate the relationship in Figure 3.2.

To understand this non-monotonic relationship between β and $\Pi_f(K, \beta)$, let us look at equation (3.2). Given a fixed p_f, increasing β can influence u_2 in two ways. On the one hand,

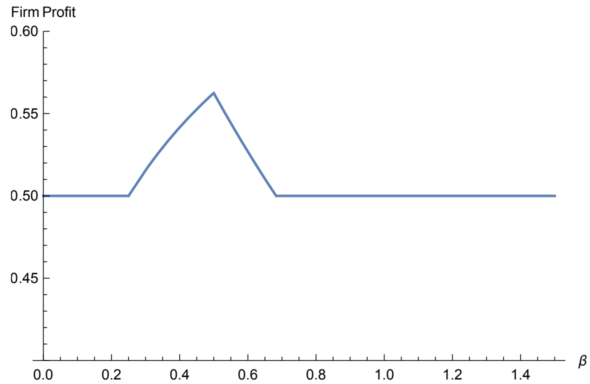

Figure 3.2: The Effect of β on Firm's Profit

Notes. $K = \frac{3}{2}$.

when β is small, a larger β corresponds to a larger scalpers' supply and a smaller firm's supply in period 2. Thus, an increase in β decreases consumers' chance of getting the product at the low price in the primary market. On the other hand, when β is large, further increasing β will increase the available units for (re)sale in period 2. Intuitively, when the supply is larger, scalpers set a lower price ($p_s^* = \frac{2-K}{2-D_{1c}}$ is increasing in D_{1c}, and D_{1c} is decreasing in β when a larger β corresponds to a severer product shortage for early purchasers). Recall that when the early consumers' expected utility of waiting is smaller (larger), they are willing to pay a higher (lower) price p_f right away, which positively (negatively) influences the firm's profitability. Thus, allowing scalpers to acquire more products increases the firm's profit when β is small and decreases it when β is large.

Given that the firm's profit can decrease in β, firms have the incentive to limit scalping, even without taking into account the negative effect of consumer complaints on the firm's reputation and profitability. This result supports the firm's practice of limiting scalping, even if doing so is costly (e.g., Ticketmaster takes costly action to verify the "real fans" of musicians to give them priority access to concert tickets). But given that the firm's profit can increase in β, the firm may sometimes prefer to have more scalping in the market, even absent the consideration of regulation cost (to reduce or eliminate scalping). This preference explains why firms sometimes make scalping easier (e.g., allow purchasing in bulk, verify the authenticity of products in the resale market), even though the opposite practices should not be too costly. To sum up, Proposition 3.2 provides a simple explanation for the firms' seemingly contradictory practices: they place (weak) restrictions on scalping, but do not impose strict ones even if they are not costly. Though other considerations (e.g., consumer surplus, reputation, cost of imposing restrictions) can play a role, the objective of maximizing profit is enough to obtain the optimality of an intermediate level of restriction.

Apart from the effect of scalping on the firm's payoff, we can also examine its effect on scalpers' profit. Not surprisingly, both the scalpers' individual and total profits can increase in β when the firm switches from only selling in the spot period to advanced selling, and vice versa. Moreover, the individual profit decreases in β conditional on the firm selling in advance, which results from the competition among scalpers. However, the effect of β on scalpers' total profit is a bit trickier. As Figure 3.3 shows, conditional on the firm deciding to sell in advance, scalpers as a whole can first benefit from and then be hurt by a weaker constraint on scalping. This result comes from two opposing effects of increased β on scalpers' total profit. On the positive side, a larger β increases the total number of units the scalpers can purchase and resell. On the negative side, the competition among scalpers decreases the expected return on each unit of product.

3.4.4 Optimal Choice of Capacity

In some markets, the firm's capacity is fixed, which is the case I have so far considered. For example, the number of available seats in a venue is given when musicians decide to hold concerts there. However, in other cases, the firm can choose its capacity to some extent. For example, even though a musician cannot alter the number of available seats in a given venue, he can decide on the venue in which to hold the concert. Here, one may be interested in the effect of capacity on the firm's profit. Note that the decision regarding capacity can either

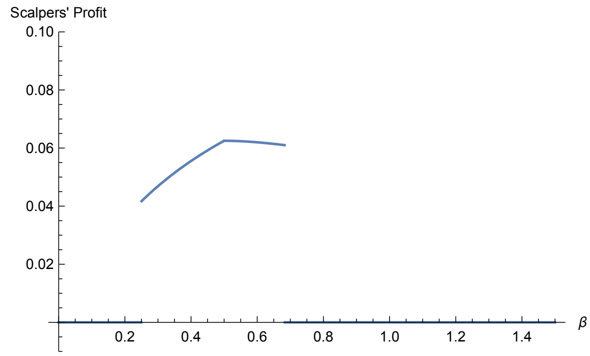

Figure 3.3: The Effect of β on Scalpers' Total Profit

Notes. $K = \frac{3}{2}$.

be made given an exogenous level of restriction on scalping, or be made in anticipation of an optimal level of scalping. I will discuss both cases as follows.

In the former case discussed above, government regulation, as opposed to the firm, may determine the level of scalping in the market. For example, the State of New Jersey requires ticket resellers to be registered and prohibits using "diggers", who are temporarily hired, to secure tickets.[12] Here, the decision on the firm's capacity K is based on an exogenous level of scalping. As Figure 3.4 show, the firm's expected profit may first increase and then decrease in K.

Note that even when the level of scalping can be determined by the firm and is set at the level that yields the highest profit for the firm (i.e., the latter cases discussed at the beginning of this subsection), this non-monotonic effect of K on the firm's profit remains. Figure 3.5 illustrates this effect. As we can see, even if β is endogenous, the firm's expected profit may be maximized at an intermediate K.

I summarize the optimal choice of K in the following proposition.

[12]See *Consumer Fraud Act (N.J.S.A.)* 56:8-26 to 56:8-38. See other laws on scalping, e.g., at https://seatgeek.com/tba/articles/ticket-resale-laws/.

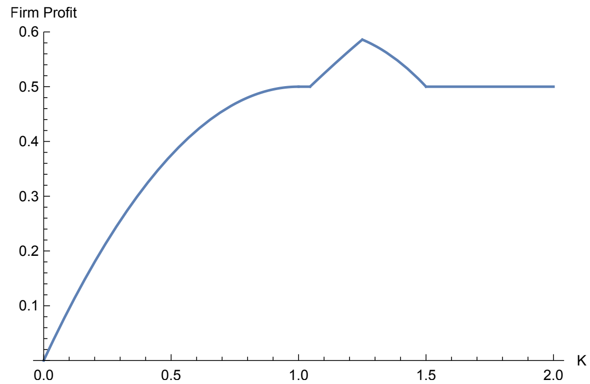

Figure 3.4: The Effect of K on Firm's Profit (Given Exogenous Level of Scalping)
Notes. $\beta = \frac{1}{4}$.

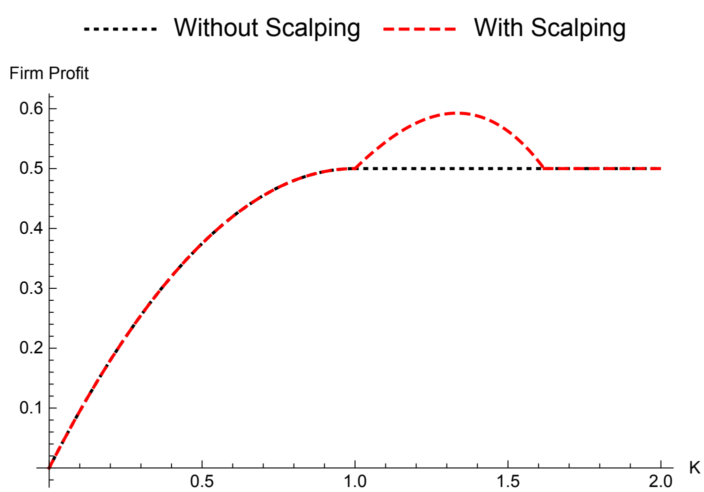

Figure 3.5: The Effect of K on Firm's Profit (Without Scalping or With the Optimal Level of Scalping)
Notes. In the case with scalping, the restriction on scalping (i.e., β) is chosen at the optimal level.

Proposition 3.3. *An intermediate capacity is optimal for the firm, even when the unit cost of production is zero, regardless of whether the restriction on scalping is exogenous. Moreover, the optimal capacity is larger when scalping is allowed.*

According to the above proposition, even if increasing the capacity is costless, the firm may find limiting its capacity optimal. Note that increasing the firm's capacity (i.e., K) has two opposing effects on the firm's profit. On the positive side, it increases the maximum number of consumers the firm can serve, leading to a potentially higher profit given a fixed price. On the negative side, increasing K alleviates the "scarcity" of the product, making consumers less eager to purchase the product right away and leading to a lower equilibrium price. Recall that consumers may be uncertain about their product valuation at the time products go on sale, and the variation in their expected valuation will increase if they choose to wait. Here, the risk of the product selling out will push some consumers to make the purchase immediately even though their true valuations may turn out to be low. After all, getting the product with some risk of disliking it may be better than not getting it at all. Moreover, their willingness to pay right away is increasing in the "scarcity" of the product. Due to the opposing effects discussed above, an intermediate value of K is optimal even if increasing K is costless, regardless of whether the restriction on scalping is exogenous.

Note that compared with the no-scalping case, when scalping is allowed, the positive effect discussed above will remain, but the negative effect will be attenuated. The reason is that though the units acquired by the scalpers will be put back in the market, they are likely to be pricier than the products sold in the primary market. Thus, even under the assumption of the efficient-rationing rule as in the main model, though the total number of units sold in the later period remains the same, consumers' expected utility is lower. In practice, we can expect some lower-valuation consumers to be served by the firm first (e.g., low-income people may have both low willingness to pay and low opportunity cost to stand in the line). In this more realistic case, the products become scarcer for high-valuation consumers in the presence of scalping. To sum up, when scalping is allowed, due to the attenuated negative effect of increasing K, the firm's optimal capacity will be larger than the one without scalping. This result is also illustrated in Figure 3.5.

3.5 Model Variations

In the main model, I made some assumptions for parsimony. In this section, I consider several model variations and extensions to see how the results are robust and obtain some other insights.

In the following subsection (section 3.5.1), I extend the main model by considering alternative rationing rules. I take the proportional-rationing rule as an example for detailed discussion, but the similar analysis and results can be applied to other rationing rules. In section 3.5.2, I allow scalpers to coordinate with each other and maximize the joint payoff. This variation and the main model show that the optimality of an intermediate level of scalping can hold under the two extremes of the scalpers' coordination/competition level: either perfect coordination or perfect competition. Given this result, we can expect the main implication of this paper to hold under the more realistic case, where scalpers have an intermediate level of coordination and competition. In addition to confirming the robustness of the main result, the analysis of this model variation shows that the coordination between scalpers can alleviate the negative effect of weakening the restrictions on scalping. In section 3.5.3, I allow the firm to have some flexibility in setting the price. That is, the firm can vary the price over time. Though this increased flexibility can diminish the positive effect of scalping on firm's profit, an intermediate level of scalping can still be strictly optimal as long as the scalpers can adjust the price more than the firm.

3.5.1 Alternative Rationing Rules

In the main model, I assume consumers with the highest willingness to pay are served first. To illustrate that this assumption is made purely for simplification, I show in this section that the main result is robust to alternative rationing rules, for example, the proportional-rationing rule. Here, when the supply falls short of demand, consumers have an equal probability of being served regardless of their valuation. All other assumptions are consistent with the main model.

In period 2, consumers with $v \geq p_f$ would like to purchase from the firm. Though the mass of consumers desiring to purchase from the firm is $M(1-p_f)$, only mass Q_{2f} of them can be served by the firm, and the remaining mass $M(1-p_f) - Q_{2f}$ need to choose between purchasing from the scalpers or leaving the market. Again, with perfect competition among the scalpers, p_s will be the market-clearing price. Under the proportional-rationing rule, it is equivalent to $Q_{2s} = \frac{M(1-p_f)-Q_{2f}}{1-p_f}(1-p_s)$. Thus, $p_s^* = \frac{2-K+Q_{2s}p_f-Mp_f}{2-K+Q_{2s}-Mp_f} = \frac{2-K+(D_{1c}+D_{1s}-2)p_f}{2-K+D_{1s}-(2-D_{1c})p_f}$.

In period 1, each individual scalper purchases one unit of product as long as the expected margin is positive and the upper bound β is not reached. The early consumers decides whether to purchase immediately by comparing the expected utility of purchasing right away, $u_1 = \frac{1}{2} - p_f$, and that of waiting, $u_2 = \int_{p_f}^{1} \frac{Q_{2f}}{M(1-p_f)}(v-p_f)dv + \int_{p_s^*}^{1} \frac{M(1-p_f)-Q_{2f}}{M(1-p_f)}(v-p_s^*)dv$. In anticipation of scalpers' and consumers' responses, the firm chooses p_f to maximize its total payoff over the two periods.

The detailed analysis is relegated to the Appendix. Note that the higher probability of lower-valuation consumers being served first increases the firm's expected profit. The reason is that when some consumers with intermediate valuation are served by the firm, the high-valuation consumers are more likely to be exploited by the scalpers, leading to a lower expected utility of waiting.

Again, an intermediate level of restriction on scalping can be strictly optimal for the firm. Moreover, given the optimal level of restriction on scalping, an intermediate value of capacity can be strictly optimal for the firm. These findings show that the main results are robust to the alternative rationing rules.

Although I only discuss the proportional-rationing rule in detail here, the analysis and results can be easily applied to other rationing rules (e.g., consumers with the lowest willingness to pay are served first).

3.5.2 When Scalpers Coordinate on the Price

In the main model, I assume scalpers compete with each other and maximize individual payoff. Thus, when the restriction on scalping is weak (i.e., β is large), the intense competition drives down the price in the resale market, leading to consumers' lower willingness to pay in period 1. In practice, we observe that scalpers often know and interact with each other, which makes cooperative pricing possible. That is, scalpers can sustain a high-enough price in the resale market even if the supply is large. To show the assumption of competing scalpers is made for simplification and does not drive the result, I show in this section that the main results continue to hold when scalpers coordinate with each other to maximize the joint payoff.

Again, denote the scalpers' and consumers' demand in period 1 by D_{1s} and D_{1c}, respectively. Thus, in period 2, the mass of consumers is $M = 2 - D_{1c}$, the scalpers' capacity is $Q_{2s} = D_{1s}$, and firm's capacity is $Q_{2f} = K - D_{1s} - D_{1c} = M + K - 2 - Q_{2s}$. Consumers will purchase from the firm first because scalpers charge a higher price in equilibrium, and only consumers with $v \leq 1 - \frac{Q_{2f}}{M}$ consider purchasing in the resale market under the efficient-rationing rule. Thus, the scalpers' expected profit in period 2 is

$$\begin{aligned} \pi_{2s} &= p_s \times \min\{Q_{2s}, M(1-p_s) - Q_{2f}\} \\ &= p_s \times \min\{Q_{2s}, -K + 2 + Q_{2s} - M \cdot p_s\}. \end{aligned} \tag{3.3}$$

To maximize the expected profit, the scalpers set

$$p_s^* = \max\{\frac{2-K}{M}, \frac{2-K+Q_{2s}}{2M}\} = \max\{\frac{2-K}{2-D_{1c}}, \frac{2-K+D_{1s}}{2(2-D_{1c})}\}.$$

Note that, the former case (i.e., $p_s^* = \frac{2-K}{M}$) applies when the scalpers' capacity Q_{2s} is small, and the price is such that the demand (i.e., $M(1-p_s) - Q_{2f}$) is equal to the supply (i.e., Q_{2s}). By contrast, the latter case (i.e., $p_s^* = \frac{2-K+Q_{2s}}{2M}$) applies when Q_{2s} is large. Here, the scalpers who coordinate with each other to maximize joint profit intentionally leave some units unsold to ensure a high-enough margin.

In period 1, scalpers choose how many units ($D_{1s} \leq \beta$) to purchase to maximize the expected total payoff over the two periods. That is, they solve for the problem:

$$\max_{D_{1s} \leq \beta} \Pi_s(D_{1s}) = \begin{cases} \frac{2-K}{2-D_{1c}} \times D_{1s} - p_f \times D_{1s}, & \text{if } D_{1s} \leq 2-K \\ \frac{(2-K+D_{1s})^2}{4(2-D_{1c})} - p_f \times D_{1s}, & \text{if } D_{1s} > 2-K \end{cases}. \tag{3.4}$$

As we can see, the scalpers' expected payoff is non-monotonic in D_{1s}. For example, when D_{1c} is such that the firm's expected profit is maximized, that is, $D_{1c} = \min\{1, K - D_{1s}\}$, the scalpers' expected profit first increases and then decreases in D_{1s} as long as $p_f \geq \frac{1}{4}$ (which always holds in equilibrium). Note that, increasing D_{1s} can positively influence the scalpers' profit in two ways. For one thing, it allows them to (re)sell more products. For another, when D_{1s} is intermediate, it reduces Q_{2f}, the available units in the primary market, which leads to a higher price in the resale market. However, increasing D_{1s} can also negatively influence the scalpers' profit in two ways. First, when D_{1s} is large, the scalpers sometimes will intentionally keep some units unsold to ensure the high margin, where the benefit of an extra unit of inventory is diminished. Second, when D_{1s} is large, further increasing it will lead to a higher supply and a lower price in period 2, hurting scalpers' profit.

Similar to the main model, an early consumer chooses whether to purchase immediately or wait until period 2 by comparing u_1 and u_2. The firm maximizes its total profit in anticipation of scalpers' and consumers' decisions. In case of multiple equilibria, I focus on the one corresponding to the firm's highest profitability. The details of the analysis are relegated to the Appendix.

Proposition 3.4. *In equilibrium, when scalpers coordinate with each other and maximize joint payoff, they may intentionally limit the number of units acquired and (or) intentionally limit the number of units (re)sold in the market. The firm's equilibrium profit first (weakly) increases and then (weakly) decreases in the level of scalping, and can be strictly maximized at an intermediate level of scalping.*

Moreover, the coordination among scalpers leads to (weakly) higher firm profitability.

Different from the main model, the scalpers may not aggressively acquire the products in a large quantity even with very large β (i.e., very weak restriction on scalping) when they coordinate with each other. That is, given the equilibrium firm's price, the scalpers find it optimal to purchase fewer than β units in order to maximize the joint payoff. The reason is that, in some cases, acquiring an additional unit of product will lower the expected price they can charge in the resale market. Moreover, the scalpers may intentionally keep some of the units unsold to ensure a high margin. Here, even though the scalpers do not expect to sell the entire capacity, they still choose to acquire the (larger) inventory so as to decrease the available supply in the primary market.

According to Proposition 3.4, an intermediate level of restriction on scalping can be strictly optimal for the firm even when scalpers coordinate with each other.[13] Note that, in the two models I have discussed so far, I consider the two extremes of scalpers' competition level: they either have perfect competition or perfect coordination. The realistic case is likely to be somewhere between these two extremes. Because the main result is shown to hold under the two extremes, we can expect it to carry through with an intermediate level of competition and coordination.

According to the first part of Proposition 3.4, we already know the coordination among scalpers does not qualitatively change the effect of restrictions with regards to scalping on

[13]Here, following the main model, I assume the scalpers cannot sell in period 2. This assumption is not crucial: even if the coordinating scalpers can sell in both periods, an intermediate level of restriction on scalping can still be strictly optimal for the firm.

firm profit. Here, a natural question concerns whether the level of competition/coordination can quantitatively influence the firm's profit. The answer is yes. The intuition is that the less intense the competition among scalpers, the higher the price in the resale market and the lower the consumers' expected utility of waiting. This effect leads to the consumers' higher willingness to pay before resolving the valuation uncertainty, and the firm's higher profit. This result is summarized in the second part of Proposition 3.4.

Following the main model, we can also discuss the effect of the firm's capacity on its expected profit. Again, increasing K can increase the firm's profit by increasing the maximum number of consumers it can serve. On the other hand, it can decrease the firm's profit by alleviating the "scarcity" of the product. Due to these two opposing effects, an intermediate value of K can be strictly optimal for the firm. Note that when scalpers coordinate with each other to maintain the margin, they may intentionally leave some units unsold, decreasing the total number of units available to the consumers. This approach mitigates the negative effect of increasing K, resulting in a larger optimal capacity. Indeed, when K is chosen at the optimal level, in equilibrium, scalpers acquire a large inventory in period 1 (to limit the firm's supply in period 2) but only sell part of the inventory in period 2 (to keep the price high).

3.5.3 When the Firm is Able to Price Dynamically and Scalpers Have Market Power

In the previous models, I assumed the firm cannot vary the price over time. This assumption is consistent with firms' practices in many cases. For example, Apple sets a fixed price for its new model during the first year, and Super Bowl organizers pre-announce that year's ticket prices and do not vary them as the event approaches. Given that the firm cannot adjust the price, it cannot take advantage of the increased variation in consumer valuation and charge a higher price. One may ask what if the firm is able to set different prices for uninformed (early) and informed (late) consumers. This section shows how the main result holds when the firm is able to price dynamically.

Recall that the main characteristic I focus on regarding scalpers in this paper is their higher flexibility in setting prices. To capture this characteristic, in this section with the firm's dynamic pricing possibility, I allow scalpers to have some market power. First, they can coordinate with each other. Here, to simplify the analysis, I assume perfect coordination (to maximize the joint payoff). Second, scalpers have some information about consumers' valuation and can charge different prices accordingly. Here, for simplicity of analysis, I assume scalpers can identify whether a consumer's valuation v is larger or lower than $1-\delta$, where δ is assumed to be small (i.e., $\delta \leq \frac{1}{3}$).[14]

Different from the firm, which charges a single price (i.e, p_{f1} in period 1 and p_{f2} in period 2) for all consumers purchasing in the same period (either due to regulations or not being able to distinguish between different types of consumers), the scalpers can set different prices for different consumers. That is, by identifying whether a consumer is a high type (by the excitement on her face or the fan t-shirt she wears) or not, scalpers can charge price p_{sh} or p_{sl} accordingly.

Similar to the main model, the firm sets prices to maximize the expected total revenue in both periods. Scalpers decide how many units (up to β) to purchase in period 1 and what prices to set in period 2 to maximize the scalpers' joint payoff. To simplify the analysis, I assume the firm's second-period price p_{f2} is set before the scalpers' prices. Each consumer chooses whether, when, and where to make the purchase to maximize her expected utility. When the supply is less than the demand, consumers with the highest willingness to pay are

[14]If we assume scalpers have perfect information about consumers' valuation and can achieve first-degree price discrimination, and if the firm can extract all scalpers' profit, as in Karp and Perloff (2005), the firm may prefer to have an infinite number of scalpers. However, perfect price discrimination is rarely (if ever) achieved in reality. This paper studies the case in which scalpers can only achieve imperfect price discrimination. Note that this information about the consumers does not change the analysis and results of the main model. That is, even with some information about the consumer type, the (perfect) competition among scalpers will drive the price down to the level as if no consumer information is available.

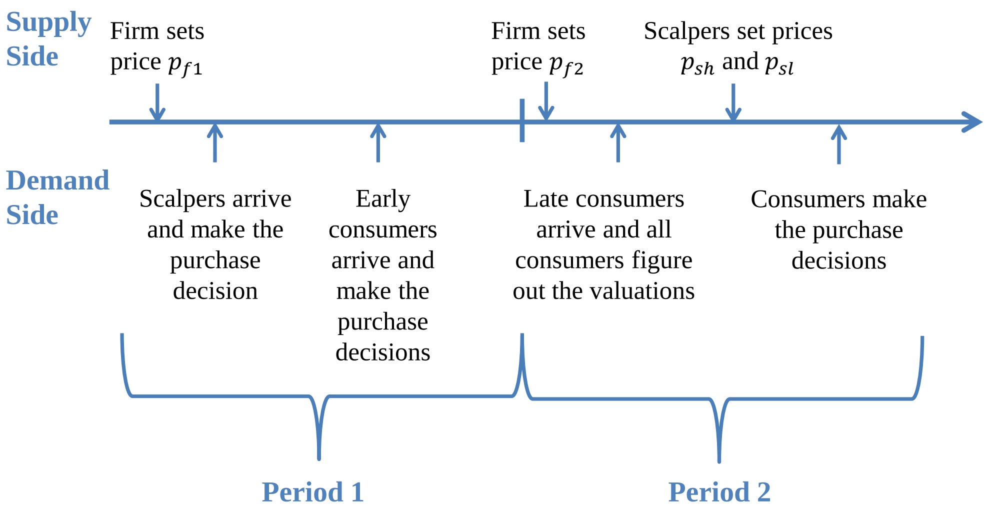

Figure 3.6: Sequence of Events

served first. The timing is summarized in Figure 3.6. I relegate the detailed analysis in the Appendix and only discuss the intuition in the main text.

In period 2, a consumer's utility from purchasing from the firm is $u_{2,f} = v - p$, whereas her utility from purchasing from the scalpers is

$$u_{2,s} = \begin{cases} v - p_{sh}, & \text{if } v \geq 1 - \delta \\ v - p_{sl}, & \text{if } v < 1 - \delta \end{cases} .$$

Given the consumers' purchase decisions and the firm's second-period price p_{f2}, the scalpers choose p_{sh} and p_{sl} to maximize expected profit. Note that scalpers do not always find it optimal to undercut the firm's price p_{f2}. Here, the scalpers will only set $p_{sh} \leq p_{f2}$ if p_{f2} is large. When p_{f2} is small, on the other hand, scalpers rather set $p_{sh} = \max\{1-\delta, 1-\frac{Q_{2f}+Q_{2s}}{M}\}$, serving the high-type consumers unable to be served by the firm due to the limited capacity at a high price. In anticipation of scalpers' pricing decision, the firm determines p_{f2} to maximize the expected payoff in period 2.

Some interesting features of the equilibrium in this sub-game are worth mentioning. First, the firm may not sell its entire capacity in period 2. Recall that I have shown in the previous models that scalpers may not always sell all the units they have. Here, when the firm can vary the price over time, it may not sell its entire capacity either. In fact, the firm does not sell the entire capacity when Q_{2f} is large and Q_{2s} is intermediate. Intuitively, when the firm has a large capacity at the beginning of second period, it has a high incentive to limit the quantity sold in order to keep a high margin. However, this incentive will be diminished if scalpers' capacity is low (and thus potential demand for the firm is high) or high (and thus most consumers can be served by the scalpers with lower prices). Second, despite the competition between the firm and scalpers, the equilibrium prices may be $p_{sh} > p_{f2} > p_{sl}$. That is, the scalpers may set a high price for the high-type consumers, knowing some of them cannot be served by the firm due to the firm's limited capacity. Also, the firm may set a price higher than the expected value of p_{sl}, knowing that although low-type consumers prefer to purchase from the scalpers, the high type will be attracted to visit the firm first. Note that even if only high-type consumers end up buying from the firm, the firm will not increase the price to $1-\delta$, because it needs to scare the scalpers away from undercutting its price by having a mediocre margin.

The decisions in period 1 are similar to those discussed before. The scalpers choose how many units (up to β) to purchase to maximize the expected total payoff. The early consumers choose whether to purchase immediately or wait. The firm sets the first-period price p_{f1} in anticipation of scalpers' and early consumers' responses to maximize its expected profit.

Again, the level of scalping has a non-monotonic effect on the firm's expected profit. As Figure 3.7 shows, the firm's profitability can first decrease, then increase, and then decrease in β, the upper bound of the number of units scalpers can acquire. Similar to the fixed-pricing case, increasing β can positively influence the firm's profit Π_f by making early consumers more eager to purchase immediately, and it can negatively influence Π_f by increasing the supply in the later market. In addition, another negative effect can take place when the

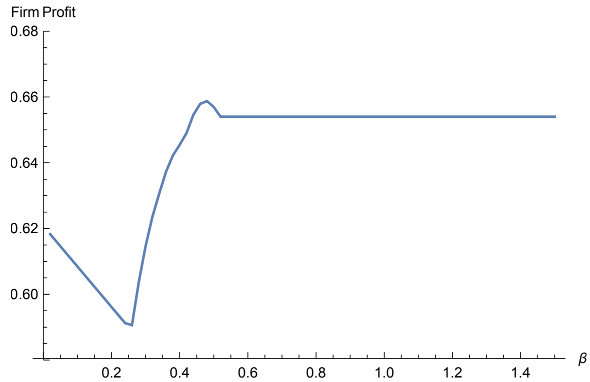

Figure 3.7: The Effect of β on Π_f with Firm's Dynamic Pricing Ability and Scalpers' Market Power

Notes. $K = \frac{3}{2}$ and $\delta = \frac{1}{4}$.

firm is allowed to vary the price. Note that when the firm sets different prices in different periods, it cares when the purchases occur. In this case, scalpers' purchases in period 1 at the discounted price (p_{f1}) decrease the firm's maximal number of units to be sold at the higher price (p_{f2}), decreasing the firm's profit.

As expected, the additional negative effect of scalping discussed above on the firm's profit will reduce the firm's ability to take advantage of scalping. However, despite this additional negative effect, an intermediate level of restriction may still be optimal for the firm. I report this result in the following proposition.

Proposition 3.5. *Even when the firm can vary the price over time, an intermediate level of restriction on scalping can still be strictly optimal for the firm if scalpers have greater flexibility in setting prices.*

Proposition 3.5 shows that an intermediate level of scalping can still be optimal for the firm even if the firm can vary the price over time, as long as the scalpers have some market power. In fact, this case occurs when both δ and K are intermediate. Recall that δ is the proportion of high-type consumers in the market, who are identifiable to the scalpers and may be charged a higher price than others. Here, if δ is too small or too large, the scalpers are not much better than the firm in price discriminating the consumers, because a small δ corresponds to a small probability that a consumer needs to pay a high price, and a large δ corresponds to a large probability that the firm will charge a similarly high price as well. The requirement for an intermediate δ for the main result to sustain emphasizes the premise I focus on: scalpers can increase the firm's profit by decreasing consumers' expected utility from waiting, due to their greater flexibility in setting prices.

Though I assume perfect coordination among scalpers in this model variation to simplify the analysis, the results can be generalized to the situation without (perfect) coordination. Note that even if scalpers do not coordinate with each other, the collusive prices can be achieved due to market friction in a Diamond-Paradox-like manner (Diamond 1971). Intuitively, when scalpers cannot announce their prices as much as the firm (either due to cost constraints or regulations), and consumers have search costs, scalpers may not have the incentive to lower the price. Thus, the logic of the above analysis and the main results can follow even without the assumption of coordinated scalpers.

3.6 Conclusion

In many markets where supply is limited, scalpers purchase the products for reselling them later at inflated prices. Driven by profit, scalpers often rush to purchase the (expected-to-be) popular products once they are on sale. And thanks to their proficiency in getting the products (e.g., with the help of scalping bots or better knowledge about when to start standing in line), they often have better access than consumers to the limited supply. When a non-negligible part of the firm's supply is acquired by scalpers, consumers who do not (or cannot) obtain the products in the primary market often end up paying a significant premium to scalpers in the resale market. Out of fear of being exploited by scalpers, a consumer may tend to purchase the product without having enough information about how much she will like it. Given these market characteristics, this paper studies the effect of scalping on the firm's profit, and aims to explain the firms' puzzling and seemingly contradictory practices: firms impose some but not complete restrictions on scalping.

By considering how the level of scalping will shift supply and demand, as well as shape consumers' expectation, this paper first finds that the firm's profit may first increase and then decrease in the severity of restrictions on scalping. This pattern results from two opposing effects of increased scalping on the firm's profit. On the positive side, it increases the possibility that products will be sold out in the primary market, where the price will often be lower than in the resale market, thus increasing consumers' willingness to pay right away. On the negative side, it increases the total supply in the later period, leading to consumers' higher expected utility from waiting until the valuation uncertainty is resolved. This effect forces the firm to lower the price so as to encourage immediate purchase. Due to these two opposing effects, an intermediate level of restriction on scalping can be strictly optimal even if the firm only aims to maximize its profit and does not consider other issues (e.g., consumer welfare, reputation, the cost or ability to restrict scalping).

Second, the firm's profit may first increase and then decrease in its capacity. Intuitively, a larger capacity increases the maximum number of consumers the firm can serve, leading to a potentially higher profit given a fixed price. However, it can lower the equilibrium price due to the decreased "scarcity." Oftentimes, the fear of insufficient supply drives consumers to purchase products before they have enough information to form a valuation of those products. And when the supply is larger, consumers are less eager to pay right away. Due to the two opposing effects of increased capacity on profit, the firm may prefer an intermediate level of capacity even if the unit cost of production is zero. Moreover, because scalping – similar to scarce supply – can push consumers to purchase immediately, the optimal capacity is larger with the existence of scalpers. In this regard, a restricted level of scalping can increase the number of units consumed, leading to higher social welfare.

Though several assumptions are made to simplify the analysis and discussion, the main results of this paper can be generalized to less constrained cases. For example, a restricted level of scalping can push the demand forward as long as scalpers are somehow better able than the firm to exploit consumers, and too many scalpers are not desirable as long as they will decrease consumers' purchases in the advance period (when consumer valuation is more homogeneous) and increase the supply in the spot period (when the variation in consumer valuation increases). In addition, as for maintaining the scalpers' margin and ensuring the high price in the resale market, the market friction can act in a similar way as the coordination among scalpers. Therefore, some information about consumer valuation can enable the scalpers to exploit consumers more than the firm, and leads to this paper's main implications, regardless of wheter a scalper cares about others' payoffs.

One possibility this paper does not explicitly consider is the manufacturer competition. When firms selling similar products compete with each other, a firm's price may be constrained by its competition with the other firms rather than the competition between its own current and future sales. Thus, it cannot (unilaterally) raise the price even if the existence of scalpers makes immediate purchase more appealing. In such cases, if the substitutable goods sold by the competing firms are handled by the same groups of scalpers (which is often the case in reality),[15] scalping can increase firms' profits through coordinating the different products' prices in a similar way as a dominant retailer.

Another possibility is the aggregate-level demand uncertainty. Under the simplifying assumption of constant aggregate demand, the realized market outcomes (e.g., the sellers' pricing decisions and the consumers' purchase decisions) end up to be consistent with the expectation. In reality, although people can have a guess of the product's popularity, the realized demand can be larger or smaller than the estimate. If the scalpers adjust to the realized demand so as to maximize the profit, the market price may turn out to be higher or lower than expected. Going back to the implications on firm's profit, the scalpers' higher

[15]For example, a concert ticket scalper may deal with concerts held by multiple pop singers, e.g., Taylor Swift, Rihanna, and Beyoncé.

ability to exploit consumers through adjusting to the random demand is another consideration in favor of scalping. Here, whether the firm ever has the incentive to limit scalping depends on how the profits are shared between the firm and scalpers.

APPENDIX

PROOFS AND DETAILS OF ANALYSIS

Analysis of the Main Model Without Showrooming (Section 1.4.1)

The following lemma summarizes the equilibrium decisions and outcomes in the absence of showrooming.

Lemma A.1. *Let* $\overline{r} = 2r - 1$ *and* r^* *solve* $8kr^3 + (1 - 12k - 128k^2)r^2 + (-14k + 256k^2)r + 9k - 128k^2 = 0$ *for* r*. Then,*

(a) If $k \leq \frac{1}{32}$*, in equilibrium,* $w = \frac{1}{2}, R = 0$*, and the B&M retailer sets* $p_1 = \frac{3}{4}$ *and* $s = 1$*. The manufacturer's profit is* $\frac{1}{8}$*, and the B&M retailer's profit is* $\frac{1}{16} - k$*.*

(b) If $\frac{1}{32} < k \leq \frac{9}{128}$*, or* $\frac{9}{128} < k \leq \frac{1}{9}$ *and* $r \geq r^*$*, in equilibrium,* $s^* = 1$ *and*[1]

$$\widetilde{w} = 1 - \sqrt[3]{4k}\left(\sqrt[3]{r + A} + \sqrt[3]{r - A}\right) \text{ and } \widetilde{R} = \frac{(8k - (1 - \widetilde{w})^2)^2}{64rk}, \text{ where } A \equiv \sqrt{r^2 + \tfrac{32}{27}k\overline{r}^3}. \tag{A.1.1}$$

As a result, the manufacturer's profit is $\frac{1}{8r}[1 - 8k + r - (2 - r)\widetilde{w} - \overline{r}\widetilde{w}^2]$*, and the B&M retailer's profit is* $\frac{1}{8r}[8(1 - r)k + (1 - \widetilde{w})r - (1 - \widetilde{w})^2]$*.*

(c) Otherwise ($k > 1/9$*, or* $\frac{9}{128} < k \leq \frac{1}{9}$ *and* $r < r^*$*), in equilibrium,*

$$s^* = \frac{(\overline{r}^2 + 2)\sqrt{8 + \overline{r}^2} - (10 - \overline{r}^2)\overline{r}}{256k(1 - r)^2}, w^* = \frac{4 - \overline{r} - \sqrt{8 + \overline{r}^2}}{8(1 - r)},$$
$$\text{and } R^* = \frac{r}{k}\left[\frac{(\sqrt{8 + \overline{r}^2} + 2 + \overline{r})^2 - 16\overline{r} - 20}{2^8(1 - r)^2}\right]^2.$$

---

[1] Here, when the quantity under the cubic root is a real number, one should choose the cubic root that is a real number, but when the quantity under the cubic root is complex, one should choose the root whose complex argument (i.e., the polar-coordinates' angle $a \in (-\pi, \pi]$) is one-third of the argument of x. This way, the imaginary parts of the expression for $\widetilde{w}$ always cancel out. Note also that the division by r in the expression on $\widetilde{R}$ does not imply that $\widetilde{R}$ increases as $r \to 0$ because the numerator of R actually tends to zero faster than r when $r \to 0$. An alternative characterization of $\widetilde{w}$ is that it is the (real) solution of $-1 + 8k - 8kr + (3 - 8k + 16kr)\widetilde{w} - 3\widetilde{w}^2 + \widetilde{w}^3 = 0$, which falls within $(0, 1)$.

As a result, the manufacturer's profit is $\dfrac{8+20\overline{r}^2-\overline{r}^4-(8+\overline{r}^2)\overline{r}\sqrt{8+\overline{r}^2}}{2^{13}k(1-r)^3}$, *and the B&M retailer's profit is*

$$\frac{(3-2r)(27-90r+96r^2+128r^3-112r^4+32r^5+(9-28r+24r^2-48r^3+16r^4)\sqrt{8+\overline{r}^2})}{2^{15}k(1-r)^4}.$$

Proof: *Part (a).* The B&M retailer's objective function without compensation for service is stated in Equation (1.1). The first-order condition with respect to p_1 is $s(1+w-2p_1)=0$, and the first-order condition with respect to s is $(1-p_1)(p_1-w)-2ks=0$. To make a positive profit, the B&M retailer has to set $s>0$. Note that for any positive s, it is optimal to set $p_1=\frac{1+w}{2}$, which in turn yields optimal $s=\frac{(1-w)^2}{8k}$. The value of s is bounded by 1. When $\frac{(1-w)^2}{8k}\geq 1$, the first-order derivative of B&M retailer's profit function with respect to s, $(1-p_1)(p_1-w)-2ks=\frac{(1-w)^2}{4}-2ks$, is positive for any $0\leq s\leq 1$, meaning that it is optimal to set $s=1$. In other words, if $\frac{(1-w)^2}{8k}\geq 1$, the B&M retailer is willing to set the highest service level even if the manufacturer does not provide any direct compensation for the service.

When this condition is satisfied, it is optimal for the manufacturer to set $R=0$, and maximize its expected profit $\pi_m=s(1-p_1)w-R=\frac{1-w}{2}w$. Now, the first-order condition is $\frac{d\pi_m}{dw}=-\frac{w}{2}+\frac{1-w}{2}=0$ and the second-order condition is $\frac{d^2\pi_m}{dw^2}=-1<0$. Thus, we know that the manufacturer's profit function is maximized at $w=\frac{1}{2}$. Going back to the condition for this to hold, $\frac{(1-w)^2}{8k}\geq 1$. When we substitute it with $w=\frac{1}{2}$, we obtain $k\leq\frac{1}{32}$. This completes the proof of part a of the lemma.

Parts (b) and (c). As we have proved, if $k\leq\frac{1}{32}$, the equilibrium retail service s is bounded by the feasibility condition $0\leq s\leq 1$. However, if $k>\frac{1}{32}$, we have optimal $s=\frac{(1-w)^2}{8k}<1$ if $w=\frac{1}{2}$ (the optimal wholesale price when the manufacturer ignores the dependence of retail service on wholesale price).

Thus, as stated in the main text, if $k>\frac{1}{32}$, the manufacturer maximizes its profit function $s^*\frac{1-w}{2}w-R$ subject to $R\geq\frac{k}{r}(s^*-\frac{(1-w)^2}{8k})^2$ (so that the B&M retailer does not shirk). The resulting objective function is given by Equation (1.2). Equation (1.2) yields the first-order conditions $\frac{1-w}{2}w - \frac{2k}{r}(s^* - \frac{(1-w)^2}{8k}) = 0$ and $\frac{s^*(1-2w)}{2} - \frac{1}{2r}(s^* - \frac{(1-w)^2}{8k})(1-w) = 0$. The first-order conditions have three solutions: $w = 1$ with $s^* = 0$, $w = \frac{4-\bar{r}+\sqrt{8+\bar{r}^2}}{8(1-r)}$ with $s^* = \frac{-(\bar{r}^2+2)\sqrt{8+\bar{r}^2}-(10-\bar{r}^2)\bar{r}}{256k(1-r)^2}$, and $w = \frac{4-\bar{r}-\sqrt{8+\bar{r}^2}}{8(1-r)}$ with $s^* = \frac{(\bar{r}^2+2)\sqrt{8+\bar{r}^2}-(10-\bar{r}^2)\bar{r}}{256k(1-r)^2}$. Among the three solutions, the only one that satisfies $w < 1$ is the third solution. Moreover, at this point, the Hessian matrix of the profit function $\begin{bmatrix} -s^* + \frac{1}{2r}(s^* - \frac{(1-w)^2}{8k}) - \frac{(1-w)^2}{8kr}, & \frac{1-2w}{2} - \frac{1-w}{2r} \\ \frac{1-2w}{2} - \frac{1-w}{2r}, & -\frac{2k}{r} \end{bmatrix}$ is negative definite (i.e., $-s^* + \frac{1}{2r}(s^* - \frac{(1-w)^2}{8k}) - \frac{(1-w)^2}{8kr} < 0$ and

$$\begin{vmatrix} -s^* + \frac{1}{2r}(s^* - \frac{(1-w)^2}{8k}) - \frac{(1-w)^2}{8kr}, & \frac{1-2w}{2} - \frac{1-w}{2r} \\ \frac{1-2w}{2} - \frac{1-w}{2r}, & -\frac{2k}{r} \end{vmatrix} > 0),$$

implying that the second-order conditions are satisfied.

Therefore, when $\frac{(\bar{r}^2+2)\sqrt{8+\bar{r}^2}-(10-\bar{r}^2)\bar{r}}{256k(1-r)^2} < 1$ (i.e., the condition of Part 3 of the lemma), $w = \frac{4-\bar{r}-\sqrt{8+\bar{r}^2}}{8(1-r)}$ and $s^* = \frac{(\bar{r}^2+2)\sqrt{8+\bar{r}^2}-(10-\bar{r}^2)\bar{r}}{256k(1-r)^2}$ is the maximum point for Equation (1.2). When $\frac{(\bar{r}^2+2)\sqrt{8+\bar{r}^2}-(10-\bar{r}^2)\bar{r}}{256k(1-r)^2} \geq 1$ (i.e. $\frac{1}{32} < k \leq \frac{9}{128}$, or $\frac{9}{128} < k \leq \frac{1}{9}$ and $r \geq r^*$, i.e, the condition in part 2 of the lemma), the optimal decision is the boundary solution $s^* = 1$. In this case, the manufacturer solves $\max_{w \leq 1} \frac{1-w}{2}w - \frac{k}{r}\left(1 - \frac{(1-w)^2}{8k}\right)^2$, whose first- and second-order conditions are, respectively, $\frac{1-2w}{2} - \frac{1}{2r}(1 - \frac{(1-w)^2}{8k})(1-w) = 0$ and $\frac{8k(1-2r)-3(1-w)^2}{16kr} < 0$. Thus, the maximal point that satisfies $0 < w < 1$ is the smallest root of $-1 + 8k - 8kr + (3 - 8k + 16kr)w - 3w^2 + w^3 = 0$, whose explicit expression is shown in Equation (A.1.1).

In both cases, the expression for R is given by $R = \frac{k}{r}(s^* - \frac{(1-w)^2}{8k})^2$, the expression for manufacturer's profit is given by $s^* \frac{1-w}{2}w - \frac{k}{r}\left(s^* - \frac{(1-w)^2}{8k}\right)^2$, and the expression for B&M retailer's profit is given by $s^* \frac{(1-w)^2}{4} - ks^{*2} + R$. This completes the proof of parts b and c of the lemma. $\square$

Proof of Proposition 1.1

The first-order condition of the manufacturer's objective function (Equation (1.3)) with respect to w is $s^*(1-2w)=0$, and the first-order condition with respect to s^* is $(1-w)w-\frac{2ks^*}{r}=0$. Thus, for any positive s^*, the first order conditions yield $w=\frac{1}{2}$, and in turn, $s^*=\frac{r}{8k}$.

When $\frac{r}{8k}<1$, the Hessian matrix $\begin{bmatrix} -2s^*, & 1-2w \\ 1-2w, & -\frac{2k}{r} \end{bmatrix} = \begin{bmatrix} -2s^*, & 0 \\ 0, & -\frac{2k}{r} \end{bmatrix}$ is negative definite at $w=\frac{1}{2}$ and $s^*=\frac{r}{8k}$, implying that the maximum expected profit is achieved at this point. When $\frac{r}{8k}\geq 1$, however, the condition $s^*\leq 1$ is binding. By comparing the first-order derivate of the profit function with respect to s^*, $(1-w)w-\frac{2ks^*}{r}=\frac{1}{4}-\frac{2ks^*}{r}$, to zero, we know that it is optimal to set the corner solution $s^*=1$.

Thus, the equilibrium is $w=\frac{1}{2}$ and $s^*=\min\{1,\ \frac{r}{8k}\}$. Specifically, $s^*=1$ if $r\geq 8k$ and $s^*=\frac{r}{8k}$ if $r<8k$. In both cases, the manufacturer's profit is given by $s^*(1-w)w-\frac{ks^{*2}}{r}$, and the B&M retailer's profit is given by $\frac{(1-r)ks^{*2}}{r}$. Using the value of w and s^* as given previously, the profits can be written as

$$\pi_m=\begin{cases} \frac{1}{4}-\frac{k}{r}, & \text{if } r\geq 8k \\ \frac{r}{64k}, & \text{if } r<8k \end{cases},\ \pi_{B\&M}=\begin{cases} \frac{k(1-r)}{r}, & \text{if } r\geq 8k \\ \frac{r(1-r)}{64k}, & \text{if } r<8k \end{cases} \text{ and } \pi_{ind}=\begin{cases} \frac{1}{4}-k, & \text{if } r\geq 8k \\ \frac{r(2-r)}{64k}, & \text{if } r<8k \end{cases}$$

Therefore, the derivatives of profits with respect to k and r are

$$\frac{\partial\pi_m}{\partial k}=\begin{cases} -\frac{1}{r}, & \text{if } k\leq\frac{r}{8} \\ -\frac{r}{64k^2}, & \text{if } k>\frac{r}{8} \end{cases},\ \frac{\partial\pi_{B\&M}}{\partial k}=\begin{cases} \frac{(1-r)}{r}, & \text{if } k\leq\frac{r}{8} \\ -\frac{r(1-r)}{64k^2}, & \text{if } k>\frac{r}{8} \end{cases},\ \frac{\partial\pi_{ind}}{\partial k}=\begin{cases} -1, & \text{if } k\leq\frac{r}{8} \\ -\frac{r(2-r)}{64k^2}, & \text{if } k>\frac{r}{8} \end{cases}$$

and

$$\frac{\partial\pi_m}{\partial r}=\begin{cases} \frac{1}{64k}, & \text{if } r<8k \\ \frac{k}{r^2}, & \text{if } r\geq 8k \end{cases},\ \frac{\partial\pi_{B\&M}}{\partial r}=\begin{cases} \frac{1-2r}{64k}, & \text{if } r<8k \\ -\frac{k}{r^2}, & \text{if } r\geq 8k \end{cases},\ \frac{\partial\pi_{ind}}{\partial r}=\begin{cases} \frac{2(1-r)}{64k}, & \text{if } r<8k \\ 0, & \text{if } r\geq 8k \end{cases}$$

Part (a) of the proposition is due to $\frac{\partial \pi_{B\&M}}{\partial k}$ being positive for $k \leq \frac{r}{8}$ and negative for $k > \frac{r}{8}$, and $\frac{\partial \pi_{B\&M}}{\partial r}$ being positive for $r < \min\{8k, \frac{1}{2}\}$ and negative for $r \geq \min\{8k, \frac{1}{2}\}$. Part (ii) of the proposition is due to $\frac{\partial \pi_m}{\partial k}$ and $\frac{\partial \pi_{ind}}{\partial k}$ being always negative, $\frac{\partial \pi_m}{\partial r}$ being always positive, and $\frac{\partial \pi_{ind}}{\partial r}$ being positive for $r < 8k$ and zero for $r \geq 8k$. This completes the proof of Proposition 1.1. □

Proof of Proposition 1.2

To compare profits reported with and without showrooming derived previously, define the six regions as presented in Table 1.1: $\mathbf{R_I}$: $k \leq \frac{1}{32}$ and $0 \leq r < 8k$. $\mathbf{R_{II}}$: $k \leq \frac{1}{32}$ and $8k \leq r \leq 1$. $\mathbf{R_{III}}$: $\frac{1}{32} < k < \frac{1}{9}$ and $\max\{0, r^*\} \leq r < 8k$. $\mathbf{R_{IV}}$: $\frac{1}{32} < k < \frac{1}{9}$ and $\max\{8k, r^*\} \leq r < 1$. $\mathbf{R_V}$: $k \geq \frac{9}{128}$ and $0 \leq r < \min\{8k, r^*, 1\}$. $\mathbf{R_{VI}}$: $k \geq \frac{9}{128}$ and $8k \leq r \leq \min\{r^*, 1\}$. Here, r^* solves $8kr^3 + (1 - 12k - 128k^2)r^2 + (-14k + 256k^2)r + 9k - 128k^2 = 0$ for r. In each of these regions, it is straightforward to derive the differences in the B&M retailer, manufacturer, and industry profits with and without showrooming using the results reported in Lemma A.1 and Proposition 1.1:

$$\Delta\pi_m = \begin{cases} \frac{r}{64k} - \frac{1}{8}, & \text{in } \mathbf{R_I} \\ \frac{1}{8} - \frac{k}{r}, & \text{in } \mathbf{R_{II}} \\ \frac{r}{64k} - \frac{(1-\tilde{w})\tilde{w}}{2} + \frac{k}{r}\left(1 - \frac{(1-\tilde{w})^2}{8k}\right)^2, & \text{in } \mathbf{R_{III}} \\ \frac{1}{4} - \frac{(1-\tilde{w})\tilde{w}}{2} - \frac{(1-\tilde{w})^2}{4r} + \frac{(1-\tilde{w})^4}{64kr}, & \text{in } \mathbf{R_{IV}} \\ \frac{r}{64k} - s^*\frac{(1-w^*)w^*}{2} + \frac{k}{r}\left(s^* - \frac{(1-w^*)^2}{8k}\right)^2, & \text{in } \mathbf{R_V} \\ \frac{1}{4} - \frac{k}{r} - s^*\frac{(1-w^*)w^*}{2} + \frac{k}{r}\left(s^* - \frac{(1-w^*)^2}{8k}\right)^2, & \text{in } \mathbf{R_{VI}} \end{cases}$$

$$\Delta\pi_{B\&M} = \begin{cases} \frac{r(1-r)}{64k} - \frac{1}{16} + k, & \text{in } \mathbf{R_I} \\ \frac{k}{r} - \frac{1}{16}, & \text{in } \mathbf{R_{II}} \\ \frac{r(1-r)}{64k} - \frac{(1-\tilde{w})^2}{4} + k - \frac{k}{r}(1 - \frac{(1-\tilde{w})^2}{8k})^2, & \text{in } \mathbf{R_{III}} \\ -\frac{(1-\tilde{w})^2}{4} + \frac{(1-\tilde{w})^2}{4r} - \frac{(1-\tilde{w})^4}{64kr}, & \text{in } \mathbf{R_{IV}} \\ \frac{r(1-r)}{64k} - s^*\frac{(1-w^*)^2}{4} + k - \frac{k}{r}(s^* - \frac{(1-w^*)^2}{8k})^2, & \text{in } \mathbf{R_V} \\ \frac{k(1-r)}{r} - s^*\frac{(1-w^*)^2}{4} + k - \frac{k}{r}(s^* - \frac{(1-w^*)^2}{8k})^2, & \text{in } \mathbf{R_{VI}} \end{cases}$$

and

$$\Delta\pi_{ind} = \Delta\pi_m + \Delta\pi_{B\&M} = \begin{cases} \frac{r(2-r)}{64k} - \frac{3}{16} + k, & \text{in } \mathbf{R_I} \\ \frac{1}{16}, & \text{in } \mathbf{R_{II}} \\ \frac{r(2-r)}{64k} - \frac{1-\tilde{w}^2}{4} + k, & \text{in } \mathbf{R_{III}} \\ \frac{\tilde{w}^2}{4}, & \text{in } \mathbf{R_{IV}} \\ \frac{r(2-r)}{64k} - s^*\frac{1-w^{*2}}{4} + k, & \text{in } \mathbf{R_V} \\ \frac{1}{4} - s^*\frac{1-w^{*2}}{4}, & \text{in } \mathbf{R_{VI}} \end{cases},$$

where $\widetilde{w}$, w^* and s^* are given in Lemma A.1. Based on the above, we can derive the first-order derivatives with respect to k and r:

$$\frac{\partial\Delta\pi_m}{\partial k} = \begin{cases} -\frac{r}{64k^2}, & \text{in } \mathbf{R_I} \\ -\frac{1}{r}, & \text{in } \mathbf{R_{II}} \\ -\frac{r}{64k^2} - \frac{1}{8r}(-8 - (2-r)\frac{\partial\tilde{w}}{\partial k} + 2(1-2r)\tilde{w}\frac{\partial\tilde{w}}{\partial k}, & \text{in } \mathbf{R_{III}} \\ \frac{1}{8r}(-(2-r)\frac{\partial\tilde{w}}{\partial k} + 2(1-2r)\tilde{w}\frac{\partial\tilde{w}}{\partial k}, & \text{in } \mathbf{R_{IV}} \\ -\frac{r}{64k^2} + \frac{27-4(1-r)(19-\bar{r}^2)-(8+\bar{r}^2)\bar{r}\sqrt{8+\bar{r}^2}}{2^{13}k^2(1-r)^3}, & \text{in } \mathbf{R_V} \\ -\frac{1}{r} + \frac{27-4(1-r)(19-\bar{r}^2)-(8+\bar{r}^2)\bar{r}\sqrt{8+\bar{r}^2}}{2^{13}k^2(1-r)^3}, & \text{in } \mathbf{R_{VI}} \end{cases}$$

$$\frac{\partial \Delta\pi_m}{\partial r} = \begin{cases} \frac{1}{64k}, & \text{in } \mathbf{R_I} \\ \frac{k}{r^2}, & \text{in } \mathbf{R_{II}} \\ \frac{1}{64k} - \frac{1}{8r^2}(-1+8k+2\tilde{w}-\tilde{w}^2 - r(2-r)\frac{\partial \tilde{w}}{\partial r} + 2r(1-2r)\tilde{w}\frac{\partial \tilde{w}}{\partial r}), & \text{in } \mathbf{R_{III}} \\ -\frac{1}{8r^2}(-1+2\tilde{w}-\tilde{w}^2 - r(2-r)\frac{\partial \tilde{w}}{\partial r} + 2r(1-2r)\tilde{w}\frac{\partial \tilde{w}}{\partial r}), & \text{in } \mathbf{R_{IV}} \\ \frac{1}{64k} - \frac{-64-128\bar{r}-40\bar{r}^2-8\bar{r}^3-4\bar{r}^4+\bar{r}^5+(24+40\bar{r}+20\bar{r}^2-4\bar{r}^3+\bar{r}^4)\sqrt{8+\bar{r}^2}}{2^{13}k(1-r)^4\sqrt{8+\bar{r}^2}}, & \text{in } \mathbf{R_V} \\ \frac{k}{r^2} - \frac{-64-128\bar{r}-40\bar{r}^2-8\bar{r}^3-4\bar{r}^4+\bar{r}^5+(24+40\bar{r}+20\bar{r}^2-4\bar{r}^3+\bar{r}^4)\sqrt{8+\bar{r}^2}}{2^{13}k(1-r)^4\sqrt{8+\bar{r}^2}}, & \text{in } \mathbf{R_{VI}} \end{cases}$$

$$\frac{\partial \Delta\pi_{B\&M}}{\partial k} = \begin{cases} -\frac{r(1-r)}{64k^2} + 1, & \text{in } \mathbf{R_I} \\ \frac{1}{r}, & \text{in } \mathbf{R_{II}} \\ -\frac{r(1-r)}{64k^2} - \frac{1}{8r}(8(1-r) + (2-r)\frac{\partial \tilde{w}}{\partial k} - 2\tilde{w}\frac{\partial \tilde{w}}{\partial k}), & \text{in } \mathbf{R_{III}} \\ -\frac{1}{8r}((2-r)\frac{\partial \tilde{w}}{\partial k} - 2\tilde{w}\frac{\partial \tilde{w}}{\partial k}), & \text{in } \mathbf{R_{IV}} \\ -\frac{r(1-r)}{64k^2} + \\ \qquad \frac{(3-2r)(27-90r+96r^2+128r^3-112r^4+32r^5+(9-28r+24r^2-48r^3+16r^4)\sqrt{8+\bar{r}^2})}{2^{15}k^2(1-r)^4}, & \text{in } \mathbf{R_V} \\ \frac{1-r}{r} + \frac{(3-2r)(27-90r+96r^2+128r^3-112r^4+32r^5+(9-28r+24r^2-48r^3+16r^4)\sqrt{8+\bar{r}^2})}{2^{15}k^2(1-r)^4}, & \text{in } \mathbf{R_{VI}} \end{cases}$$

$$\frac{\partial \Delta\pi_{B\&M}}{\partial r} = \begin{cases} \frac{1-2r}{64k}, & \text{in } \mathbf{R_I} \\ -\frac{k}{r^2}, & \text{in } \mathbf{R_{II}} \\ \frac{1-2r}{64k} - \frac{1}{8r^2}(1-8k-2\tilde{w}+\tilde{w}^2 + r(2-r)\frac{\partial \tilde{w}}{\partial r} - 2r\tilde{w}\frac{\partial \tilde{w}}{\partial r}), & \text{in } \mathbf{R_{III}} \\ -\frac{1}{8r^2}(9-8k-2\tilde{w}+\tilde{w}^2 + r(2-r)\frac{\partial \tilde{w}}{\partial r} - 2r\tilde{w}\frac{\partial \tilde{w}}{\partial r}), & \text{in } \mathbf{R_{IV}} \\ \frac{1-2r}{64k} - \frac{r(-240-68\bar{r}+84\bar{r}^2-34\bar{r}^3+20\bar{r}^4-6\bar{r}^5+\bar{r}^6+(84+28\bar{r}-42\bar{r}^2+16\bar{r}^3-6\bar{r}^4+\bar{r}^5)\sqrt{8+\bar{r}^2})}{2^{13}k(1-r)^5\sqrt{8+\bar{r}^2}}, & \text{in } \mathbf{R_V} \\ -\frac{k}{r^2} - \frac{r(-240-68\bar{r}+84\bar{r}^2-34\bar{r}^3+20\bar{r}^4-6\bar{r}^5+\bar{r}^6+(84+28\bar{r}-42\bar{r}^2+16\bar{r}^3-6\bar{r}^4+\bar{r}^5)\sqrt{8+\bar{r}^2})}{2^{13}k(1-r)^5\sqrt{8+\bar{r}^2}}, & \text{in } \mathbf{R_{VI}} \end{cases}$$

$$
\frac{\partial \Delta\pi_{ind}}{\partial k} = \begin{cases}
-\frac{r(2-r)}{64k^2} + 1, & \text{in } \mathbf{R_I} \\
0, & \text{in } \mathbf{R_{II}} \\
-\frac{r(2-r)}{64k^2} + \frac{\tilde{w}}{2}\frac{\partial \tilde{w}}{\partial k} + 1, & \text{in } \mathbf{R_{III}} \\
\frac{\tilde{w}}{2}\frac{\partial \tilde{w}}{\partial k}, & \text{in } \mathbf{R_{IV}} \\
-\frac{r(2-r)}{64k^2} + \frac{48+24\bar{r}+92\bar{r}^2-84\bar{r}^3-4\bar{r}^4+6\bar{r}^5-\bar{r}^6+(-8-44\bar{r}+20\bar{r}^2+6\bar{r}^4-\bar{r}^5)\sqrt{8+\bar{r}^2}}{2^{15}k^2(1-r)^4}, & \text{in } \mathbf{R_V} \\
-1 + \frac{48+24\bar{r}+92\bar{r}^2-84\bar{r}^3-4\bar{r}^4+6\bar{r}^5-\bar{r}^6+(-8-44\bar{r}+20\bar{r}^2+6\bar{r}^4-\bar{r}^5)\sqrt{8+\bar{r}^2}}{2^{15}k^2(1-r)^4}, & \text{in } \mathbf{R_{VI}}
\end{cases}
$$

$$
\frac{\partial \Delta\pi_{ind}}{\partial r} = \begin{cases}
\frac{2(1-r)}{64k}, & \text{in } \mathbf{R_I} \\
0, & \text{in } \mathbf{R_{II}} \\
\frac{2(1-r)}{64k} + \frac{\tilde{w}}{2}\frac{\partial \tilde{w}}{\partial r}, & \text{in } \mathbf{R_{III}} \\
\frac{\tilde{w}}{2}\frac{\partial \tilde{w}}{\partial r}, & \text{in } \mathbf{R_{IV}} \\
\frac{2(1-r)}{64k} - \frac{-304-372\bar{r}+104\bar{r}^2+82\bar{r}^3-10\bar{r}^4+19\bar{r}+19\bar{r}^5-6\bar{r}^8+\bar{r}^7+(108+128\bar{r}-34\bar{r}^2-50\bar{r}^3+15\bar{r}^4-6\bar{r}^5+\bar{r}^6)\sqrt{8+\bar{r}^2}}{2^{14}k(1-r)^5\sqrt{8+\bar{r}^2}}, & \text{in } \mathbf{R_V} \\
-\frac{-304-372\bar{r}+104\bar{r}^2+82\bar{r}^3-10\bar{r}^4+19\bar{r}+19\bar{r}^5-6\bar{r}^8+\bar{r}^7+(108+128\bar{r}-34\bar{r}^2-50\bar{r}^3+15\bar{r}^4-6\bar{r}^5+\bar{r}^6)\sqrt{8+\bar{r}^2}}{2^{14}k(1-r)^5\sqrt{8+\bar{r}^2}}, & \text{in } \mathbf{R_{VI}}
\end{cases}
$$

Checking the sign of these derivatives led to the results presented in Table 1.1, from which all the claims of this proposition immediately follow. This completes the proof of Proposition 1.2. □

Proof of Proposition 1.3

Comparing the profit functions in each region of Table 1.1, we have the following.

The manufacturer's profits: $\Delta\pi_m < 0$ in $\mathbf{R_I}$. $\Delta\pi_m \geq 0$ in $\mathbf{R_{II}}$. $\Delta\pi_m \geq 0$ in $\mathbf{R_{III}}$ if r is not too small. $\Delta\pi_m > 0$ in $\mathbf{R_{IV}}$. $\Delta\pi_m \geq 0$ in $\mathbf{R_V}$ if $r \geq \frac{1}{2}$. $\Delta\pi_m > 0$ in $\mathbf{R_{VI}}$.

The B&M retailer's profits: $\Delta\pi_{B\&M} \geq 0$ in $\mathbf{R_I}$ if $r \geq \frac{1}{2}(1-\sqrt{1-16k+256k^2})$. $\Delta\pi_{B\&M} \geq 0$ in $\mathbf{R_{II}}$ if $r \leq 16k$. $\Delta\pi_{B\&M} \geq 0$ in $\mathbf{R_{III}}$ if r is in an intermediate range. $\Delta\pi_{B\&M} \geq 0$ in $\mathbf{R_{IV}}$ if r is not too large. $\Delta\pi_{B\&M} < 0$ in $\mathbf{R_V}$. $\Delta\pi_{B\&M} < 0$ in $\mathbf{R_{VI}}$.

The equations on the boundary curves in Figure 1.3 are:

Line 1: $r = 16k$ ($\Delta\pi_{B\&M} = 0$ in $\mathbf{R_{II}}$).

Line 2: $r = 1 - \frac{(1-\tilde{w})^2}{16k}$ ($\Delta\pi_{B\&M} = 0$ in $\mathbf{R_{IV}}$).

Line 3: $\frac{(1-\tilde{w})^2}{4} - k + \frac{k}{r}(1 - \frac{(1-\tilde{w})^2}{8k})^2 = \frac{r(1-r)}{64k}$ ($\Delta\pi_{B\&M} = 0$ in $\mathbf{R_{III}}$).

Line 4: $r = 8k$ ($\Delta\pi_m = 0$ in $\mathbf{R_{II}}$).

Line 5: $\frac{1}{2}(1-\tilde{w})\tilde{w} - \frac{k}{r}(1 - \frac{(1-\tilde{w})^2}{8k})^2 = \frac{r}{64k}$ ($\Delta\pi_m = 0$ in $\mathbf{R_{III}}$).

This completes the proof of Proposition 1.3. $\square$

Proof of Proposition 1.4

Remind that we consider $k \leq \frac{1}{32}$ only. If showrooming is not possible, similarly to part (i) of Lemma 1, one derives that the optimal wholesale contract is $w = \frac{1}{2}$ and $R = 0$. The B&M retailer sets $p_1 = \frac{3}{4}$ and $s = 1$. Since the B&M retailer always sets the highest service level, there is no need for the manufacturer to detect (i.e., $r = 0$). As a result, the manufacturer's profit is $\frac{1}{8}$, and the B&M retailer's profit is $\frac{1}{16} - k$.

If showrooming is possible, the retail prices are competed down to w, and the retail margin is zero. Here, the manufacturer can only obtain positive profit by incentivizing a higher service level through compensation. Note that to ensure nonzero sales, it must be that $R \geq ks^{*2}$ in the first stage (otherwise, the B&M retailer always sets $s = 0$). If $R \geq ks^{*2}$, in equilibrium, the B&M retailer randomizes between shirking $s = 0$ and not shirking $s = s^*$, and the manufacturer detects probabilistically ($r \in (0, 1)$).

Let us denote the probability that B&M retailer shirks by $x \in (0,1)$. In equilibrium, the B&M retailer is indifferent between shirking and not shirking, i.e. $(1-r)R = R - ks^{*2}$. Thus, $r = \frac{ks^{*2}}{R}$. Moreover, the manufacturer chooses the value of r to maximize its expected profit $(1-x)s^*(1-w)w - (1-xr)R - dr$. The manufacturer's indifference between detecting and not detecting yields $xR = d$, i.e., $x = \frac{d}{R}$.

Now, in the first stage, the manufacturer maximizes its expected profit $(1-x)s^*(1-w)w - R = (1-\frac{d}{R})s^*(1-w)w - R$ subject to $0 \leq w \leq 1$, $0 \leq s^* \leq 1$, $0 \leq \frac{ks^{*2}}{R} \leq 1$ and $0 \leq \frac{d}{R} \leq 1$. The optimal wholesale price is $w = \frac{1}{2}$. Moreover, when $0 \leq \frac{ks^{*2}}{R} \leq 1$ and $0 \leq \frac{d}{R} \leq 1$ are not binding, it is optimal to set $s^* = 1$ and $R = \frac{\sqrt{d}}{2}$ $(4k^2 < d < \frac{1}{4})$. As a result, the manufacturer's profit is $\frac{1}{4} - \sqrt{d}$, and the B&M retailer's profit is $\frac{\sqrt{d}}{2} - k$. To ensure that the manufacturer earns nonnegative profit, we should have $d \leq \frac{1}{16}$. When $d \leq 4k^2$, on the other hand, it is optimal to choose the corner solution $R = ks^{*2}$ and solve $\max_{0 \leq s^* \leq 1} \frac{1}{4}(1 - \frac{d}{ks^{*2}})s^* - ks^{*2}$. Since the first-order derivative $\frac{1}{4}(1 + \frac{d}{ks^{*2}} - 8ks^*) \geq \frac{1}{4}(1 + 0 - 8k) > 0$ for all $k \leq \frac{1}{32}$, $d \leq 4k^2$ and $0 \leq s^* \leq 1$, it is optimal to set $s^* = 1$. As a result, the manufacturer's profit is $\frac{1}{4} - k - \frac{d}{4k}$, and the B&M retailer's profit is 0. Thus, the differences in profits are

$$\Delta\pi_m = \begin{cases} \frac{1}{8} - k - \frac{d}{4k}, & \text{if } 0 \leq d \leq 4k^2 \\ \frac{1}{8} - \sqrt{d}, & \text{if } 4k^2 < d \leq \frac{1}{16} \\ -\frac{1}{8}, & \text{otherwise} \end{cases} \text{ and } \Delta\pi_{B\&M} = \begin{cases} k - \frac{1}{16}, & \text{if } 0 \leq d \leq 4k^2 \\ \frac{\sqrt{d}}{2} - \frac{1}{16}, & \text{if } 4k^2 < d \leq \frac{1}{16} \\ k - \frac{1}{16}, & \text{otherwise} \end{cases},$$

where $0 \leq k \leq \frac{1}{32}$. It is easy to see that $\Delta\pi_m \geq 0$ iff $0 \leq d \leq \frac{1}{64}$, and $\Delta\pi_{B\&M} \geq 0$ iff $\frac{1}{64} \leq d \leq \frac{1}{16}$.

This completes the proof of Proposition 1.4. □

Analysis of the Model Variation With Smooth Cost of Service (Section 1.5.2)

Similar to the main model, when showrooming is not possible, the manufacturer's profit maximization problem is

$$\begin{aligned} \max_{w\leq 1, s^*\geq 0, R\geq 0} \quad & s^*\frac{1-w}{2}w - R \\ \text{s.t.} \quad & s^*\frac{(1-w)^2}{4} - c(s^*) + R \geq s'\frac{(1-w)^2}{4} - c(s') + (1-r)R. \\ & s' = arg\,max_{s'\geq 0}\left(s'\frac{(1-w)^2}{4} - c(s')\right) \end{aligned}$$

Moreover, given that $c(s) = k_1 s + k_2 s^2$, we can rewrite the manufacturer's problem as

$$\max_{w\leq 1, s^*\geq 0} s^*\frac{1-w}{2}w - \frac{k_2}{r}\left(s^* - \frac{(1-w)^2 - 4k_1}{8k_2}\right)^2.$$

Here, as long as $k_1 \leq \frac{1}{4}$, the manufacturer's profit is maximized when w is the first root of the equation $1-4k_1+(-6+8k_1+2r)w+(9-6r)w^2+(-4+4r)w^3 = 0$, and $s^* = \frac{(1-w)^2-4k_1+2r(1-w)w}{8k_2}$. Note that as long as k_2 is large enough, we have $\frac{(1-w)^2-4k_1}{8k_2} < 1$ and $s^* < 1$. In other words, for large enough k_2, it would never be practical for the B&M retailer to set a service level larger than 1, although we do not put an upper bound for s. The equilibrium B&M retailer's profit is $s^*\frac{(1-w)^2}{4} - k_1 s^* - k_2 s^{*2} + \frac{k_2}{r}(s^* - \frac{(1-w)^2-4k_1}{8k_2})^2$.

Similarly, when showrooming is allowed, the manufacturer's expected profit is $s(1-w)w - \frac{1}{r}(k_1 s + k_2 s^2)$, which is maximized at $w = \frac{1}{2}$ and $s = s^* = \max\{0, \frac{r-4k_1}{8k_2}\}$. As long as $k_1 \leq \frac{r}{4}$, the equilibrium manufacturer's profit is $\frac{(r-4k_1)^2}{64k_2 r}$, and the equilibrium B&M retailer's profit is $\frac{(1-r)(r^2-16k_1^2)}{64k_2 r}$.

Analysis of the Model Variation With Discrete Service Levels (Section 1.5.2)

The manufacturer may decide not to suggest the high service level s_h if the required w and R are too costly for it. Equivalently, we can assume that the manufacturer always suggest $s^* = s_h$, but would choose $R = 0$ if incentivizing the high service level is not worthy.

When consumers cannot showroom, as in the main model, it is optimal for the B&M retailer to set $p_1 = \frac{1+w}{2}$. It can obtain $s_h(1-p_1)(p_1-w) - C + R = s_h\frac{(1-w)^2}{4} - C + R$ by choosing $s = s_h$, and $s_l(1-p_1)(p_1-w) + (1-r)R = s_l\frac{(1-w)^2}{4} + (1-r)R$ by choosing $s = s_l$. In anticipation of the B&M retailer's best response, the manufacturer's profit is $s_h\frac{1-w}{2}w - R$ if $(s_h - s_l)\frac{(1-w)^2}{4} + rR \geq C$, and $s_l\frac{1-w}{2}w - (1-r)R$ otherwise. Depending on the parameter values, it may be optimal for the manufacturer to either (1) not incentivize high service as the retailer will still choose $s = s_h$, or (2) incentivize high service through w only, or (3) incentivize high service through both w and R, or (4) not incentivize high service even though it results in $s = s_l$. The equilibrium outcomes when showrooming is not possible are reported in Table A.1.1.

When showrooming is allowed, the retail prices are driven down to the wholesale price, i.e., $p_1 = p_2 = w$. The B&M retailer can obtain $-C + R$ by choosing $s = s_h$, and $(1-r)R$ by choosing $s = s_l$. In anticipation of the B&M retailer's best response, the manufacturer's profit is $s_h(1-w)w - R$ if $rR \geq C$, and $s_l(1-w)w - (1-r)R$ otherwise. Depending on the parameter values, it may be optimal for the manufacturer to either (1) incentivize the high service level through R, or (2) not incentivize the high service level even though it results in $s = s_l$. The equilibrium outcomes when showrooming is allowed are reported in Table A.1.2.

We use the following example to illustrate how the main result is obtained in this model variation:

Let $s_l = \frac{2}{5}$, $s_h = 1$, $r = \frac{1}{2}$ and $C = \frac{3}{80}$. Then, $C \leq \frac{s_h - s_l}{16}$ and $C \leq \frac{r(s_h - s_l)}{4}$. Thus, in the no-showrooming case, $\pi_m = \frac{1}{8}$ and $\pi_{B\&M} = \frac{1}{40}$; whereas in the showrooming case, $\pi_m = \frac{7}{40}$ and $\pi_{B\&M} = \frac{3}{80} > \frac{1}{40}$.

Analysis of the Model Variation Where Consumers Have Lower Valuations Online (Section 1.5.3)

Let us denote the offline wholesale price by w_1, and the online wholesale price by w_2.

Table A.1.1: Discrete Service Level (No Showrooming)

Range	$C \leq \frac{s_h - s_l}{16}$	$\begin{cases} 0 \leq r \leq \frac{s_h-s_l}{2s_h} + \frac{1}{2}\sqrt{\frac{s_h-s_l}{s_h}} \\ \frac{s_h-s_l}{16} < C \leq \frac{(2s_h-s_l)(s_h-s_l)}{16s_h} + \frac{1}{8}\sqrt{\frac{(s_h-s_l)^3}{s_h}} \end{cases}$ or $\begin{cases} \frac{s_h-s_l}{2s_h} + \frac{1}{2}\sqrt{\frac{s_h-s_l}{s_h}} < r \leq 1 \\ \frac{s_h-s_l}{16} < C \leq \frac{s_h^2(s_h-s_l)r^2}{4((2r-1)s_h+s_l)^2} \end{cases}$
Wholesale Price	$\frac{1}{2}$	$1 - \sqrt{\frac{4C}{s_h - s_l}}$
Compensation R	0	0
Offline Retail Price	$\frac{3}{4}$	$1 - \sqrt{\frac{C}{s_h - s_l}}$
Service Level	s_h	s_h
Manufacturer Profit	$\frac{s_h}{8}$	$s_h\sqrt{\frac{C}{s_h - s_l}} - \frac{2s_h C}{s_h - s_l}$
B&M Retailer Profit	$\frac{s_h}{16} - C$	$\frac{s_l C}{s_h - s_l}$
Range	$\begin{cases} \frac{s_h-s_l}{2s_h} + \frac{1}{2}\sqrt{\frac{s_h-s_l}{s_h}} < r \leq 1 \\ \frac{s_h^2(s_h-s_l)r^2}{4((2r-1)s_h+s_l)^2} < C \leq \frac{r(2rs_l+s_l)(s_h-s_l)}{8((2r-1)s_h+s_l)} \end{cases}$	$\begin{cases} 0 \leq r \leq \frac{s_h-s_l}{2s_h} + \frac{1}{2}\sqrt{\frac{s_h-s_l}{s_h}} \\ C > \frac{(2s_h-s_l)(s_h-s_l)}{16s_h} + \frac{1}{8}\sqrt{\frac{(s_h-s_l)^3}{s_h}} \end{cases}$ or $\begin{cases} \frac{s_h-s_l}{2s_h} + \frac{1}{2}\sqrt{\frac{s_h-s_l}{s_h}} < r \leq 1 \\ C > \frac{r(2rs_l+s_l)(s_h-s_l)}{8((2r-1)s_h+s_l)} \end{cases}$
Wholesale Price	$\frac{(r-1)s_h + s_l}{(2r-1)s_h + s_l}$	$\frac{1}{2}$
Compensation R	$\frac{C}{r} - \frac{r^2 s_h^2 (s_h - s_l)}{4r((2r-1)s_h + s_l)^2}$	0
Offline Retail Price	$\frac{(3r-2)s_h + 2s_l}{2((2r-1)s_h + s_l)}$	$\frac{3}{4}$
Service Level	s_h	s_l
Manufacturer Profit	$\frac{r^2 s_h^2}{4r((2r-1)s_h + s_l)} - \frac{C}{r}$	$\frac{s_l}{8}$
B&M Retailer Profit	$\frac{r^2 s_h^2((r-1)s_h + s_l)}{4r((2r-1)s_h + s_l)^2} + \frac{(1-r)C}{r}$	$\frac{s_l}{16}$

Given the service level s, the B&M retailer chooses p_1 to maximize its expected profit

$$\pi_{B\&M} = \begin{cases} -ks^2, & \text{if } \frac{p_1-p_2}{1-\delta} \geq 1 \\ s(1 - \frac{p_1-p_2}{1-\delta})(p_1 - w_1) - ks^2, & \text{if } 1 > \frac{p_1-p_2}{1-\delta} \geq \frac{p_2}{\delta} \\ s(1-p_1)(p_1 - w_1) - ks^2, & \text{if } \frac{p_1-p_2}{1-\delta} < \frac{p_2}{\delta} \end{cases},$$

Table A.1.2: Discrete Service Level (Showrooming)

Range	Wholesale Price	Compensation	Service Level	Manufacturer Profit	B&M Retailer Profit
$C \leq \frac{r(s_h - s_l)}{4}$	$w = \frac{1}{2}$	$R = \frac{C}{r}$	$s = s_h$	$\frac{s_h}{4} - \frac{C}{r}$	$\frac{(1-r)C}{r}$
$C > \frac{r(s_h - s_l)}{4}$	$w = \frac{1}{2}$	$R = 0$	$s = s_l$	$\frac{s_l}{4}$	0

and the online retailer chooses p_2 to maximize its expected profit

$$\pi_{online} \begin{cases} s(1-\frac{p_2}{\delta})(p_2 - w_2), & \text{if } \frac{p_1 - p_2}{1-\delta} \geq 1 \\ s(\frac{p_1-p_2}{1-\delta} - \frac{p_2}{\delta})(p_2 - w_2), & \text{if } 1 > \frac{p_1-p_2}{1-\delta} \geq \frac{p_2}{\delta} \\ 0, & \text{if } \frac{p_1-p_2}{1-\delta} < \frac{p_2}{\delta} \end{cases}.$$

By solving this optimization problem, we know that $p_1 = w_1$ and $p_2 = \frac{\delta + w_2}{2}$ if $w_1 \geq \frac{2-\delta+w_2}{2}$, $p_1 = w_1$ and $p_2 = w_1 - 1 + \delta$ if $\frac{2-\delta+w_2}{2} > w_1 \geq \frac{2(1-\delta)+w_2}{2-\delta}$, $p_1 = \frac{2(1-\delta)+2w_1+w_2}{4-\delta}$ and $p_2 = \frac{\delta(1-\delta)+\delta w_1 + 2w_2}{4-\delta}$ if $\frac{2(1-\delta)+w_2}{2-\delta} > w_1 \geq \frac{-\delta(1-\delta)+(2-\delta)w_2}{\delta}$, $p_1 = \frac{w_2}{\delta}$ and $p_2 = w_2$ if $\frac{-\delta(1-\delta)+(2-\delta)w_2}{\delta} > w_1 \geq \frac{2w_2 - \delta}{\delta}$, and $p_1 = \frac{1+w_1}{2}$ if $w_1 < \frac{2w_2-\delta}{\delta}$. Thus, the B&M retailer's expected profit is

$$\pi_{B\&M}(s, w_1, w_2) = \begin{cases} 0, & \text{if } w_1 \geq \frac{2(1-\delta)+w_2}{2-\delta} \\ s\frac{(2(1-\delta)-(2-\delta)w_1+w_2)^2}{(1-\delta)(4-\delta)^2}, & \text{if } \frac{2(1-\delta)+w_2}{2-\delta} > w_1 \geq \frac{-\delta(1-\delta)+(2-\delta)w_2}{\delta} \\ s\frac{(\delta-w_2)(w_2-\delta w_1)}{\delta^2}, & \text{if } \frac{-\delta(1-\delta)+(2-\delta)w_2}{\delta} > w_1 \geq \frac{2w_2-\delta}{\delta} \\ s\frac{(1-w_1)^2}{4}, & \text{if } w_1 < \frac{2w_2-\delta}{\delta} \end{cases}, \quad \text{(A.1.2)}$$

and the expected offline and online demands are, respectively,

$$D_{B\&M}(s, w_1, w_2) = \begin{cases} 0, & \text{if } w_1 \geq \frac{2(1-\delta)+w_2}{2-\delta} \\ s\frac{2(1-\delta)-(2-\delta)w_1+w_2}{(1-\delta)(4-\delta)}, & \text{if } \frac{2(1-\delta)+w_2}{2-\delta} > w_1 \geq \frac{-\delta(1-\delta)+(2-\delta)w_2}{\delta} \\ s(1-\frac{w_2}{\delta}), & \text{if } \frac{-\delta(1-\delta)+(2-\delta)w_2}{\delta} > w_1 \geq \frac{2w_2-\delta}{\delta} \\ s\frac{1-w_1}{2}, & \text{if } w_1 < \frac{2w_2-\delta}{\delta} \end{cases}, \quad \text{(A.1.3)}$$

and

$$D_{online}(s, w_1, w_2) = \begin{cases} s\frac{\delta - w_2}{2\delta}, & \text{if } w_1 \geq \frac{2-\delta+w_2}{2} \\ s\frac{1-w_1}{\delta}, & \text{if } \frac{2-\delta+w_2}{2} > w_1 \geq \frac{2(1-\delta)+w_2}{2-\delta} \\ s\frac{\delta(1-\delta)+\delta w_1-(2-\delta)w_2}{\delta(1-\delta)(4-\delta)}, & \text{if } \frac{2(1-\delta)+w_2}{2-\delta} > w_1 \geq \frac{-\delta(1-\delta)+(2-\delta)w_2}{\delta} \\ 0, & \text{if } w_1 < \frac{-\delta(1-\delta)+(2-\delta)w_2}{\delta} \end{cases} \tag{A.1.4}$$

As for the choice of s, the B&M retailer can obtain $\pi_{B\&M}(s^*, w_1, w_2) + R$ by choosing the suggested service level R, and $\max_{s \neq s^*} \pi_{B\&M}(s, w_1, w_2) + (1-r)R$ by choosing another service level.

Case: The manufacturer has to offer the product at the same wholesale price to different retailers

When the manufacturer has to offer the product at the same wholesale price to different retailers, we have $w_1 = w_2 = w$. The manufacturer's problem can be written as

$$\begin{aligned} \max_{0\leq w\leq 1, 0\leq s^*\leq 1, R\geq 0} \quad & D_{industry}(s^*, w)\cdot w - R \\ \text{s.t.} \quad & \pi_{B\&M}(s^*, w) + R \geq \max_{s\neq s^*} \pi_{B\&M}(s, w) + (1-r)R \\ & s^* \geq \arg\max_{0\leq s\leq 1} \pi_{B\&M}(s, w) \end{aligned}$$

where

$$\pi_{B\&M}(s, w) = \begin{cases} s\frac{(1-\delta)(2-w)^2}{(4-\delta)^2} - ks^2, & \text{if } w \leq \frac{\delta}{2} \\ s\frac{(\delta-w)(1-\delta)w}{\delta^2} - ks^2, & \text{if } \frac{\delta}{2} < w \leq \frac{\delta}{2-\delta} \\ s\frac{(1-w)^2}{4} - ks^2, & \text{if } w > \frac{\delta}{2-\delta} \end{cases},$$

and

$$D_{industry}(s, w) = D_{B\&M}(s, w) + D_{online}(s, w) = \begin{cases} s\frac{3\delta-(2+\delta)w}{\delta(4-\delta)}, & \text{if } w \leq \frac{\delta}{2} \\ s(1-\frac{w}{\delta}), & \text{if } \frac{\delta}{2} < w \leq \frac{\delta}{2-\delta} \\ s\frac{(1-w)}{2}, & \text{if } w > \frac{\delta}{2-\delta} \end{cases}.$$

Thus, we can rewrite the expected manufacturer's profit given the optimal R as

$$\pi_m(s^*,w)=\begin{cases} s^*\frac{3\delta-(2+\delta)w}{\delta(4-\delta)}w-\frac{k}{r}(s^*-\min\{1,\frac{1}{2k}\cdot\frac{(1-\delta)(2-w)^2}{(4-\delta)^2}\})^2, & \text{if } w\le\frac{\delta}{2}\\ s^*(1-\frac{w}{\delta})w-\frac{k}{r}(s^*-\min\{1,\frac{1}{2k}\cdot\frac{(\delta-w)(1-\delta)w}{\delta^2}\})^2, & \text{if } \frac{\delta}{2}<w\le\frac{\delta}{2-\delta}\\ s^*\frac{(1-w)}{2}w-\frac{k}{r}(s^*-\min\{1,\frac{1}{2k}\cdot\frac{(1-w)^2}{4}\})^2, & \text{if } w>\frac{\delta}{2-\delta}\end{cases}$$

Though the full analysis is complex due to the existence of multiple cases, we can use the following examples to show that showrooming can increase B&M retailer's profit:

Let $k=\frac{1}{20}$, $\delta=\frac{11}{20}$ and $r=1$. Then, in the no-showrooming case, $\pi_{B\&M}\approx 0.0308$ and $\pi_m\approx 0.1209$, whereas in the showrooming case, $\pi_{B\&M}=\frac{1}{16}=0.0625$ and $\pi_m=\frac{11}{80}=0.1375$.

The result can hold even if the manufacturer cannot incentivize service through direct compensation, and can use wholesale-price contracts only. In this case, $R=0$, and the manufacturer's profit is

$$\pi_m(w)=\begin{cases} \min\{1,\frac{1}{2k}\cdot\frac{(1-\delta)(2-w)^2}{(4-\delta)^2}\}\times\frac{3\delta-(2+\delta)w}{\delta(4-\delta)}w, & \text{if } w\le\frac{\delta}{2}\\ \min\{1,\frac{1}{2k}\cdot\frac{(\delta-w)(1-\delta)w}{\delta^2}\}\times(1-\frac{w}{\delta})w, & \text{if } \frac{\delta}{2}<w\le\frac{\delta}{2-\delta}\\ \min\{1,\frac{1}{2k}\cdot\frac{(1-w)^2}{4}\}\times\frac{(1-w)}{2}w, & \text{if } w>\frac{\delta}{2-\delta}\end{cases}$$

For example, when $k=\frac{1}{20}$, $\delta=\frac{11}{20}$ and $r=0$, then in the no-showrooming case, $\pi_{B\&M}=\frac{1}{20}=0.05$ and $\pi_m=\frac{\sqrt{10}}{10}-\frac{1}{5}\approx 0.1162$, whereas in the showrooming case, $\pi_{B\&M}=\frac{1}{16}=0.0625$ and $\pi_m=\frac{11}{80}=0.1375$.

Case: The manufacturer is allowed to set different wholesale prices to different retailers

When the manufacturer is allowed to set different wholesale prices to the B&M retailer and online retailer, the manufacturer's problem can be written as

$$\begin{aligned} \max_{w_1\le 1,w_2\le 1,0\le s^*\le 1,R\ge 0}\quad & D_{B\&M}(s^*,w_1,w_2)w_1+D_{online}(s^*,w_1,w_2)w_2-R\\ \text{s.t.}\quad & \pi_{B\&M}(s^*,w_1,w_2)+R\ge\max_{s\ne s^*}\pi_{B\&M}(s,w_1,w_2)+(1-r)R\end{aligned}$$

Here, to avoid the arbitrage between the offline and online retailers, we need to have $w_1 \leq p_2$ and $w_2 \leq p_1$. Thus, by substituting $\pi_{B\&M}$, $D_{B\&M}$ and D_{online} in different cases according to Equation (A.1.2), (A.1.3), and (A.1.4), we can rewrite the expected manufacturer's profit given the optimal R as

$$
\begin{cases}
s^* \frac{\delta - w_2}{2\delta} w_2 - \frac{k}{r} s^{*2}, & \text{if } w_1 \geq \frac{2-\delta+w_2}{2}, w_1 \leq \frac{\delta+w_2}{2} \\
& \quad \text{and } w_2 \leq w_1 \\
s^* \frac{1-w_1}{\delta} w_2 - \frac{k}{r} s^{*2}, & \text{if } \frac{2-\delta+w_2}{2} > w_1 \geq \frac{2(1-\delta)+w_2}{2-\delta}, \\
& \quad w_1 \leq w_1 - 1 + \delta, \\
& \quad \text{and } w_2 \leq w_1 \\
s^* \frac{2(1-\delta)-(2-\delta)w_1+w_2}{(1-\delta)(4-\delta)} w_1 + s^* \frac{\delta(1-\delta)+\delta w_1-(2-\delta)w_2}{\delta(1-\delta)(4-\delta)} w_2 & \\
\qquad - \frac{k}{r}(s^* - \min\{1, \frac{1}{2k} \cdot \frac{(2(1-\delta)-(2-\delta)w_1+w_2)^2}{(1-\delta)(4-\delta)^2}\})^2, & \text{if } \frac{2(1-\delta)+w_2}{2-\delta} > w_1 \geq \frac{-\delta(1-\delta)+(2-\delta)w_2}{\delta}, \\
& \quad w_1 \leq \frac{\delta(1-\delta)+\delta w_1+2w_2}{4-\delta}, \\
& \quad \text{and } w_2 \leq \frac{2(1-\delta)+2w_1+w_2}{4-\delta} \\
s^*(1 - \frac{w_2}{\delta}) w_1 - \frac{k}{r}(s^* - \min\{1, \frac{1}{2k} \cdot \frac{(\delta-w_2)(w_2-\delta w_1)}{\delta^2}\})^2, & \text{if } \frac{-\delta(1-\delta)+(2-\delta)w_2}{\delta} > w_1 \geq \frac{2w_2-\delta}{\delta}, \\
& \quad w_1 \leq w_2, \\
& \quad \text{and } w_2 \leq \frac{w_2}{\delta} \\
s^* \frac{1-w_1}{2} w_1 - \frac{k}{r}(s^* - \min\{1, \frac{1}{2k} \cdot \frac{(1-w_1)^2}{4}\})^2, & \text{if } w_1 < \frac{2w_2-\delta}{\delta}, w_1 \leq w_2, \\
& \quad \text{and } w_2 \leq \frac{1+w_1}{2}
\end{cases}
$$

when $r > 0$, and

$$
\begin{cases}
\min\{1, \frac{1}{2k} \cdot \frac{(2(1-\delta)-(2-\delta)w_1+w_2)^2}{(1-\delta)(4-\delta)^2}\}\times & \\
\quad \left(\frac{2(1-\delta)-(2-\delta)w_1+w_2}{(1-\delta)(4-\delta)}w_1 + \frac{\delta(1-\delta)+\delta w_1-(2-\delta)w_2}{\delta(1-\delta)(4-\delta)}w_2\right), & \text{if } \frac{2(1-\delta)+w_2}{2-\delta} > w_1 \geq \frac{-\delta(1-\delta)+(2-\delta)w_2}{\delta}, \\
& w_1 \leq \frac{\delta(1-\delta)+\delta w_1+2w_2}{4-\delta}, \\
& \text{and } w_2 \leq \frac{2(1-\delta)+2w_1+w_2}{4-\delta} \\
\min\{1, \frac{1}{2k} \cdot \frac{(\delta-w_2)(w_2-\delta w_1)}{\delta^2}\} \times (1-\frac{w_2}{\delta})w_1, & \text{if } \frac{-\delta(1-\delta)+(2-\delta)w_2}{\delta} > w_1 \geq \frac{2w_2-\delta}{\delta}, \\
& w_1 \leq w_2, \text{ and } w_2 \leq \frac{w_2}{\delta} \\
\min\{1, \frac{1}{2k} \cdot \frac{(1-w_1)^2}{4}\} \times \frac{1-w_1}{2}w_1, & \text{if } w_1 < \frac{2w_2-\delta}{\delta}, w_1 \leq w_2 \text{ and } w_2 \leq \frac{1+w_1}{2}
\end{cases}
$$

when $r = 0$.

Though the full analysis is complex due to the existence of multiple cases, we can use the following example to show that showrooming can increase B&M retailer's profit:

When $k = \frac{1}{20}$, $\delta = \frac{11}{20}$ and $r = 0$, then in the no-showrooming case, $\pi_{B\&M} = \frac{1}{20} = 0.05$ and $\pi_m = \frac{\sqrt{10}}{10} - \frac{1}{5} \approx 0.1162$, whereas in the showrooming case, $\pi_{B\&M} = \frac{1}{16} = 0.0625$ and $\pi_m = \frac{11}{80} = 0.1375$.

Analysis of the Model Variation With Hotelling-like Retailer Differentiation (Section 1.5.3)

We only present the case when $r = 0$, i.e., the manufacturer is restricted to wholesale-price-only contracts, since this is sufficient to establish the results we are looking for.

When consumers cannot showroom, they will purchase from the B&M retailer only if $V - tx - p_1 \geq 0$. Therefore, the consumer demand is $s\frac{(1-p_1)^2}{2t}$ if $p_1 + t \geq 1$ and $s(1 - p_1 - \frac{t}{2})$ if $p_1 + t < 1$. Thus, the B&M retailer's expected profit is

$$
\pi_{B\&M} = \begin{cases} s\frac{(1-p_1)^2}{2t}(p_1 - w) - ks^2, & \text{if } p_1 \geq 1 - t \\ s(1 - p_1 - \frac{t}{2})(p_1 - w) - ks^2, & \text{if } p_1 < 1 - t \end{cases}.
$$

Thus, when $w \geq \frac{2-3t}{2}$, we have $p_1 = \frac{1+2w}{3}$ and $s = \min\{1, \frac{(1-w)^3}{27kt}\}$, and when $w \geq \frac{2-3t}{2}$, we have $p_1 = \frac{2+2w-t}{4}$ and $s = \min\{1, \frac{(2-2w-t)^2}{32k}\}$. In turn, the manufacturer's expected profit is

$$\pi_m = \begin{cases} \min\{1, \frac{(1-w)^3}{27kt}\} \times \frac{2(1-w)^2 w}{9t}, & \text{if } w \geq \frac{2-3t}{2} \\ \min\{1, \frac{(2-2w-t)^2}{32k}\} \times \frac{2-2w-t}{4} w, & \text{if } w < \frac{2-3t}{2} \end{cases}.$$

When showrooming is allowed, consumers will purchase from the B&M retailer only if $V - tx - p_1 \geq \max\{0, V - t(1-x) - p_2\}$, and they will purchase from the online retailer only if $V - t(1-x) - p_2 \geq \max\{0, V - tx - p_1\}$. The expected consumer demand is reported in Table A.1.3.

Table A.1.3: Expected Consumer Demand (Showrooming)

Case	Offline	Online
$p_1 + p_2 \geq 2 - t$	$s\frac{(1-p_1)^2}{2t}$	$s\frac{(1-p_2)^2}{2t}$
$p_1 + p_2 < 2 - t$	$s\frac{(t+p_2-p_1)(4-t-p_2-3p_1)}{8t}$	$s\frac{(t+p_1-p_2)(4-t-p_1-3p_2)}{8t}$

Here, the B&M retailer's expected profit is

$$\pi_{B\&M} = \begin{cases} s\frac{(1-p_1)^2}{2t}(p_1 - w_1) - ks^2, & \text{if } p_1 \geq 2 - t - p_2 \\ s\frac{(t+p_2-p_1)(4-t-p_2-3p_1)}{8t}(p_1 - w_1) - ks^2, & \text{if } p_1 < 2 - t - p_2 \end{cases},$$

whereas the online retailer's expected profit is

$$\pi_{online} = \begin{cases} s\frac{(1-p_2)^2}{2t}(p_2 - w_2), & \text{if } p_2 \geq 2 - t - p_1 \\ s\frac{(t+p_1-p_2)(4-t-p_1-3p_2)}{8t}(p_2 - w_2), & \text{if } p_2 < 2 - t - p_1 \end{cases}.$$

Given the expected response of the retailers, the manufacturer chooses the offline wholesale price w_1 and the online wholesale price w_2 to maximize its expected profit.

Though the full analysis is complex due to the high degree of the polynomials, we use the following example to show how the possibility of wholesale-price discrimination can benefit the B&M retailer more under showrooming:

Let $k = \frac{1}{40}$ and $t = 1$. Then, though showrooming does not increase B&M retailer's profit when wholesale-price discrimination is not allowed, it increases B&M retailer's profit from $\pi_{B\&M} = \frac{78125}{4251528} \approx 0.0184$ to $\pi_{B\&M} \approx 0.0250$ when wholesale-price discrimination is allowed.

Proof of Proposition 1.5

The analysis of Sections 1.5.3 we presented above provides parameter values under which showrooming can increase B&M retailer's profit even if $r = 0$, i.e., the manufacturer is restricted to wholesale-price-only contracts. Furthermore, the examples provided in Sections 1.5.3 show that the B&M retailer profit may increase even when the manufacturer is able to set different wholesale prices to the two retailers. This completes the proof of Proposition 1.5. $\square$

Analysis of the Model Variation With the Possibility of Online Returns and Offline Shopping Costs (Section 1.5.4)

When showrooming is not possible, the online retailer can earn positive profit only if the product return service is provided. Moreover, since consumers will utilize the return service only if they intend to purchase online, providing the online return service is a (weakly) dominant strategy. Now, if the offline service level is $s = 1$, all shoppers will check the offline retail price p_1, and the non-shoppers will visit the offline retailer and check the offline retail price only if $V - \hat{p_1} - t \geq \max\{0, \delta V - p_2\}$, where $\hat{p_1}$ is the expected offline price. Once consumers arrive at the offline retailer, they will decide to purchase offline if $V - p_1 \geq \max\{0, \delta V - p_2\}$, and purchase online if $\delta V - p_2 > \max\{0, V - p_1\}$. On the other hand, if a consumer does not visit the offline retailer (either because the offline service $s = 0$ or she is a "non-shopper" and $V - \hat{p_1} - t < \max\{0, \delta V - p_2\}$) she will purchase online if $\delta V - p_2 \geq 0$.

Thus, when $s = 1$, we have

$$\pi_{online} = \begin{cases} \lambda(\frac{p_1-p_2}{1-\delta} - \frac{p_2}{\delta})(p_2 - w - u) + (1-\lambda)(\frac{\hat{p_1}-p_2+t}{1-\delta} - \frac{p_2}{\delta})(p_2 - w - u), & \\ & \text{if } \frac{p_1-p_2}{1-\delta} \geq \frac{p_2}{\delta} \text{ and } p_1 \leq \hat{p_1} + t \\ (\frac{p_1-p_2}{1-\delta} - \frac{p_2}{\delta})(p_2 - w - u), & \text{if } \frac{p_1-p_2}{1-\delta} \geq \frac{p_2}{\delta} \text{ and } p_1 > \hat{p_1} + t \\ (1-\lambda)(\frac{\hat{p_1}-p_2+t}{1-\delta} - \frac{p_2}{\delta})(p_2 - w - u), & \text{if } \frac{\hat{p_1}-p_2+t}{1-\delta} \geq \frac{p_2}{\delta} > \frac{p_1-p_2}{1-\delta} \\ 0, & \text{if otherwise} \end{cases} \cdot$$

and

$$\pi_{B\&M} = \begin{cases} \lambda(1 - \frac{p_1-p_2}{1-\delta})(p_1 - w) + (1-\lambda)(1 - \frac{\hat{p_1}-p_2+t}{1-\delta})(p_1 - w) - C + R, & \\ & \text{if } \frac{p_1-p_2}{1-\delta} \geq \frac{p_2}{\delta} \text{ and } p_1 \leq \hat{p_1} + t \\ (1 - \frac{p_1-p_2}{1-\delta})(p_1 - w) - C + R, & \text{if } \frac{p_1-p_2}{1-\delta} \geq \frac{p_2}{\delta} \text{ and } p_1 > \hat{p_1} + t \\ \lambda(1-p_1)(p_1 - w) + (1-\lambda)(1 - \frac{\hat{p_1}-p_2+t}{1-\delta})(p_1 - w) - C + R, & \\ & \text{if } \frac{\hat{p_1}-p_2+t}{1-\delta} \geq \frac{p_2}{\delta} > \frac{p_1-p_2}{1-\delta} \\ \lambda(1-p_1)(p_1 - w) + (1-\lambda)(1 - \hat{p_1} - t)(p_1 - w) - C + R, & \\ & \text{if } \frac{p_2}{\delta} > \frac{\hat{p_1}-p_2+t}{1-\delta} \text{ and } p_1 \leq \hat{p_1} + t \\ (1-p_1)(p_1 - w) - C + R, & \text{if } \frac{p_2}{\delta} > \frac{\hat{p_1}-p_2+t}{1-\delta} \text{ and } p_1 > \hat{p_1} + t \end{cases} .$$

When $s = 0$, on the other hand, we have

$$\pi_{online} = \begin{cases} (1 - \frac{p_2}{\delta})(p_2 - w - u), & \text{if } 1 \geq \frac{p_2}{\delta} \\ 0, & \text{if } 1 < \frac{p_2}{\delta} \end{cases},$$

and $\pi_{B\&M} = (1-r)R$. Note that in equilibrium, we have $\hat{p_1} = p_1$.

Moreover, by comparing the retailer's expected profit in different cases, we know that when $s = 1$, the online retailer's best response given p_1 is

$$B_{online}(p_1) = \begin{cases} \frac{\delta(p_1+t(1-\lambda))+w+u}{2}, & \text{if } p_1 \geq \frac{w+u}{\delta} + t\sqrt{1-\lambda} \\ \frac{\delta(p_1+t)+w+u}{2}, & \text{if } \frac{w+u}{\delta} + t\sqrt{1-\lambda} > p_1 \geq \frac{w+u}{\delta} - t \\ w+u, & \text{if } p_1 < \frac{w+u}{\delta} - t \end{cases} \tag{A.1.5}$$

whereas the B&M retailer's best response given p_2 is

$$B_{B\&M}(p_2) = \begin{cases} \frac{1-t(1-\lambda)+w\lambda+p_2-\delta}{1+\lambda}, & \text{if } \frac{1-t(1-\lambda)+w\lambda}{1+\lambda} \geq \frac{p_2}{\delta} - t,\ \frac{1-t(1-\lambda)+w\lambda+p_2(1-\lambda)-\delta(1+w\lambda)}{1+(1-2\delta)\lambda} \geq \frac{p_2}{\delta}, \\ & \qquad \text{and } \frac{1-t(1-\lambda)+w\lambda+p_2-\delta}{1+\lambda} \geq \frac{p_2}{\delta} \\ \frac{p_2}{\delta}, & \text{if } \frac{1-t(1-\lambda)+w\lambda}{1+\lambda} \geq \frac{p_2}{\delta} - t,\ \frac{1-t(1-\lambda)+w\lambda+p_2(1-\lambda)-\delta(1+w\lambda)}{1+(1-2\delta)\lambda} \geq \frac{p_2}{\delta} \\ & \qquad \text{and } \frac{1-t(1-\lambda)+w\lambda+p_2-\delta}{1+\lambda} < \frac{p_2}{\delta} \\ \frac{1-t(1-\lambda)+w\lambda+p_2(1-\lambda)-\delta(1+w\lambda)}{1+(1-2\delta)\lambda}, & \\ & \text{if } \frac{1-t(1-\lambda)+w\lambda}{1+\lambda} \geq \frac{p_2}{\delta} - t \text{ and } \frac{p_2}{\delta} > \frac{1-t(1-\lambda)+w\lambda+p_2(1-\lambda)-\delta(1+w\lambda)}{1+(1-2\delta)\lambda} \\ & \qquad \geq \frac{p_2}{\delta} - t \text{ and } \frac{1-t(1-\lambda)+w\lambda+p_2-\delta}{1+\lambda} < \frac{p_2}{\delta} \\ \frac{p_2}{\delta} - t, & \text{if } \frac{1-t(1-\lambda)+w\lambda}{1+\lambda} \geq \frac{p_2}{\delta} - t,\ \frac{1-t(1-\lambda)+w\lambda+p_2(1-\lambda)-\delta(1+w\lambda)}{1+(1-2\delta)\lambda} < \frac{p_2}{\delta} - t \\ & \qquad \text{and } \frac{1-t(1-\lambda)+w\lambda+p_2-\delta}{1+\lambda} < \frac{p_2}{\delta} \\ \frac{1-t(1-\lambda)+w\lambda}{1+\lambda}, & \text{if } \frac{1-t(1-\lambda)+w\lambda}{1+\lambda} < \frac{p_2}{\delta} - t,\ \frac{1-t(1-\lambda)+w\lambda+p_2(1-\lambda)-\delta(1+w\lambda)}{1+(1-2\delta)\lambda} < \frac{p_2}{\delta} - t \\ & \qquad \text{and } \frac{1-t(1-\lambda)+w\lambda+p_2-\delta}{1+\lambda} < \frac{p_2}{\delta} \end{cases} \tag{A.1.6}$$

In anticipation of the retailers' best responses, the manufacturer chooses w and R to maximize its expected profit.

When showrooming is allowed, offering the online return service is still a (weakly) dominant strategy for the online retailer. When the B&M service is $s = 1$, all shoppers will examine the product offline. We have

$$\pi_{online} = \begin{cases} \lambda(\frac{p_1-p_2}{1-\delta} - \frac{p_2}{\delta})(p_2 - w) + (1-\lambda)(\frac{\hat{p_1}-p_2+t}{1-\delta} - \frac{p_2}{\delta})(p_2 - w - u), \\ \qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad \text{if } \frac{p_1-p_2}{1-\delta} \geq \frac{p_2}{\delta} \text{ and } p_1 \leq \hat{p_1} + t \\ \lambda(\frac{p_1-p_2}{1-\delta} - \frac{p_2}{\delta})(p_2 - w) + (1-\lambda)(\frac{p_1-p_2}{1-\delta} - \frac{\hat{p_1}-p_2+t}{1-\delta})(p_2 - w) \\ \qquad\qquad\qquad +(1-\lambda)(\frac{\hat{p_1}-p_2+t}{1-\delta} - \frac{p_2}{\delta})(p_2 - w - u), \\ \qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad \text{if } \frac{p_1-p_2}{1-\delta} \geq \frac{p_2}{\delta} \text{ and } p_1 > \hat{p_1} + t \\ (1-\lambda)(\frac{\hat{p_1}-p_2+t}{1-\delta} - \frac{p_2}{\delta})(p_2 - w - u), & \text{if } \frac{\hat{p_1}-p_2+t}{1-\delta} \geq \frac{p_2}{\delta} > \frac{p_1-p_2}{1-\delta} \\ 0, & \text{if otherwise} \end{cases}.$$

and

$$\pi_{B\&M} = \begin{cases} \lambda(1 - \frac{p_1-p_2}{1-\delta})(p_1 - w) + (1-\lambda)(1 - \frac{\hat{p_1}-p_2+t}{1-\delta})(p_1 - w) - C + R, \\ \qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad \text{if } \frac{p_1-p_2}{1-\delta} \geq \frac{p_2}{\delta} \text{ and } p_1 \leq \hat{p_1} + t \\ (1 - \frac{p_1-p_2}{1-\delta})(p_1 - w) - C + R, & \text{if } \frac{p_1-p_2}{1-\delta} \geq \frac{p_2}{\delta} \text{ and } p_1 > \hat{p_1} + t \\ \lambda(1 - p_1)(p_1 - w) + (1-\lambda)(1 - \frac{\hat{p_1}-p_2+t}{1-\delta})(p_1 - w) - C + R, \\ \qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad \text{if } \frac{\hat{p_1}-p_2+t}{1-\delta} \geq \frac{p_2}{\delta} > \frac{p_1-p_2}{1-\delta} \\ \lambda(1 - p_1)(p_1 - w) + (1-\lambda)(1 - \hat{p_1} - t)(p_1 - w) - C + R, \\ \qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad \text{if } \frac{p_2}{\delta} > \frac{\hat{p_1}-p_2+t}{1-\delta} \text{ and } p_1 \leq \hat{p_1} + t \\ (1 - p_1)(p_1 - w) - C + R, & \text{if } \frac{p_2}{\delta} > \frac{\hat{p_1}-p_2+t}{1-\delta} \text{ and } p_1 > \hat{p_1} + t \end{cases}.$$

When the offline service is $s = 0$, we have

$$\pi_{online} = \begin{cases} (1 - \frac{p_2}{\delta})(p_2 - w - u), & \text{if } 1 \geq \frac{p_2}{\delta} \\ 0, & \text{if } 1 < \frac{p_2}{\delta} \end{cases},$$

and $\pi_{B\&M} = (1-r)R$. By comparing the retailer's expected profit in different cases, we know that when $s = 1$, the online retailer's best response given p_1 is

$$B_{online}(p_1) = \begin{cases} \frac{\delta(p_1+t(1-\lambda))+w+u(1-\lambda)}{2}, & \text{if } p_1 \geq \frac{w}{\delta} + \frac{(\delta t+u)}{\delta}\sqrt{1-\lambda} \\ \frac{\delta(p_1+t)+w+u}{2}, & \text{if } \frac{w}{\delta} + \frac{(\delta t+u)}{\delta}\sqrt{1-\lambda} > p_1 \geq \frac{w+u}{\delta} - t \\ w+u, & \text{if } p_1 < \frac{w+u}{\delta} - t \end{cases} \tag{A.1.7}$$

whereas the B&M retailer's best response is the same as in the no-showrooming case, i.e., Equation (A.1.6).

As we can see, whether showrooming is possible or not, as long as $s = 1$ in equilibrium, the online retailer's best response function given p_1 has a discontinuity point. As a result, for some medium value of w, there may not be pure-strategy equilibrium in the subgame of retailers' price competition. Though the full analysis is complicated due to the possibility of mix-strategy pricing equilibrium, we can use the following example to show that the main result remains to hold here:

Let $\delta = \frac{3}{4}$, $r = \frac{2}{5}$, $u = \frac{1}{100}$, $\lambda = \frac{4}{5}$, $t = \frac{1}{1000}$ and $C = \frac{2}{25}$. We first describe the equilibrium decisions. Then, we prove that no player would like to deviate from the proposed equilibrium decisions under the subgame perfect refinement. Last, we compare the equilibrium B&M retailer's profit with/without showooming and show that showrooming can benefit the B&M retailer.

Step 1: Equilibrium decisions in the no-showrooming case.

When showrooming is not possible, the equilibrium decisions are: $w = \frac{82846}{269375} \approx 0.307549$, $R = \frac{358882479}{23220125000} \approx 0.0154557$, $p_1 = \frac{990073}{2155000} \approx 0.459431$, and $p_2 = \frac{356799}{1077500} \approx 0.331136$. As a result, $\pi_{B\&M} = \frac{1076647437}{116100625000} \approx 0.0092734$.

Step 2: Prove that no player would like to deviate from the equilibrium decisions described in Step 1.

According to the retailers' best response functions listed in Equation (A.1.5) and (A.1.6), it is trivial that neither of the retailers would like to choose another retail price. Also, given

w, R and the expected sales, the B&M retailer would not like to deviate to $s = 0$. Now, it suffice to show that the manufacturer would not deviate, either. Note that based on the equilibrium decisions we proposed, the manufacturer's profit is $\pi_m = \frac{210524551}{1346875000} \approx 0.156306$.

First, we consider the cases with pure-strategy equilibrium in the subgame of retailers' price competition. The possible cases are:

(1) The manufacturer chooses $w < \frac{629}{1600} - \frac{19}{8000\sqrt{5}}$ and

$$R \geq -\frac{1}{r}\left(\lambda(1 - \frac{p_1^* - p_2^*}{1-\delta})(p_1^* - w) + (1-\lambda)(1 - \frac{p_1^* - p_2^* + t}{1-\delta})(p_1^* - w) - C\right),$$

where $p_1^* = \frac{2(1-\delta)-(2-\delta)(1-\lambda)t+(1+2\lambda)w+u}{2(1+\lambda)-\delta}$ and $p_2^* = \frac{\delta(1-\delta)+\lambda(1-\lambda)\delta t+(1+\lambda+\delta\lambda)w+(1+\lambda)u}{2(1+\lambda)-\delta}$. The expected manufacturer's profit is $(1 - \frac{p_2^*}{\delta})w - R$.

(2) The manufacturer chooses $w < \frac{629}{1600} - \frac{19}{8000\sqrt{5}}$ and

$$R < -\frac{1}{r}\left(\lambda(1 - \frac{p_1^* - p_2^*}{1-\delta})(p_1^* - w) + (1-\lambda)(1 - \frac{p_1^* - p_2^* + t}{1-\delta})(p_1^* - w) - C\right),$$

where $p_1^* = \frac{2(1-\delta)-(2-\delta)(1-\lambda)t+(1+2\lambda)w+u}{2(1+\lambda)-\delta}$ and $p_2^* = \frac{\delta(1-\delta)+\lambda(1-\lambda)\delta t+(1+\lambda+\delta\lambda)w+(1+\lambda)u}{2(1+\lambda)-\delta}$. The expected manufacturer's profit is $(1 - \frac{p_2^{**}}{\delta})w - R$, where $p_2^{**} = \frac{\delta+w+u}{2}$.

(3) The manufacturer chooses $\frac{48775-21\sqrt{5}}{80000} \leq w < \frac{611}{1000}$ and

$$R \geq -\frac{1}{r}\left(\lambda(1 - p_1^*)(p_1^* - w) + (1-\lambda)(1 - \frac{p_1^* - p_2^* + t}{1-\delta})(p_1^* - w) - C\right),$$

where $p_1^* = \frac{2(1-\delta)-(2-\delta)(1-\lambda)t+(1+\lambda-2\delta\lambda)w+(1-\lambda)u}{2(1+\lambda)-\delta(1+3\lambda)}$ and $p_2^* = \frac{\delta(1-\delta)(1+2\lambda t)+(1+\lambda-\delta\lambda-\delta^2\lambda)w+(1+\lambda-2\delta\lambda)u}{2(1+\lambda)-\delta(1+3\lambda)}$. The expected manufacturer's profit is $\lambda(1 - p_1^*)w + (1-\lambda)(1 - \frac{p_2^*}{\delta})w - R$.

(4) The manufacturer chooses $\frac{48775-21\sqrt{5}}{80000} \leq w < \frac{611}{1000}$ and

$$R < -\frac{1}{r}\left(\lambda(1 - p_1^*)(p_1^* - w) + (1-\lambda)(1 - \frac{p_1^* - p_2^* + t}{1-\delta})(p_1^* - w) - C\right),$$

where $p_1^* = \frac{2(1-\delta)-(2-\delta)(1-\lambda)t+(1+\lambda-2\delta\lambda)w+(1-\lambda)u}{2(1+\lambda)-\delta(1+3\lambda)}$ and $p_2^* = \frac{\delta(1-\delta)(1+2\lambda t)+(1+\lambda-\delta\lambda-\delta^2\lambda)w+(1+\lambda-2\delta\lambda)u}{2(1+\lambda)-\delta(1+3\lambda)}$. The expected manufacturer's profit is $(1 - \frac{p_2^{**}}{\delta})w - R$, where $p_2^{**} = \frac{\delta+w+u}{2}$.

(5) The manufacturer chooses $\frac{611}{1000} \leq w \leq 1$ and

$$R \geq = -\frac{1}{r}\left(\lambda(1-p_1^*)(p_1^*-w) + (1-\lambda)(1-p_1^*-t)(p_1^*-w) - C\right),$$

where $p_1^* = \frac{1-t(1-\lambda)+\lambda w}{1+\lambda}$ and $p_2 = w+u$. The expected manufacturer's profit is $\lambda(1-p_1^*)w + (1-\lambda)(1-p_1^*-t)w - R$.

(6) The manufacturer chooses $\frac{611}{1000} \leq w \leq \frac{37}{50}$ and

$$R <= -\frac{1}{r}\left(\lambda(1-p_1^*)(p_1^*-w) + (1-\lambda)(1-p_1^*-t)(p_1^*-w) - C\right),$$

where $p_1^* = \frac{1-t(1-\lambda)+\lambda w}{1+\lambda}$ and $p_2 = w+u$. The expected manufacturer's profit is $(1-\frac{p_2^{**}}{\delta})w - R$, where $p_2^{**} = \frac{\delta+w+u}{2}$.

By checking each of the above cases, we can conclude that the manufacturer cannot earn a higher profit by deviating from the equilibrium decisions proposed in Step 1. Especially, it is trivial to exclude Case (2), (4) and (6), because the manufacturer's expected profit under $s=0$ case satisfies $\pi_m \leq (1-\frac{\frac{\delta+w+u}{2}}{\delta})w = \frac{\delta-w-u}{2\delta}w \leq \frac{(\delta-u)^2}{8\delta} = \frac{1369}{15000} \approx 0.0912667$.

Then, we show that for the parameter values we consider, the manufacturer would not deviate to the region of w such that retailers would choose mixed pricing strategy, i.e., $\frac{629}{1600} - \frac{19}{8000\sqrt{5}} < w \leq \frac{48775-21\sqrt{5}}{80000}$. To prove this, we show that the manufacturer's highest possible profit through choosing $w \in (\frac{629}{1600} - \frac{19}{8000\sqrt{5}}, \frac{48775-21\sqrt{5}}{80000}]$ is smaller than the expected profit based on the equilibrium decisions we proposed, i.e., $\pi_m = \frac{210524551}{1346875000} \approx 0.156306$. As discussed above, it suffices to consider the case with $s=1$.

Note that the manufacturer's expected profit is $D_{industry}(w) \cdot w - R$. When $w \in (\frac{629}{1600} - \frac{19}{8000\sqrt{5}}, \frac{48775-21\sqrt{5}}{80000}]$, the highest possible $D_{industry}(w)$ is achieved when the retailers' price competition is intense. To get the lowest possible prices, we use a variation of Equation(A.1.5):

$$LowPrice_{online}(p_1) = \begin{cases} \frac{\delta(p_1+t(1-\lambda))+w+u}{2}, & \text{if } p_1 \geq \frac{w+u}{\delta} - t \\ w+u, & \text{if } p_1 < \frac{w+u}{\delta} - t \end{cases} \tag{A.1.8}$$

By combining Equation (A.1.8) and (A.1.6), we can obtain the lowest possible prices $p_{1,low}(w)$ and $p_{2,low}(w)$. Then, $D_{industry} \leq \lambda(1 - \min\{p_{1,low}(w), \frac{p_{2,low}(w)}{\delta}\}) + (1-\lambda)(1 - \min\{p_{1,low}(w) + t, \frac{p_{2,low}(w)}{\delta}\})$. The lowest possible R, on the other hand, is achieved when the retailers' price competition is not that intense, and the value of R is just enough to ensure $s = 1$. Here, to get the highest possible prices, we use another variation of Equation (A.1.5):

$$HighPrice_{online}(p_1) = \begin{cases} \frac{\delta(p_1+t)+w+u}{2}, & \text{if } p_1 \geq \frac{w+u}{\delta} - t \\ w+u, & \text{if } p_1 < \frac{w+u}{\delta} - t \end{cases} \tag{A.1.9}$$

By combining Equation (A.1.9) and (A.1.6), we can obtain the highest possible prices $p_{1,high}(w)$ and $p_{2,high}(w)$. Then,

$$R \geq -\frac{1}{r}[\lambda(1 - p_{1,high}(w))(p_{1,high}(w) - w) + (1-\lambda)(1 - \frac{p_{1,high}(w) - p_{2,high}(w) + t}{1-\delta})(p_{1,high}(w) - w) - C].$$

We can show that when $w \in (\frac{629}{1600} - \frac{19}{8000\sqrt{5}}, \frac{48775 - 21\sqrt{5}}{80000}]$,

$$\begin{aligned} \pi_m \leq{} & \lambda(1 - \min\{p_{1,low}(w), \frac{p_{2,low}(w)}{\delta}\}) + (1-\lambda)(1 - \min\{p_{1,low}(w) + t, \frac{p_{2,low}(w)}{\delta}\}) \cdot w + \frac{1}{r} \times \\ & [\lambda(1 - p_{1,high}(w))(p_{1,high}(w) - w) + (1-\lambda)(1 - \frac{p_{1,high}(w) - p_{2,high}(w) + t}{1-\delta})(p_{1,high}(w) - w) - C] \\ \leq{} & \frac{138966\sqrt{5} + 268514255}{1800000000} \approx 0.149347. \end{aligned}$$

This completes the proof that no player would like to deviate from the equilibrium decisions described in Step 1.

Step 3: Equilibrium decisions in the showrooming case.

When showrooming is possible, the equilibrium decisions are: $w = \frac{168527}{538750} \approx 0.312811$, $R = \frac{203076818439}{8382465125000} \approx 0.0242264$, $p_1 = \frac{19007947}{40945000} \approx 0.464231$, and $p_2 = \frac{6766461}{20472500} \approx 0.330515$. As a result, $\pi_{B\&M} = \frac{609230455317}{41912325625000} \approx 0.0145358$.

Step 4: Prove that no player would like to deviate from the equilibrium decisions described in Step 3. This proof is similar to Step 2.

Step 5: Comparison. Showrooming increases the B&M retailer's profit from $\pi_{B\&M,NS} = \frac{1076647437}{116100625000} \approx 0.0092734$ to $\pi_{B\&M,S} = \frac{609230455317}{41912325625000} \approx 0.0145358$.

Details of Other Extensions (Section 1.5.5)

Some Consumers do not Need a Showroom

As we are interested in the profit implications of showrooming, we have assumed in the main model an extreme case where all consumers need the informational service provided by the brick-and-mortar retailer in order to be interested in the product category, and all consumers would engage in showrooming unless such ability is prevented by the retailer. In reality, of course, it may be that not all consumers need information to value a purchase. If consumers are prevented from showrooming, some consumers may buy directly online without visiting the brick-and-mortar retailer, while others may decide to shop at the brick-and-mortar retailer. The relevant market for the brick-and-mortar retailer is then the segment of consumers who would purchase from it in the absence of showrooming. Let us normalize the size of this segment to one.

The simplest assumption of the online-offline competition in this case is the assumption that online market is competitive by itself, i.e., has two or more undifferentiated etailers.[2] In the absence of showrooming, the brick-and-mortar retailer only cares about the consumers who need showroom enough to buy from it. Let us assume that the resulting demand from this segment is $s(1-p)$ where s is the service level at the brick-and-mortar retailer and p is the price there. Thus, we are back to the setup we had in the main model, with the only difference that (if the manufacturer cannot price discriminate between the online and offline retailers), the optimal wholesale price may be affected by the online channel. Let us denote the size of the consumers who do not need a showroom by α. These consumers choose where to buy based on the price alone regardless of whether showrooming is possible and thus, end up buying online due to the lower price there. Thus, the online demand is $\alpha(1-w)$ where

[2] If there is only one online retailer, the solution is not analytically tractable because the retail price equilibrium is in mixed strategies.

w is the online price reduced to the wholesale price due to competition, then the optimal wholesale price in both channels happens to be $w = 1/2$. Thus, for $k \leq 1/32$, the solution of the main model applies with no changes. But for larger k, when the manufacturer would like to incentivize higher service level than the B&M retailer would choose given $w = 1/2$ and no direct compensation, it is more costly for the manufacturer to incentivize service through reducing wholesale price since it has an added cost of reducing price in the online channel below the online-optimal. Thus, both the B&M retailer and the manufacturer's profits will be below the ones in the main model.

With showrooming, the brick-and-mortar retailer becomes undifferentiated from the online ones (after consumers use its free service) and we are exactly back to the main model setup with showrooming. It then immediately follows that the brick-and-mortar retailer and the manufacturer would be better off due to showrooming in the respective ranges they were better off in the main model. Thus the potential for the positive profitability implications of showrooming is even bigger if online marketplace presents a serious competition to the brick-and-mortar retailers even in the absence of showrooming.

Note that in the above variations, while showrooming could be beneficial to the retailer depending on the market (parameter) conditions, the added competition from the online marketplace is not. In other words, the effect of competition is different from the effect of showrooming. Conceptually, the reason for the difference is that showrooming is costly and necessary to enhance demand, and therefore, the manufacturer is willing to provide incentives for it.

Two-segment Online/Offline Differentiation

As an alternative to Section 1.5.3, another way to model online-offline differentiation is by considering two-segment heterogeneity, where one segment is indifferent between shopping online and offline, and the other one is loyal to the B&M store (i.e., has zero value of online purchases). Assume in addition that the cost of service to the B&M retailer is proportional to the realized demand (there are at least two reasons to expect this: first, consumers who end up making the purchase in either channel are more interested in examining the product, and thus, spend longer time in the store, ask a lot of questions and/or use after-sale support; second, only the consumers who expect to buy a product visit the physical store to examine the products). Let us denote the fraction of consumers preferring to shop offline by $\rho \in (0,1)$. Then, the demand for B&M retailer under showrooming is $s\rho(1-p_1)$ if $p_1 > p_2$, and $s(1-p_1)$ if $p_1 \leq p_2$, where s is the service level, p_1 is the offline retail price, and p_2 is the online retail price. The demand for the online channel under showrooming, on the other hand, is $s(1-\rho)(1-p_2)$ if $p_1 > p_2$, and 0 if $p_1 \leq p_2$. To simplify the illustration, we assume that the online market is competitive, i.e., the online retail price p_2 will be driven down to w.

To capture the idea that the service cost is increasing in the realized demand, assume it to be kD, where D is the realized demand. That is, the cost of service is $k(1-p_1)s^2$ if showrooming is not possible or $p_1 \leq p_2$, and $k[\rho(1-p_1)+(1-\rho)(1-p_2)]s^2$ when showrooming is allowed and $p_1 > p_2$.

When consumers cannot showroom, the B&M retailer maximizes $s(1-p_1)(p_1-w)-k(1-p_1)s^2$ subject to $p_1 \in [w,1]$ and $s \in [0,1]$. The B&M retailer chooses $s=1$ and $p_1 = \frac{1+k+w}{2}$ if $0 < w \leq 1-3k$, and chooses $s = \frac{1-w}{3k}$ and $p_1 = \frac{2+w}{3}$ if $w > 1-3k$. Knowing this, the manufacturer seeks to

$$\max_{0\leq w\leq 1} s(1-p_1)w = \max_{0\leq w\leq 1} \begin{cases} \dfrac{1-w-k}{2}w, & \text{if } 0 < w \leq 1-3k \\ \dfrac{(1-w)^2 w}{9k}, & \text{if } w > 1-3k \end{cases}.$$

The equilibrium outcomes when showrooming is not possible are reported in Table A.1.4.

When showrooming is allowed, the B&M retailer maximizes $s\rho(1-p_1)(p_1-w)-k[\rho(1-p_1)+(1-\rho)(1-w)]s^2$ and the manufacturer maximizes $s[\rho(1-p_1)+(1-\rho)(1-w)]w$ given the optimal retailer response to w. The equilibrium outcomes are reported in Table A.1.5.

Table A.1.4: Equilibrium with Two-segment Online/Offline Differentiation (No Showrooming)

Range	Wholesale Price	Retail Price	Service Level	Manufacturer Profit	B&M Retailer Profit
$0 < k \leq \frac{1}{5}$	$w = \frac{1-k}{2}$	$p_1 = \frac{3+k}{4}$	$s = 1$	$\frac{(1-k)^2}{8}$	$\frac{(1-k)^2}{16}$
$\frac{1}{5} < k \leq \frac{2}{9}$	$w = 1 - 3k$	$p_1 = 1 - k$	$s = 1$	$k - 3k^2$	k^2
$\frac{2}{9} < k$	$w = \frac{1}{3}$	$p_1 = \frac{7}{9}$	$s = \frac{2}{9k}$	$\frac{2^2}{3^5 k}$	$\frac{2^3}{3^6 k}$

Table A.1.5: Equilibrium with Two-segment Online/Offline Differentiation (With Showrooming)

Range	$0 < k \leq \tilde{k}_1$	$\tilde{k}_1 < k \leq \tilde{k}_2$	$\tilde{k}_2 < k$
Wholesale Price	$\frac{2-\rho-\rho k}{2(2-\rho)}$	$\frac{\rho-k(B-2\rho+4)}{\rho}$	$\frac{1}{3}$
Retail Price	$\frac{3(2-\rho)+k(4-3\rho)}{4(2-\rho)}$	$\frac{2\rho-k(B-3\rho+4)}{2\rho}$	$\frac{-B+4\rho+4}{9\rho}$
Service Level	$s = 1$	$s = 1$	$\frac{2\left(\rho^2+(2-\rho)B+8\rho-8\right)}{9k\rho(B-\rho+2)}$
Manufacturer Profit	$\frac{(2-\rho-\rho k)^2}{8(2-\rho)}$	$\frac{k}{2\rho^2}[(\rho^2+(2-\rho)B-8\rho+8)\cdot$ $(-kB+2k\rho-4k+\rho)]$	$\frac{2\left(\rho^2+(2-\rho)B+8\rho-8\right)}{3^5 k\rho}$
B&M Retailer Profit	$\frac{k^2\rho^3+2k\left(\rho^3-10\rho^2+24\rho-16\right)+(2-\rho)^2\rho}{16(2-\rho)^2}$	$\frac{k^2\left(\rho^2-(\rho-2)B-8\rho+8\right)}{2\rho}$	$\frac{(-B+\rho+4)^2(B+5\rho-4)^2}{2^2 3^6 k\rho^2(B-\rho+2)}$

Notes. $B \equiv \sqrt{\rho^2 - 16\rho + 16}$, $\tilde{k}_1 \equiv \frac{-3\rho^3+22\rho^2-48\rho+32-2(2-\rho)^2 B}{\rho(5\rho^2-16\rho+16)}$ and $\tilde{k}_2 \equiv \frac{4(2-\rho)-2B}{9\rho}$.

Note that Table A.1.4 can be obtained by substituting $\rho = 1$ in Table A.1.5. Thus, we only need to prove Table A.1.5:

The first-order condition on the B&M retailer's profit maximization with respect to s under showrooming is $\rho(1-p_1)(p_1-w) - 2k[\rho(1-p_1) + (1-\rho)(1-w)]s = 0$, which implies $s = \frac{\rho(1-p_1)(p_1-w)}{2k[\rho(1-p_1)+(1-\rho)(1-w)]}$. Note that s is bounded by 1. Therefore, when w is small ($\frac{\rho(1-p_1)(p_1-w)}{2k[\rho(1-p_1)+(1-\rho)(1-w)]} \geq 1$), the B&M retailer chooses $s = 1$ and solves $\max_{0 \leq p_1 \leq 1} \rho(1-p_1)(p_1 - w) - k[\rho(1-p_1) + (1-\rho)(1-w)]$, where the first-order condition is $\rho(1 + w + k - 2p_1) =$

0. Thus, $p_1 = \frac{1+k+w}{2}$. Now, $\frac{\rho(1-p_1)(p_1-w)}{2k[\rho(1-p_1)+(1-\rho)(1-w)]} \geq 1$ yields $w \leq \frac{\rho-2k(2-\rho)-k\sqrt{\rho^2-16\rho+16}}{\rho}$. When w is large ($w > \frac{\rho-2k(2-\rho)-k\sqrt{\rho^2-16\rho+16}}{\rho}$), on the other hand, the first-order conditions $\rho(1-p_1)(p_1-w) - 2k[\rho(1-p_1)+(1-\rho)(1-w)]s = 0$ and $s\rho(1+w-2p_1)+k\rho s^2 = 0$ yield $s = \frac{(1-w)(4-2\rho-\sqrt{\rho^2-16\rho+16})}{3k\rho}$ and $p_1 = \frac{\left(4-\sqrt{\rho^2-16\rho+16}\right)(1-w)+\rho(5w+1)}{6\rho}$.

Knowing this, the manufacturer seeks to

$$\max_{0\leq w\leq 1} s[\rho(1-p_1)+(1-\rho)(1-w)]w =$$

$$= \begin{cases} \frac{[(2-\rho)(1-w)-\rho k]w}{2}, & \text{if } 0 < w \leq \frac{\rho-2k(2-\rho)-k\sqrt{\rho^2-16\rho+16}}{\rho} \\ \frac{\left(\rho^2+8\rho-8+(2-\rho)\sqrt{\rho^2-16\rho+16}\right)(1-w)^2 w}{18k\rho}, & \text{otherwise} \end{cases}$$

In the case of small w, the first-order condition is $\frac{2-\rho-\rho k-2(2-\rho)w}{2} = 0$, and the second-order condition is $-2+\rho \leq 0$. In the case of large w, the first-order condition is $\frac{\left(\rho^2+8\rho-8+(2-\rho)\sqrt{\rho^2-16\rho+16}\right)}{18k\rho}(1-w)(1-3w) = 0$, and the second-order condition is $\frac{\left(\rho^2+8\rho-8+(2-\rho)\sqrt{\rho^2-16\rho+16}\right)}{18k\rho}(6w-4) \leq 0$. Thus, when the boundary conditions are not binding, the optimal point for the small w case is $w = \frac{2-\rho-\rho k}{2(2-\rho)}$, and the optimal point for the large w case is $w = \frac{1}{3}$.

It is easy to check that when $k < \tilde{k}_1$ and $k > \tilde{k}_2$ (where the thresholds $\tilde{k}_1$ and $\tilde{k}_2$ are defined in Table A.1.5), the boundary conditions are not binding, and the results are shown in Columns 2 and 4 of Table A.1.5. When $\tilde{k}_1 < k < \tilde{k}_2$, the boundary condition is binding, and it is optimal to set $w = \frac{\rho-k\left(\sqrt{\rho^2-16\rho+16}-2\rho+4\right)}{\rho}$ (corner solution).

This proves Table A.1.5.

Note that the thresholds $\tilde{k}_1$ and $\tilde{k}_2$ in Table A.1.5 are both increasing in the value of ρ, the proportion of consumers retained by the B&M retailer when showrooming is possible. When $0 < k \leq \tilde{k}_1$, the wholesale price, the B&M retailer price and the manufacturer profit decrease in ρ, while the B&M retailer profit increases in ρ. When $\tilde{k}_1 < k \leq \tilde{k}_2$, the wholesale price, the B&M retailer price and the manufacturer profit increase in ρ, while the B&M retailer profit decreases in ρ. When $\tilde{k}_2 < k$, the B&M retail price, service level, the manufacturer profit and the B&M retailer profit all increase in ρ.

The manufacturer's profit is decreasing in ρ for small k because of the double-marginalization problem in the offline channel. When the full service is always achieved without extra incentive from the manufacturer (i.e. lowered wholesale price), the manufacturer prefers a larger online channel. When k is large, on the other hand, the manufacturer profit is increasing in ρ because the cost of the service-underprovision problem under showrooming outweighs the benefit of the reduced double-marginalization problem.

By comparing Table A.1.4 and Table A.1.5, we know that two-segment online/offline differentiation, our main result hold even if the manufacturer is limited to the wholesale-price-only contracts. As shown in Figure A.1.1, the consumer ability to showroom increases the B&M retailer's profit when ρ is large and k is in an intermediate range, and increases the manufacturer's profit when k is small. Moreover, the consumer ability to showroom increases both the B&M retailer's and the manufacturer's profit if ρ is large and k is in an intermediate range.

The equations on the boundary curves in Figure A.1.1 are:

Line 1: $0 < k \leq \frac{1}{5}$ and $\frac{(1-k)^2}{16} = \frac{(-B+\rho+4)^2(B+5\rho-4)^2}{2^2 3^6 k\rho^2(B-\rho+2)}$.

Line 2: $0 < k \leq \frac{1}{5}$ and $\frac{(1-k)^2}{8} = \frac{2\left(\rho^2+(2-\rho)B+8\rho-8\right)}{3^5 k\rho}$.

Line 3: $k - 3k^2 = \frac{2\left(\rho^2+(2-\rho)B+8\rho-8\right)}{3^5 k\rho}$.

Line 4: $\frac{(1-k)^2}{16} = \frac{k^2\left(\rho^2-(\rho-2)B-8\rho+8\right)}{2\rho}$.

Here, B is defined in Table A.1.5 and ρ^* is the second smallest root of $2304-2720\rho-2095\rho^2+2496\rho^3=0$.

Differentiation with Discrete Consumer Valuations

Another way to simplify the model while keeping the main insights in the base model is to assume discrete consumer valuation. Let us denote a consumer's valuation of getting the best-fitting product offline and online by V_1 and V_2 respectively.

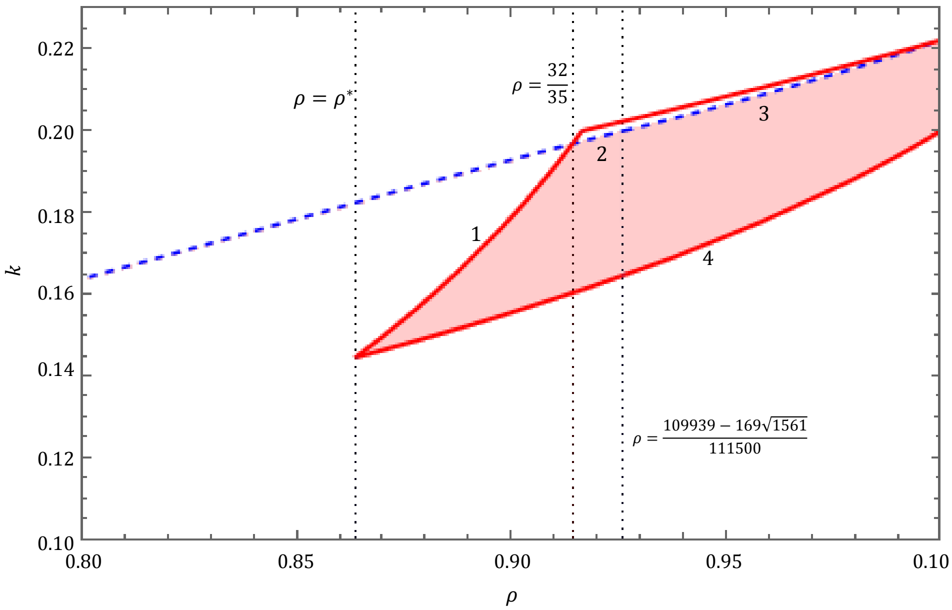

Figure A.1.1: Effect of Showrooming on Profits with Two-segment Online/Offline Differentiation

Notes. Showrooming increases B&M retailer's profit when the parameters are in the region within the thick/red curve. Showrooming increases manufacturer's profit when the parameters are in the region below the dashed/blue curve. The shaded area represents the region of parameters where both the B&M retailer and the manufacturer are better off due to showrooming.

Instead of assuming $V_1 = V_2 \sim U[0, 1]$ as in the base model, we now assume that V_i ($i = 1, 2$) independently follows a two-point distribution. Specifically, $V_i = \overline{v}$ with probability q, and $V_i = \underline{v}$ with probability $1 - q$. Thus, the consumer knowing the best-fitting product would fall into one of the four groups as listed in Table A.1.6. We remain to allow continuous service level and assume a quadratic service cost function $c(s) = ks^2$. Moreover, similar to the variation with smooth cost of service, in this model variation, we remove the limit on s, and only restrict it to be nonnegative. To simplify the analysis, we assume that the online market is competitive.

If the Direct Compensation for Service is allowed

Table A.1.6: Consumer Valuation Segments

Segment	Fraction	Valuation if Shopping Offline	Valuation if Shopping Online
A	q^2	$\overline{v}$	$\overline{v}$
B	$q(1-q)$	$\overline{v}$	$\underline{v}$
C	$q(1-q)$	$\underline{v}$	$\overline{v}$
D	$(1-q)^2$	$\underline{v}$	$\underline{v}$

Following the differentiated retailers' model in the main text, we first discuss the case when the manufacturer can use the direct compensation R to incentivize a suggested service level s^*.

When showrooming is not allowed, the B&M retailer is a monopoly. A consumer knowing her best-fitting product will make the purchase if and only if $V_1 - p_1 \geq 0$, where $V_1 = \overline{v}$ with probability q, and $V_1 = \underline{v}$ with probability $1-q$. Therefore, the B&M retailer will choose between (1) setting $p_1 = \overline{v}$ and only selling to the high-valuation consumers, and (2) setting $p_1 = \underline{v}$ and selling to all the consumers. Specifically, the optimal retail price is $p_1 = \overline{v}$ if $w \geq \frac{\underline{v}-q\overline{v}}{1-q}$, and $p_1 = \underline{v}$ otherwise. In the former case, the B&M retailer's profit can be written as

$$\max\{sq(\overline{v}-w) - ks^2 + (1-r)R, s^*q(\overline{v}-w) - ks^{*2} + R\},$$

whereas in the latter case, the B&M retailer's profit is

$$\max\{s(\underline{v}-w) - ks^2 + (1-r)R, s^*(\underline{v}-w) - ks^{*2} + R\}.$$

The B&M retailer chooses the optimal retail price and the service level given the manufacturer's decisions. And in expectation of the B&M retailer's response, the manufacturer chooses the wholesale price w, the suggested service level s^* and the direct compensation R

to maximize its expected profit. The manufacturer's expected profit can be written as:

$$\begin{cases} s^*qw - \frac{k}{r}(s^* - \frac{q(\overline{v}-w)}{2k})^2, & \text{if } \overline{v} \geq w \geq \frac{\underline{v}-q\overline{v}}{1-q} \\ s^*w - \frac{k}{r}(s^* - \frac{\underline{v}-w}{2k})^2, & \text{if } w < \frac{\underline{v}-q\overline{v}}{1-q} \end{cases}.$$

We report the equilibrium outcomes when showrooming is not allowed in Table A.1.7.

When showrooming is possible, consumers can choose to purchase from the competitive online market, where the retail price $p_2 = w$. We have three possible cases. First, if $w = \overline{v}$, then the B&M retailer cannot obtain positive margin from sales, and in order to get compensation from the manufacturer, the B&M retailer sets $p_1 = w = \overline{v}$. Second, if $\overline{v} > w > \underline{v}$, then $p_2 = w > \underline{v}$, and as long as $p_1 > w$, consumers in Segment A and C as listed in Table A.1.6 all purchase online. To obtain Segment B, it is optimal for the B&M retailer to set $p_1 = \overline{v}$. Third, if $w \leq \underline{v}$, then $p_2 = w \leq \underline{v}$, and as long as $p_1 > w$, consumers in Segment A, C and D all purchase online. To compete with the online retailers for Segment B, it is optimal for the B&M retailer to set $p_1 = \overline{v} - \underline{v} + w$. Again, in expectation of the B&M retailer's response, the manufacturer chooses w, s^* and R to maximize its expected profit. The manufacturer's expected profit can be written as:

$$\begin{cases} s^*(2q - q^2)w - \frac{k}{r}(s^* - \frac{q(1-q)(\overline{v}-w)}{2k})^2, & \text{if } \overline{v} \geq w > \underline{v} \\ s^*w - \frac{k}{r}(s^* - \frac{q(1-q)(\overline{v}-\underline{v})}{2k})^2, & \text{if } w \leq \underline{v} \end{cases}$$

We report the equilibrium outcomes when showrooming is possible in Table A.1.8.

When compare the profits with and without showrooming, we obtain that consumer ability to showroom may increase both the B&M retailer's and the manufacturer's profit. Moreover, it can happen when some consumers previously purchased offline switch to the online market. For example, let $\underline{v} = \frac{1}{4}$, $\overline{v} = 1$, $r = \frac{1}{2}$ and $q = \frac{1}{5}$. Then, when showrooming is not allowed, Case 1 as listed in Table A.1.7 applies. Consumers in Segment A and B purchase offline, and the equilibrium profits are $\pi_m = \frac{q^2\overline{v}^2}{4k(2-r)} = \frac{1}{150k}$ and $\pi_{B\&M} = \frac{(1-r)q^2\overline{v}^2}{4k(2-r)^2} = \frac{1}{450k}$.

Table A.1.7: Discrete Consumer Valuation (No Showrooming)

Range	$\begin{cases} \frac{\underline{v}}{2-r} \geq \frac{\underline{v}-q\overline{v}}{1-q} \\ M_{ns,1} > M_{ns,2} \end{cases}$	$\begin{cases} \frac{\underline{v}}{2-r} \geq \frac{\underline{v}-q\overline{v}}{1-q} \\ M_{ns,1} \leq M_{ns,2} \end{cases}$	$\frac{\underline{v}}{2-r} < \frac{\underline{v}-q\overline{v}}{1-q}$
Wholesale Price	$\frac{\overline{v}}{2-r}$	$\frac{\underline{v}-q\overline{v}}{1-q}$	$\frac{\underline{v}}{2-r}$
Offline Retail Price	$\overline{v}$	$\underline{v}$	$\underline{v}$
Suggested Service Level	$\frac{q\overline{v}}{2k(2-r)}$	$\frac{q(\overline{v}-\underline{v})+r(\underline{v}-q\overline{v})}{2k(1-q)}$	$\frac{\underline{v}}{2k(2-r)}$
Potential Service Level (if retailer deviates)	$\frac{(1-r)q\overline{v}}{2k(2-r)}$	$\frac{q(\overline{v}-\underline{v})}{2k(1-q)}$	$\frac{(1-r)\underline{v}}{2k(2-r)}$
Compensation R	$\frac{rq^2\overline{v}^2}{4k(2-r)^2}$	$\frac{r(\underline{v}-q\overline{v})^2}{4k(1-q)^2}$	$\frac{r\underline{v}^2}{4k(2-r)^2}$
Manufacturer Profit	$M_{ns,1} = \frac{q^2\overline{v}^2}{4k(2-r)}$	$M_{ns,2} = \frac{2q(\overline{v}-\underline{v})(\underline{v}-q\overline{v})+r(\underline{v}-q\overline{v})^2}{4k(1-q)^2}$	$\frac{\underline{v}^2}{4k(2-r)}$
B&M Retailer Profit	$\frac{(1-r)q^2\overline{v}^2}{4k(2-r)^2}$	$\frac{q^2(\overline{v}-\underline{v})^2+r(1-r)(\underline{v}-q\overline{v})^2}{4k(1-q)^2}$	$\frac{(1-r)\underline{v}^2}{4k(2-r)^2}$
Industry Profit	$\frac{(3-2r)q^2\overline{v}^2}{4k(2-r)^2}$	$\frac{(1-q)^2\underline{v}^2-(1-r)^2(\underline{v}-q\overline{v})^2}{4k(1-q)^2}$	$\frac{(3-2r)\underline{v}^2}{4k(2-r)^2}$
Segments	A and B	A, B, C and D	A, B, C and D

Table A.1.8: Discrete Consumer Valuation (Showrooming)

Range	$\begin{cases} \frac{\overline{v}(1-q)}{2(1-q)-r(2-q)} > \overline{v} \\ M_{s,1} > M_{s,3} \end{cases}$	$\begin{cases} \underline{v} < \frac{\overline{v}(1-q)}{2(1-q)-r(2-q)} \leq \overline{v} \\ M_{s,2} > M_{s,3} \end{cases}$	$\begin{cases} \frac{\overline{v}(1-q)}{2(1-q)-r(2-q)} > \overline{v} \\ M_{s,1} \leq M_{s,3} \end{cases}$, or $\begin{cases} \underline{v} < \frac{\overline{v}(1-q)}{2(1-q)-r(2-q)} \leq \overline{v} \\ M_{s,2} \leq M_{s,3} \end{cases}$, or $\frac{\overline{v}(1-q)}{2(1-q)-r(2-q)} \leq \underline{v}$
Wholesale Price	$\overline{v}$	$\frac{\overline{v}(1-q)}{2(1-q)-r(2-q)}$	$\underline{v}$
Offline Retail Price	$\overline{v}$	$\overline{v}$	$\overline{v}$
Suggested Service Level	$\frac{rq(2-q)\overline{v}}{2k}$	$\frac{q(1-q)^2\overline{v}}{2k(2(1-q)-r(2-q))}$	$\frac{q(1-q)(\overline{v}-\underline{v}+r\underline{v})}{2k}$
Potential Service Level (if retailer deviates)	0	$\frac{q(1-q)(1-q-r(2-q))\overline{v}}{2k(2(1-q)-r(2-q))}$	$\frac{q(1-q)(\overline{v}-\underline{v})}{2k}$
Compensation R	$\frac{rq^2(2-q)^2\overline{v}^2}{4k}$	$\frac{rq^2(2-q)^2(1-q)^2\overline{v}^2}{4k(2(1-q)-r(2-q))^2}$	$\frac{r\underline{v}^2}{4k}$
Manufacturer Profit	$M_{s,1} = \frac{rq^2(2-q)^2\overline{v}^2}{4k}$	$M_{s,2} = \frac{(2-q)q^2(1-q)^2\overline{v}^2}{4k(2(1-q)-r(2-q))}$	$M_{s,3} = \frac{2q(1-q)(\overline{v}-\underline{v})\underline{v}+r\underline{v}^2}{4k}$
B&M Retailer Profit	$\frac{rq^2(2-q)^2\overline{v}^2}{4k}$	$\frac{q^2(1-q)^2((1-q)^2+r(2q-q^2))\overline{v}^2}{4k(2(1-q)-r(2-q))^2}$	$\frac{q^2(1-q)^2(\overline{v}-\underline{v})^2+r(1-r)\underline{v}^2}{4k}$
Industry Profit	$\frac{rq^2(2-q)^2\overline{v}^2}{2k}$	$\frac{q^2(1-q)^3(5-3q-2r(2-q))\overline{v}^2}{4k(2(1-q)-r(2-q))^2}$	$\frac{((q-q^2)\overline{v}+(1-q+q^2)\underline{v})^2-(1-r)^2\underline{v}^2}{2k}$
Segments	Segment B offline Segment C online Segment A offline or online	Segment B offline Segment A and C online	Segment B offline Segment A, C and D online

When showrooming is possible, on the other hand, Case 3 as listed in Table A.1.8 applies. Consumers in Segment B purchase offline, and consumers in Segment A, C and D purchase online. Moreover, the equilibrium profits are $\pi_m = \frac{2q(1-q)(\overline{v}-\underline{v})\underline{v}+r\underline{v}^2}{4k} = \frac{73}{3200k} > \frac{1}{150k}$ and $\pi_{B\&M} = \frac{q^2(1-q)^2(\overline{v}-\underline{v})^2+r(1-r)\underline{v}^2}{4k} = \frac{1201}{160000k} > \frac{1}{450k}$.

To show that this result is not due to the restriction of setting homogeneous wholesale prices across two channels, we further report the results for the case when wholesale-price discrimination is allowed.

In anticipation of the retailers' response, the manufacturer's expected profit can be written as:

$$\begin{cases} s^*q(1-q)w_1 + s^*(1-q+q^2)w_2 - \frac{k}{r}(s^* - \frac{q(1-q)(\overline{v}-\underline{v}+w_2-w_1)}{2k})^2, \\ \qquad\qquad \text{if } w_2 \le \underline{v} \text{ and } w_1 \ge \frac{w_2(1-2q+2q^2)-q(1-q)(\overline{v}-\underline{v})}{1-2q+2q^2} \\ s^*(1-q+q^2)w_1 + s^*(q-q^2)w_2 - \frac{k}{r}(s^* - \frac{(1-q+q^2)(w_2-w_1)}{2k})^2, \\ \qquad\qquad \text{if } w_2 \le \underline{v} \text{ and } \frac{w_2(1-2q+2q^2)-q(1-q)(\overline{v}-\underline{v})}{1-2q+2q^2} > w_1 \ge \frac{q(1-q)w_2-\overline{v}+\underline{v}}{q(1-q)} \\ s^*w_1 - \frac{k}{r}(s^* - \frac{w_2-\overline{v}+\underline{v}-w_1}{2k})^2, \\ \qquad\qquad \text{if } w_2 \le \underline{v} \text{ and } w_1 < \frac{q(1-q)w_2-\overline{v}+\underline{v}}{q(1-q)} \\ s^*q(1-q)w_1 + s^*qw_2 - \frac{k}{r}(s^* - \frac{q(1-q)(\overline{v}-w_1)}{2k})^2, \\ \qquad\qquad \text{if } \overline{v} \ge w_2 > \underline{v} \text{ and } q(1-q)(\overline{v}-w_1) \ge MUnitRev(w_1,w_2) \\ s^*qw_1 + s^*q(1-q)w_2 - \frac{k}{r}(s^* - \frac{q(w_1-w_1)}{2k})^2, \\ \qquad\qquad \text{if } \overline{v} \ge w_2 > \underline{v} \text{ and } q(w_2-w_1) \ge MUnitRev(w_1,w_2) \\ s^*(1-q+q^2)w_1 + s^*q(1-q)w_2 - \frac{k}{r}(s^* - \frac{(1-q+q^2)(\underline{v}-w_1)}{2k})^2, \\ \qquad\qquad \text{if } \overline{v} \ge w_2 > \underline{v} \text{ and } (1-q+q^2)(\underline{v}-w_1) \ge MUnitRev(w_1,w_2) \\ s^*w_1 - \frac{k}{r}(s^* - \frac{w_2-\overline{v}+\underline{v}-w_1}{2k})^2, \\ \qquad\qquad \text{if } \overline{v} \ge w_2 > \underline{v} \text{ and } w_2-\overline{v}+\underline{v}-w_1 \ge MUnitRev(w_1,w_2) \end{cases}$$

where $MUnitRev(w_1, w_2) = \max\{q(1-q)(\overline{v}-w_1), q(w_2-w_1), (1-q+q^2)(\underline{v}-w_1), w_2-\overline{v}+\underline{v}-w_1\}$ is the manufacturer's sales revenue at a unit service level. By checking the inner and corner solutions, we conclude that the equilibrium outcomes will fall into one of the following cases:

Case 1 (which holds only if $\frac{\underline{v}(1-2q+2q^2)-q(1-q)(\overline{v}-\underline{v})}{1-2q+2q^2} \le \frac{q(1-q)\overline{v}-(1-q+q^2)(1-r)\underline{v}}{q(1-q)(2-r)}$)

Here, $w_1 = \frac{q(1-q)\overline{v}-(1-q+q^2)(1-r)\underline{v}}{q(1-q)(2-r)}$, $w_2 = \underline{v}$, $s^* = \frac{q(1-q)\overline{v}+(1-q+q^2)\underline{v}}{2k(2-r)}$ and $R = \frac{r((q-q^2)\overline{v}+(1-q+q^2)\underline{v})^2}{4k(2-r)^2}$. The manufacturer's profit is $\frac{((q-q^2)\overline{v}+(1-q+q^2)\underline{v})^2}{4k(2-r)}$, and the B&M retailer's profit is $\frac{(1-r)((q-q^2)\overline{v}+(1-q+q^2)\underline{v})^2}{4k(2-r)^2}$.

Case 2 (which holds only if $\frac{\underline{v}(1-2q+2q^2)-q(1-q)(\overline{v}-\underline{v})}{1-2q+2q^2} > \frac{q(1-q)\overline{v}-(1-q+q^2)(1-r)\underline{v}}{q(1-q)(2-r)}$)

Here, $w_1 = \frac{q(1-q)\overline{v}-(1-q+q^2)(1-r)\underline{v}}{q(1-q)(2-r)}$, $w_2 = \underline{v}$, $s^* = \frac{q(1-q)(1-q+q^2)(\overline{v}-\underline{v})+r((1-q+q^2)^2\underline{v}-q^2(1-q)^2\overline{v})}{2k(1-2q+2q^2)}$ and $R = \frac{r((1-q+q^2)^2\underline{v}-q^2(1-q)^2\overline{v})^2}{4k(1-2q+2q^2)^2}$. The manufacturer's profit is $\frac{((1-q+q^2)^2\underline{v}-(q-q^2)^2\overline{v})(2q(1-q)(1-q+q^2)(\overline{v}-\underline{v})+r((1-q+q^2)^2\underline{v}-(q-q^2)^2\overline{v}))}{4k(1-2q+2q^2)^2}$, and the B&M retailer's profit is $\frac{(\overline{v}-\underline{v})^2(1-q+q^2)^2q^2(1-q)^2+r(1-r)((1-q+q^2)^2\underline{v}-(q-q^2)^2\overline{v})^2}{4k(1-2q+2q^2)^2}$.

Case 3: (which holds only if $\frac{(q+r-qr)\overline{v}}{2-r} \ge \frac{\underline{v}-q\overline{v}}{1-q}$)

Here, $w_1 = \frac{(q+r-qr)\overline{v}}{2-r}$, $w_2 = \overline{v}$, $s^* = \frac{(2-q)q\overline{v}}{2k(2-r)}$ and $R = \frac{rq^2(2-q)^2\overline{v}^2}{4k(2-r)^2}$. The manufacturer's profit is $\frac{(2-q)^2q^2\overline{v}^2}{4k(2-r)}$, and the B&M retailer's profit is $\frac{(1-r)(2-q)^2q^2\overline{v}^2}{4k(2-r)^2}$.

Case 4: (which holds only if $\frac{(q+r-qr)\overline{v}}{2-r} < \frac{\underline{v}-q\overline{v}}{1-q}$)

Here, $w_1 = \frac{\underline{v}-q\overline{v}}{1-q}$, $w_2 = \overline{v}$, $s^* = \frac{q(\overline{v}-\underline{v})+rq((1-3q+q^2)\overline{v}+\underline{v})}{2k(1-q)}$, and $R = \frac{q^2r((1-3q+q^2)\overline{v}+\underline{v})^2}{4k(1-q)^2}$. The manufacturer's profit is $\frac{q^2((1-3q+q^2)\overline{v}+\underline{v})(2(\overline{v}-\underline{v})+r((1-3q+q^2)\overline{v}+\underline{v}))}{4k(1-q)^2}$, and the B&M retailer's profit is $\frac{q^2((\overline{v}-\underline{v})^2+r(1-r)((1-3q+q^2)\overline{v}+\underline{v})^2)}{4k(1-q)^2}$.

Case 5: (which holds only if $\frac{(1-q+q^2)(-1+4q-4q^2+2q^3+q(2-4q+3q^2-q^3)r)\underline{v}}{q(2-2q+q^2)(2-2q+2q^2-q(2-2q+q^2)r)} \le \frac{(1-q+q^2)\underline{v}-(q-q^2)\overline{v}}{1-2q+2q^2}$ and $\frac{(1-q+q^2)(1-2q+5q^2-4q^3+2q^4-q(2-4q+5q^2-3q^3+q^4)r)\underline{v}}{q^2(2-2q+q^2)(2-2q+2q^2-q(2-2q+q^2)r)} \le \frac{(1-q)\overline{v}+q^2\underline{v}}{1-q+q^2}$)

Here, $w_1 = \frac{(1-q+q^2)(-1+4q-4q^2+2q^3+q(2-4q+3q^2-q^3)r)\underline{v}}{q(2-2q+q^2)(2-2q+2q^2-q(2-2q+q^2)r)}$,
$w_2 = \frac{(1-q+q^2)(1-2q+5q^2-4q^3+2q^4-q(2-4q+5q^2-3q^3+q^4)r)\underline{v}}{q^2(2-2q+q^2)(2-2q+2q^2-q(2-2q+q^2)r)}$, $s^* = \frac{(1-q+q^2)^2\underline{v}}{2kq(2-2q+q^2)(2-2q+2q^2-q(2-2q+q^2)r)}$ and $R = \frac{(1-q+q^2)^2r\underline{v}^2}{4k(-2+2q-2q^2+q(2-2q+q^2)r)^2}$. The manufacturer's profit is $\frac{(1-q+q^2)^2\underline{v}^2}{4kq(2-2q+q^2)(2-2q+2q^2-q(2-2q+q^2)r)}$, and the B&M retailer's profit is
$\frac{(1-q+q^2)^2((1-q+q^2)^2-q(4-12q+18q^2-14q^3+6q^4-q^5)r)\underline{v}^2}{4kq^2(2-2q+q^2)^2(-2+2q-2q^2+q(2-2q+q^2)r)^2}$.

Case 6: (which holds only if $\frac{(1-q+q^2)(-1+4q-4q^2+2q^3+q(2-4q+3q^2-q^3)r)\underline{v}}{q(2-2q+q^2)(2-2q+2q^2-q(2-2q+q^2)r)} > \frac{(1-q+q^2)\underline{v}-(q-q^2)\overline{v}}{1-2q+2q^2}$)

Here, $w_1 = \frac{(1-q+q^2)\underline{v}-(q-q^2)\overline{v}}{1-2q+2q^2}$, $w_2 = \frac{(1-q)^3\overline{v}+q(1-q+q^2)\underline{v}}{1-2q+2q^2}$,
$s^* = \frac{q(1-2q+2q^2-q^3)(\overline{v}-\underline{v})+((1-2q+2q^2)\underline{v}+q^2(-2+4q-3q^2+q^3(\overline{v}-\underline{v})))r}{2k(1-2q+2q^2)}$ and
$R = \frac{((1-2q+2q^2)\underline{v}+q^2(-2+4q-3q^2+q^3(\overline{v}-\underline{v})))^2r}{4k(1-2q+2q^2)^2}$. The manufacturer's profit is
$\frac{((1-2q+2q^2)\underline{v}+q^2(-2+4q-3q^2+q^3(\overline{v}-\underline{v})))(2q(1-2q+2q^2-q^3)(\overline{v}-\underline{v})+((1-2q+2q^2)\underline{v}+q^2(-2+4q-3q^2+q^3(\overline{v}-\underline{v})r)}{4k(1-2q+2q^2)^2}$, and the B&M retailer's profit is $s^*(1-q+q^2)(\underline{v}-w_1)-ks^{*2}+R$.

Case 7: (which holds only if
$\underline{v} < \frac{(1-q+q^2)(q(1-q)(1-q+q^2)\underline{v}-(1+q^2)(2-2q+q^2)\overline{v}+(1-q+2q^2-2q^3+q^4)(\overline{v}-(q-q^2)\underline{v})r)}{r(1-q+2q^2-2q^3+q^4)^2-2(1-q+q^2)(1-q+2q^2-2q^3+q^4)} \leq \frac{(1-q)\overline{v}+q^2\underline{v}}{1-q+q^2}$)

Here, $w_1 = \frac{(1-q+q^2)((q-q^2)\overline{v}-(1-q+3q^2-4q^3+2q^4)\underline{v})+q(1-q)(1-q+2q^2-2q^3+q^4)((q-q^2)\underline{v}-\overline{v})r}{r(1-q+2q^2-2q^3+q^4)^2-2(1-q+q^2)(1-q+2q^2-2q^3+q^4)}$,
$w_2 = \frac{(1-q+q^2)(q(1-q)(1-q+q^2)\underline{v}-(1+q^2)(2-2q+q^2)\overline{v}+(1-q+2q^2-2q^3+q^4)(\overline{v}-(q-q^2)\underline{v})r)}{r(1-q+2q^2-2q^3+q^4)^2-2(1-q+q^2)(1-q+2q^2-2q^3+q^4)}$,
$s^* = \frac{(1-q+q^2)^2((q-q^2)\overline{v}+(1-q+q^2)\underline{v})}{2k(1-q+2q^2-2q^3+q^4)(2-2q+2q^2-(1-q+2q^2-2q^3+q^4)r)}$ and $R = \frac{(1-q+q^2)^2((q-q^2)\overline{v}+(1-q+q^2)\underline{v})^2r}{4k(2-2q+2q^2-(1-q+2q^2-2q^3+q^4)r)^2}$. The manufacturer's profit is $\frac{(1-q+q^2)^2((q-q^2)\overline{v}+(1-q+q^2)\underline{v})^2}{4k(1-q+2q^2-2q^3+q^4)(2-2q+2q^2-(1-q+2q^2-2q^3+q^4)r)}$, and the B&M retailer's profit is $\frac{(1-q+q^2)^2((q-q^2)\overline{v}+(1-q+q^2)\underline{v})^2((1-q+q^2)^2-(1-q+2q^3-q^4)(1-q+2q^2-2q^3+q^4)r)}{4k(1-q+2q^2-2q^3+q^4)^2(2-2q+2q^2-(1-q+2q^2-2q^3+q^4)r)^2}$.

Case 8: (which holds only if
$\frac{(1-q)\overline{v}+q^2\underline{v}}{1-q+q^2} < \frac{(1-q+q^2)(q(1-q)(1-q+q^2)\underline{v}-(1+q^2)(2-2q+q^2)\overline{v}+(1-q+2q^2-2q^3+q^4)(\overline{v}-(q-q^2)\underline{v})r)}{r(1-q+2q^2-2q^3+q^4)^2-2(1-q+q^2)(1-q+2q^2-2q^3+q^4)} \leq \overline{v}$)

Here, $w_1 = \frac{(1-q+q^2)\underline{v}-(q-q^2)\overline{v}}{1-2q+2q^2}$, $w_2 = \frac{(1-2q+2q^2)\overline{v}-q^2(1-q)^2(\overline{v}-\underline{v})}{1-2q+2q^2}$,
$s = \frac{q(1-q)(1-q+q^2)(\overline{v}-\underline{v})+((1-2q+2q^2)\underline{v}-q^2(1-q)^2(1+q-q^2)(\overline{v}-\underline{v}))r}{2k(1-2q+2q^2)}$ and
$R = \frac{((1-2q+2q^2)\underline{v}-q^2(1-q)^2(1+q-q^2)(\overline{v}-\underline{v}))^2r}{4k(1-2q+2q^2)^2}$. The manufacturer's profit is
$\frac{(((1-2q+2q^2)\underline{v}-q^2(1-q)^2(1+q-q^2)(\overline{v}-\underline{v}))(2q(1-q)(1-q+q^2)(\overline{v}-\underline{v})+r(1-2q+2q^2)\underline{v}-q^2(1-q)^2(1+q-q^2)(\overline{v}-\underline{v}))}{4k(1-2q+2q^2)^2}$, and the B&M retailer's profit is $s^*(1-q+q^2)(\underline{v}-w_1)-ks^{*2}+R$.

Case 9: (which holds only if $\frac{\underline{v}}{2-r} \leq \frac{\underline{v}-q\overline{v}}{1-q}$)

Here, $w_1 = \frac{\underline{v}}{2-r}$, $w_2 = \overline{v}$, $s^* = \frac{\underline{v}}{2k(2-r)}$ and $R = \frac{r\underline{v}^2}{4k(2-r)^2}$. The manufacturer's profit is $\frac{\underline{v}^2}{4k(2-r)}$, and the B&M retailer's profit is $\frac{(1-r)\underline{v}^2}{4k(2-r)^2}$.

Case 10: (which holds only if $\frac{\underline{v}}{2-r} > \frac{\underline{v}-q\overline{v}}{1-q}$)

Here, $w_1 = \frac{\underline{v}-q\overline{v}}{1-q}$, $w_2 = \overline{v}$, $s^* = \frac{q(\overline{v}-\underline{v})+r(\underline{v}-q\overline{v})}{2k(1-q)}$ and $R = \frac{r(\underline{v}-q\overline{v})^2}{4k(1-q)^2}$. The manufacturer's profit is $\frac{2q(\overline{v}-\underline{v})(\underline{v}-q\overline{v})+r(\underline{v}-q\overline{v})^2}{4k(1-q)^2}$, and the B&M retailer's profit is $\frac{q^2(\overline{v}-\underline{v})^2+r(1-r)(\underline{v}-q\overline{v})^2}{4k(1-q)^2}$.

By comparing w_1 and w_2 in the above cases, it can be shown that w_1 may be smaller than w_2. In other words, when the wholesale price discrimination is allowed, the manufacturer may charge an even lower wholesale price in the offline channel. In fact, using the same example as discussed above (i.e., $\underline{v} = \frac{1}{4}$, $\overline{v} = 1$, $r = \frac{1}{2}$ and $q = \frac{1}{5}$), we can show that the possibility of wholesale-price discrimination may benefit the B&M retailer profit more in the showrooming case:

When $\underline{v} = \frac{1}{4}$, $\overline{v} = 1$, $r = \frac{1}{2}$ and $q = \frac{1}{5}$, the manufacturer's profits in the above 10 cases are, respectively, $\frac{1369}{60000k}$, 0, $\frac{27}{1250k}$,0, 0, 0, 0, $\frac{317919889}{14450000000k}$, 0, and $\frac{13}{2048k}$.[3] Here, the first case yields the highest expected profit. Thus, in equilibrium, Case 1 applies. Moreover, the equilibrium outcomes are $w_1 = \frac{q(1-q)\overline{v}-(1-q+q^2)(1-r)\underline{v}}{q(1-q)(2-r)} = \frac{11}{48} < \underline{v} = w_2$, and $\pi_{B\&M} = \frac{(1-r)((q-q^2)\overline{v}+(1-q+q^2)\underline{v})^2}{4k(2-r)^2} = \frac{1369}{180000k} > \frac{1201}{160000k} > \frac{1}{450k}$. Here, $\frac{1}{450k}$ is the equilibrium B&M retailer's profit when showrooming is not possible, whereas $\frac{1201}{160000k}$ is the equilibrium B&M retailer's profit when showrooming is allowed and the manufacturer charges the same wholesale price in online/offline channels.

Wholesale-Price-Only Contract

We now discuss the case when the manufacturer cannot use direct compensation to incentivize the suggested (higher) service level, and it is restricted to the wholesale-price-only contract.

This case can be thought as the extreme case of the previous subsection, where incentivizing a higher service level through direct compensation is too costly (i.e., $r \to 0$), and the manufacturer incentivizes the service provision solely through adjusting wholesale price. Not surprisingly, the equilibrium outcomes with wholesale-price-only contract exactly correspond to the $r = 0$ case as listed in the previous subsection.

We use the following examples to show that the consumer ability to showroom may increase both the B&M retailer's and the manufacturer's profit under wholesale-price-contract:

[3]For these parameter values, the conditions of cases 2, 4, 5, 6, 7, and 9 cannot be satisfied.

If wholesale price discrimination is not allowed, let $\underline{v} = \frac{1}{4}$, $\overline{v} = 1$ and $q = \frac{1}{4}$. Then, when showrooming is not possible, $\pi_m = \frac{1}{2^7 k}$ and $\pi_{B\&M} = \frac{1}{2^8 k}$; whereas when showrooming is possible, $\pi_m = \frac{3^2}{2^9 k}$ and $\pi_{B\&M} = \frac{3^4}{2^{14} k}$.

If wholesale price discrimination is allowed, let $\underline{v} = \frac{1}{4}$, $\overline{v} = 1$ and $q = \frac{2}{5}$. Then, when showrooming is not possible, $\pi_m = \frac{1}{50k}$ and $\pi_{B\&M} = \frac{1}{100k}$; whereas when showrooming is possible, $\pi_m = \frac{2^5}{5^4 k}$ and $\pi_{B\&M} = \frac{2^4}{5^4 k}$.

Proof of Lemma 2.1

Note that $\min_j d(x_e, l_j)$ is distributed uniformly on $(0, \frac{1}{2n})$. If we denote the quality of the product by q, the probability of positive expert opinion conditional on the product quality is

$$\begin{aligned} Prob(\text{positive}|q) &= Prob(q - \min_j d(x_e, l_j) \cdot t > u_0 | q) \\ &= Prob(\min_j d(x_e, l_j) < \frac{q - u_0}{t}) \\ &= \begin{cases} 0, & \text{if } q \le u_0 \\ \frac{2n(q-u_0)}{t}, & \text{if } 0 < n \le \frac{t}{2(q-u_0)} \\ 1, & \text{if } n > \frac{t}{2(q-u_0)} > 0 \end{cases} . \end{aligned}$$

Therefore, the probability of a positive opinion is

$$\begin{aligned} Prob(\text{positive}) &= Prob(\text{pos}|q = 1) \cdot Prob(q = 1) + Prob(\text{pos}|q = q_0) \cdot Prob(q = q_0) \\ &= \frac{1}{2} Prob(\text{pos}|q = 1) + \frac{1}{2} Prob(\text{pos}|q = q_0). \end{aligned}$$

If $q_0 > u_0$, we have

$$Prob(\text{positive}) = \begin{cases} \frac{1}{2} \times \frac{2n(1-u_0)}{t} + \frac{1}{2} \times \frac{2n(q_0-u_0)}{t}, & \text{if } n \leq \frac{t}{2(1-u_0)} \\ \frac{1}{2} \times 1 + \frac{1}{2} \times \frac{2n(q_0-u_0)}{t}, & \text{if } \frac{t}{2(1-u_0)} < n \leq \frac{t}{2(q_0-u_0)} \\ \frac{1}{2} \times 1 + \frac{1}{2} \times 1, & \text{if } n > \frac{t}{2(q_0-u_0)} \end{cases}$$

$$= \begin{cases} \frac{n(1+q_0-2u_0)}{t}, & \text{if } n \leq \frac{t}{2(1-u_0)} \\ \frac{t+2n(q_0-u_0)}{2t}, & \text{if } \frac{t}{2(1-u_0)} < n \leq \frac{t}{2(q_0-u_0)} \\ 1, & \text{if } n > \frac{t}{2(q_0-u_0)} \end{cases}.$$

If $q_0 \leq u_0$, on the other hand, we have

$$Prob(\text{positive}) = \begin{cases} \frac{1}{2} \times \frac{2n(1-u_0)}{t} + \frac{1}{2} \times 0, & \text{if } n \leq \frac{t}{2(1-u_0)} \\ \frac{1}{2} \times 1 + \frac{1}{2} \times 0, & \text{if } n > \frac{t}{2(1-u_0)} \end{cases}$$

$$= \begin{cases} \frac{n(1-u_0)}{t}, & \text{if } n \leq \frac{t}{2(1-u_0)} \\ \frac{1}{2}, & \text{if } n > \frac{t}{2(1-u_0)} \end{cases}.$$

This completes the proof of Lemma 2.1.

Proof of Lemma 2.2

First, we derive the case of $q_0 > u_0$. By Bayes' Theorem, the probability that the product quality is high conditional on a positive expert opinion is

$$Prob(q = 1|\text{positive}) = \frac{Prob(\text{positive}|q = 1) \cdot Prob(q = 1)}{Prob(\text{positive})} = \frac{1}{2} \cdot \frac{Prob(\text{positive}|q = 1)}{Prob(\text{positive})}.$$

By substituting in it with the expressions obtained in the proof of Lemma 2.1, we have

$$Prob(q=1|\text{positive}) = \begin{cases} \frac{1}{2}\times\frac{\frac{2n(1-u_0)}{t}}{\frac{n(1+q_0-2u_0)}{t}}, & \text{if } n \le \frac{t}{2(1-u_0)} \\ \frac{1}{2}\times\frac{1}{\frac{t+2n(q_0-u_0)}{2t}}, & \text{if } \frac{t}{2(1-u_0)} < n \le \frac{t}{2(q_0-u_0)} \\ \frac{1}{2}\times\frac{1}{1}, & \text{if } n > \frac{t}{2(q_0-u_0)} \end{cases}$$

$$= \begin{cases} \frac{1-u_0}{1+q_0-2u_0}, & \text{if } n \le \frac{t}{2(1-u_0)} \\ \frac{t}{t+2n(q_0-u_0)}, & \text{if } \frac{t}{2(1-u_0)} < n \le \frac{t}{2(q_0-u_0)} \\ \frac{1}{2}, & \text{if } n > \frac{t}{2(q_0-u_0)} \end{cases}.$$

As a result, the expected product quality conditional on a positive opinion is

$$\begin{aligned} \hat{q}_p &= 1\cdot Prob(q=1|\text{positive}) + q_0\cdot Prob(q=q_0|\text{positive}) \\ &= 1\cdot Prob(q=1|\text{positive}) + q_0\cdot(1-Prob(q=1|\text{positive})) \\ &= \begin{cases} 1\times\frac{1-u_0}{1+q_0-2u_0} + q_0\times(1-\frac{1-u_0}{1+q_0-2u_0}), & \text{if } n \le \frac{t}{2(1-u_0)} \\ 1\times\frac{t}{t+2n(q_0-u_0)} + q_0\times(1-\frac{t}{t+2n(q_0-u_0)}), & \text{if } \frac{t}{2(1-u_0)} < n \le \frac{t}{2(q_0-u_0)} \\ 1\times\frac{1}{2} + q_0\times(1-\frac{1}{2}), & \text{if } n > \frac{t}{2(q_0-u_0)} \end{cases} \\ &= \begin{cases} \frac{1+q_0^2-u_0(1+q_0)}{1+q_0-2u_0}, & \text{if } n \le \frac{t}{2(1-u_0)} \\ \frac{t+2nq_0(q_0-u_0)}{t+2n(q_0-u_0)}, & \text{if } \frac{t}{2(1-u_0)} < n \le \frac{t}{2(q_0-u_0)} \\ \frac{1+q_0}{2}, & \text{if } n > \frac{t}{2(q_0-u_0)} \end{cases}. \end{aligned}$$

On the other hand, if the expert opinion is negative,

$$\begin{aligned} Prob(q=1|\text{negative}) &= \frac{Prob(\text{negative}|q=1)\cdot Prob(q=1)}{Prob(\text{negative})} \\ &= \frac{1}{2}\cdot\frac{1-Prob(\text{positive}|q=1)}{1-Prob(\text{positive})} \\ &= \begin{cases} \frac{1}{2}\times\frac{1-\frac{2n(1-u_0)}{t}}{1-\frac{n(1+q_0-2u_0)}{t}} = \frac{t-2n(1-u_0)}{2(t-n(1+q_0-2u_0))}, & \text{if } n \le \frac{t}{2(1-u_0)} \\ 0, & \text{if } n > \frac{t}{2(1-u_0)} \end{cases}, \end{aligned}$$

and

$$\hat{q}_n = 1 \cdot Prob(q=1|\text{negative}) + q_0 \cdot (1 - Prob(q=1|\text{negative}))$$

$$= \begin{cases} 1 \times \frac{t-2n(1-u_0)}{2(t-n(1+q_0-2u_0))} + q_0 \times (1 - \frac{t-2n(1-u_0)}{2(t-n(1+q_0-2u_0))}), & \text{if } n \leq \frac{t}{2(1-u_0)} \\ 1 \times 0 + q_0 \times (1-0), & \text{if } n > \frac{t}{2(1-u_0)} \end{cases}$$

$$= \begin{cases} \frac{t(1+q_0)-2n(1+q_0^2-u_0(1+q_0))}{2(t-n(1+q_0-2u_0))}, & \text{if } n \leq \frac{t}{2(1-u_0)} \\ q_0, & \text{if } n > \frac{t}{2(1-u_0)} \end{cases}.$$

Similarly, if $q_0 \leq u_0$,

$$Prob(q=1|\text{positive}) = 1,$$

$$\hat{q}_p = 1,$$

$$Prob(q=1|\text{negative}) = \begin{cases} \frac{1}{2} \times \frac{1-\frac{2n(1-u_0)}{t}}{1-\frac{n(1-u_0)}{t}} = \frac{t-2n(1-u_0)}{2t-2n(1-u_0)}, & \text{if } n \leq \frac{t}{2(1-u_0)} \\ 0, & \text{if } n > \frac{t}{2(1-u_0)} \end{cases},$$

and

$$\hat{q}_n = \begin{cases} 1 \times \frac{t-2n(1-u_0)}{2t-2n(1-u_0)} + q_0 \times (1 - \frac{t-2n(1-u_0)}{2t-2n(1-u_0)}), & \text{if } n \leq \frac{t}{2(1-u_0)} \\ 1 \times 0 + q_0 \times (1-0), & \text{if } n > \frac{t}{2(1-u_0)} \end{cases}$$

$$= \begin{cases} \frac{(1+q_0)t-2n(1-u_0)}{2(t-n(1-u_0))}, & \text{if } n \leq \frac{t}{2(1-u_0)} \\ q_0, & \text{if } n > \frac{t}{2(1-u_0)} \end{cases}.$$

This completes the proof of Lemma 2.2.

Proof of Proposition 2.1

We can derive the first-order derivative of $E[\pi(n)]$ with respect to n as follows.

If $q_0 > u_0$,

$$\frac{dE[\pi(n)]}{dn} = \begin{cases} \frac{(1-q_0)^4 t}{16(t-n(1+q_0-2u_0))^2(1+q_0-2u_0)V}, & \text{if } n \leq \frac{t}{2(1-u_0)} \\ -\frac{(1-q_0)^2(q_0-u_0)t}{4(t+2n(q_0-u_0))^2 V}, & \text{if } \frac{t}{2(1-u_0)} < n \leq \frac{t}{2(q_0-u_0)} \\ 0, & \text{if } n > \frac{t}{2(q_0-u_0)} \end{cases}.$$

If $q_0 \leq u_0$, on the other hand,

$$\frac{dE[\pi(n)]}{dn} = \begin{cases} \frac{(1-q_0)^2(1-u_0)t}{16V(t-n(1-u_0))^2}, & \text{if } n \leq \frac{t}{2(1-u_0)} \\ 0, & \text{if } n > \frac{t}{2(1-u_0)} \end{cases}$$

The relationship between $E[\pi(n)]$ and n stated in the first part of Proposition 2.1 can be proved since $\frac{dE[\pi(n)]}{dn} > 0$ if $n \leq \frac{t}{2(1-u_0)}$, $\frac{dE[\pi(n)]}{dn} < 0$ if $\frac{t}{2(1-u_0)} < n \leq \frac{t}{2(q_0-u_0)}$, and $\frac{dE[\pi(n)]}{dn} = 0$ otherwise.

As for the optimal number of product variants, we know that if $q_0 > u_0$, $E[\pi(n)]$ is first increasing in n, then decreasing in n, and then constant in n. Thus, the maximal value of $E[\pi(n)]$ is achieved at $n = \frac{t}{2(1-u_0)}$. If $q_0 \leq u_0$, on the other hand, $E[\pi(n)]$ is first increasing in n and then constant in n. As a result, the maximal value of $E[\pi(n)]$ is achieved if $n \geq \frac{t}{2(1-u_0)}$.

This completes the proof of Proposition 2.1.

Proof of Corollary 2.1

This corollary immediately follows from the optimal number of variants stated in Proposition 2.1.

Proof of Proposition 2.2

With zero cost of providing a new product variant, the low-quality firm can be weakly better off by mimicking the high-quality firm on the decision of n. When the equilibrium decicion of n is pooling, as we have discussed in the main text, the high-quality firm achieves the maximum expected profit at $n = \frac{t}{2(1-u_0)}$ if $q_0 > u_0$, and $n \geq \frac{t}{2(1-u_0)}$ if $q_0 \leq u_0$. Thus, under the concept of undefeated equilibrium, $n_u^* = \frac{t}{2(1-u_0)}$ if $q_0 > u_0$, and $n_u^* \geq \frac{t}{2(1-u_0)}$ if $q_0 \leq u_0$, which coincides with the equilibrium n when q is unknown to the firm as listed in Proposition 2.1.

Moreover, as we have argued in the main text, with a pre-determined n, the firm always finds it optimal to set the n product variants equidistant from each other.

This completes the proof of Proposition 2.2.

Proof of Proposition 2.3

The first part of Proposition 2.3 is proved by observing that the expected revenue is constant in n but the cost increases in n.

We prove the second part of Proposition 2.3 in the following four steps.

Step 1: $n_u^* \leq \frac{t}{2(1-u_0)}$.

In the proof of Proposition 2.1, we have shown that $\frac{dE[R(n)]}{dn} < 0$ if $\frac{t}{2(1-u_0)} < n \leq \frac{t}{2(q_0-u_0)}$, and $\frac{dE[R(n)]}{dn} = 0$ if $n > \max\{\frac{t}{2(1-u_0)}, \frac{t}{2(q_0-u_0)}\}$. As a result, here, $\frac{dE[\pi(n)]}{dn} = \frac{d(E[R(n)]-C\cdot n)}{dn} = \frac{dE[R(n)]}{dn} - C < 0$ if $n > \frac{t}{2(1-u_0)}$, i.e., $E[\pi(n)]$ is decreasing in n if $n > \frac{t}{2(1-u_0)}$. Therefore, we prove that the maximum point of $E[\pi(n)] = E[R(n)] - C \cdot n$ satisfies $n_u^* \leq \frac{t}{2(1-u_0)}$.

Step 2: $\frac{dE[R(n)]}{dn}$ *is strictly increasing in* n *in the range of* $n \leq \frac{t}{2(1-u_0)}$.

When $n \leq \frac{t}{2(1-u_0)}$, if $q_0 > u_0$, $\frac{d^2E[R(n)]}{dn^2} = \frac{(1-q_0)^4 t}{8V(t-n(1+q_0-2u_0))^3} > 0$, and if $q_0 \leq u_0$, $\frac{d^2E[R(n)]}{dn^2} = \frac{(1-q_0)^2(1-u_0)^2 t}{8V(t-n(1-u_0))^3} > 0$. Thus, $\frac{dE[R(n)]}{dn}$ is strictly increasing in n if $n \leq \frac{t}{2(1-u_0)}$.

Step 3: $n_u^* = 1$ *or* $n_u^* = \frac{t}{2(1-u_0)}$.

Suppose to the contrary, $E[\pi(n)]$ is maximized at $n_{u0} \in (1, \frac{t}{2(1-u_0)})$. Then, we should have $\frac{dE[\pi(n)]}{dn}|_{n=n_{u0}} = \frac{dE[R(n)]}{dn}|_{n=n_{u0}} - C \geq 0$. Since $\frac{dE[R(n)]}{dn}$ is strictly increasing in n, for any value of $n' \in (n_{u0}, \frac{t}{2(1-u_0)}]$, we have $\frac{dE[\pi(n)]}{dn}|_{n=n'} = \frac{dE[R(n)]}{dn}|_{n=n'} - C > \frac{dE[R(n)]}{dn}|_{n=n_{u0}} - C \geq 0$. In other words, $E[\pi(n)]$ is strictly increasing in n if $n_{u0} < n \leq \frac{t}{2(1-u_0)}$, and thus $E[\pi(\frac{t}{2(1-u_0)})] > E[\pi(n_{u0})]$, which is a contradiction. As a result, $n_u^* = 1$ or $n_u^* = \frac{t}{2(1-u_0)}$.

Step 4: $n_u^* = \frac{t}{2(1-u_0)}$ if $\frac{t}{2(1-u_0)} > 1$ and c is small, and $n_u^* = 1$ otherwise.

By Step 3, to find the maximal value of $E[\pi(n)]$, we only need to compare $E[\pi(\frac{t}{2(1-u_0)})]$ and $E[\pi(1)]$. If $\frac{t}{2(1-u_0)} > 1$, $E[\pi(\frac{t}{2(1-u_0)})] > E[\pi(1)]$ is equivalent to $E[R(\frac{t}{2(1-u_0)})] - C \cdot \frac{t}{2(1-u_0)} > E[R(1)] - C$, i.e.,

$$C < \frac{E[R(\frac{t}{2(1-u_0)})] - E[R(1)]}{\frac{t}{2(1-u_0)} - 1}.$$

Thus, if $\frac{t}{2(1-u_0)} > 1$ and C is small, we have $n_u^* = \frac{t}{2(1-u_0)}$. Otherwise, $n_u^* = 1$.

By comparing the result here with Proposition 2.1, we complete the proof of Proposition 2.3.

Proof of Lemma 2.3

Similar to the proof of Proposition 2.3, $E[\pi(n)|q = 1]$ is maximized at $n_h = \frac{t}{2(1-u_0)}$ if $\frac{t}{2(1-u_0)} > 1$ and $C < C_h = \frac{E[R(\frac{t}{2(1-u_0)}|q=1)]-E[R(1|q=1)]}{\frac{t}{2(1-u_0)}-1}$, and $n_h = 1$ otherwise.

As for $E[\pi(n)|q = q_0] = E[R(n)|q = q_0] - C \cdot n$, we know that, $E[R(n)|q = q_0]$ is decreasing in n if $n \leq \frac{t}{2(1-u_0)}$, increasing in n if $\frac{t}{2(1-u_0)} < n \leq \frac{t}{2(q_0-u_0)}$, and constant in n otherwise. As a result, when $q_0 \leq u_0$, $E[\pi(n)|q = q_0]$ is maximized at $n_l = 1$. When $q_0 > u_0$, on the other hand, we have $E[R(\frac{t}{2(q_0-u_0)})|q = q_0] > E[R(1)|q = q_0]$. As a result, when $C < C_l$, $E[\pi(n)|q = q_0]$ is maximized at $n_l = n' \in (\frac{t}{2(1-u_0)}, \frac{t}{2(q_0-u_0)}]$.

In conclusion, when $\frac{t}{2(1-u_0)} > 1$ and c is small enough, $n_l = 1 < \frac{t}{2(1-u_0)} = n_h$ if $q_0 \leq u_0$ and $n_l > \frac{t}{2(1-u_0)} = n_h$ if $q_0 > u_0$.

This completes the proof of Lemma 2.3.

Proof of Proposition 2.4

Proof of Part (ii):

In the case of separating equilibrium, with positive fixed product cost, the low-quality firm, who can be perfectly identified as the low type, has no incentive to increase n, and thus $n_l^* = 1 < n_h^*$.

The separating equilibrium holds if one of the following cases holds:

(A1.1) $q_0 > u_0$, $\frac{t}{2(q_0-u_0)} > n_h^* \geq \frac{t}{2(1-u_0)}$, $n_h^* > 1$, and $\frac{1}{4V}(1+V-\frac{t}{4})^2 - C \cdot n_h^* \geq \frac{1}{4V}(q_0+V-\frac{t}{4})^2 - C \geq \frac{1}{4Vt}(2n_h^*(q_0-u_0)(1+V-\frac{t}{4})^2 + (t-2n_h^*(q_0-u_0))(q_0+V-\frac{t}{4})^2) - C \cdot n_h^*$.

(A1.2) $q_0 \leq u_0$, $n_h^* = \frac{t}{2(1-u_0)} > 1$ or $n_h^* = 2 > 1 \geq \frac{t}{2(1-u_0)}$, and $\frac{1}{4V}(1+V-\frac{t}{4})^2 - C \cdot n_h^* \geq \frac{1}{4Vt}(2n(1-u_0)(1+V-\frac{t}{4})^2 + (t-2n(1-u_0))(q_0+V-\frac{t}{4})^2) - C \cdot n$ for any $n < n_h^*$.

Thus, we have separating equilibrium with $n_h^* > n_l^* = 1$ if $q_0 > u_0$ and c is intermediate. or $q_0 < u_0$ and c is small.

To prove that $n_l^* < n_u^* < n_h^*$ is possible, and n_h^* may be strictly higher than $\max\{\frac{t}{2(1-u_0)}, 1\}$, we provide an example.

Example: If $t = 4$, $V = 5$, $u_0 = 0$, $q_0 = \frac{1}{50}$, and $C = \frac{1}{150}$, we have $\frac{t}{2(1-u_0)} = 2$, $n_u^* = 2$, $n_h^* = 3$ and $n_l^* = 1$. When q is unknown to the firm, the expected revenue is $\frac{1}{4Vt}(n(1+q_0-2u_0)(\frac{1+q_0^2-(1+q_0)u_0}{1+q_0-2u_0}+V-\frac{t}{4})^2+(t-n(1+q_0-2u_0)(\frac{t(1+q_0)-2n(1+q_0^2-u_0(1+q_0))}{2(t-n(1+q_0-2u_0))}+V-\frac{t}{4})^2)$ if $n \leq \frac{t}{2(1-u_0)} = 2$, which equals to $\frac{1551409}{1519800}$ if $n = 1$, and $\frac{104911}{102000}$ if $n = 2$. Since $C = \frac{1}{150} < \frac{104911}{102000} - \frac{1551409}{1519800} = \frac{117649}{15198000}$, we have $n_u^* = 2$. As for the case where q is known to the firm, first, we show that $n_h^* = 3$ and $n_l^* = 1$ can be an equilibrium. When consumers believe that $n = 3$ is chosen by the high-quality firm, the low-quality firm does not want to set $n_l = n_h^* = 3$. In fact, if $n_l = 3$, it will be recognized as the low type as long as the expert opinion is negative, and thus its expected revenue is $\frac{1}{4V}((1+V-\frac{t}{4})^2 \cdot \frac{2n(q_0-u_0)}{t} + (q_0+V-\frac{t}{4})^2 \cdot (1-\frac{2n(q_0-u_0)}{t}) = \frac{4106397}{5000000}$. Conversely, it obtains $\frac{1}{4V}(q_0+V-\frac{t}{4})^2 = \frac{40401}{50000}$ if $n_l = 1$. Since $2C = \frac{1}{75} > \frac{4106397}{5000000} - \frac{40401}{50000} = \frac{66297}{5000000}$, the low-quality firm does not want to deviate. The high-quality firm does not want to deviate either. The equilibrium expected revenue for the high-quality firm is $\frac{1}{4V}(1+V-\frac{t}{4})^2 = \frac{5}{4}$. However, if it deviates to $n_h = 1$ or 2, it will be considered as the low type and receive the expected revenue $\frac{40401}{50000}$. Since $2C = \frac{1}{75} < \frac{5}{4} - \frac{40401}{50000} = \frac{22099}{50000}$, the high-quality firm does not want to deviate. As a result, we prove that $n_h^* = 3$ and $n_l^* = 1$ can be an equilibrium. Second, we show that $n_h = 2$ and $n_l = 1$ cannot be an equilibrium. When consumers believe that $n_h = 2$ is chosen by the high-quality firm, the low-quality firm can obtain $\frac{1}{4V}((1+V-\frac{t}{4})^2 \cdot \frac{2n_h(q_0-u_0)}{t} + (q_0+V-\frac{t}{4})^2 \cdot (1-\frac{2n_h(q_0-u_0)}{t}) = \frac{2042149}{2500000}$ by deviating to $n_l = 2$. Since $C < \frac{2042149}{2500000} - \frac{40401}{50000}$, the deviation is profitable. As a result, $n_h = 2$ and $n_l = 1$ cannot be an equilibrium. Thirdly, we prove that the high-quality firm earns a higher profit in separating equilibrium than in pooling equilibrium. If both types of firms choose the same value of n, the equilibrium profit for the high-quality firm is $\frac{1}{4V}(\frac{2n(1-u_0)}{t}(\frac{1+q_0^2-(1+q_0)u_0}{1+q_0-2u_0}+V-\frac{t}{4})^2 + (1-\frac{2n(1-u_0)}{t})(\frac{t(1+q_0)-2n(1+q_0^2-u_0(1+q_0))}{2(t-n(1+q_0-2u_0))}+V-\frac{t}{4})^2)$, which equals to $\frac{3155810217701}{2887240050000}$ if $n=1$, and $\frac{161315401}{130050000}$ if $n=2$. Since $C = \frac{1}{150} < \frac{5}{4} - \frac{161315401}{130050000}$ and $2C = \frac{1}{75} < \frac{5}{4} - \frac{3155810217701}{2887240050000}$, the high-quality firm earns a higher expected profit in the separating equilibrium. To sum up, $n_u^* = 2$, $n_h^* = 3$ and $n_l^* = 1$.

Proof of Part (i):

Using the concept of undefeated equilibrium, we know that the equilibrium value of n is such that the optimal profit of the high-quality firm is achieved. In other words, either $n_h^* = n_l^* = 1$ or $n_h^* = n_l^* = \frac{t}{2(1-u_0)} > 1$.

Especially, $n_p^* = n_h^* = n_l^* = 1$ holds if one of the following cases holds:

(A2.1) $\frac{t}{2(1-u_0)} \leq 1$,

(A2.2.1) $\frac{t}{2(1-u_0)} > 1$, $q_0 > u_0$ and $E[R(1)|q=1] - C \geq E[R(\frac{t}{2(1-u_0)})|q=1] - C \cdot \frac{t}{2(1-u_0)}$,

(A2.2.2) $\frac{t}{2(1-u_0)} > 1$, $q_0 \leq u_0$, and $E[R(1)|q=1] - C \geq \frac{2n(1-u_0)}{t} \cdot \frac{(1+V-\frac{t}{4})^2}{4V} + (1-\frac{2n(1-u_0)}{t}) \cdot \frac{(q_0+V-\frac{t}{4})^2}{4V} - C \cdot n$ for any $n \leq \frac{t}{2(1-u_0)}$.

Consider the function $f(n) = \frac{2n(1-u_0)}{t} \cdot \frac{(1+V-\frac{t}{4})^2}{4V} + (1-\frac{2n(1-u_0)}{t}) \cdot \frac{(q_0+V-\frac{t}{4})^2}{4V}$. Note that $f(n)$ is linear in n, and $\frac{df(n)}{dn} = \frac{(1-q_0)(1-u_0)(1+q_0+2(V-\frac{t}{4}))}{2Vt} > 0$. Thus, $f(n) - C \cdot n$ is maximized at either 1 or $\frac{t}{2(1-u_0)}$. Clearly, as long as $\frac{t}{2(1-u_0)} > 1$, we have

$$
\begin{aligned}
E[R(1)|q=1] &= \frac{2(1-u_0)}{t} \cdot \frac{(1+V-\frac{t}{4})^2}{4V} + (1 - \frac{2(1-u_0)}{t}) \cdot \frac{(\frac{(1+q_0)t-2(1-u_0)}{2(t-(1-u_0))} + V - \frac{t}{4})^2}{4V} \\
&> \frac{2(1-u_0)}{t} \cdot \frac{(1+V-\frac{t}{4})^2}{4V} + (1 - \frac{2(1-u_0)}{t}) \cdot \frac{(q_0 + V - \frac{t}{4})^2}{4V} \\
&= f(1).
\end{aligned}
$$

Moreover, $f(\frac{t}{2(1-u_0)}) = \frac{(1+V-\frac{t}{4})^2}{4V} = E[R(\frac{t}{2(1-u_0)})|q=1]$. Therefore, conditions (A2.2.1) and (A2.2.2) can be combined into:

(A2.2) $\frac{t}{2(1-u_0)} > 1$ and $E[R(1)|q=1] - C \geq E[R(\frac{t}{2(1-u_0)})|q=1] - C \cdot \frac{t}{2(1-u_0)}$.

The equilibrium with $n_p^* = n_h^* = n_l^* = \frac{t}{2(1-u_0)} > 1$, on the other hand, holds if all of the following conditions hold:

(A3.1) $\frac{t}{2(1-u_0)} > 1$,

(A3.2) $q_0 > u_0$,

(A3.3) $E[R(\frac{t}{2(1-u_0)})|q=1] - C \cdot \frac{t}{2(1-u_0)} > E[R(1)|q=1] - C$,

(A3.4) $E[R(\frac{t}{2(1-u_0)})|q=q_0] - C \cdot \frac{t}{2(1-u_0)} \geq \frac{(q_0+V-\frac{t}{4})^2}{4V} - C$.

To sum up, we have pooling equilibrium if $q_0 > u_0$ and C is small, or C is large.

To prove that n_p^* may be strictly higher than n_u^*, we provide an example.

Example: If $t = 4$, $V = 5$, $u_0 = 0$, $q_0 = \frac{1}{2}$, and $C = \frac{1}{50}$, we can prove that $n_u^* = 1$ and $n_h^* = n_l^* = 2$. When q is unknown to the firm, the expected revenue is $\frac{1}{4Vt}(n(1 + q_0 - 2u_0)(\frac{1+q_0^2-(1+q_0)u_0}{1+q_0-2u_0} + V - \frac{t}{4})^2 + (t - n(1 + q_0 - 2u_0)(\frac{t(1+q_0)-2n(1+q_0^2-u_0(1+q_0))}{2(t-n(1+q_0-2u_0))} + V - \frac{t}{4})^2)$ if $n \leq \frac{t}{2(1-u_0)} = 2$, which equals to $\frac{677}{600}$ if $n = 1$, and $\frac{271}{240}$ if $n = 2$. Since $C = \frac{1}{50} > \frac{271}{240} - \frac{677}{600} = \frac{1}{1200}$, we have $n_u^* = 1$. As for the case where q is known to the firm, we first show that $n_l^* = n_h^* = 2$ is an equilibrium. The low-quality firm will not unilaterally deviate from $n_l^* = n_h^* = 2$ to $n_l = 1$. The expected revenue for the low-quality firm when $n_l^* = n_h^* = 2$ is $\frac{1}{4V(1-u_0)}[(q_0 - u_0)(\frac{1+q_0^2-(1+q_0)u_0}{1+q_0-2u_0} + V - \frac{t}{4})^2 + (1 - q_0)(q_0 + V - \frac{t}{4})^2] = \frac{157}{144}$. However, when it deviates to $n_l = 1$, the expected revenue is $\frac{1}{4V}(q_0 + V - \frac{t}{4})^2 = \frac{81}{80}$. Since $C = \frac{1}{50} < \frac{157}{144} - \frac{81}{80} = \frac{7}{90}$, the low-quality firm does not have the incentive to deviate. Clearly, the high-quality firm will not unilaterally deviate to $n_h = 1$, either, since its expected loss of deviating is higher than the low-quality firm. As a result, $n_l^* = n_h^* = 2$ is an equilibrium. Secondly, we show that $n_l^* = n_h^* = 2$ is the undefeated pooling equilibrium. The expected revenue for the high-quality firm in pooling equilibrium is $\frac{1}{4Vt}(2n(q_0 - u_0)(\frac{1+q_0^2-(1+q_0)u_0}{1+q_0-2u_0} + V - \frac{t}{4})^2 + (t - 2n(q_0 - u_0))(\frac{t(1+q_0)-2n(1+q_0^2-u_0(1+q_0))}{2(t-n(1+q_0-2u_0))} + V - \frac{t}{4})^2)$ if $n \leq 2$, which is $\frac{20453}{18000}$ if $n = 1$, and $\frac{841}{720}$ if $n = 2$. Since $C = \frac{1}{50} < \frac{841}{720} - \frac{20453}{18000} = \frac{143}{4500}$, we know that $n_l^* = n_h^* = 2$ is an undefeated pooling equilibrium. Thirdly, by checking Condition (A1.2), we can conclude that separating equilibrium does not exist in this case. To sum up, $n_u^* = 1$ and $n_h^* = n_l^* = 2$.

This completes the proof of Proposition 2.4.

Proof of Corollary 2.2

Corollary 2.2 follows directly from Proposition 2.4.

Proof of Proposition 2.5

Recall that, when $q_0 > u_0$ and $\lambda = 0$, the expected profit of the firm, who is uncertain about its own quality level, is first strictly increasing, then strictly decreasing, and then constant in n. Because the profit function is stepwise polynomial in λ, both the profit function itself and its first-order-derivative with respect to n are continuous in λ. As a result, as long as λ is small enough, the expected profit is still firstly increasing in n when n is small and strictly decreasing in n when n is intermediate; moreover, although with positive λ, the expected profit is increasing in n when n is large, the local maximum (achieved at $n = +\infty$) is not larger than that achieved at a finite n, as long as λ is small.

This completes the proof of Proposition 2.5.

Proof of Proposition 2.6

Proof of Part (i):

Firstly, $n_p^* = n_h^* = n_l^*(= +\infty)$ is the unique equilibrium if $q_0 < u_0$. If $q_0 < u_0$, the expected profit of the high-quality firm conditional on the pooling equilibrium is strictly increasing in n as long as $\lambda > 0$. As a result, $n_h^* = n_l^* = +\infty$ is an undefeated pooling equilibrium. Moreover, with $n_p = +\infty$, the firm type can be perfectly identified by the expert product opinion, and neither firm has the incentive to choose a smaller n. As a result, $n_p^* = n_h^* = n_l^* = +\infty$ is the unique equilibrium if $q_0 < u_0$.

Secondly, $n_p^* = n_h^* = n_l^* < +\infty$ is the unique equilibrium if $q_0 > u_0$ and λ is small. Similar to the proof of Part (i), this follows from the fact that a finite n_p^* is optimal for both high- and low-quality firms when $\lambda = 0$ and that the profit functions are continuous in λ.

Thirdly, $n_p^* = n_h^* = n_l^* = +\infty$ is the unique equilibrium if both q_0 and λ are large. Note that when $q_0 > u_0$, $n_p^* = n_h^* = n_l^* = +\infty$ is an undefeated pooling equilibrium if $\frac{1}{4V}(\frac{1+q_0}{2} + V - \frac{1-\lambda}{4}t)^2 > \frac{1}{4V}(\frac{t+2nq_0(q_0-u_0)}{t+2n(q_0-u_0)} + V - \frac{n-(n-1)\lambda}{4n}t)^2$ for any $n \in [\frac{t}{2(1-u_0)}, \frac{t}{2(q_0-u_0)}]$, which is true if both q_0 and λ are large. Moreover, the separating equilibrium can be sustained in the case of $q_0 > u_0$ only if the following condition holds:

(A4) There exists $n_h^* \in [\frac{t}{2(1-u_0)}, \frac{t}{2(q_0-u_0)})$, s.t. $\frac{1}{4V}(1+V-\frac{n_h^*-(n_h^*-1)\lambda}{4n_h^*}t)^2 \geq \frac{1}{4V}(q_0+V-\frac{1-\lambda}{4}t)^2 \geq \frac{1}{4V}[\frac{2n_h^*(q_0-u_0)}{t}(1+V-\frac{n_h^*-(n_h^*-1)\lambda}{4n_h^*}t)^2 + \frac{t-2n_h^*(q_0-u_0)}{t}(q_0+V-\frac{n_h^*-(n_h^*-1)\lambda}{4n_h^*}t)^2]$.

The above condition cannot be true if both q_0 and λ are large. As a result, $n_p^* = n_h^* = n_l^* = +\infty$ is the unique equilibrium if both q_0 and λ are large.

In conclusion, we have pooling equilibrium with $n_p^* = n_h^* = n_l^*$ if either q_0 or λ is small, or both q_0 and λ are large.

To prove that $n_p^* < n_u^* < +\infty$ is possible, we provide an example.

Example: If $t = 1$, $V = 1$, $u_0 = \frac{1}{2}$, $q_0 = \frac{51}{100}$, and $\lambda = \frac{1}{100}$, we have $n_p^* = n_h^* = n_l^* = 1$ and $n_u^* \in (1, +\infty)$. When q is unknown to the firm, the expected firm profit is $E[\pi(n)] = \frac{1}{4V}[(\frac{1}{2}+\frac{n(q_0-u_0)}{t})(\frac{t+2nq_0(q_0-u_0)}{t+2n(q_0-u_0)}+V-\frac{n-(n-1)\lambda}{4n}t)^2+(\frac{1}{2}-\frac{n(q_0-u_0)}{t})(q_0+V-\frac{n-(n-1)\lambda}{4n}t)^2]$ if $n \in [\frac{t}{2(1-u_0)}, \frac{t}{2(q_0-u_0)}] = [1, 50]$, and $E[\pi(n)] = \frac{1}{4V}(\frac{1+q_0}{2} + V - \frac{n-(n-1)\lambda}{4n}t)^2$ if $n > \frac{t}{2(q_0-u_0)} =$

50. Since $E[\pi(2)] = \frac{19337317}{33280000} > E[\pi(1)] = \frac{47383}{81600} > E[\pi(+\infty)] = \frac{363609}{640000}$, we know that $n_u^* \in (1, +\infty)$. When q is known to the firm, we first prove that $n_p^* = n_h^* = n_l^* = 1$ is a pooling equilibrium. Note that the low-quality does not want to deviate because it obtains $\frac{1}{4V}[\frac{2n(q_0-u_0)}{t}(\frac{t+2nq_0(q_0-u_0)}{t+2n(q_0-u_0)} + V - \frac{n-(n-1)\lambda}{4n}t)^2 + (1 - \frac{2n(q_0-u_0)}{t})(q_0 + V - \frac{n-(n-1)\lambda}{4n}t)^2] = \frac{21021637}{52020000}$ by choosing $n_l^* = 1$, and $\frac{1}{4V}(q_0 + V - \frac{1-\lambda}{4}t)^2 = \frac{10201}{25600} < \frac{21021637}{52020000}$ by choosing $n_l = +\infty$. Then, we prove that $n_p^* = n_h^* = n_l^* = 1$ is the unique (undefeated) equilibrium. Clearly, separating equilibrium cannot be sustained here, since the high-quality firm cannot further decrease n_h so as to signal its high type. Denote the expected profit of the high-quality firm conditional on the pooling equilibrium by $E[\pi(n)|q = 1]$. By checking the sign of the first-order derivative, $E[\pi(n)|q = 1]$ is increasing in n if $1 \leq n \leq 50$ and increasing in n if $n > 50$. Moreover, $E[\pi(1)|q = 1] = \frac{1}{4V}(\frac{t+2nq_0(q_0-u_0)}{t+2n(q_0-u_0)} + V - \frac{n-(n-1)\lambda}{4n}t)^2|_{n=1} = \frac{4923961}{6502500} > E[\pi(+\infty)|q = 1] = \frac{363609}{640000}$, meaning that $n_p^* = n_h^* = n_l^* = 1$ is the undefeated equilibrium. To sum up,$n_p^* = n_h^* = n_l^* = 1$ and $n_u^* \in (1, +\infty)$.

Proof of Part (ii):

In the case of separating equilibrium, with $\lambda > 0$, the low-quality firm, who can be perfectly identified as the low type, has no incentive to decrease n, and thus $n_h^* < n_l^* = +\infty$.

By checking condition (A4), we know that separating equilibrium can be sustained when both q_0 and λ are intermediate. To prove that the separating equilibrium can be the undefeated equilibrium, and that $n_h^* < n_u^* < n_l^* = +\infty$, we provide an example.

Example: If $t = 1$, $V = 1$, $u_0 = \frac{1}{2}$, $q_0 = \frac{51}{100}$, and $\lambda = \frac{1}{8}$, we have $n_h^* = 1$, $n_l^* = +\infty$ and $n_u^* \in (1, +\infty)$. When q is unknown to the firm, since $E[\pi(10)] = \frac{3671791}{6144000} > E[\pi(+\infty) = \frac{1510441}{2560000}] > E[\pi(1)] = \frac{47383}{81600}$, we know that $n_u^* \in (1, +\infty)$. When q is known to the firm, we first prove that $n_h^* = 1$ and $n_l^* = +\infty$ is an equilibrium. Note that the low-quality firm does not want to deviate because it obtains $\frac{1}{4V}(q_0 + V - \frac{1-\lambda}{4}t)^2 = \frac{1067089}{2560000}$ by choosing $n_l^* = +\infty$, and $\frac{1}{4V}[\frac{2n(q_0-u_0)}{t}(1 + V - \frac{n-(n-1)\lambda}{4n}t)^2 + (1 - \frac{2n(q_0-u_0)}{t})(q_0 + V - \frac{n-(n-1)\lambda}{4n}t)^2] = \frac{808549}{2000000} < \frac{1067089}{2560000}$ by choosing $n_l = 1$. The high-quality firm does not want to deviate either, because it obtains $\frac{1}{4V}(1+V-\frac{n-(n-1)\lambda}{4n}t)^2 = \frac{49}{64}$ by choosing $n_h^* = 1$, and $\frac{1}{4V}(q_0+V-\frac{1-\lambda}{4}t)^2 = \frac{1067089}{2560000} < \frac{49}{64}$ by choosing $n_h = +\infty$. Secondly, we prove that $n_h = 2$ is not enough for the high-quality firm to signal its high type because if so, the low-quality firm would obtain $\frac{1}{4V}[\frac{2n_h(q_0-u_0)}{t}(1+V-\frac{n_h-(n_h-1)\lambda}{4n_h}t)^2 + (1-\frac{2n_h(q_0-u_0)}{t})(q_0+V-\frac{n_h-(n_h-1)\lambda}{4n_h}t)^2] = \frac{107956969}{256000000} > \frac{1067089}{2560000}$ by deviating to $n_l = n_h = 2$. Thirdly, we prove that the separating equilibrium $n_h^* = 1$ and $n_l^* = +\infty$ leads to a higher equilibrium profit for the high-quality firm as compared with the pooling equilibrium. Denote the expected profit of the high-quality firm conditional on the pooling equilibrium by $E[\pi(n)|q=1]$. By checking the first-order derivative, we know that $E[\pi(n)|q=1]$ is maximized at either $n = \frac{50}{27}$ or $n = +\infty$. Since $E[\pi(\frac{50}{27})|q=1] = \frac{1}{4V}(\frac{t+2nq_0(q_0-u_0)}{t+2n(q_0-u_0)} + V - \frac{n-(n-1)\lambda}{4n}t)^2|_{n=\frac{50}{27}} = \frac{312481}{409600} < \frac{49}{64}$ and $E[\pi(+\infty)|q=1] = \frac{1}{4V}(\frac{1+q_0}{2} + V - \frac{1-\lambda}{4}t) = \frac{1510441}{2560000} < \frac{49}{64}$, we know that the separating equilibrium is more favorable to the high-quality firm. To sum up, $n_h^* = 1$, $n_l^* = +\infty$ and $n_u^* \in (1, +\infty)$.

This completes the proof of Proposition 2.6.

Proof of Proposition 2.7

The proof consists of two steps. In the first step, we solve for the optimal p given n and derive the firm's expected profit $E[\pi(n)]$. In the second step, we show how $n = \frac{t}{2(1-u_0)}$ is optimal for the firm. Moreover, it is strictly optimal if $q_0 > u_0$ and c is intermediate.

First, let us look at the optimal pricing decision given the product variety n (and thus the probability of a positive expert opinion $Prob(pos.)$, the expected quality given the positive expert opinion $\hat{q}_p$, and the expected quality given the negative expert opinion $\hat{q}_n$).

The expected firm profit is given in Equation (2.23). By checking the signs of the first-order derivatives, we know that:

(i) $f_1(p) = p - c$ increases in p;

(ii) $f_2(p) = 1 \times (p-c) \times Prob(pos.) + \frac{V+\hat{q}_n-\frac{t}{4}-p}{V} \times (p-c) \times Prob(neg.)$ increases in p if $p \leq \frac{\frac{V}{Prob(neg.)}+\hat{q}_n-\frac{t}{4}+c}{2}$ and decreases in p otherwise;

(iii) $f_3(p) = \frac{V+\hat{q}_p-\frac{t}{4}-p}{V} \times (p-c) \times Prob(pos.) + \frac{V+\hat{q}_n-\frac{t}{4}-p}{V} \times (p-c) \times Prob(neg.)$ increases in p if $p \leq \frac{V+\frac{1+q_0}{2}-\frac{t}{4}+c}{2}$ and decreases in p otherwise;

(iv) $f_4(p) = \frac{V+\hat{q}_p-\frac{t}{4}-p}{V} \times (p-c) \times Prob(pos.) + 0$ increases in p if $p \leq \frac{v+\hat{q}_p-\frac{t}{4}+c}{2}$ and decreases in p otherwise;

(v) $f_5(p) = 0$ is constant in p.

When V is large (e.g., $V \geq \frac{3}{2}$), which is consistent with the assumption in the main model, we can show that:

(a) If $c \leq 2\hat{q}_n - \hat{q}_p - \frac{t}{4} + V$, then $\frac{\frac{V}{Prob(neg.)}+\hat{q}_n-\frac{t}{4}+c}{2} > \hat{q}_p - \frac{t}{4}$, $\hat{q}_p - \frac{t}{4} < \frac{V+\frac{1+q_0}{2}-\frac{t}{4}+c}{2} \leq V + \hat{q}_n - \frac{t}{4}$, and $\frac{V+\hat{q}_p-\frac{t}{4}+c}{2} \leq V + \hat{q}_n - \frac{t}{4}$. Thus, $E[\pi(n,p)]$ increases in p if $p \leq \frac{V+\frac{1+q_0}{2}-\frac{t}{4}+c}{2}$ and decreases in p if $p > \frac{V+\frac{1+q_0}{2}-\frac{t}{4}+c}{2}$. In other words, the optimal price is $p^* = \frac{V+\frac{1+q_0}{2}-\frac{t}{4}+c}{2}$, and the expected firm's profit is $E[\pi(n)] = f_3(p^*) = \frac{(V+\frac{1+q_0}{2}-\frac{t}{4}-c)^2}{4V}$.

(b) If $2\hat{q}_n - \hat{q}_p - \frac{t}{4} + V < c \leq 2\hat{q}_n - \frac{1+q_0}{2} - \frac{t}{4} + V$, then $\frac{\frac{V}{Prob(neg.)}+\hat{q}_n-\frac{t}{4}+c}{2} > \hat{q}_p - \frac{t}{4}$, $\hat{q}_p - \frac{t}{4} < \frac{V+\frac{1+q_0}{2}-\frac{t}{4}+c}{2} \leq V + \hat{q}_n - \frac{t}{4}$, and $V + \hat{q}_n - \frac{t}{4} < \frac{V+\hat{q}_p-\frac{t}{4}+c}{2} \leq V + \hat{q}_p - \frac{t}{4}$. Thus, $E[\pi(n,p)]$ increases in p if $p \leq \frac{V+\frac{1+q_0}{2}-\frac{t}{4}+c}{2}$, decreases in p if $\frac{V+\frac{1+q_0}{2}-\frac{t}{4}+c}{2} < p \leq V + \hat{q}_n - \frac{t}{4}$, increases in p if $V + \hat{q}_n - \frac{t}{4} < p \leq \frac{V+\hat{q}_p-\frac{t}{4}+c}{2}$, and then decreases in p if $p > \frac{V+\hat{q}_p-\frac{t}{4}+c}{2}$. In this case, the maximum profit is obtained at either $p = \frac{V+\frac{1+q_0}{2}-\frac{t}{4}+c}{2}$ ($E[\pi(n,p)] = \frac{(V+\frac{1+q_0}{2}-\frac{t}{4}-c)^2}{4V}$) or $p = \frac{V+\hat{q}_p-\frac{t}{4}+c}{2}$ (which corresponds to $E[\pi(n,p)] = \frac{(V+\hat{q}_p-\frac{t}{4}-c)^2}{4V} Prob(pos.)$). By comparing the firm's expected profits at the two critical points, we know that if $2\hat{q}_n - \hat{q}_p - \frac{t}{4} + V < c \leq \frac{(\frac{1+q_0}{2}-\hat{q}_p\sqrt{Prob(pos.)})(1+\sqrt{Prob(pos.)})}{Prob(neg.)} - \frac{t}{4} + V$, then $p^* = \frac{V+\frac{1+q_0}{2}-\frac{t}{4}+c}{2}$ and $E[\pi(n)] = f_3(p^*) = \frac{(V+\frac{1+q_0}{2}-\frac{t}{4}-c)^2}{4V}$. If $\frac{(\frac{1+q_0}{2}-\hat{q}_p\sqrt{Prob(pos.)})(1+\sqrt{Prob(pos.)})}{Prob(neg.)} - \frac{t}{4} + V < c \leq 2\hat{q}_n - \frac{1+q_0}{2} - \frac{t}{4} + V$, on the other hand, then $p^* = \frac{V+\hat{q}_p-\frac{t}{4}+c}{2}$ and $E[\pi(n)] = f_4(p^*) = \frac{(V+\hat{q}_p-\frac{t}{4}-c)^2}{4V} Prob(pos.)$.

(c) If $2\hat{q}_n - \frac{1+q_0}{2} - \frac{t}{4} + V < c \leq V + \hat{q}_p - \frac{t}{4}$, then $\frac{\frac{V}{Prob(neg.)}+\hat{q}_n-\frac{t}{4}+c}{2} > \hat{q}_p - \frac{t}{4}$, $\frac{V+\frac{1+q_0}{2}-\frac{t}{4}+c}{2} > V + \hat{q}_n - \frac{t}{4}$, and $V + \hat{q}_n - \frac{t}{4} < \frac{V+\hat{q}_p-\frac{t}{4}+c}{2} \leq V + \hat{q}_p - \frac{t}{4}$. Thus, $E[\pi(n,p)]$ increases in p if $p \leq$

$\frac{V+\hat{q}_p-\frac{t}{4}+c}{2}$ and decreases in p if $p > \frac{V+\hat{q}_p-\frac{t}{4}+c}{2}$. In other words, the optimal price is $p^* = \frac{V+\hat{q}_p-\frac{t}{4}+c}{2}$, and the expected firm's profit is $E[\pi(n)] = f_4(p^*) = \frac{(V+\hat{q}_p-\frac{t}{4}-c)^2}{4V} Prob(pos.)$.

(d) If $c > V + \hat{q}_p - \frac{t}{4}$, then $\frac{\frac{V}{Prob(neg.)}+\hat{q}_n-\frac{t}{4}+c}{2} > \hat{q}_p - \frac{t}{4}$, $\frac{V+\frac{1+q_0}{2}-\frac{t}{4}+c}{2} > V + \hat{q}_n - \frac{t}{4}$, and $\frac{V+\hat{q}_p-\frac{t}{4}+c}{2} > V + \hat{q}_p - \frac{t}{4}$. Thus, $E[\pi(n,p)]$ increases in p. In other words, the optimal price is $p^* > V + \hat{q}_p - \frac{t}{4}$, and the expected firm's profit is $E[\pi(n)] = f_5(p^*) = 0$.

The above completes the analysis of the optimal pricing decision and summarizes the expected firm's profit given the product variety decision n, i.e.,

$$E[\pi(n)] = \begin{cases} \frac{(V+\frac{1+q_0}{2}-\frac{t}{4}-c)^2}{4V}, & \text{if } c \leq \frac{(\frac{1+q_0}{2}-\hat{q}_p\sqrt{Prob(pos.)})(1+\sqrt{Prob(pos.)})}{Prob(neg.)} - \frac{t}{4} + V \\ \frac{(V+\hat{q}_p-\frac{t}{4}-c)^2}{4V} Prob(pos.), & \text{if } \frac{(\frac{1+q_0}{2}-\hat{q}_p\sqrt{Prob(pos.)})(1+\sqrt{Prob(pos.)})}{Prob(neg.)} - \frac{t}{4} + V < c \\ & \qquad \leq V + \hat{q}_p - \frac{t}{4} \\ 0, & \text{if } c > V + \hat{q}_p - \frac{t}{4} \end{cases}. \tag{A.2.1}$$

This completes the first step of the proof.

Second, let us show how the firm's profit is maximized at $n = \frac{t}{2(1-u_0)}$, and strictly maximized at $n = \frac{t}{2(1-u_0)}$ if $q_0 > u_0$ and c is intermediate. As follows, we show the detailed analysis for the case $q_0 > u_0$. The analysis is similar for the case $q_0 \leq u_0$.

Note that

$$\frac{(\frac{1+q_0}{2} - \hat{q}_p\sqrt{Prob(pos.)})(1 + \sqrt{Prob(pos.)})}{Prob(neg.)} - \frac{t}{4} + V$$

$$= \begin{cases} \frac{(\frac{1+q_0}{2} - \frac{1+q_0^2-u_0(1+q_0)}{\sqrt{\frac{(1+q_0-2u_0)t}{n}}})(1+\sqrt{\frac{n(1+q_0-2u_0)}{t}})}{1-\frac{n(1+q_0-2u_0)}{t}} - \frac{t}{4} + V, & \text{if } n \leq \frac{t}{2(1-u_0)} \\ q_0 - \frac{1-q_0}{2}\sqrt{\frac{2t}{t+2n(q_0-u_0)}} - \frac{t}{4} + V, & \text{if } \frac{t}{2(1-u_0)} < n \leq \frac{t}{2(q_0-u_0)} \\ \frac{3q_0-1}{2} - \frac{t}{4} + V, & \text{if } n > \frac{t}{2(q_0-u_0)} \end{cases}.$$

By checking the sign of the first derivative with respect to n, we know that

$$\frac{(\frac{1+q_0}{2} - \hat{q}_p\sqrt{Prob(pos.)})(1 + \sqrt{Prob(pos.)})}{Prob(neg.)} - \frac{t}{4} + V$$

decreases in n when $n \leq \frac{t}{2(1-u_0)}$, increases in n when $\frac{t}{2(1-u_0)} < n \leq \frac{t}{2(q_0-u_0)}$, and stays constant in n when $n > \frac{t}{2(q_0-u_0)}$. In turn, we know that its minimum value is $q_0 - (1 - q_0)\sqrt{\frac{1-u_0}{2(1+q_0-2u_0)}} - \frac{t}{4} + V$ and the maximum value is $\frac{1+q_0}{2} - \frac{t}{4} + V$.

Besides, Note that (1)

$$\hat{q}_p - \frac{t}{4} + V = \begin{cases} \frac{1+q_0^2-u_0(1+q_0)}{1+q_0-2u_0} - \frac{t}{4} + V, & \text{if } n \leq \frac{t}{2(1-u_0)} \\ \frac{t+2nq_0(q_0-u_0)}{t+2n(q_0-u_0)} - \frac{t}{4} + V, & \text{if } \frac{t}{2(1-u_0)} < n \leq \frac{t}{2(q_0-u_0)} \\ \frac{1+q_0}{2} - \frac{t}{4} + V, & \text{if } n > \frac{t}{2(q_0-u_0)} \end{cases}$$

is continuous and weakly decreasing in n. Its minimum value is $\frac{1+q_0}{2} - \frac{t}{4} + V$ and its maximum value is $\frac{1+q_0^2-u_0(1+q_0)}{1+q_0-2u_0} - \frac{t}{4} + V$.

Based on the above results and Equation (A.2.1), if $\begin{cases} q_0 > u_0 \\ c > \frac{1+q_0^2-u_0(1+q_0)}{1+q_0-2u_0} - \frac{t}{4} + V \end{cases}$, then $c > V + \hat{q}_p - \frac{t}{4}$, and $E[\pi(n)] = 0$ is constant in n.

If $\begin{cases} q_0 > u_0 \\ \frac{1+q_0}{2} - \frac{t}{4} + V < c < \frac{1+q_0^2-u_0(1+q_0)}{1+q_0-2u_0} - \frac{t}{4} + V \end{cases}$, then $c > \frac{(\frac{1+q_0}{2} - \hat{q}_p\sqrt{Prob(pos.)})(1+\sqrt{Prob(pos.)})}{Prob(neg.)} - \frac{t}{4} + V$, and $E[\pi(n)] = \begin{cases} \frac{(V+\hat{q}_p-\frac{t}{4}-c)^2}{4V} Prob(pos.), & \text{if } c \leq V + \hat{q}_p - \frac{t}{4} \\ 0, & \text{if } c > V + \hat{q}_p - \frac{t}{4} \end{cases}$. Note that (1) $\hat{q}_p - \frac{t}{4} + V$ is continuous and weakly decreasing in n, and (2) $\hat{q}_p - \frac{t}{4} + V = \frac{1+q_0^2-u_0(1+q_0)}{1+q_0-2u_0} - \frac{t}{4} + V > c$ if $n = \frac{t}{2(1-u_0)}$ and $\hat{q}_p - \frac{t}{4} + V = \frac{1+q_0}{2} - \frac{t}{4} + V < c$ if $n = \frac{t}{2(q_0-u_0)}$. Thus, there exists a value $\tilde{n}$, such that $\frac{t}{2(1-u_0)} < \tilde{n} < \frac{t}{2(q_0-u_0)}$ and $\hat{q}_p(\tilde{n}) - \frac{t}{4} + V = c$. Moreover, $\frac{1+q_0}{2} - \frac{t}{4} + V < c \leq \hat{q}_p - \frac{t}{4} + V$ if

$n \leq \tilde{n}$, and $\hat{q}_p - \frac{t}{4} + V < c < \frac{1+q_0^2-u_0(1+q_0)}{1+q_0-2u_0} - \frac{t}{4} + V$ if $n > \tilde{n}$. That is,

$$E[\pi(n)] = \begin{cases} \frac{(V+\hat{q}_p-\frac{t}{4}-c)^2}{4V} Prob(pos.), & \text{if } n \leq \tilde{n} \\ 0, & \text{if } n > \tilde{n} \end{cases}$$

$$= \begin{cases} \frac{(V+\frac{1+q_0^2-u_0(1+q_0)}{1+q_0-2u_0}-\frac{t}{4}-c)^2}{4V} \times \frac{n(1+q_0-2u_0)}{t}, & \text{if } n \leq \frac{t}{2(1-u_0)} \\ \frac{(V+\frac{t+2nq_0(q_0-u_0)}{t+2n(q_0-u_0)}-\frac{t}{4}-c)^2}{4V} \times \frac{t+2n(q_0-u_0)}{2t}, & \text{if } \frac{t}{2(1-u_0)} < n \leq \tilde{n} \\ 0, & \text{if } n > \tilde{n} \end{cases}$$

Since $\frac{dE[\pi(n)]}{dn} > 0$ if $n \leq \hat{n}$, $\frac{dE[\pi(n)]}{dn} < 0$ if $\frac{t}{2(1-u_0)} < n \leq \hat{n}$, and $\frac{dE[\pi(n)]}{dn} = 0$ if $n > \hat{n}$, the expected firm's profit is strictly maximized at $n^* = \frac{t}{2(1-u_0)}$.

If $\begin{cases} q_0 > u_0 \\ q_0 - (1-q_0)\sqrt{\frac{1-u_0}{2(1+q_0-2u_0)}} - \frac{t}{4} + V \leq c \leq \frac{1+q_0}{2} - \frac{t}{4} + V \end{cases}$, then similar to the above analysis, the expected firm's profit is strictly maximized at $n^* = \frac{t}{2(1-u_0)}$.

If $\begin{cases} q_0 > u_0 \\ c < q_0 - (1-q_0)\sqrt{\frac{1-u_0}{2(1+q_0-2u_0)}} - \frac{t}{4} + V \end{cases}$, then $c < \frac{(\frac{1+q_0}{2}-\hat{q}_p\sqrt{Prob(pos.)})(1+\sqrt{Prob(pos.)})}{Prob(neg.)} - \frac{t}{4} + V$, and $E[\pi(n)] = \frac{(V+\frac{1+q_0}{2}-\frac{t}{4}-c)^2}{4V}$ is constant in n.

As above, we show that firm's profit is maximized at $n = \frac{t}{2(1-u_0)}$, and strictly maximized at $n = \frac{t}{2(1-u_0)}$ if c is intermediate for the case $q_0 > u_0$. Similarly, we can prove that the firm's profit is maximized at $n = \frac{t}{2(1-u_0)}$ for the case $q_0 \leq u_0$.

This completes the proof of Proposition 2.7.

Proof of Proposition 2.8

If the expert opinion is positive if and only if $q - \min_j d(x_e, l_j) \cdot t - p > u_0'$, then similar to the proofs of Lemma 2.1 and 2.2, we can derive Equation (2.24), (2.25), and (2.26).

To prove that the expected firm profit may be (strictly) maximized at an intermediate number of product variants, $n^* = \frac{t}{2(1-u_0'-p^*)}$, we first show in the more general case that

$n^*(p) = \frac{t}{2(1-u_0'-p)}$ is optimal given a fixed price p, and then use an example to show that $n^* = \frac{t}{2(1-u_0'-p^*)}$ can be strictly optimal.

First, we show that $n^*(p) = \frac{t}{2(1-u_0'-p)}$ is the optimal number of product variants given a fixed price p.

Note that when V is large (which follows the assumption in the main model), the firm never finds it optimal for the market to be fully covered. Thus, only cases (3), (4), and (5) in Equation (2.23) are relevant. Here, if case (3) applies, firm's expected profit is $E[\pi(n,p)] = \frac{V+\frac{1+q_0}{2}-\frac{t}{4}-p}{V} \times (p-c)$, which is constant in n; and if case (5) applies, firm's expected profit is $E[\pi(n,p)] = 0$, which is also constant in n. Thus, it suffices to show that $n^*(p) = \frac{t}{2(1-u_0'-p)}$ maximizes the firm's expected profit in case (4), i.e., $E[\pi(n,p)] = \frac{V+\hat{q}_p-\frac{t}{4}-p}{V} \times (p-c) \times Prob(pos.)$. Equivalently, it suffices to show that $n^*(p) = \frac{t}{2(1-u_0'-p)}$ maximizes $g(n) = (V + \hat{q}_p - \frac{t}{4} - p) \times Prob(pos.)$.

Note that when $q_0 \geq u_0' + p$,

$$g(n) = \begin{cases} (V + \frac{1+q_0^2-(u_0'+p)(1+q_0)}{1+q_0-2u_0'-2p} - \frac{t}{4} - p) \times \frac{n(1+q_0-2u_0'-2p)}{t}, & \text{if } n \leq \frac{t}{2(1-u_0'-p)} \\ (V + \frac{t+2nq_0(q_0-u_0'-p)}{t+2n(q_0-u_0'-p)} - \frac{t}{4} - p) \times \frac{t+2n(q_0-u_0'-p)}{2t}, & \text{if } \frac{t}{2(1-u_0'-p)} < n \leq \frac{t}{2(q_0-u_0'-p)} \\ (V + \frac{1+q_0}{2} - \frac{t}{4} - p) \times 1, & \text{if } n > \frac{t}{2(q_0-u_0'-p)} \end{cases},$$

and its first-order derivative with respect to n

$$\frac{dg(n)}{dn} = \begin{cases} (V + \frac{1+q_0^2-(u_0'+p)(1+q_0)}{1+q_0-2u_0'-2p} - \frac{t}{4} - p) \times \frac{1+q_0-2u_0'-2p}{t}, & \text{if } n \leq \frac{t}{2(1-u_0'-p)} \\ \frac{(q_0-u_0'-p)(V+q_0-\frac{t}{4}-p)}{t}, & \text{if } \frac{t}{2(1-u_0'-p)} < n \leq \frac{t}{2(q_0-u_0'-p)} \\ 0, & \text{if } n > \frac{t}{2(q_0-u_0'-p)} \end{cases}.$$

Since $\frac{dg(n)}{dn} > 0$ if $n < \frac{t}{2(1-u_0'-p)}$, $\frac{dg(n)}{dn} < 0$ if $\frac{t}{2(1-u_0'-p)} < n < \frac{t}{2(q_0-u_0'-p)}$, and $\frac{dg(n)}{dn} = 0$ if $n > \frac{t}{2(q_0-u_0'-p)}$, we know that $n^*(p) = \frac{t}{2(1-u_0'-p)}$ strictly maximizes $g(n)$ when $q_0 \geq u_0' + p$.

When $q_0 < u_0' + p$, on the other hand,

$$g(n) = \begin{cases} (V + 1 - \frac{t}{4} - p) \times \frac{n(1-u_0'-p)}{t}, & \text{if } n \leq \frac{t}{2(1-u_0'-p)} \\ (V + 1 - \frac{t}{4} - p) \times \frac{1}{2}, & \text{if } n > \frac{t}{2(1-u_0'-p)} \geq 0 \\ (V + 1 - \frac{t}{4} - p) \times 0, & \text{if } n \geq 0 > \frac{t}{2(1-u_0'-p)} \end{cases},$$

and its first-order derivative with respect to n

$$\frac{dg(n)}{dn} = \begin{cases} (V + 1 - \frac{t}{4} - p) \times \frac{1-u_0'-p}{t}, & \text{if } n \leq \frac{t}{2(1-u_0'-p)} \\ 0, & \text{if } n > \frac{t}{2(1-u_0'-p)} \end{cases}.$$

Since $\frac{dg(n)}{dn} > 0$ if $n < \frac{t}{2(1-u_0'-p)}$ and $\frac{dg(n)}{dn} = 0$ if $n > \frac{t}{2(1-u_0'-p)}$, we know that $n^*(p) = \frac{t}{2(1-u_0'-p)}$ maximizes $g(n)$ when $q_0 < u_0' + p$.

As above, we prove that $n^*(p) = \frac{t}{2(1-u_0'-p)}$ is optimal. This result leads to

$$E[\pi(p)] = E[\pi(n^*(p), p)] = \begin{cases} \frac{V + \frac{1+q_0^2-(u_0'+p)(1+q_0)}{1+q_0-2u_0'-2p} - \frac{t}{4} - p}{V} \times \frac{1+q_0-2u_0'-2p}{2(1-u_0'-p)} \times (p-c), \\ \qquad \text{if } p \leq V + \frac{1+q_0^2-(u_0'+p)(1+q_0)}{1+q_0-2u_0'-2p} - \frac{t}{4} \text{ and } p \leq q_0 - u_0' \\ \frac{V+1-\frac{t}{4}-p}{V} \times \frac{n(1-u_0'-p)}{t} \times (p-c), \\ \qquad \text{if } q_0 - u_0' < p \leq V + 1 - \frac{t}{4} \\ 0, \qquad \text{otherwise} \end{cases} \tag{A.2.2}$$

Although the expression of the optimal price p^* and in turn the expression of n are complicated due to the high order of p in the above function, in the next step (i.e., the second step), we derive the optimal decisions in an example and how that $n^*(p) = \frac{t}{2(1-u_0'-p^*)}$ can be strictly optimal for the firm.

Example: If $t = 4$, $V = 4$, $u_0' = -\frac{7}{2}$, $q_0 = \frac{3}{5}$, and $c = \frac{18}{5}$, then we can rewrite Equation (A.2.2) as

$$E[\pi(p)] = \begin{cases} \frac{(5p-18)(819-405p+50p^2)}{500(9-2p)}, & \text{if } p \leq \frac{39}{10} \\ 0, & \text{otherwise} \end{cases}.$$

The above expression is strictly maximized at p^*, where p^* is the first root of the equation $-72981 + 52650p - 12600p^2 + 1000p^3 = 0$, of which the numerical value is around 3.74037. Note that in this example, we have $q_0 \geq u'_0 + p^*$. As proven in step 1, the firm's expected profit is strictly maximized at $n^* = \frac{t}{2(1-u'_0-p^*)}$.

This completes the proof of Proposition 2.8.

Detailed Analysis of Section 3.4.2

This analysis contains three steps. First, I derive the equilibrium decisions conditional on positive consumer demand in period 1. Second, I derive the equilibrium firm profit conditional on zero consumer demand in period 1. Last, I compare the firm's expected payoff in the above two cases and summarize the equilibrium.

Step 1: Derive the equilibrium decisions conditional on positive consumer demand in period 1.

The following analysis is subject to the constraint of $K < 2$ (otherwise the firm's capacity will be enough to serve all the consumers in the market and all early consumers wait until period 2). Let me start from showing that the firm's problem is equivalent to maximizing p_f.

Claim A.1. *When the expected mass of early purchasers is positive, scalpers purchase positive number of units in period 1. The firm's expected payoff is $\Pi_f = p_f \times K$. Thus, the firm maximizes p_f.*

Proof of Claim A.1: Recall that,

$$\begin{aligned}
u_2 &= \int_{1-\frac{Q_{2f}}{M}}^{1} (v - p_f)^+ dv + \int_{0}^{1-\frac{Q_{2f}}{M}} (v - p_s^*)^+ dv \\
&= \int_{1-\frac{K-D_{1c}-D_{1s}}{2-D_{1c}}}^{1} (v - p_f) dv + \int_{\frac{2-K}{2-D_{1c}}}^{1-\frac{K-D_{1c}-D_{1s}}{2-D_{1c}}} (v - \frac{2-K}{2-D_{1c}}) dv \\
&= \frac{1}{2} - p_f - \frac{(2-K)^2 - 2(2-D_{1c})(2-K)p_f + 2(2-K-2p_f+D_{1c}\cdot p_f)D_{1s}}{2(2-D_{1c})^2}
\end{aligned}.$$

To ensure positive consumer demand in period 1, we need to have $u_1 = \frac{1}{2} - p_f \geq u_2$, which corresponds to $p_f \leq \frac{(2-K)(2-K+2D_{1s})}{2(2-D_{1c})(2-K+D_{1s})} \leq \frac{2-K}{2-D_{1c}}$. Since $\Pi_{2s} = \frac{2-K}{M} \times Q_{2s} - p_f \times Q_{2s} = \frac{2-K}{2-D_{1c}} \times D_{1s} - p_f \times D_{1s}$ when $D_{1s} \leq 2-K$, we know that scalpers will purchase positive number of units if they expect positive consumer demand in period 1. Note that the scalpers can only make positive profit if firm's capacity is not enough to serve all consumers with $v \geq p_f$. In other words, the firm will sell its entire capacity, and $\Pi_f = p_f \times K$. Here, to maximize the expected payoff, the firm maximizes p_f. This completes the proof of Claim A.1.

Given $p_f \leq \frac{(2-K)(2-K+2D_{1s})}{2(2-D_{1c})(2-K+D_{1s})} \leq \frac{2-K}{2-D_{1c}}$, an individual scalper expects positive profit if he acquires one unit of the product, and thus he will purchase it as long as the upper limit β is not reached. In other words, $D^*_{1s} = \beta$. Based on Claim A.1, I derive the equilibrium decisions in the following two cases:

Case: $\beta \leq K-1$: An early consumer is willing to pay at most $\frac{(2-K)(2-K+2D_{1s})}{2(2-D_{1c})(2-K+D_{1s})}$ right away if he expects a mass D_{1c} of early consumers to purchase in period 1. Note that multiple equilibria exist here, depending on the expected value of D_{1c}. Here, following the literature (e.g., Bagwell and Riordan 1991, Pesendorfer 1995), I assume that the firm can coordinate the consumers and achieve the equilibrium yielding the highest firm's profit, i.e., $D^*_{1c} = 1$. Therefore, the equilibrium decisions are: $p^*_f = \frac{(2-K)(2-K+2\beta)}{2(2-K+\beta)}$, $D^*_{1s} = \beta$, $D^*_{1c} = 1$ and $p^*_s = 2-K$. The expected firm's and individual scalper's profits are: $\Pi_f = \frac{(2-K)(2-K+2\beta)K}{2(2-K+\beta)}$ and $\Pi_{s,individual} = \frac{(2-K)^2}{2(2-K+\beta)}$. The expected consumer surplus is: $CS_{early} = CS_{late} = \frac{(2-K)(K-1)+\beta(2K-3)}{2(2-K+\beta)}$.

Case: $K-1 < \beta \leq K$: Here, the feasible D_{1c} yielding the highest firm's profit is $D^*_{1c} = K - D_{1s} = K - \beta$, meaning $M^* = 2 - D^*_{1c} = 2 - K + \beta$. Therefore, the equilibrium decisions are: $p^*_f = \frac{(2-K)(2-K+2\beta)}{2(2-K+\beta)^2}$, $D^*_{1s} = \beta$, $D^*_{1c} = K - \beta$ and $p^*_s = \frac{2-K}{2-K+\beta}$. The expected firm's and individual scalper's profits are: $\Pi_f = \frac{(2-K)(2-K+2\beta)K}{2(2-K+\beta)^2}$ and $\Pi_{s,individual} = \frac{(2-K)^2}{2(2-K+\beta)^2}$. The expected consumer surplus is: $CS_{early} = CS_{late} = \frac{\beta^2}{2(2-K+\beta)^2}$.

Step 2: Derive the equilibrium firm profit conditional on zero consumer demand in period 1.

The result is summarized in Claim A.2.

Claim A.2. *If no early consumer purchases in period 1, then the equilibrium firm profit is*

$$\Pi_f = \begin{cases} \frac{(2-K)K}{2}, & \textit{if } K \leq 1 \\ \frac{1}{2}, & \textit{if } K > 1 \end{cases}$$

Proof of Claim A.2: Note that when no early consumer purchases in the first period, the mass of consumers in the second-period market is $M = 2$. Thus,

$$D_{1s}^{*} = \begin{cases} 0, & \text{if } p_f \geq \frac{2-K}{2} \\ \beta, & \text{if } p_f < \frac{2-K}{2} \end{cases}.$$

This corresponds to the expected firm's profit

$$\Pi_f = \begin{cases} p_f \times \min\{K, 2(1-p)\}, & \text{if } p_f \geq \frac{2-K}{2} \\ p_f \times K, & \text{if } p_f < \frac{2-K}{2} \end{cases}$$

$$= \begin{cases} p_f \times 2(1-p_f), & \text{if } p_f \geq \frac{2-K}{2} \\ p_f \times K, & \text{if } p_f < \frac{2-K}{2} \end{cases}$$

Here, when $p_f \geq \frac{2-K}{2}$, Π_f is maximized at $p = \max\{\frac{1}{2}, \frac{2-K}{2}\}$, corresponding to

$$\Pi_f = \begin{cases} \frac{(2-K)K}{2}, & \text{if } K \leq 1 \\ \frac{1}{2}, & \text{if } K > 1 \end{cases};$$

whereas when $p_f < \frac{2-K}{2}$, $\Pi_f < \frac{(2-K)K}{2} \leq \frac{1}{2}$. This completes the proof of Claim A.2.

Step 3: Compare the firm's expected payoffs derived in Step 1 and 2, and obtain the equilibrium.

Here, we only need to compare the expected firm's payoff derived in Step 1 and Step 2, respectively,

$$\Pi_{f,\text{Step 1}} = \begin{cases} \frac{(2-K)(2-K+2\beta)K}{2(2-K+\beta)}, & \text{if } \beta \leq K-1 \\ \frac{(2-K)(2-K+2\beta)K}{2(2-K+\beta)^2}, & \text{if } K-1 < \beta \leq K \end{cases}$$

and

$$\Pi_{f,\text{Step 2}} = \begin{cases} \frac{(2-K)K}{2}, & \text{if } K \leq 1 \\ \frac{1}{2}, & \text{if } K > 1 \end{cases}. \tag{A.12}$$

Here, the former is preferred over the latter if and only if

$$\begin{cases} 1 < K \leq \frac{1+\sqrt{5}}{2} \\ \frac{(2-K)(K-1)^2}{-1+4K-2K^2} \leq \beta \leq (2-K)(K-1+\sqrt{K(K-1)}) \end{cases}.$$

Thus, the (unconditional) equilibrium decisions are:

(1) $p_f^* = \frac{(2-K)(2-K+2\beta)}{2(2-K+\beta)}$, $D_{1s}^* = \beta$, $D_{1c}^* = 1$ and $p_s^* = 2-K$ if $\begin{cases} 1 < K \leq \frac{1+\sqrt{5}}{2} \\ \frac{(2-K)(K-1)^2}{-1+4K-2K^2} \leq \beta \leq K-1 \end{cases}$.

(2) $p_f^* = \frac{(2-K)(2-K+2\beta)}{2(2-K+\beta)^2}$, $D_{1s}^* = \beta$, $D_{1c}^* = K-\beta$ and $p_s^* = \frac{2-K}{2-K+\beta}$ if $\begin{cases} 1 < K \leq \frac{1+\sqrt{5}}{2} \\ K-1 < \beta \leq (2-K)(K-1+\sqrt{K(K-1)}) \end{cases}$.

(3) $p_f^* = \max\{\frac{2-K}{2}, \frac{1}{2}\}$, $D_{1s}^* = 0$ and $D_{1c}^* = 0$ otherwise.

This completes detailed analysis of Section 3.4.2.

Proof of Proposition 3.1

Based on the equilibrium decisions derived in the above analysis of Section 3.4.2, the firm chooses to sell in period 1 (with a price such that early consumers or scalpers are willing to pay immediately) if and only if $\frac{(2-K)(K-1)^2}{-1+4K-2K^2} \leq \beta \leq (2-K)(K-1+\sqrt{K(K-1)})$, i.e., there is restricted level of scalping. This proves Proposition 3.1.

Proof of Proposition 3.2

Based on the analysis of Section 3.4.2, the firm's expected payoff is $\Pi_f^* = \frac{(2-K)K}{2}$ if $K \leq 1$,

$$\Pi_f^* = \begin{cases} \frac{1}{2}, & \text{if } \beta \leq \frac{(2-K)(K-1)^2}{-1+4K-2K^2} \\ \frac{(2-K)(2-K+2\beta)K}{2(2-K+\beta)}, & \text{if } \frac{(2-K)(K-1)^2}{-1+4K-2K^2} < \beta \leq K-1 \\ \frac{(2-K)(2-K+2\beta)K}{2(2-K+\beta)^2}, & \text{if } K-1 < \beta \leq (2-K)(K-1+\sqrt{K(K-1)}) \\ \frac{1}{2}, & \text{if } (2-K)(K-1+\sqrt{K(K-1)}) < \beta \leq K \end{cases}$$

if $1 < K \leq \frac{1+\sqrt{5}}{2}$, and $\Pi_f^* = \frac{1}{2}$ if $K > \frac{1+\sqrt{5}}{2}$. Here, it is easy to see that $\frac{d\Pi_f^*}{d\beta} \geq 0$ if $\beta \leq K-1$, and $\frac{d\Pi_f^*}{d\beta} \leq 0$ if $\beta \geq K-1$. This shows that the firm's equilibrium profit is first increasing and then decreasing in β.

Note that when $1 < K < \frac{1+\sqrt{5}}{2}$, Π_f^* achieves its strict maximum at $\beta = K-1$. This shows that an intermediate value of β is strictly optimal for the firm when K is intermediate.

This completes the proof of Proposition 3.2.

Proof of Proposition 3.3

If scalping is allowed and the optimal β is chosen (i.e., $\beta = K-1$ for $1 < K \leq \frac{1+\sqrt{5}}{2}$, and β equals any positive value otherwise), then the firm's expected profit is

$$\begin{cases} \frac{(2-K)K}{2}, & \text{if } K \leq 1 \\ \frac{(2-K)K^2}{2}, & \text{if } 1 < K \leq \frac{1+\sqrt{5}}{2} \\ \frac{1}{2}, & \text{if } K > \frac{1+\sqrt{5}}{2} \end{cases},$$

which achieves the strict maximum at $K = \frac{4}{3}$. On the other hand, if there is no scalping, the firm's expected profit is $\begin{cases} \frac{(2-K)K}{2}, & \text{if } K \leq 1 \\ \frac{1}{2}, & \text{if } K > 1 \end{cases}$, which achieves the maximum at $K = 1$.

To sum up, an intermediate value of K is optimal for the firm even with zero unit cost production. Moreover, when scalping is allowed, the optimal value of K (i.e., $\frac{4}{3}$) is larger than that when scalping is not allowed (i.e., 1).

This completes the proof of Proposition 3.3.

Detailed Analysis of Section 3.5.1

Similar to the analysis of main model, I first derive the expected firm profit conditional on positive consumer demand in period 1. Then, I compare it with the expected firm profit conditional on zero consumer demand in period 1, i.e., $\begin{cases} \frac{(2-K)K}{2}, & \text{if } K \leq 1 \\ \frac{1}{2}, & \text{if } K > 1 \end{cases}$. Last, I show that the main results remain to hold under the proportional-rationing rule.

As in the main text, the early consumers' expected utility of waiting is

$$\begin{aligned} u_2 &= \int_{p_f}^{1} \frac{Q_{2f}}{M(1-p_f)}(v-p_f)dv + \int_{p_s^*}^{1} \frac{M(1-p_f)-Q_{2f}}{M(1-p_f)}(v-p_s^*)dv \\ &= \int_{p_f}^{1} \frac{M+K-2-Q_{2s}}{M(1-p_f)}(v-p_f)dv + \int_{\frac{2-K+Q_{2s}p_f-Mp_f}{2-K+Q_{2s}-Mp_f}}^{1} \frac{2-K+Q_{2s}-Mp_f}{M(1-p_f)}(v-\frac{2-K+Q_{2s}p_f-Mp_f}{2-K+Q_{2s}-Mp_f})dv \\ &= \frac{(1-p_f)Q_{2s}^2}{2M(2-K+Q_{2s}-Mp_f)} - \frac{(1-p_f)(2-K+Q_{2s}-M)}{2M} \end{aligned}$$

Similar to the main model, when expecting positive consumer demand in period 1 ($u_1 \geq u_2$), the scalpers will purchase β units in period 1. The firm's expected payoff is $\Pi_f = p_f \times K$. Moreover, in the equilibrium yielding the highest firm's profit, the expected mass of early purchasers is $D_{1c}^* = \min\{1, K-\beta\}$.

Thus, if $\beta \leq K-1$, then the equilibrium decisions are: $p_f^* = \frac{8+6\beta-2K(3+\beta)+K^2-\sqrt{(8+6\beta-2K(3+\beta)+K^2)^2-4(2-K)(3-K+\beta)(2-K+2\beta)}}{6-2K+2\beta}$, $D_{1s}^* = \beta$, $D_{1c}^* = 1$ and $p_s^* = \frac{(2-K)^2+2(3-K)\beta-\sqrt{(2-K)^4+4(2-K)^3\beta+4(5-4K+K^2)\beta^2}}{2\beta}$. The expected firm's and individual scalper's profits are: $\Pi_f = \frac{8+6\beta-2K(3+\beta)+K^2-\sqrt{(8+6\beta-2K(3+\beta)+K^2)^2-4(2-K)(3-K+\beta)(2-K+2\beta)}}{6-2K+2\beta}K$ and $\Pi_{s,individual} = \frac{(3-K)(2-K)^2+2(7-5K+K^2)\beta-(3-K)\sqrt{(2-K)^4+4(2-K)^3\beta+4(5-4K+K^2)\beta^2}}{2(3-K+\beta)\beta}$. The expected consumer surplus is: $CS_{early} = CS_{late} = \frac{-5+5K-K^2-(5-2K)\beta+\sqrt{(2-K)^4+4(2-K)^3\beta+4(5-4K+K^2)\beta^2}}{2(3-K+\beta)}$.

If $K-1<\beta\leq K$, on the other hand, the equilibrium decisions are: $p_f^*=\frac{(2-K)(2-K+2\beta)}{2(2-K+\beta)^2}$, $D_{1s}^*=\beta$, $D_{1c}^*=K-\beta$ and $p_s^*=2-K$. The expected firm's and individual scalper's profits are: $\Pi_f=\frac{(2-K)(2-K+2\beta)K}{2(2-K+\beta)^2}$ and $\Pi_{s,individual}=\frac{(2-K)^2}{2(2-K+\beta)^2}$. The expected consumer surplus is: $CS_{early}=CS_{late}=\frac{\beta^2}{2(2-K+\beta)^2}$.

That is, the expected firm profit conditional on positive consumer demand in period 1 is

$$\begin{cases}\frac{8+6\beta-2K(3+\beta)+K^2-\sqrt{(8+6\beta-2K(3+\beta)+K^2)^2-4(2-K)(3-K+\beta)(2-K+2\beta)}}{6-2K+2\beta}K, & \text{if } \beta\leq K-1\\ \frac{(2-K)(2-K+2\beta)K}{2(2-K+\beta)^2}, & \text{if } K-1<\beta\leq K\end{cases}.$$

Recall that the expected firm profit conditional on zero consumer demand in period 1 is $\begin{cases}\frac{(2-K)K}{2}, & \text{if } K\leq 1\\ \frac{1}{2}, & \text{if } K>1\end{cases}$. Thus, the (unconditional) equilibrium firm profit is $\Pi_f^*=\frac{(2-K)K}{2}$ if $K\leq 1$,

$$\Pi_f^*=\begin{cases}\frac{8+6\beta-2K(3+\beta)+K^2-\sqrt{(8+6\beta-2K(3+\beta)+K^2)^2-4(2-K)(3-K+\beta)(2-K+2\beta)}}{6-2K+2\beta}K, & \\ & \text{if } \beta\leq K-1\\ \frac{(2-K)(2-K+2\beta)K}{2(2-K+\beta)^2}, & \text{if } K-1<\beta\leq(2-K)(K-1+\sqrt{K(K-1)})\\ \frac{1}{2}, & \text{if } (2-K)(K-1+\sqrt{K(K-1)})<\beta\leq K\end{cases}$$

if $1<K\leq\frac{3}{2}$,

$$\Pi_f^*=\begin{cases}\frac{1}{2}, & \text{if } \beta\leq\frac{(K-1)(3-2K)(1-4K+2K^2)}{1-12K+20K^2-8K^3}\\ \frac{8+6\beta-2K(3+\beta)+K^2-\sqrt{(8+6\beta-2K(3+\beta)+K^2)^2-4(2-K)(3-K+\beta)(2-K+2\beta)}}{6-2K+2\beta}K, & \\ & \text{if } \frac{(K-1)(3-2K)(1-4K+2K^2)}{1-12K+20K^2-8K^3}<\beta\leq K-1\\ \frac{(2-K)(2-K+2\beta)K}{2(2-K+\beta)^2}, & \text{if } K-1<\beta\leq(2-K)(K-1+\sqrt{K(K-1)})\\ \frac{1}{2}, & \text{if } (2-K)(K-1+\sqrt{K(K-1)})<\beta\leq K\end{cases}$$

if $\frac{3}{2}<K\leq\frac{1+\sqrt{5}}{2}$, and $\Pi_f^*=\frac{1}{2}$ if $K>\frac{1+\sqrt{5}}{2}$. Here, it is easy to see that $\frac{d\Pi_f^*}{d\beta}\geq 0$ if $\beta\leq K-1$, and $\frac{d\Pi_f^*}{d\beta}\leq 0$ if $\beta\geq K-1$. This shows that the firm's equilibrium profit is first increasing and then decreasing in β.

Note that when $1 < K < \frac{1+\sqrt{5}}{2}$, Π_f^* achieves its strict maximum at $\beta = K - 1$. This shows that an intermediate value of β is strictly optimal for the firm when K is intermediate.

Moreover, when β is chosen at the level optimal for the firm, the firm's expected profit is $\begin{cases} \frac{(2-K)K}{2}, & \text{if } K \leq 1 \\ \frac{(2-K)K^2}{2}, & \text{if } 1 < K \leq \frac{1+\sqrt{5}}{2} \\ \frac{1}{2}, & \text{if } K > \frac{1+\sqrt{5}}{2} \end{cases}$, which achieves the strict maximum at $K = \frac{4}{3}$. This shows that an intermediate value of K is optimal for the firm even with zero unit cost of production.

This completes the analysis of Section 3.5.1 and proves that the main results hold under the proportional-rationing rule.

Detailed Analysis of Section 3.5.2

Here, I derive the equilibrium decisions when scalpers coordinate with each other on price and maximize the joint payoff. Similar to the analysis of main model, I first derive the expected firm profit conditional on positive consumer demand in period 1. I focus on the case of $1 < K < 2$. (Similar to the main model, the firm would rather not to sell in period 1 if $K \leq 1$ or $K \geq 2$).

Similar to Claim A.1, when expecting positive consumer demand in period 1, the scalpers purchase positive number of units, and the firm maximizes p_f. In the equilibrium yielding the highest firm's profit, $D_{1c}^* = \max\{1, K - D_{1s}\}$. Thus, we can rewrite the scalpers' problem (Equation (3.4)) for the $K \leq \frac{3}{2}$ case as

$$\max_{D_{1s} \leq \beta} \Pi_s(D_{1s}) = \begin{cases} (2-K) \times D_{1s} - p_f \times D_{1s}, & \text{if } D_{1s} \leq K - 1 \\ \frac{2-K}{2-K+D_{1s}} \times D_{1s} - p_f \times D_{1s}, & \text{if } K - 1 < D_{1s} \leq 2 - K \\ \frac{2-K+D_{1s}}{4} - p_f \times D_{1s}, & \text{if } D_{1s} > 2 - K \end{cases} \tag{A.13}$$

and that for the $\frac{3}{2} < K < 2$ case as

$$\max_{D_{1s}\leq\beta} \Pi_s(D_{1s}) = \begin{cases} (2-K)\times D_{1s} - p_f \times D_{1s}, & \text{if } D_{1s} \leq 2-K \\ \frac{(2-K+D_{1s})^2}{4} - p_f \times D_{1s}, & \text{if } 2-K < D_{1s} \leq K-1 \\ \frac{2-K+D_{1s}}{4} - p_f \times D_{1s}, & \text{if } D_{1s} > K-1 \end{cases} \tag{A.14}$$

Recall that

$$\begin{aligned} u_2 &= \int_{1-\frac{Q_{2f}}{M}}^{1} (v-p_f)^+ dv + \int_0^{1-\frac{Q_{2f}}{M}} (v-p_s^*)^+ dv \\ &= \begin{cases} \int_{1-\frac{M+K-2-Q_{2s}}{M}}^{1} (v-p_f)dv + \int_{\frac{2-K}{M}}^{1-\frac{M+K-2-Q_{2s}}{M}} (v - \frac{2-K}{M})dv, & \text{if } Q_{2s} \leq 2-K \\ \int_{1-\frac{M+K-2-Q_{2s}}{M}}^{1} (v-p_f)dv + \int_{\frac{2-K+Q_{2s}}{2M}}^{1-\frac{M+K-2-Q_{2s}}{M}} (v - \frac{2-K+Q_{2s}}{2M})dv, & \text{if } Q_{2s} > 2-K \end{cases} \\ &= \begin{cases} \frac{1}{2} - p_f - \frac{(2-K)^2 - 2Mp_f(2-K) + 2(2-K-Mp_f)Q_{2s}}{2M^2}, & \text{if } Q_{2s} \leq 2-K \\ \frac{1}{2} - p_f - \frac{3(2-K)^2 - 8Mp_f(2-K) + 2(6-3K-4Mp_f)Q_{2s} + 3Q_{2s}^2}{8M^2}, & \text{if } Q_{2s} > 2-K \end{cases} \end{aligned} \tag{A.15}$$

Thus, to ensure positive consumer demand in period 1 ($u_1 = \frac{1}{2} - p_f \geq u_2$), we need to have $p_f \leq \frac{(2-K)(2-K+2D_{1s})}{2(2-D_{1c}^*)(2-K+D_{1s})}$ for $D_{1s} \leq 2-K$ and $p_f \leq \frac{3(2-K+D_{1s})}{8(2-D_{1c}^*)}$ for $D_{1s} > 2-K$.

Case: $\beta \leq K-1$ and $\beta \leq 2-K$: Here, $D_{1s} \leq \beta \leq 2-K$, and thus $p_f \leq \frac{(2-K)(2-K+2D_{1s})}{2(2-D_{1c}^*)(2-K+D_{1s})} \leq \frac{2-K}{2-D_{1c}^*}$, where $D_{1c}^* = 1$. Given $\pi_{2s} = (2-K)\times D_{1s} - p_f \times D_{1s}$, we have $D_{1s}^* = \beta$. Therefore, the equilibrium decisions are: $p_f^* = \frac{(2-K)(2-K+2\beta)}{2(2-K+\beta)}$, $D_{1s}^* = \beta$, $D_{1c}^* = 1$ and $p_s^* = 2-K$. The expected firm's and scalpers' profits are: $\Pi_f = \frac{(2-K)(2-K+2\beta)K}{2(2-K+\beta)}$ and $\Pi_s = \frac{(2-K)^2\beta}{2(2-K+\beta)}$. The expected consumer surplus is: $CS_{early} = CS_{late} = \frac{(2-K)(K-1)+\beta(2K-3)}{2(2-K+\beta)}$.

Case: $K-1 < \beta \leq 2-K$: Here, to maximize the expected joint payoff, the scalpers choose $D_{1s} = K-1$ if $p_f \geq (2-K)^2$, $D_{1s} = (\frac{1}{\sqrt{p_f}-1})(2-K)$ if $\begin{cases} \frac{1}{4} < p_f < (2-K)^2 \\ \beta \geq (\frac{1}{\sqrt{p_f}-1})(2-K) \end{cases}$,

and $D_{1s} = \beta$ if $p_f \leq \frac{1}{4}$ or $\begin{cases} \frac{1}{4} < p_f < (2-K)^2 \\ \beta < (\frac{1}{\sqrt{p_f}-1})(2-K) \end{cases}$. Note that the highest price p_f to ensure early consumers to purchase immediately is $p_f = \frac{K(2-K)}{2}$ when $D_{1s} = K-1$, $p_f = \frac{4}{9}$ when $D_{1s} = (\frac{1}{\sqrt{p_f}-1})(2-K) = \frac{2-K}{2}$, and $p_f = \frac{(2-K)(2-K+2\beta)}{2(2-K+\beta)^2}$ when $D_{1s} = \beta$. To sum up, if $\begin{cases} 1 < K \leq \frac{4}{3} \\ K-1 < \beta \leq \frac{2-K}{2} \end{cases}$, the equilibrium decisions are: $p_f^* = \frac{(2-K)(2-K+2\beta)}{2(2-K+\beta)^2}$, $D_{1s}^* = \beta$, $D_{1c}^* = K-\beta$ and $p_s^* = 2-K$. The expected firm's and scalpers' profits are: $\Pi_f = \frac{(2-K)(2-K+2\beta)K}{2(2-K+\beta)^2}$ and $\Pi_s = \frac{(2-K)^2\beta}{2(2-K+\beta)^2}$. The expected consumer surplus is: $CS_{early} = CS_{late} = \frac{\beta^2}{2(2-K+\beta)^2}$. If $\begin{cases} 1 < K \leq \frac{4}{3} \\ \frac{2-K}{2} < \beta \leq 2-K \end{cases}$, the equilibrium decisions are: $p_f^* = \frac{4}{9}$, $D_{1s}^* = \frac{2-K}{2}$, $D_{1c}^* = \frac{3K-2}{2}$ and $p_s^* = \frac{2}{3}$. The expected firm's and scalpers' profits are: $\Pi_f = \frac{4}{9}K$ and $\Pi_s = \frac{2-K}{9}$. The expected consumer surplus is: $CS_{early} = CS_{late} = \frac{1}{18}$. And if $\begin{cases} \frac{4}{3} < K \leq \frac{3}{2} \\ K-1 < \beta \leq 2-K \end{cases}$, the equilibrium decisions are: $p_f^* = \frac{(2-K)K}{2}$, $D_{1s}^* = K-1$, $D_{1c}^* = 1$ and $p_s^* = 2-K$. The expected firm's and scalpers' profits are: $\Pi_f = \frac{(2-K)K^2}{2}$ and $\Pi_s = \frac{(K-1)(2-K)^2}{2}$. The expected consumer surplus is: $CS_{early} = CS_{late} = \frac{(K-1)^2}{2}$.

Case: $K-1 \leq 2-K < \beta$: Here, to maximize the expected joint payoff, the scalpers choose $D_{1s} = K-1$ if $p_f \geq (2-K)^2$, $D_{1s} = (\frac{1}{\sqrt{p_f}-1})(2-K)$ if $\frac{1}{4} < p_f < (2-K)^2$, and $D_{1s} = \beta$ if $p_f \leq \frac{1}{4}$. Note that the highest price p_f to ensure early consumers to purchase immediately is $p_f = \frac{K(2-K)}{2}$ when $D_{1s} = K-1$, $p_f = \frac{4}{9}$ when $D_{1s} = (\frac{1}{\sqrt{p_f}-1})(2-K) = \frac{2-K}{2}$, and $p_f = \frac{3}{8}$ when $D_{1s} = \beta$. To sum up, if $\begin{cases} 1 < K \leq \frac{4}{3} \\ K-1 \leq 2-K < \beta \end{cases}$, the equilibrium decisions are: $p_f^* = \frac{4}{9}$, $D_{1s}^* = \frac{2-K}{2}$, $D_{1c}^* = \frac{3K-2}{2}$ and $p_s^* = \frac{2}{3}$. The expected firm's and scalpers' profits are: $\Pi_f = \frac{4}{9}K$ and $\Pi_s = \frac{2-K}{9}$. The expected consumer surplus is: $CS_{early} = CS_{late} = \frac{1}{18}$. If $\begin{cases} \frac{4}{3} < K \leq \frac{3}{2} \\ K-1 \leq 2-K < \beta \end{cases}$, on the other hand, the equilibrium decisions are: $p_f^* = \frac{(2-K)K}{2}$, $D_{1s}^* = K-1$, $D_{1c}^* = 1$ and $p_s^* = 2-K$. The expected firm's and scalpers' profits are:

$\Pi_f = \frac{(2-K)K^2}{2}$ and $\Pi_s = \frac{(K-1)(2-K)^2}{2}$. The expected consumer surplus is: $CS_{early} = CS_{late} = \frac{(K-1)^2}{2}$.

Case: $2-K < \beta \leq K-1$*:* Here, the scalpers' joint payoff is maximized at $D_{1s} = \beta$ if $p \leq \frac{(2-K+\beta)^2}{4\beta}$, or $D_{1s} = 0$ if $p > \frac{(2-K+\beta)^2}{4\beta}$. To sell in the first period, the maximum price the firm can set is $p_f = \min\{\frac{(2-K+\beta)^2}{4\beta}, \frac{3(2-K+Q_{2s})}{8M}\}$. To sum up, if $\begin{cases} \frac{3}{2} < K \leq \frac{5}{3} \\ 2-K < \beta \leq K-1 \end{cases}$ or $\begin{cases} \frac{5}{3} < K < 2 \\ 2-K < \beta \leq 4-2K \end{cases}$, the equilibrium decisions are: $p_f^* = \frac{3(2-K+\beta)}{8}$, $D_{1s}^* = \beta$, $D_{1c}^* = 1$ and $p_s^* = \frac{2-K+\beta}{2}$. The expected firm's and scalpers' profits are: $\Pi_f = \frac{3(2-K+\beta)K}{8}$, $\Pi_s = \frac{(2-K+\beta)^2}{4} - \frac{3(2-K+\beta)}{8}$. The expected consumer surplus is: $CS_{early} = CS_{late} = \frac{2-3K+3\beta}{8}$. If $\begin{cases} \frac{5}{3} < K < 2 \\ 4-2K < \beta \leq K-1 \end{cases}$, on the other case, the equilibrium decisions are: $p_f^* = \frac{(2-K+\beta)^2}{4\beta}$, $D_{1s}^* = \beta$, $D_{1c}^* = 1$ and $p_s^* = \frac{2-K+\beta}{2}$. The expected firm's and scalpers' profits are: $\Pi_f = \frac{(2-K+\beta)^2 K}{4\beta}$, $\Pi_s = 0$. The expected consumer surplus is: $CS_{early} = \frac{2\beta-(2-K+\beta)^2}{4\beta}$ and $CS_{late} = \frac{-2(K-1)(2-K)^2+(8-8K+3K^2)\beta-2\beta^2-\beta^3}{8\beta}$.

Case: $2-K < K-1 < \beta$*:* Here, to maximize the expected joint payoff, the scalpers choose $D_{1s} = 0$ if $p_f > \frac{1}{4(K-1)}$, $D_{1s} = K-1$ if $\frac{1}{4} < p_f \leq \frac{1}{4(K-1)}$, and $D_{1s} = \beta$ if $p_f \leq \frac{1}{4}$. Note that the highest price p_f to ensure early consumers to purchase immediately is $\frac{3}{8}$ when $D_{1s} \geq 2-K$. Thus, $p_f^* = \min\{\frac{3}{8}, \frac{1}{4(K-1)}\}$. To sum up, if $\begin{cases} \frac{3}{2} < K \leq \frac{5}{3} \\ 2-K < K-1 < \beta \end{cases}$, the equilibrium decisions are: $p_f^* = \frac{3}{8}$, $D_{1s}^* = K-1$, $D_{1c}^* = 1$ and $p_s^* = \frac{1}{2}$. The expected firm's and scalpers' profits are: $\Pi_f = \frac{3K}{8}$, $\Pi_s = \frac{5-3K}{8}$. The expected consumer surplus is: $CS_{early} = CS_{late} = \frac{1}{8}$. If $\begin{cases} \frac{5}{3} < K < 2 \\ 2-K < K-1 < \beta \end{cases}$, on the other hand, the equilibrium decisions are: $p_f^* = \frac{1}{4(K-1)}$,

$D^*_{1s} = K-1$, $D^*_{1c} = 1$ and $p^*_s = \frac{1}{2}$. The expected firm's and scalpers' profits are: $\Pi_f = \frac{K}{4(K-1)}$, $\Pi_s = 0$. The expected consumer surplus is: $CS_{early} = \frac{2K-3}{4(K-1)}$ and $CS_{late} = \frac{1}{8}$.

This completes the analysis conditional on positive consumer demand in period 1. Now, we only need to compare the expected firm's payoff derived above with $\begin{cases} \frac{(2-K)K}{2}, & \text{if } K \leq 1 \\ \frac{1}{2}, & \text{if } K > 1 \end{cases}$. If the former is larger, the equilibrium is the one specified above. If the latter is larger, on the other hand, the equilibrium is the one specified in the proof of Claim A.2.

This completes the derivation of the equilibrium decisions when scalpers coordinate with each other on price and maximize the joint payoff.

Proof of Proposition 3.4

Based on the analysis of Section 3.5.2, we know that in equilibrium, the scalpers intentionally limit the number of units acquired (i.e., $D^*_{1s} < \beta$) if $\begin{cases} 1 < K \leq \frac{4}{3} \\ \beta > \frac{2-K}{2} \end{cases}$ or $\begin{cases} \frac{4}{3} < K < 2 \\ \beta > K-1 \end{cases}$. Moreover, the scalpers intentionally limit the number of units (re)sold (i.e., $2-K < D^*_{1s}$) if $\begin{cases} \frac{3}{2} < K < 2 \\ \beta > 2-K \end{cases}$.

To prove the effect of scalping on firm's profit summarized in Proposition 3.4, let me first list the firm's equilibrium profit based on the analysis of Section 3.5.2: $\Pi^*_f = \frac{(2-K)K}{2}$ if $K \leq 1$,

$$\Pi^*_f = \begin{cases} \frac{1}{2}, & \text{if } \beta \leq \frac{(2-K)(K-1)^2}{-1+4K-2K^2} \\ \frac{(2-K)(2-K+2\beta)K}{2(2-K+\beta)}, & \text{if } \frac{(2-K)(K-1)^2}{-1+4K-2K^2} < \beta \leq K-1 \\ \frac{(2-K)(2-K+2\beta)K}{2(2-K+\beta)^2}, & \text{if } K-1 < \beta \leq (2-K)(K-1+\sqrt{K(K-1)}) \\ \frac{1}{2}, & \text{if } (2-K)(K-1+\sqrt{K(K-1)}) < \beta \leq K \end{cases}$$

if $1 < K \leq \frac{9}{8}$,

$$\Pi_f^* = \begin{cases} \frac{1}{2}, & \text{if } \beta \leq \frac{(2-K)(K-1)^2}{-1+4K-2K^2} \\ \frac{(2-K)(2-K+2\beta)K}{2(2-K+\beta)}, & \text{if } \frac{(2-K)(K-1)^2}{-1+4K-2K^2} < \beta \leq K-1 \\ \frac{(2-K)(2-K+2\beta)K}{2(2-K+\beta)^2}, & \text{if } K-1 < \beta \leq \frac{2-K}{2} \\ \frac{4K}{9}, & \text{if } \frac{2-K}{2} < \beta \leq K \end{cases}$$

if $\frac{9}{8} < K \leq \frac{4}{3}$,

$$\Pi_f^* = \begin{cases} \frac{1}{2}, & \text{if } \beta \leq \frac{(2-K)(K-1)^2}{-1+4K-2K^2} \\ \frac{(2-K)(2-K+2\beta)K}{2(2-K+\beta)}, & \text{if } \frac{(2-K)(K-1)^2}{-1+4K-2K^2} < \beta \leq K-1 \\ \frac{(2-K)K^2}{2}, & \text{if } K-1 < \beta \leq K \end{cases}$$

if $\frac{4}{3} < K \leq \frac{3}{2}$,

$$\Pi_f^* = \begin{cases} \frac{1}{2}, & \text{if } \beta \leq \frac{(2-K)(K-1)^2}{-1+4K-2K^2} \\ \frac{(2-K)(2-K+2\beta)K}{2(2-K+\beta)}, & \text{if } \frac{(2-K)(K-1)^2}{-1+4K-2K^2} < \beta \leq 2-K \\ \frac{3(2-K+\beta)K}{8}, & \text{if } 2-K < \beta \leq K-1 \\ \frac{3K}{8}, & \text{if } K-1 < \beta \leq K \end{cases}$$

if $\frac{3}{2} < K \leq \frac{3+\sqrt{3}}{3}$,

$$\Pi_f^* = \begin{cases} \frac{1}{2}, & \text{if } \beta \leq \frac{4-6K+3K^2}{3K} \\ \frac{3(2-K+\beta)K}{8}, & \text{if } \frac{4-6K+3K^2}{3K} < \beta \leq K-1 \\ \frac{3K}{8}, & \text{if } K-1 < \beta \leq K \end{cases}$$

if $\frac{3+\sqrt{3}}{3} < K \leq \frac{5}{3}$,

$$\Pi_f^* = \begin{cases} \frac{1}{2}, & \text{if } \beta \leq \frac{4-6K+3K^2}{3K} \\ \frac{3(2-K+\beta)K}{8}, & \text{if } \frac{4-6K+3K^2}{3K} < \beta \leq 4-2K \\ \frac{(2-K+\beta)^2K}{4\beta}, & \text{if } 4-2K < \beta < K-1 \\ \frac{K}{4(K-1)}, & \text{if } K-1 < \beta \leq K \end{cases}$$

if $\frac{5}{3} < K \leq \frac{3+\sqrt{5}}{3}$,

$$\Pi_f^* = \begin{cases} \frac{1}{2}, & \text{if } \beta \leq \frac{(K-1)^2+\sqrt{1-4K+2K^2}}{K} \\ \frac{(2-K+\beta)^2 K}{4\beta}, & \text{if } \frac{(K-1)^2+\sqrt{1-4K+2K^2}}{K} < \beta < K-1 \\ \frac{K}{4(K-1)}, & \text{if } K-1 < \beta \leq K \end{cases}$$

if $\frac{3+\sqrt{5}}{3} < K \leq 2$, and $\Pi_f^* = \frac{1}{2}$ if $K > 2$.

Here, it is easy to see that $\frac{d\Pi_f^*}{d\beta} \geq 0$ if $\beta \leq K-1$, and $\frac{d\Pi_f^*}{d\beta} \leq 0$ if $\beta \geq K-1$. This shows that the firm's equilibrium profit is first increasing and then decreasing in β. Moreover, when $1 < K < \frac{4}{3}$, Π_f^* achieves its strict maximum at $\beta = K-1$. This shows that an intermediate value of β is strictly optimal for the firm when K is intermediate.

This proves the first part of Proposition 3.4.

The second part of Proposition 3.4 naturally follows the comparison between the equilibrium firm's profit in Section 3.5.2 and that in the main model.

Detailed Analysis of Section 3.5.3

Similar to the main model, I start with the decisions in period 2 and then analyze the decisions in period 1.

Decisions in Period 2

If $Q_{2f} + Q_{2s} \leq M \cdot \delta$, the firm's and the scalpers' capacity is not enough to serve any low-valuation consumers (i.e., $v < 1-\delta$). The firm and the scalpers compete for the high-valuation consumers (i.e., $v \geq 1-\delta$). The equilibrium prices will be the market-clearing price, i.e., $p_{sh}^* = p_{sl}^* = p_{f2}^* = 1 - \frac{Q_{2f}+Q_{2s}}{M}$. The rest discussion is based on $Q_{2f} + Q_{2s} > M \cdot \delta$.

First, notice that it suffices to consider a limited cases of possible prices: (I) $p_{sh} = 1-\delta \geq p_{sl} > p_{f2}$, (II) $p_{sh} = 1-\delta > p_{f2} \geq p_{sl}$, and (III) $1-\delta \geq p_{f2} = p_{sh} \geq p_{sl}$. This naturally follows the following three observations: (1) it is never strictly optimal for scalpers to set p_{sh} and p_{sl} such that $p_{sh} < p_{sl}$,(2) $\max\{p_{sh}, p_{f2}\} \leq 1-\delta$, and (3) given $p_{f2} \leq 1-\delta$, it is optimal for the scalpers to set $p_{sh} = 1-\delta$ or $p_{sh} = p_{f2}$.

To prove the first observation, note that any $p^{'}_{sh}$ and $p^{'}_{sl}$ such that $p^{'}_{sh} < p^{'}_{sl}$ is weakly dominated by $p^{''}_{sh} = p^{''}_{sl} = p^{'}_{sh}$ or $p^{''}_{sh} = p^{''}_{sl} = p^{'}_{sl}$. There is no incentive for scalpers to charge higher-valuation consumers a lower price, given that the competitor (i.e., the firm) charges a single price.

To prove the second and the third observation, note that the scalpers want to maximize the profit generated from selling to high-valuation consumers (since they pay a higher price). If $p_{f2} > 1-\delta \geq \frac{1}{2}$, the scalpers maximize

$$\begin{cases} p_{sh} \times \min\{Q_{2s}, M(1-\frac{Q_{2f}}{M} - p_{sh}), M\cdot\delta - Q_{2f}\}, & \text{if } p_{sh} > p_{f2} \\ p_{sh} \times \min\{Q_{2s}, M(1-p_{sh}), M\cdot\delta\}, & \text{if } p_{sh} \leq p_{f2} \end{cases} \tag{A.16}$$

by setting $p_{sh} = \min\{\max\{\frac{1}{2}, 1-\frac{Q_{2s}}{M}, 1-\delta\}, p_{f2}\}$, which leads to zero firm's profit in period 2. Thus, $p_{f2} \leq 1-\delta$. Moreover, given $p_{f2} \leq 1-\delta$, the scalpers again maximize Equation (A.16). The optimal p_{sh} here is $1-\delta$ or p_{f2}.

Based on the three observations discussed above, we only need to consider a limited cases of possible prices: (I) $p_{sh} = 1-\delta \geq p_{sl} > p_{f2}$, (II) $p_{sh} = 1-\delta > p_{f2} \geq p_{sl}$, and (III) $1-\delta \geq p_{f2} = p_{sh} \geq p_{sl}$. As follows, I start with the scalpers' pricing decisions given δ, M, Q_{2f}, Q_{2s} and p_{f2}. For each of the three cases listed above, I fix p_{sh} and solve for the (locally) optimal p_{sl} under the constraints $p_{sl} > p_{f2}$ or $p_{sl} \leq p_{f2}$. Then, I derive the scalpers' (globally) optimal pricing decisions in period 2 by comparing across the three cases. After obtaining the scalpers' best response to p_{f2}, I derive the firm's optimal pricing decision in period 2.

Step 1: Derive the Scalpers' Pricing Decisions in Period 2

Case (I): $p_{sh} = 1-\delta \geq p_{sl} > p_{f2}$

Given the consumers' purchase decisions, the scalpers' second-period profit under Case (I) is:

$$\pi_{2s,I} = \begin{cases} (1-\delta)\times(M\cdot\delta - Q_{2f}) + p_{sl}\times\min\{Q_{2f}+Q_{2s}-M\cdot\delta, M(1-\delta-p_{sl})\}, & \\ & \text{if } Q_{2f}+Q_{2s} > M\cdot\delta \geq Q_{2f} \\ p_{sl}\times\min\{Q_{2s}, M(1-\frac{Q_{2f}}{M}-p_{sl})\}, & \text{if } Q_{2f} > M\cdot\delta \text{ and } p_{f2} < 1-\frac{Q_{2f}}{M} \\ 0, & \text{if } Q_{2f} > M\cdot\delta \text{ and } p_{f2} \geq 1-\frac{Q_{2f}}{M} \end{cases}$$

This leads to

$$p^*_{sl,I} = \max\{1-\frac{Q_{2f}+Q_{2s}}{M}, \min\{\frac{1-\delta}{2}, \frac{1-\frac{Q_{2f}}{M}}{2}\}, p_{f2}+\epsilon\}.$$

where ϵ is positive and infinitely small. The results are summarized in Table A.3.1.

Case (II): $p_{sh} = 1-\delta > p_{f2} \geq p_{sl}$

The scalpers' second-period profit under Case (II) is:

$$\pi_{2s,II} = \begin{cases} (1-\delta)\times(M\cdot\delta - Q_{2f}) + p_{sl}\times\min\{Q_{2f}+Q_{2s}-M\cdot\delta, M(1-\delta-p_{sl})\}, & \\ & \text{if } Q_{2f}+Q_{2s} > M\cdot\delta \geq Q_{2f} \\ p_{sl}\times\min\{Q_{2s}, M(1-\delta-p_{sl})\}, & \text{if } Q_{2f} > M\cdot\delta \end{cases}$$

This leads to

$$p^*_{sl,II} = \min\{\max\{1-\frac{Q_{2f}+Q_{2s}}{M}, 1-\delta-\frac{Q_{2s}}{M}, \frac{1-\delta}{2}\}, p_{f2}\}.$$

The results are summarized in Table A.3.2.

Case (III): $1-\delta \geq p_{f2} = p_{sh} \geq p_{sl}$

The scalpers' second-period profit under Case (III) is:

$$\pi_{2s,III} = \begin{cases} p_{f2}\times Q_{2s}, & \text{if } Q_{2f}+Q_{2s} > M\cdot\delta \geq Q_{2s} \\ p_{f2}\times M\cdot\delta + & \\ \quad p_{sl}\times\min\{Q_{2s}-M\cdot\delta, M(1-\delta-p_{sl})\}, & \text{if } Q_{2s} > M\cdot\delta \end{cases}$$

This leads to

$$p^*_{sl,III} = \min\{\max\{1 - \frac{Q_{2s}}{M}, \frac{1-\delta}{2}\}, p_{f2}\}.$$

The results are summarized in Table A.3.3.

To obtain the scalpers' pricing decisions in period 2 given δ, M, Q_{2f}, Q_{2s} and p_{f2}, we only need to compare the scalpers' expected second-period profits across the above three cases. I summarize the results in Table A.3.4 through Table A.3.7.

Step 2: Derive the Firm's Pricing Decision in Period 2

In anticipation of scalpers' pricing strategy, the firm sets p_{f2} to maximize its expected profit in the second period. I summarize the results in Table A.3.8 through Table A.3.12. (Note: I focus on the case $\delta \leq \frac{1}{3}$.)

Decisions in Period 1

I start with early consumers' and scalpers' purchase decisions. Then, I analyze the firm's decisions on how to coordinate the early consumers (i.e., what D^*_{1c} to have) and what first-period price to charge (i.e., what p_{f1} to have). Note that the closed-form solutions are difficult to obtained since there are too many cases to discuss. Thus, I explain the algorithm of deriving the equilibrium decisions numerically.

An early consumer compares the expected utility of purchasing immediately ($u_1 = \frac{1}{2} - p_{f1}$) and waiting ($u_2$). To derive u_2, note that, if $Q_{2f} + Q_{2s} \leq M \cdot \delta$, we already know that $p^*_{sh} = p^*_{sl} = p^*_{f2} = 1 - \frac{Q_{2f}+Q_{2s}}{M}$. If $Q_{2f} + Q_{2s} \leq M \cdot \delta$, on the other hand, based on Table A.3.8 through Table A.3.12, we know that $p^*_{f2} = p^*_{sh} = p^*_{sl}$ if $\begin{cases} Q_{2f} \geq \frac{M}{3} \\ M - 2Q_{2f} \leq Q_{2s} < Q_{2f} \end{cases}$, and $p^*_{sl} \leq p^*_{f2} < p^*_{sh}$ otherwise. Moreover, the prices will be such that all consumers with $v \geq p^*_{sl}$ can be served. To sum up, when an early consumer expects a total mass of D_{1c} early consumers will purchase immediately, her expected utility of waiting is

$$u_2 = \begin{cases} \int_{1-\frac{Q_{2f}}{M}}^{1} (v - p_{f2})dv + \int_{1-\delta}^{1-\frac{Q_{2f}}{M}} (v - p_{sh})dv + \int_{p_{sl}}^{1-\delta} (v - p_{sl})dv, \\ \qquad\qquad\qquad\qquad \text{if } M \cdot \delta - Q_{2s} \leq Q_{2f} < M \cdot \delta \\ \int_{1-\frac{Q_{2f}}{M}}^{1} (v - p_{f2})dv + \int_{p_{sl}}^{1-\frac{Q_{2f}}{M}} (v - p_{sl})dv, \\ \qquad\qquad\qquad\qquad \text{if } M \cdot \delta < Q_{2f} < \frac{M}{3} \text{ or } \frac{M}{3} \leq Q_{2f} < \max\{Q_{2s}, \frac{M-Q_{2s}}{2}\} \\ \int_{p_{f2}}^{1} (v - p_{f2})dv, \qquad \text{if } Q_{2f} < M \cdot \delta - Q_{2s} \text{ or } Q_{2f} \geq \max\{\frac{M}{3}, Q_{2s}, \frac{M-Q_{2s}}{2}\} \end{cases}, \tag{A.17}$$

where $M = 2 - D_{1c}$, $Q_{2s} = D_{1s}$, and $Q_{2f} = K - D_{1s} - D_{1c}$. I summarize the value of u_2 in Table A.3.8 through Table A.3.12. Here, an early consumer will purchase immediately only if $u_1 = \frac{1}{2} - p_{f1} \geq u_2$.

The scalpers choose $D_{1s} \leq \beta$ to maximize the expected profit $\Pi_s(D_{1s}) = \pi_{2s} - p_{f1} \times D_{1s}$ in anticipation of early consumers' purchase decisions. Here, the expressions of π_{2s} are given in Table A.3.8 through Table A.3.12.

In anticipation of the scalpers' and the early consumers' purchase decisions, the firm chooses p_{f1} to maximize its total payoff over the two periods. Note that the closed-form solutions are difficult to obtained since there are too many cases to discuss. Thus, I explain the algorithm of deriving the equilibrium decisions (numerically) as follows:

1. Fix p_{f1} and D_{1s}, find out the value of $D_{1c} = D_{1c}^*(p_{f1}, D_{1s})$, subject to $D_{1c} = 0$ or $\begin{cases} 0 < D_{1c} \leq 1 \\ u_1 \geq u_2 \end{cases}$, that maximizes $\Pi_f = p_{f1} \times (D_{1s} + D_{1c}) + \pi_{2f}$, where π_{2f} is given in Table A.3.8 through Table A.3.12.

2. Obtain the scalpers' expected total payoff $\Pi_s = \pi_{2s} - p_{f1} \times D_{1s}$ given p_{f1}, D_{1s} and $D_{1c} = D_{1c}^*(p_{f1}, D_{1s})$ (which is obtained in Step 1). Here, π_{2s} is given in Table A.3.8 through Table A.3.12.

3. Fix p_{f1}, find out the value of $D_{1s} = D^*_{1s}(p_{f1}, D^*_{1c})$, subject to $D_{1s} \leq \beta$, that maximizes $\Pi_s = \pi_{2s} - p_{f1} \times D_{1s}$, the value of which is obtained in Step 2.

4. Obtain the firm's expected total payoff $\Pi_f = p_{f1} \times (D_{1s} + D_{1c}) + \pi_{2f}$ given p_{f1}, $D_{1s} = D^*_{1s}(p_{f1}, D^*_{1c})$ (which is obtained in Step 3) and $D_{1c} = D^*_{1c}(p_{f1}, D^*_{1s})$ (which is obtained in Step 1).

5. Find out the value of $p_{f1} = p^*_{f1}(D^*_{1s}, D^*_{1c})$ that maximizes $\Pi_f = p_{f1} \times (D_{1s} + D_{1c}) + \pi_{2f}$, the value of which is obtained in Step 5.

As above, I explain how to derive the equilibrium decisions. The equilibrium payoffs can be obtained accordingly. This completes the analysis of Section 3.5.3.

Proof of Proposition 3.5

In the previous analysis, I derive the equilibrium decisions when the firm is able to charge different prices in different periods, and the scalpers have market power. Proposition 3.5 can then be proved by observing that the firm's equilibrium profit can be strictly optimized at an intermediate β (e.g., when $K = \frac{3}{2}$ and $\delta = \frac{1}{4}$, as illustrated in Figure 3.7).

Table A.3.1: Scalpers' Pricing Decision When $p_{sh} = 1 - \delta \geq p_{sl} > p_{f2}$

(a) $Q_{2f} < M \cdot \delta$

Range of Q_{2s}	$M \cdot \delta - Q_{2f} \leq Q_{2s} < \frac{M(1+\delta)}{2} - Q_{2f}$		$Q_{2s} \geq \frac{M(1+\delta)}{2} - Q_{2f}$	
Range of p_{f2}	$p_{f2} < 1 - \frac{Q_{2f}+Q_{2s}}{M}$	$p_{f2} \geq 1 - \frac{Q_{2f}+Q_{2s}}{M}$	$p_{f2} < \frac{1-\delta}{2}$	$p_{f2} \geq \frac{1-\delta}{2}$
Pricing Decision	$p_{sh} = 1 - \delta$ $p_{sl} = 1 - \frac{Q_{2f}+Q_{2s}}{M}$	$p_{sh} = 1 - \delta$ $p_{sl} = p_{f2} + \epsilon$	$p_{sh} = 1 - \delta$ $p_{sl} = \frac{1-\delta}{2}$	$p_{sh} = 1 - \delta$ $p_{sl} = p_{f2} + \epsilon$
Sellers' Profits	$\pi_{2s,I} = (1-\delta) \cdot (M \cdot \delta - Q_{2f})$ $+(1 - \frac{Q_{2f}+Q_s}{M}) \cdot (Q_{2f} + Q_{2s} - M \cdot \delta)$ $\pi_{2f,I} = p_{f2} \times Q_{2f}$	$\pi_{2s,I} = (1-\delta) \cdot (M \cdot \delta - Q_{2f})$ $+(p_{f2} + \epsilon) \cdot M(1 - \delta - p_{f2} - \epsilon)$ $\pi_{2f,I} = p_{f2} \times Q_{2f}$	$\pi_{2s,I} = (1-\delta) \cdot (M \cdot \delta - Q_{2f})$ $+\frac{M(1-\delta)^2}{4}$ $\pi_{2f,I} = p_{f2} \times Q_{2f}$	$\pi_{2s,I} = (1-\delta) \cdot (M \cdot \delta - Q_{2f})$ $+(p_{f2} + \epsilon) \cdot M(1 - \delta - p_{f2} - \epsilon)$ $\pi_{2f,I} = p_{f2} \times Q_{2f}$

(b) $Q_{2f} \geq M \cdot \delta$

Range of Q_{2s}	$Q_{2s} \leq \frac{M-Q_{2f}}{2}$			$Q_{2s} > \frac{M-Q_{2f}}{2}$		
Range of p_{f2}	$p_{f2} < 1 - \frac{Q_{2f}+Q_{2s}}{M}$	$1 - \frac{Q_{2f}+Q_{2s}}{M} \leq p_{f2} < 1 - \frac{Q_{2f}}{M}$	$p_{f2} \geq 1 - \frac{Q_{2f}}{M}$	$p_{f2} < \frac{1-\frac{Q_{2f}}{M}}{2}$	$\frac{1-\frac{Q_{2f}}{M}}{2} \leq p_{f2} < 1 - \frac{Q_{2f}}{M}$	$p_{f2} \geq 1 - \frac{Q_{2f}}{M}$
Pricing Decision	$p_{sh} = 1 - \delta$ $p_{sl} = 1 - \frac{Q_{2f}+Q_{2s}}{M}$	$p_{sh} = 1 - \delta$ $p_{sl} = p_{f2} + \epsilon$	$p_{sh} = 1 - \delta$ $p_{sl} \in (p_{f2}, 1 - \delta]$	$p_{sh} = 1 - \delta$ $p_{sl} = \frac{1-\frac{Q_{2f}}{M}}{2}$	$p_{sh} = 1 - \delta$ $p_{sl} = p_{f2} + \epsilon$	$p_{sh} = 1 - \delta$ $p_{sl} \in (p_{f2}, 1 - \delta]$
Sellers' Profits	$\pi_{2s,I} = (1 - \frac{Q_{2f}+Q_{2s}}{M}) \times Q_{2s}$ $\pi_{2f,I} = p_{f2} \times Q_{2f}$	$\pi_{2s,I} = (M - Q_{2f})(p_{f2} + \epsilon)$ $-M(p_{f2} + \epsilon)^2$ $\pi_{2f,I} = p_{f2} \times Q_{2f}$	$\pi_{2s,I} = 0$ $\pi_{2f,I} = p_{f2} \cdot M(1 - p_{f2})$	$\pi_{2s,I} = \frac{M(1-\frac{Q_{2f}}{M})^2}{4}$ $\pi_{2f,I} = p_{f2} \times Q_{2f}$	$\pi_{2s,I} = (M - Q_{2f})(p_{f2} + \epsilon)$ $-M(p_{f2} + \epsilon)^2$ $\pi_{2f,I} = p_{f2} \times Q_{2f}$	$\pi_{2s,I} = 0$ $\pi_{2f,I} = p_{f2} \cdot M(1 - p_{f2})$

Table A.3.2: Scalpers' Pricing Decision When $p_{sh} = 1 - \delta > p_{f2} \geq p_{sl}$

(a) $Q_{2f} < M \cdot \delta$

Range of Q_{2s}	$M \cdot \delta - Q_{2f} \leq Q_{2s} < \frac{M(1+\delta)}{2} - Q_{2f}$		$Q_s \geq \frac{M(1+\delta)}{2} - Q_{2f}$		
Range of p_{f2}	$p_{f2} < 1 - \frac{Q_{2f}+Q_{2s}}{M}$	$p_{f2} \geq 1 - \frac{Q_{2f}+Q_{2s}}{M}$	$p_{f2} < 1 - \frac{Q_{2f}+Q_{2s}}{M}$	$1 - \frac{Q_{2f}+Q_{2s}}{M} \leq p_{f2} < \frac{1-\delta}{2}$	$p_{f2} \geq \frac{1-\delta}{2}$
Pricing Decision	$p_{sh} = 1 - \delta$ $p_{sl} = p_{f2}$	$p_{sh} = 1 - \delta$ $p_{sl} = 1 - \frac{Q_{2f}+Q_{2s}}{M}$	$p_{sh} = 1 - \delta$ $p_{sl} = p_{f2}$	$p_{sh} = 1 - \delta$ $p_{sl} = p_{f2}$	$p_{sh} = 1 - \delta$ $p_{sl} = \frac{1-\delta}{2}$
Sellers' Profits	$\pi_{2s,II} = (1-\delta) \cdot (M \cdot \delta - Q_{2f})$ $+p_{f2} \cdot (Q_{2f} + Q_{2s} - M \cdot \delta)$ $\pi_{2f,II} = p_{f2} \times Q_{2f}$	$\pi_{2s,II} = (1-\delta) \cdot (M \cdot \delta - Q_{2f})$ $+(1 - \frac{Q_{2f}+Q_{2s}}{M}) \cdot (Q_{2f} + Q_{2s} - M \cdot \delta)$ $\pi_{2f,II} = p_{f2} \times Q_{2f}$	$\pi_{2s,II} = (1-\delta) \cdot (M \cdot \delta - Q_{2f})$ $+p_{f2} \cdot (Q_{2f} + Q_{2s} - M \cdot \delta)$ $\pi_{2f,II} = p_{f2} \times Q_{2f}$	$\pi_{2s,II} = (1-\delta) \cdot (M \cdot \delta - Q_{2f})$ $+p_{f2} \cdot M(1 - \delta - p_{f2})$ $\pi_{2f,II} = p_{f2} \times Q_{2f}$	$\pi_{2s,II} = (1-\delta) \cdot (M \cdot \delta - Q_{2f})$ $+\frac{M(1-\delta)^2}{4}$ $\pi_{2f,II} = p_{f2} \times Q_{2f}$

(b) $Q_{2f} \geq M \cdot \delta$

Range of Q_{2s}	$Q_{2s} < \frac{M(1-\delta)}{2}$		$Q_{2s} \geq \frac{M(1-\delta)}{2}$		
Range of p_{f2}	$p_{f2} < 1 - \delta - \frac{Q_{2s}}{M}$	$p_{f2} \geq 1 - \delta - \frac{Q_{2s}}{M}$	$p_{f2} \leq 1 - \delta - \frac{Q_{2s}}{M}$	$1 - \delta - \frac{Q_{2s}}{M} < p_{f2} < \frac{1-\delta}{2}$	$p_{f2} \geq \frac{1-\delta}{2}$
Pricing Decision	$p_{sh} = 1 - \delta$ $p_{sl} = p_{f2}$	$p_{sh} = 1 - \delta$ $p_{sr} = 1 - \delta - \frac{Q_{2s}}{M}$	$p_{sh} = 1 - \delta$ $p_{sl} = p_{f2}$	$p_{sh} = 1 - \delta$ $p_{sl} = p_{f2}$	$p_{sh} = 1 - \delta$ $p_{sl} = \frac{1-\delta}{2}$
Sellers' Profits	$\pi_{2s,II} = p_{f2} \times Q_{2s}$ $\pi_{2f,II} = p_{f2} \times \min\{Q_{2f},$ $M(1 - p_{f2}) - Q_{2s}\}$	$\pi_{2s,II} = (1 - \delta - \frac{Q_{2s}}{M}) \times Q_{2s}$ $\pi_{2f,II} = p_{f2} \times M \cdot \delta$	$\pi_{2s,II} = p_{f2} \times Q_{2s}$ $\pi_{2f,II} = p_{f2} \times \min\{Q_{2f},$ $M(1 - p_{f2}) - Q_{2s}\}$	$\pi_{2s,II} = p_{f2} \times M(1 - \delta - p_{f2})$ $\pi_{2f,II} = p_{f2} \times M \cdot \delta$	$\pi_{2s,II} = \frac{M(1-\delta)^2}{4}$ $\pi_{2f,II} = p_{f2} \times M \cdot \delta$

Table A.3.3: Scalpers' Pricing Decision When $1 - \delta \geq p_{f2} = p_{sh} \geq p_{sl}$

Range of Q_{2s}	$M \cdot \delta - Q_{2f} \leq Q_{2s} < M \cdot \delta$	$M \cdot \delta \leq Q_{2s} < \frac{M(1+\delta)}{2}$	
Range of p_{f2}	p_{f2}	$p_{f2} < 1 - \frac{Q_{2s}}{M}$	$p_{f2} \geq 1 - \frac{Q_{2s}}{M}$
Pricing Decision	$p_{sh} = p_{f2}$ $p_{sl} \in [0, p_{f2}]$	$p_{sh} = p_{f2}$ $p_{sl} = p_{f2}$	$p_{sh} = p_{f2}$ $p_{sl} = 1 - \frac{Q_{2s}}{M}$
Sellers' Profits	$\pi_{2s,III} = p_{f2} \times Q_{2s}$ $\pi_{2f,III} = p_{f2} \times \min\{Q_{2f}, M(1 - p_{f2}) - Q_{2s}\}$		$\pi_{2s,III} = p_{f2} \cdot M \cdot \delta$ $+(1 - \frac{Q_{2s}}{M})(Q_{2s} - M \cdot \delta)$ $\pi_{2f,III} = 0$

Range of Q_{2s}	$Q_{2s} \geq \frac{M(1+\delta)}{2}$		
Range of p_{f2}	$p_{f2} < 1 - \frac{Q_{2s}}{M}$	$1 - \frac{Q_{2s}}{M} \leq p_{f2} < \frac{1-\delta}{2}$	$p_{f2} \geq \frac{1-\delta}{2}$
Pricing Decision	$p_{sh} = p_{f2}$ $p_{sl} = p_{f2}$	$p_{sh} = p_{f2}$ $p_{sl} = p_{f2}$	$p_{sh} = p_{f2}$ $p_{sl} = \frac{1-\delta}{2}$
Sellers' Profits	$\pi_{2s,III} = p_{f2} \times Q_{2s}$ $\pi_{2f,III} = p_{f2} \times \min\{Q_{2f},$ $M(1 - p_{f2}) - Q_{2s}\}$	$\pi_{2s,III} = p_{f2} \cdot M(1 - p_{f2})$ $\pi_{2f,III} = 0$	$\pi_{2s,III} = p_{f2} \cdot M \cdot \delta$ $+\frac{M(1-\delta)^2}{4}$ $\pi_{2f,III} = 0$

Table A.3.4: Scalpers' Pricing Decisions When $Q_{2f} < \frac{M(1-\sqrt{1-2\delta+2\delta^2})}{2}$

Range of Q_{2s}	$M\cdot\delta - Q_{2f} \leq Q_{2s} < \frac{(M\cdot\delta-Q_{2f})^2}{M\cdot\delta-2Q_{2f}}$			$\frac{(M\cdot\delta-Q_{2f})^2}{M\cdot\delta-2Q_{2f}} \leq Q_{2s} < \frac{M(1+\delta)}{2} - Q_{2f}$	
Range of p_{f2}	$p_{f2} \leq p_{f2,A}$	$p_{f2,A} < p_{f2} \leq 1 - \frac{Q_{2s}}{M}$	$p_{f2} > 1 - \frac{Q_{2s}}{M}$	$p_{f2} \leq p_{f2,B}$	$p_{f2} > p_{f2,B}$
Pricing Decision	$p_{sh} = 1-\delta$ $p_{sl} = 1 - \frac{Q_{2f}+Q_{2s}}{M}$	$p_{sh} = p_{f2}$ $p_{sl} = p_{f2}$	$p_{sh} = p_{f2}$ $p_{sl} = p_{sl}^{'}$	$p_{sh} = 1-\delta$ $p_{sl} = 1 - \frac{Q_{2f}+Q_{2s}}{M}$	$p_{sh} = p_{f2}$ $p_{sl} = p_{sl}^{'}$
Sellers' Profits	$\pi_{2s} = (1-\delta)\cdot(M\cdot\delta - Q_{2f})$ $+(1-\frac{Q_{2f}+Q_{2s}}{M})\cdot(Q_{2f}+Q_{2s}-M\cdot\delta)$ $\pi_{2f} = p_{f2} \times Q_{2f}$	$\pi_{2s} = p_{f2} \times Q_{2s}$ $\pi_{2f} = p_{f2} \times \min\{Q_{2f},$ $M(1-p_{f2}) - Q_{2s}\}$	$\pi_{2f} = 0$	$\pi_{2s} = (1-\delta)\cdot(M\cdot\delta - Q_{2f})$ $+(1-\frac{Q_{2f}+Q_{2s}}{M})\cdot(Q_{2f}+Q_{2s}-M\cdot\delta)$ $\pi_{2f} = p_{f2} \times Q_{2f}$	$\pi_{2f} = 0$

Range of Q_{2s}	$\frac{M(1+\delta)}{2} - Q_{2f} \leq Q_{2s} < \frac{M(1+\delta)}{2}$		$Q_{2s} \geq \frac{M(1+\delta)}{2}$	
Range of p_{f2}	$p_{f2} \leq p_{f2,C}$	$p_{f2} > p_{f2,C}$	$p_{f2} \leq \frac{(1-\delta)(M\cdot\delta-Q_{2f})}{M\cdot\delta}$	$p_{f2} > \frac{(1-\delta)(M\cdot\delta-Q_{2f})}{M\cdot\delta}$
Pricing Decision	$p_{sh} = 1-\delta$ $p_{sl} = \frac{1-\delta}{2}$	$p_{sh} = p_{f2}$ $p_{sl} = p_{sl}^{'}$	$p_{sh} = 1-\delta$ $p_{sl} = \frac{1-\delta}{2}$	$p_{sh} = p_{f2}$ $p_{sl} = p_{sl}^{'}$
Sellers' Profits	$\pi_{2s} = (1-\delta)\cdot(M\cdot\delta - Q_{2f})$ $+\frac{M(1-\delta)^2}{4}$ $\pi_{2f} = p_{f2} \times Q_{2f}$	$\pi_{2f} = 0$	$\pi_{2s} = (1-\delta)\cdot(M\cdot\delta - Q_{2f})$ $+\frac{M(1-\delta)^2}{4}$ $\pi_{2f} = p_{f2} \times Q_{2f}$	$\pi_{2f} = 0$

Notes. $p_{f2,A} = \frac{M(1-\delta)(M\cdot\delta - Q_{2f}) + (Q_{2f}+Q_{2s}-M\cdot\delta)(M-Q_{2f}-Q_{2s})}{M(1-\delta)Q_{2s}}$

$$p_{f2,B} = 1 - \frac{(M\cdot\delta - Q_{2f})^2 + 2Q_{2f}Q_{2s}}{M^2\delta}$$

$$p_{f2,C} = \frac{(\frac{M(1+\delta)}{2} - Q_{2s})^2 + M(1-\delta)(M\cdot\delta - Q_{2f})}{M^2\delta}$$

$$p_{sl}^{'} = \min\{p_{f2}, \max\{\frac{1-\delta}{2}, 1 - \frac{Q_{2s}}{M}\}\}$$

Table A.3.5: Scalpers' Pricing Decisions When $\frac{M(1-\sqrt{1-2\delta+2\delta^2})}{2} \leq Q_{2f} < \frac{M\cdot\delta}{2}$

Range of Q_{2s}	$M\cdot\delta - Q_{2f} \leq Q_{2s} < \frac{M(1+\delta)}{2} - Q_{2f}$			$\frac{M(1+\delta)}{2} - Q_{2f} \leq Q_{2s} < \frac{M+\sqrt{3M^2\delta^2-2M^2\delta+4M(1-\delta)Q_{2f}}}{2}$		
Range of p_{f2}	$p_{f2} \leq p_{f2,A}$	$p_{f2,A} < p_{f2} \leq 1 - \frac{Q_{2s}}{M}$	$p_{f2} > 1 - \frac{Q_{2s}}{M}$	$p_{f2} \leq \frac{(1-\delta)(M+3M\delta-4Q_{2f})}{4Q_{2s}}$	$\frac{(1-\delta)(M+3M\delta-4Q_{2f})}{4Q_{2s}} < p_2 \leq 1 - \frac{Q_{2s}}{M}$	$p_2 > 1 - \frac{Q_{2s}}{M}$
Pricing Decision	$p_{sh} = 1-\delta$ $p_{sl} = 1 - \frac{Q_{2f}+Q_{2s}}{M}$	$p_{sh} = p_{f2}$ $p_{sl} = p_{f2}$	$p_{sh} = p_{f2}$ $p_{sl} = p_{sl}^{'}$	$p_{sh} = 1-\delta$ $p_{sl} = \frac{1-\delta}{2}$	$p_{sh} = p_{f2}$ $p_{sl} = p_{f2}$	$p_{sh} = p_{f2}$ $p_{sl} = p_{sl}^{'}$
Sellers' Profits	$\pi_{2s} = (1-\delta)\cdot(M\cdot\delta - Q_{2f})$ $+(1-\frac{Q_{2f}+Q_{2s}}{M})\cdot(Q_{2f}+Q_{2s}-M\cdot\delta)$ $\pi_{2f} = p_{f2}\times Q_{2f}$	$\pi_{2s} = p_{f2}\times Q_{2s}$ $\pi_{2f} = p_{f2}\times \min\{Q_{2f},$ $M(1-p_{f2})-Q_{2s}\}$	$\pi_{2f} = 0$	$\pi_{2s} = (1-\delta)\cdot(M\cdot\delta - Q_{2f})$ $+\frac{M(1-\delta)^2}{4}$ $\pi_{2f} = p_{f2}\times Q_{2f}$	$\pi_{2s} = p_{f2}\times Q_{2s}$ $\pi_{2f} = p_{f2}\times \min\{Q_{2f},$ $M(1-p_{f2})-Q_{2s}\}$	$\pi_{2f} = 0$

Range of Q_{2s}	$\frac{M+\sqrt{3M^2\delta^2-2M^2\delta+4M(1-\delta)Q_{2f}}}{2} \leq Q_{2s} < \frac{M(1+\delta)}{2}$		$Q_{2s} \geq \frac{M(1+\delta)}{2}$	
Range of p_{f2}	$p_{f2} \leq p_{f2,C}$	$p_{f2} > p_{f2,C}$	$p_{f2} \leq \frac{(1-\delta)(M\cdot\delta-Q_{2f})}{M\cdot\delta}$	$p_{f2} > \frac{(1-\delta)(M\cdot\delta-Q_{2f})}{M\cdot\delta}$
Pricing Decision	$p_{sh} = 1-\delta$ $p_{sl} = \frac{1-\delta}{2}$	$p_{sh} = p_{f2}$ $p_{sl} = p_{sl}^{'}$	$p_{sh} = 1-\delta$ $p_{sl} = \frac{1-\delta}{2}$	$p_{sh} = p_{f2}$ $p_{sl} = p_{sl}^{'}$
Sellers' Profits	$\pi_{2s} = (1-\delta)\cdot(M\cdot\delta - Q_{2f})$ $+\frac{M(1-\delta)^2}{4}$ $\pi_{2f} = p_{f2}\times Q_{2f}$	$\pi_{2f} = 0$	$\pi_{2s} = (1-\delta)\cdot(M\cdot\delta - Q_{2f})$ $+\frac{M(1-\delta)^2}{4}$ $\pi_{2f} = p_{f2}\times Q_{2f}$	$\pi_{2f} = 0$

Notes. $p_{f2,A} = \frac{M(1-\delta)(M\cdot\delta - Q_{2f}) + (Q_{2f}+Q_{2s}-M\cdot\delta)(M-Q_{2f}-Q_{2s})}{M(1-\delta)Q_{2s}}$

$$p_{f2,C} = \frac{(\frac{M(1+\delta)}{2} - Q_{2s})^2 + M(1-\delta)(M\cdot\delta - Q_{2f})}{M^2\delta}$$

$$p_{sl}^{'} = \min\{p_{f2}, \max\{\frac{1-\delta}{2}, 1 - \frac{Q_{2s}}{M}\}\}$$

Table A.3.6: Scalpers' Pricing Decisions When $\frac{M\cdot\delta}{2} \leq Q_{2f} < M\cdot\delta$

Range of Q_{2s}	$M\cdot\delta - Q_{2f} \leq Q_{2s} < \frac{M(1+\delta)}{2} - Q_{2f}$			$\frac{M(1+\delta)}{2} - Q_{2f} \leq Q_{2s} < \frac{M+\sqrt{3M^2\delta^2-2M^2\delta+4M(1-\delta)Q_{2f}}}{2}$		
Range of p_{f2}	$p_{f2} \leq p_{f2,A}$	$p_{f2,A} < p_{f2} \leq 1-\frac{Q_{2s}}{M}$	$p_{f2} > 1-\frac{Q_{2s}}{M}$	$p_{f2} \leq \frac{(1-\delta)(M+3M\delta-4Q_{2f})}{4Q_{2s}}$	$\frac{(1-\delta)(M+3M\delta-4Q_{2f})}{4Q_{2s}} < p_2 \leq 1-\frac{Q_{2s}}{M}$	$p_2 > 1-\frac{Q_{2s}}{M}$
Pricing Decision	$p_{sh} = 1-\delta$ $p_{sl} = 1-\frac{Q_{2f}+Q_{2s}}{M}$	$p_{sh} = p_{f2}$ $p_{sl} = p_{f2}$	$p_{sh} = p_{f2}$ $p_{sl} = p_{sl}^{'}$	$p_{sh} = 1-\delta$ $p_{sl} = \frac{1-\delta}{2}$	$p_{sh} = p_{f2}$ $p_{sl} = p_{f2}$	$p_{sh} = p_{f2}$ $p_{sl} = p_{sl}^{'}$
Sellers' Profits	$\pi_{2s} = (1-\delta)\cdot(M\cdot\delta - Q_{2f})$ $+(1-\frac{Q_{2f}+Q_{2s}}{M})\cdot(Q_{2f}+Q_{2s}-M\cdot\delta)$ $\pi_{2f} = p_{f2}\times Q_{2f}$	$\pi_{2s} = p_{f2}\times Q_{2s}$ $\pi_{2f} = p_{f2}\times\min\{Q_{2f},$ $M(1-p_{f2})-Q_{2s}\}$	$\pi_{2f} = 0$	$\pi_{2s} = (1-\delta)\cdot(M\cdot\delta - Q_{2f})$ $+\frac{M(1-\delta)^2}{4}$ $\pi_{2f} = p_{f2}\times Q_{2f}$	$\pi_{2s} = p_{f2}\times Q_{2s}$ $\pi_{2f} = p_{f2}\times\min\{Q_{2f},$ $M(1-p_{f2})-Q_{2s}\}$	$\pi_{2f} = 0$

Range of Q_{2s}	$Q_{2s} \geq \frac{M+\sqrt{3M^2\delta^2-2M^2\delta+4M(1-\delta)Q_{2f}}}{2}$	
Range of p_{f2}	$p_{f2} \leq p_{f2,D}$	$p_{f2} > p_{f2,D}$
Pricing Decision	$p_{sh} = 1-\delta$ $p_{sl} = \frac{1-\delta}{2}$	$p_{sh} = p_{f2}$ $p_{sl} = p_{sl}^{'}$
Sellers' Profits	$\pi_{2s} = (1-\delta)\cdot(M\cdot\delta - Q_{2f}) + \frac{M(1-\delta)^2}{4}$ $\pi_{2f} = p_{f2}\times Q_{2f}$	$\pi_{2f} = 0$

Notes. $p_{f2,A} = \dfrac{M(1-\delta)(M\cdot\delta - Q_{2f}) + (Q_{2f}+Q_{2s}-M\cdot\delta)(M-Q_{2f}-Q_{2s})}{M(1-\delta)Q_{2s}}$

$$p_{2,D} = \frac{M - \sqrt{3M^2\delta^2 - 2M^2\delta + 4M(1-\delta)Q_{2f}}}{2M}$$

$$p_{sl}^{'} = \min\{p_{f2}, \max\{\frac{1-\delta}{2}, 1-\frac{Q_{2s}}{M}\}\}$$

Table A.3.7: Scalpers' Pricing Decisions When $Q_{2f} \geq M \cdot \delta$

Range of Q_{2s}	$Q_{2s} < \frac{M-Q_{2f}}{2}$			$\frac{M-Q_{2f}}{2} \leq Q_{2s} < \frac{M+\sqrt{2MQ_{2f}-Q_{2f}^2}}{2}$		
Range of p_{f2}	$p_{f2} \leq 1 - \frac{Q_{2f}+Q_{2s}}{M}$	$1 - \frac{Q_{2f}+Q_{2s}}{M} < p_{f2} \leq 1 - \frac{Q_{2s}}{M}$	$p_{f2} > 1 - \frac{Q_{2s}}{M}$	$p_{f2} \leq \frac{(M-Q_{2f})^2}{4MQ_{2s}}$	$\frac{(M-Q_{2f})^2}{4MQ_{2s}} < p_{f2} \leq 1 - \frac{Q_{2s}}{M}$	$p_{f2} > 1 - \frac{Q_{2s}}{M}$
Pricing Decision	$p_{sh} = 1 - \delta$ $p_{sl} = 1 - \frac{Q_{2f}+Q_{2s}}{M}$	$p_{sh} = p_{f2}$ $p_{sl} = p_{f2}$	$p_{sh} = p_{f2}$ $p_{sl} = p_{sl}^{'}$	$p_{sh} = 1 - \delta$ $p_{sl} = \frac{1-\frac{Q_{2f}}{M}}{2}$	$p_{sh} = p_{f2}$ $p_{sl} = p_{f2}$	$p_{sh} = p_{f2}$ $p_{sl} = p_{sl}^{'}$
Sellers' Profits	$\pi_{2s} = (1 - \frac{Q_{2f}+Q_{2s}}{M}) \times Q_{2s}$ $\pi_{2f} = p_{f2} \times Q_{2f}$	$\pi_{2s} = p_{f2} \times Q_{2s}$ $\pi_{2f} = p_{f2} \times \min\{Q_{2f}, M(1-p_{f2}) - Q_{2s}\}$	$\pi_{2f} = 0$	$\pi_{2s} = \frac{M(1-\frac{Q_{2f}}{M})^2}{4}$ $\pi_{2f} = p_{f2} \times Q_{2f}$	$\pi_{2s} = p_{f2} \times Q_{2s}$ $\pi_{2f} = p_{f2} \times \min\{Q_{2f}, M(1-p_{f2}) - Q_{2s}\}$	$\pi_{2f} = 0$

Range of Q_{2s}	$Q_{2s} \geq \frac{M+\sqrt{2MQ_{2f}-Q_{2f}^2}}{2}$	
Range of p_{f2}	$p_{f2} \leq \frac{M-\sqrt{2MQ_{2f}-Q_{2f}^2}}{2M}$	$p_{f2} > \frac{M-\sqrt{2MQ_{2f}-Q_{2f}^2}}{2M}$
Pricing Decision	$p_{sh} = 1 - \delta$ $p_{sl} = \frac{1-\frac{Q_{2f}}{M}}{2}$	$p_{sh} = p_{f2}$ $p_{sl} = p_{sl}^{'}$
Sellers' Profits	$\pi_{2s} = \frac{M(1-\frac{Q_f}{M})^2}{4}$ $\pi_{2f} = p_{f2} \times Q_{2f}$	$\pi_{2f} = 0$

Notes. $p_{sl}^{'} = \min\{p_{f2}, \max\{\frac{1-\delta}{2}, 1 - \frac{Q_{2s}}{M}\}\}$.

Table A.3.8: Firm's Second-Period Pricing Decision When $Q_{2f} < \frac{M(1-\sqrt{1-2\delta+2\delta^2})}{2}$

Range of Q_{2s}	$M \cdot \delta - Q_{2f} \leq Q_{2s} < \frac{(M\cdot\delta - Q_{2f})^2}{M\cdot\delta - 2Q_{2f}}$	$\frac{(M\cdot\delta - Q_{2f})^2}{M\cdot\delta - 2Q_{2f}} \leq Q_{2s} < \frac{M(1+\delta)}{2} - Q_{2f}$
Firm's Pricing Decision	$p_{f2} = p_{f2,A}$	$p_{f2} = p_{f2,B}$
Scalpers' Pricing Decision	$p_{sh} = 1-\delta$ $p_{sl} = 1 - \frac{Q_{2f}+Q_{2s}}{M}$	$p_{sh} = 1-\delta$ $p_{sl} = 1 - \frac{Q_{2f}+Q_{2s}}{M}$
Sellers' Profits	$\pi_{2s} = (1-\delta)\cdot(M\cdot\delta - Q_{2f})$ $+(1-\frac{Q_{2f}+Q_{2s}}{M})\cdot(Q_{2f}+Q_{2s}-M\cdot\delta)$ $\pi_{2f} = p_{f2,A} \times Q_{2f}$	$\pi_{2s} = (1-\delta)\cdot(M\cdot\delta - Q_{2f})$ $+(1-\frac{Q_{2f}+Q_{2s}}{M})\cdot(Q_{2f}+Q_{2s}-M\cdot\delta)$ $\pi_{2f} = p_{f2,B} \times Q_{2f}$
u_2	$\frac{(Q_{2f}+Q_{2s})(2M^2\delta^2 - 2M\delta(2Q_{2f}+Q_{2s}) + 2Q_{2f}^2 + 3Q_{2f}Q_{2s} + Q_{2s}^2)}{2M^2Q_{2s}}$	$\frac{2M^3\delta^3 - 2M^2\delta^2(Q_{2f}+Q_{2s}) + M\delta(-3Q_{2f}^2 + 2Q_{2f}Q_{2s} + Q_{2s}^2) + 2Q_{2f}^2(Q_{2f}+2Q_{2s})}{2M^3\delta}$

Range of Q_{2s}	$\frac{M(1+\delta)}{2} - Q_{2f} \leq Q_{2s} < \frac{M(1+\delta)}{2}$	$Q_{2s} \geq \frac{M(1+\delta)}{2}$
Firm's Pricing Decision	$p_{f2} = p_{f2,C}$	$p_{f2} = \frac{(1-\delta)(M\cdot\delta - Q_{2f})}{M\cdot\delta}$
Scalpers' Pricing Decision	$p_{sh} = 1-\delta$ $p_{sl} = \frac{1-\delta}{2}$	$p_{sh} = 1-\delta$ $p_{sl} = \frac{1-\delta}{2}$
Sellers' Profits	$\pi_{2s} = (1-\delta)\cdot(M\cdot\delta - Q_{2f})$ $+\frac{M(1-\delta)^2}{4}$ $\pi_{2f} = p_{f2,C} \times Q_{2f}$	$\pi_{2s} = (1-\delta)\cdot(M\cdot\delta - Q_{2f})$ $+\frac{M(1-\delta)^2}{4}$ $\pi_{2f} = \frac{(1-\delta)(M\cdot\delta - Q_{2f})}{M\cdot\delta} \times Q_{2f}$
u_2	$\frac{1}{8M^3\delta}(M^3(\delta - 2\delta^2 + 5\delta^3) - 2M^2(1+\delta)^2Q_{2f}$ $+M(8\delta(Q_{2s} - Q_{2f}) + 8(Q_{2f} + Q_{2}s)Q_{2f} - 8Q_{2f}Q_{2s}^2))$	$\frac{1-2\delta+5\delta^2}{8} + \frac{(1-\delta)Q_{2f}^2}{M^2 d\delta}$

Notes. $p_{f2,A} = \frac{M(1-\delta)(M\cdot\delta - Q_{2f}) + (Q_{2f}+Q_{2s}-M\cdot\delta)(M - Q_{2f} - Q_{2s})}{M(1-\delta)Q_{2s}}$

$p_{f2,B} = 1 - \frac{(M\cdot\delta - Q_{2f})^2 + 2Q_{2f}Q_{2s}}{M^2\delta}$

$p_{f2,C} = \frac{(\frac{M(1+\delta)}{2} - Q_{2s})^2 + M(1-\delta)(M\cdot\delta - Q_{2f})}{M^2\delta}$

Table A.3.9: Firm's Second-Period Pricing Decision When $\frac{M(1-\sqrt{1-2\delta+2\delta^2})}{2} \leq Q_{2f} < \frac{M\cdot\delta}{2}$

Range of Q_{2s}	$M \cdot \delta - Q_{2f} \leq Q_{2s} < \frac{M(1+\delta)}{2} - Q_{2f}$	$\frac{M(1+\delta)}{2} - Q_{2f} \leq Q_{2s} < \frac{M+\sqrt{3M^2\delta^2-2M^2\delta+4M(1-\delta)Q_{2f}}}{2}$
Firm's Pricing Decision	$p_{f2} = p_{f2,A}$	$p_{f2} = \frac{(1-\delta)(M+3M\delta-4Q_{2f})}{4Q_{2s}}$
Scalpers' Pricing Decision	$p_{sh} = 1-\delta$ $p_{sl} = 1 - \frac{Q_{2f}+Q_{2s}}{M}$	$p_{sh} = 1-\delta$ $p_{sl} = \frac{1-\delta}{2}$
Sellers' Profits	$\pi_{2s} = (1-\delta)\cdot(M\cdot\delta - Q_{2f}) + (1 - \frac{Q_{2f}+Q_{2s}}{M})\cdot(Q_{2f}+Q_{2s}-M\cdot\delta)$ $\pi_{2f} = p_{f2,A} \times Q_{2f}$	$\pi_{2s} = (1-\delta)\cdot(M\cdot\delta - Q_{2f}) + \frac{M(1-\delta)^2}{4}$ $\pi_{2f} = \frac{(1-\delta)(M+3M\delta-4Q_{2f})}{4Q_{2s}} \times Q_{2f}$
u_2	$\frac{(Q_{2f}+Q_{2s})(2M^2\delta^2-2M\delta(2Q_{2f}+Q_{2s})+2Q_{2f}^2+3Q_{2f}Q_{2s}+Q_{2s}^2)}{2M^2Q_{2s}}$	$\frac{1}{8MQ_{2s}}(M((-2-4\delta+6\delta^2)Q_{2f}+(1-2\delta+5\delta^2)Q_{2s}) + 8(1-\delta)Q_{2f}(Q_{2f}+Q_{2s}))$

Range of Q_{2s}	$\frac{M+\sqrt{3M^2\delta^2-2M^2\delta+4M(1-\delta)Q_{2f}}}{2} \leq Q_{2s} < \frac{M(1+\delta)}{2}$	$Q_{2s} \geq \frac{M(1+\delta)}{2}$
Firm's Pricing Decision	$p_{f2} = p_{f2,C}$	$p_{f2} = \frac{(1-\delta)(M\cdot\delta-Q_{2f})}{M\cdot\delta}$
Scalpers' Pricing Decision	$p_{sh} = 1-\delta$ $p_{sl} = \frac{1-\delta}{2}$	$p_{sh} = 1-\delta$ $p_{sl} = \frac{1-\delta}{2}$
Sellers' Profits	$\pi_{2s} = (1-\delta)\cdot(M\cdot\delta - Q_{2f}) + \frac{M(1-\delta)^2}{4}$ $\pi_{2f} = p_{f2,C} \times Q_{2f}$	$\pi_{2s} = (1-\delta)\cdot(M\cdot\delta - Q_{2f}) + \frac{M(1-\delta)^2}{4}$ $\pi_{2f} = \frac{(1-\delta)(M\cdot\delta-Q_{2f})}{M\cdot\delta} \times Q_{2f}$
u_2	$\frac{1}{8M^3\delta}(M^3(\delta-2\delta^2+5\delta^3) - 2M^2(1+\delta)^2Q_{2f} + M(8\delta(Q_{2s}-Q_{2f}) + 8(Q_{2f}+Q_2s)Q_{2f} - 8Q_{2f}Q_{2s}^2))$	$\frac{1-2\delta+5\delta^2}{8} + \frac{(1-\delta)Q_{2f}^2}{M^2 d\delta}$

Notes. $p_{f2,A} = \dfrac{M(1-\delta)(M\cdot\delta - Q_{2f}) + (Q_{2f}+Q_{2s}-M\cdot\delta)(M-Q_{2f}-Q_{2s})}{M(1-\delta)Q_{2s}}$

$p_{f2,C} = \dfrac{(\frac{M(1+\delta)}{2} - Q_{2s})^2 + M(1-\delta)(M\cdot\delta - Q_{2f})}{M^2\delta}$

Table A.3.10: Firm's Second-Period Pricing Decision When $\frac{M\cdot\delta}{2} \leq Q_{2f} < M\cdot\delta$

Range of Q_{2s}	$M\cdot\delta - Q_{2f} \leq Q_{2s} < \frac{M(1+\delta)}{2} - Q_{2f}$	$\frac{M(1+\delta)}{2} - Q_{2f} \leq Q_{2s}$ $< \frac{M+\sqrt{3M^2\delta^2-2M^2\delta+4M(1-\delta)Q_{2f}}}{2}$	$Q_{2s} \geq \frac{M+\sqrt{3M^2\delta^2-2M^2\delta+4M(1-\delta)Q_{2f}}}{2}$
Firm's Pricing Decision	$p_{f2} = p_{f2,A}$	$p_{f2} = \frac{(1-\delta)(M+3M\delta-4Q_{2f})}{4Q_{2s}}$	$p_{f2} = p_{f2,D}$
Scalpers' Pricing Decision	$p_{sh} = 1-\delta$ $p_{sl} = 1 - \frac{Q_{2f}+Q_{2s}}{M}$	$p_{sh} = 1-\delta$ $p_{sl} = \frac{1-\delta}{2}$	$p_{sh} = 1-\delta$ $p_{sl} = \frac{1-\delta}{2}$
Sellers' Profits	$\pi_{2s} = (1-\delta)\cdot(M\cdot\delta - Q_{2f})$ $+(1-\frac{Q_{2f}+Q_{2s}}{M})\cdot(Q_{2f}+Q_{2s}-M\cdot\delta)$ $\pi_{2f} = p_{f2,A}\times Q_{2f}$	$\pi_{2s} = (1-\delta)\cdot(M\cdot\delta - Q_{2f}) + \frac{M(1-\delta)^2}{4}$ $\pi_{2f} = \frac{(1-\delta)(M+3M\delta-4Q_{2f})}{4Q_{2s}} \times Q_{2f}$	$\pi_{2s} = (1-\delta)\cdot(M\cdot\delta - Q_{2f}) + \frac{M(1-\delta)^2}{4}$ $\pi_{2f} = p_{f2,D}\times Q_{2f}$
u_2	$\frac{(Q_{2f}+Q_{2s})(2M^2\delta^2-2M\delta(2Q_{2f}+Q_{2s})+2Q_{2f}^2+3Q_{2f}Q_{2s}+Q_{2s}^2)}{2M^2Q_{2s}}$	$\frac{1}{8MQ_{2s}}(M((-2-4\delta+6\delta^2)Q_{2f}+(1-2\delta+5\delta^2)Q_{2s})$ $+8(1-\delta)Q_{2f}(Q_{2f}+Q_{2s}))$	$\frac{1-2\delta+5\delta^2}{8} + \frac{(1-2\delta)Q_{2f}}{2M} + \frac{Q_{2f}\sqrt{3M^2\delta^2-2M^2\delta+4M(1-\delta)Q_{2f}}}{2M^2}$

Notes. $p_{f2,A} = \dfrac{M(1-\delta)(M\cdot\delta - Q_{2f}) + (Q_{2f}+Q_{2s}-M\cdot\delta)(M-Q_{2f}-Q_{2s})}{M(1-\delta)Q_{2s}}$

$$p_{2,D} = \frac{M - \sqrt{3M^2\delta^2 - 2M^2\delta + 4M(1-\delta)Q_{2f}}}{2M}$$

Table A.3.11: Firm's Second-Period Pricing Decision When $M \cdot \delta \leq Q_{2f} < \frac{M}{3}$

Range of Q_{2s}	$Q_{2s} < \frac{M-Q_{2f}}{2}$	$\frac{M-Q_{2f}}{2} \leq Q_{2s} < \frac{M+\sqrt{2MQ_{2f}-Q_{2f}^2}}{2}$	$Q_{2s} \geq \frac{M+\sqrt{2MQ_{2f}-Q_{2f}^2}}{2}$
Firm's Pricing Decision	$p_{f2} = 1 - \frac{Q_{2f}+Q_{2s}}{M}$	$p_{f2} = \frac{(M-Q_{2f})^2}{4MQ_{2s}}$	$p_{f2} = \frac{M-\sqrt{2MQ_{2f}-Q_{2f}^2}}{2M}$
Scalpers' Pricing Decision	$p_{sh} = 1 - \delta$ $p_{sl} = 1 - \frac{Q_f+Q_s}{M}$	$p_{sh} = 1 - \delta$ $p_{sl} = \frac{1-\frac{Q_{2f}}{M}}{2}$	$p_{sh} = 1 - \delta$ $p_{sl} = \frac{1-\frac{Q_{2f}}{M}}{2}$
Sellers' Profits	$\pi_{2s} = (1 - \frac{Q_f+Q_s}{M}) \times Q_{2s}$ $\pi_{2f} = (1 - \frac{Q_f+Q_s}{M}) \times Q_{2f}$	$\pi_{2s} = \frac{M(1-\frac{Q_{2f}}{M})^2}{4}$ $\pi_{2f} = \frac{(M-Q_{2f})^2}{4MQ_{2s}} \times Q_{2f}$	$\pi_{2s} = \frac{M(1-\frac{Q_{2f}}{M})^2}{4}$ $\pi_{2f} = \frac{M-\sqrt{2MQ_{2f}-Q_{2f}^2}}{2M} \times Q_{2f}$
u_2	$\frac{(Q_{2f}+Q_{2s})^2}{2M^2}$	$\frac{1}{2} - \frac{(2Q_{2f}+3Q_{2s})(M-Q_{2f})^2}{8M^2Q_{2s}}$	$\frac{(M+3Q_{2f})(M-Q_{2f})+4Q_{2f}\sqrt{2MQ_{2f}-Q_{2f}^2}}{8M^2}$

Table A.3.12: Firm's Second-Period Pricing Decision When $Q_{2f} \geq \frac{M}{3}$

Range of Q_{2s}	$Q_{2s} < M - 2Q_{2f}$	$M - 2Q_{2f} \leq Q_{2s} < Q_{2f}$	$Q_{2f} \leq Q_{2s} < \frac{M+\sqrt{2MQ_{2f}-Q_{2f}^2}}{2}$	$Q_s \geq \frac{M+\sqrt{2MQ_{2f}-Q_{2f}^2}}{2}$
Firm's Pricing Decision	$p_{f2} = 1 - \frac{Q_{2f}+Q_{2s}}{M}$	$p_{f2} = \frac{1-\frac{Q_{2s}}{M}}{2}$	$p_{f2} = \frac{(M-Q_{2f})^2}{4MQ_{2s}}$	$p_{f2} = \frac{M-\sqrt{2MQ_{2f}-Q_{2f}^2}}{2M}$
Scalpers' Pricing Decision	$p_{sh} = 1 - \delta$ $p_{sl} = 1 - \frac{Q_f+Q_s}{M}$	$p_{sh} = \frac{1-\frac{Q_{2s}}{M}}{2}$ $p_{sl} = \frac{1-\frac{Q_{2s}}{M}}{2}$	$p_{sh} = 1 - \delta$ $p_{sl} = \frac{1-\frac{Q_{2f}}{M}}{2}$	$p_{sh} = 1 - \delta$ $p_{sl} = \frac{1-\frac{Q_{2f}}{M}}{2}$
Sellers' Profits	$\pi_{2s} = (1 - \frac{Q_{2f}+Q_{2s}}{M}) \times Q_{2s}$ $\pi_{2f} = (1 - \frac{Q_{2f}+Q_{2s}}{M}) \times Q_{2f}$	$\pi_{2s} = \frac{1-\frac{Q_{2s}}{M}}{2} \times Q_{2s}$ $\pi_{2f} = \frac{M(1-\frac{Q_{2s}}{M})^2}{4}$	$\pi_{2s} = \frac{M(1-\frac{Q_{2f}}{M})^2}{4}$ $\pi_{2f} = \frac{(M-Q_{2f})^2}{4MQ_{2s}} \times Q_{2f}$	$\pi_{2s} = \frac{M(1-\frac{Q_{2f}}{M})^2}{4}$ $\pi_{2f} = \frac{M-\sqrt{2MQ_{2f}-Q_{2f}^2}}{2M} \times Q_{2f}$
u_2	$\frac{(Q_{2f}+Q_{2s})^2}{2M^2}$	$\frac{(M+Q_{2s})^2}{8M^2}$	$\frac{1}{2} - \frac{(2Q_{2f}+3Q_{2s})(M-Q_{2f})^2}{8M^2Q_{2s}}$	$\frac{(M+3Q_{2f})(M-Q_{2f})+4Q_{2f}\sqrt{2MQ_{2f}-Q_{2f}^2}}{8M^2}$

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