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Latent Growth Modeling for Information Systems: Theoretical Extensions and Practical Applications–Supplement

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Online Supplementary Appendix A: Basics of Latent Growth Modeling (LGM)

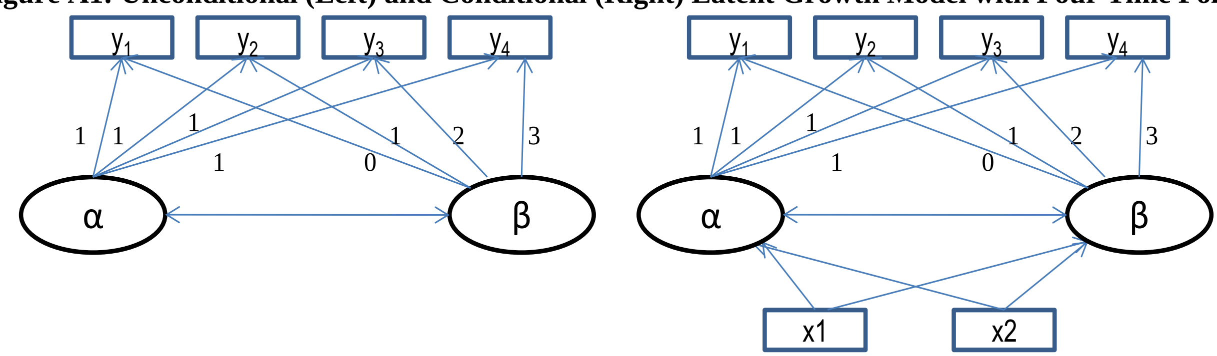
LGM has four key properties: (1) directly modeling change (trajectory) over time (within individual variables), (2) examining the effect of initial conditions on change over time, (3) identifying the exogenous variables that influence growth, and (4) examining the interplay among changes across variables.

In LGM, within-individual variation, also referred to as the *Level 1* model, captures the degree of change *within* individual variables. Between-individual (random coefficient) variation, also referred to as the *Level 2* model, captures the degree of change *between* individual variables. In the very basic LGM model, no other predictors are assumed to account for the variation in the specific parameters of the trajectories. Therefore, the *Level 2* model is also called an unconditional model. The left panel of Figure A1 depicts the simple LGM specified by Equations (A1) and (A2) with four time points, while the right panel of Figure A1 presents a path diagram of a longitudinal latent growth model with two covariates, *x1* and *x2*. For ease of presentation, the error terms were omitted from Figure A1. Please note that the loadings on α are fixed to be 1 for all four time points in Figure A1, while those of slope β are fixed as λ_t=0, 1, 2, 3 for t=1, 2, 3, 4. This simplest coding scheme is referred to as the LGM intercept-slope approach.

$$y_{it} = \alpha_i + \beta_t \lambda_t + \varepsilon_{it} \qquad \text{[A1]} \quad \text{(Level 1 Model)}$$

$$\alpha_i = \mu_\alpha + \varepsilon_{i\alpha};\ \ \beta_i = \mu_\beta + \varepsilon_{i\beta} \qquad \text{[A2]} \quad \text{(Level 2 Model)}$$

Figure A1: Unconditional (Left) and Conditional (Right) Latent Growth Model with Four Time Points



The change *within* individual variables over time may reveal some interesting longitudinal patterns (e.g., Marcoulides and Hershberger, 1997; Gottfried et al., 2007). Sample research questions that can be addressed using the unconditional model include:

RQ1: Does variable Y exhibit significant change over time?
RQ2: How does the initial value of Y affect the rate of change of Y?

RQ1 tests if the mean slope (μ_β) is significant, essentially asking whether a certain variable exhibits an increasing or decreasing rate of change over time. RQ2 is equivalent to testing if the covariance between the two random variables, intercept (α_i) and slope (β_i), is significant (depicted as a bi-directional edge between the two in Figure A1), essentially asking whether a variable's initial levels affect its future rate of change over the period under investigation.

The addition of variables that can potentially be used to predict the intercept and/or slope of LGM models requires the examination of a so-called *conditional* model (Bollen and Curran, 2006). Generally, the covariates considered in this manner are time invariant. The general form is:

$$\alpha_i = \mu_\alpha + \gamma_{\alpha 1i} x_{1i} + \gamma_{\alpha 2i} x_{2i} + \varepsilon_{i\alpha}$$
$$\beta_i = \mu_\beta + \gamma_{\beta 1i} x_{1i} + \gamma_{\beta 2i} x_{2i} + \varepsilon_{i\beta} \qquad \text{[A3]}$$

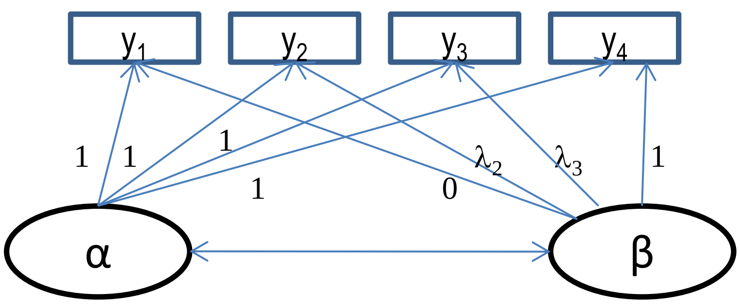
The potential of the conditional model is its ability to estimate both the level and the rate of change (slope) of the variables of interest while controlling for additional covariates. Sample research questions that can be addressed using the conditional model include:

RQ3a: Does covariate X significantly affect the initial value of Y?
RQ3b: Does covariate X significantly affect the rate of change of Y?

Using the specification of equation (A3) as an example, RQ3a tests if coefficients $\gamma_{\alpha 1i}$ and $\gamma_{\alpha 2i}$ are significant; whereas RQ3b tests whether $\gamma_{\beta 1i}$ and $\gamma_{\beta 2i}$ are significant. These research questions essentially test whether a covariate X affects the initial value and rate of change of variable Y over the given period.

The previous formulations focused on *linear* latent growth models, implying a constant rate of change over time. However, more complex trajectories, such as non-linear growth may exist, such as how can we design a non-linear model? In LGM, non-linear trajectory is achieved by a different approach via coding of time in λ_t. However, we need to know the specific non-linear shape of the rate of change to correctly design a non-linear LGM. Although there is only one way for a trajectory to be linear, there are infinite non-linear ways. Hence, not much can be said about the general non-linear form of LGM. Bollen and Curran (2006) (Chapter 4) discussed quadratic, cubic, and exponential trajectories, and interested readers could refer to Bollen and Curran for specific non-linear functional forms.

Figure A2: Level and Shape (LS) Time Coding Scheme



An alternative approach to modeling the change process is not to a priori assume a particular form of non-linearity (Raykov and Marcoulides, 2008). This general approach is called the Level/Shape (LS) strategy, where λ_t (the loadings on β_i) is normally fixed to be 0 for the first time period and 1 for the last time period, respectively. Unlike the coding in the slope-intercept approach, the λ_t's in the middle (besides the first and last) are not fixed and are treated as free parameters to be estimated from the data. This coding mechanism ensures that the Slope factor is interpreted as a change factor confined within 0 and 1. Freeing the loadings of the remaining time periods implies that the loadings reflect the part of the total change (100%) that occurs between the first and the last measurements (Duncan et al., 1999; Raykov and Marcoulides, 2008). A sample question enabled by a nonlinear growth model can be:

RQ4: What is the shape of growth of Y, linear or non-linear?

Specific to the example illustrated in Figure A2, RQ4 seeks to identify the value of λ_2 and λ_3, which capture the time growth pattern of Y.

Online Supplementary Appendix B. Linking IT Spending to Firm Performance over time
(Replicating Bharadwaj et al. (1999))

Replication with Ordinary Least Squares (OLS) Regression

We also illustrate LGM by replicating Bharadwaj's et al. (1999) seminal study that investigated the effect of IT spending on Tobin's q for the *Information Week* Top 500 firms, controlling for four industry (industry Tobin's q, capital expenditure, concentration ratio, and industry regulation[1]) and five firm (number of employees, market share, related diversification, R&D intensity, advertising intensity) variables. We obtained IT data from 2002 to 2006 from *InformationWeek* (*IW*)[2] to replicate their analysis. We first ran ordinary least square regression (OLS) as specified by Bharadwaj et al., (Table B1). Our replication with the 2002-2006 *IW* data yielded largely consistent results to Bharadwaj et al.'s 1989-1993 data in terms of the directionality of the relationships[3] and their level of significance.

Table B1. The OLS Results (Original Method used by Bharadwaj et al., 1999)

Variable	Coefficient	Standard Error	P-Value
Intercept	0.36	0.065	<0.0001
IT Budget	0.011	0.002	<0.0001
Industry Concentration	-0.198	0.07	0.0043
Employees	0.01	0.011	0.38
Diversity	-0.124	0.049	0.0125
Advertising Intensity	3.517	0.241	<0.0001
R&D Intensity	0.522	0.241	0.0302
Market Share	0.948	0.096	<0.0001
Industry Tobin's q	0.519	0.013	<0.0001
Industry Capital Intensity	0.0817	0.0134	<0.0001

(N=1614, R-square=0.159)

Replication with Latent Growth Modeling (LGM)

We then performed the data analysis with LGM. All ten variables involved are modeled as time varying, and each variable is decomposed into a level and a slope component. Figure B1 depicts the LGM graph along with the estimated path coefficients.[4] The complete results for all variables are tabulated in Table B2. Figure B1 reveals a simple conditional model: the level of firm performance (Tobin's Q) is conditioned on the level of all eight independent variables; yet, the slope of firm performance is conditioned on both the level and slope of all independent variables.

Comparing the original OLS results (Table B1) with the LGM results (Table B2) demonstrates several key distinctions between the two models. OLS analyzes the relationships between levels of dependent and independent variables as shown by Bharadwaj et al. (1999) who performed an OLS regression for each of the five years. Analysis in this manner assumes each year's data is independent of each other. In contrast, LGM enables to capture interdependencies among time periods through the slope term. For example, in both Bharadwaj et al. and our LGM replication , the level of IT is found to have a positive and significant impact on the level of Tobin's Q. However, according to our LGM analysis, the level of IT is not significant on the slope of Tobin's Q, implying that the initial level of IT spending has no significant impact on the growth of firm performance. Further, the slope of IT is not found to be significant on the slope of Tobin's Q either, suggesting that accelerating IT spending will not necessary lead to accelerated firm performance. LGM enables us to test two new hypotheses that were not addressed in the IT literature, notably:

Hypothesis A: The higher the initial level of IT spending, the higher the growth of firm performance.
Hypothesis B: The higher the growth of IT spending, the higher the growth of firm performance.

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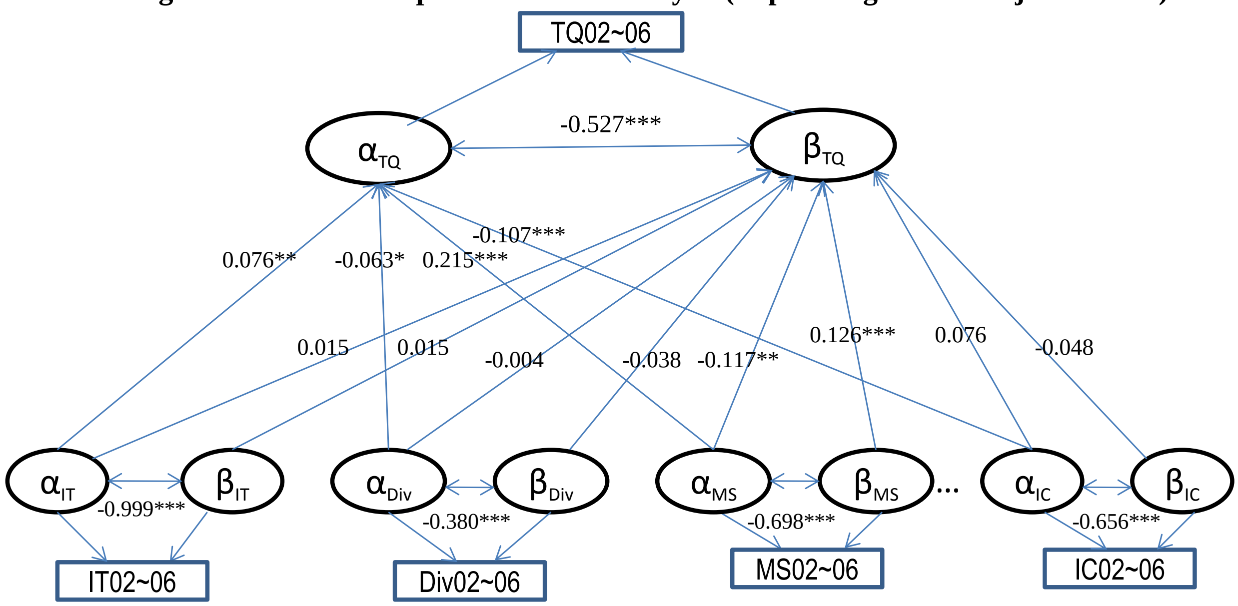
[1] We were alas not able to obtain data on *industry regulation* for the 2002-2006 period used in this study.
[2] We thank V. Sambamurthy for sharing the IW 500 IT spending data for the 2002-2006 period. We were not able to obtain the original IT spending data for 1989-1993 that was used in Bharadwaj et al. (1999).
[3] While our results show that industry capital expenditure has a positive effect on firm performance, Bharadwaj et al. (1999) found industry capital expenditure to have a negative effect.
[4] Due to the complexity of the graph, we only graphically show part of the LGM graph with four variables (IT spending, diversification, market share, and industry concentration).

In addition, in Figure B1, we allow each level variable to co-vary with the corresponding slope variable (delineated with a bi-directional arrow). For example, the path coefficient of β=-0.527 (p<0.01) between α_{TQ} and β_{TQ} implies that the higher the initial value (i.e., in 2002) of a firm's performance, the slower its growth rate it will be. This is not surprising since it is harder for high-performing firms to grow even faster. The ability of LGM to model within-variable variation also enables us to propose the following hypothesis between a variable's own initial level and its own growth potential over time, notably:

Hypothesis C: The higher the initial level of firm performance, the lower the growth of firm performance.

Figure B1. Partial Graph of the LGM Analysis (Replicating Bharadwaj et al. 1999)



(* sig. at p < 0.1, ** sig. at p<.05, * sig. at p<.01, AGFI=0.923, CFI=0.943, NFI=0.929, SRMR=0.067, RMSEA=0.047)**

Table B2. Complete LGM Results (All Variables)

Variable	TQ (Level)	TQ (Slope)
IT level	0.076**	0.015
IT Slope		0.015
Market Share level	0.215***	-0.117**
Market Share Slope		0.126***
Related Diversity level	-0.063*	0.004
Related Diversity Slope		-0.038
Number of Employee level	-0.048	-0.035
Number of Employees Slope		-0.105**
Advertising Intensity Level	0.269***	-0.204***
Advertising Intensity Slope		0.008
R&D Intensity Level	0.177***	-0.219***
R&D Intensity Slope		0.012
Industry Tobin's q Level	0.400***	0.074
Industry Tobin's q Slope		0.549***
Industry Concentration Level	-0.107***	0.076
Industry Concentration Slope		-0.048
Industry Capital Intensity Level	0.091**	0.125**
Industry Capital Intensity Slope		0.215***

Online Supplementary Appendix C: The SAS codes for the simulation

```
%macro iteration; /*developed under SAS 9.1 */
%do j=1 %to 10000;/* the sample size 10000 is a user input */

 data samplesize;
 number=ranuni(-1);
 record=250;
 if number<0.33 then record=100;
 if number>0.67 then record=1000;
 call symput('_size',record);
 run;

 data samplex;
  mu_a=100; mu_b=15; ea=3; eb=2; ex=10;
  t1=0; t2=1; t3=2; t4=3;
  %do i=1 %to &_size;
   ax=mu_a+ea*normal(-1);
   bx=mu_b+eb*normal(-2);
   x1=ax+bx*t1+ex*normal(-3);
   x2=ax+bx*t2+ex*normal(-4);
   x3=ax+bx*t3+ex*normal(-5);
   x4=ax+bx*t4+ex*normal(-6);
   output;
   %end; /*end i */
  run;

data rand;
rand2=0.01; rand1=0.05;
call symput('_randb',rand2);          call symput('_randa',rand1);
run;

data normality;
randn=ranuni(-78);
normal2=0;
if randn>=0.2 and randn<0.4 then normal2=1;
if randn>=0.4 and randn<0.6 then normal2=2;
if randn>=0.6 and randn<0.8 then normal2=3;
if randn>=0.8 then normal2=4;
call symput('_normality',normal2);
/*0=LGM 1=nonnormal 2=nonlinear 3=noniid (heterogeneity) 4=non LGM */
run;

data sampley; set samplex;

if &_normality=0 then do;
 z1=100+5*rannor(-300);
 ay=500+&_randa*z1+15*normal(-101); /* both level and slope are a function of z1 and z2 */
 by=10+&_randb*z1+2*normal(-201);
 y1=ay+40*rannor(-700);
 y2=ay+by*t1+40*rannor(-800); /*level+slope*lamda_t+b*x+error*/
 y3=ay+by*t2+40*rannor(-900);
 y4=ay+by*t3+40*rannor(-1000);
end;
```

```
if &_normality=1 then do;
 z1=100-2.5+5*ranuni(-300);
 ay=500+&_randa*z1+15*normal(-101);
 by=10+&_randb*z1+2*normal(-201);
 y1=ay-20+40*ranuni(-700);
 y2=ay+by*t1-20+40*ranuni(-800); /*use square root of 40 to make sure the variance is close*/
 y3=ay+by*t2-20+40*ranuni(-900);
 y4=ay+by*t3-20+40*ranuni(-1000);
end;

if &_normality=1 then do;
 z1=100+5*rannor(-300);
 ay=500+&_randa*z1+15*normal(-101);
 by=10+&_randb*z1+2*normal(-201);
 y1=ay+40*rannor(-700);
 y2=ay+by*exp(t1)+40*rannor(-800); /*level+slope*lamda_t+b*x+error*/
 y3=ay+by*exp(t2)+40*rannor(-900);
 y4=ay+by*exp(t3)+40*rannor(-1000);
end;

if &_normality=2 then do;
   pop=ranuni(-987);
   if pop<0.5 then
     do;
       z1=100+5*rannor(-300);
       ay=500+&_randa*z1+15*normal(-101);
       by=10+&_randb*z1+2*normal(-201);
       y1=ay+40*rannor(-700);
       y2=ay+by*exp(t1)+40*rannor(-800);
       y3=ay+by*exp(t2)+40*rannor(-900);
       y4=ay+by*exp(t3)+40*rannor(-1000);
     end;
     else  do;
       z1=100+5*rannor(-300);
       ay=550+&_randa*z1+15*normal(-101);
       by=10+&_randb*z1+2*normal(-201);
       y1=ay+40*rannor(-700);
       y2=ay+by*exp(t1)+40*rannor(-800);
       y3=ay+by*exp(t2)+40*rannor(-900);
       y4=ay+by*exp(t3)+40*rannor(-1000);
     end;

if &_normality=3 then do;
   pop=ranuni(-987);
   if pop<0.5 then
     do;
       z1=100+5*rannor(-300);
       ay=500+&_randa*z1+15*normal(-101);
       by=10+&_randb*z1+2*normal(-201);
       y1=ay+40*rannor(-700);
       y2=ay+by*exp(t1)+40*rannor(-800);
       y3=ay+by*exp(t2)+40*rannor(-900);
       y4=ay+by*exp(t3)+40*rannor(-1000);
     end;
```

```
    else  do;
     z1=100+5*rannor(-300);
     ay=550+&_randa*z1+15*normal(-101);
     by=10+&_randb*z1+2*normal(-201);

     y1=ay+40*rannor(-700);
     y2=ay+by*exp(t1)+40*rannor(-800);
     y3=ay+by*exp(t2)+40*rannor(-900);
     y4=ay+by*exp(t3)+40*rannor(-1000);
    end;

if &_normality=4 then do;
 z1=100+5*rannor(-300);
  ay=500+&_randa*z1+15*normal(-10);
  by=10+&_randb*z1+2*normal(-11);
 by2=5+0.005*z1*z1+normal(-80);

 /*the underlying model is not a LGM model */
 y1=ay+by+40*rannor(-700)+by2;
 y2=ay+by*t1+40*rannor(-800)+by2; /*level+slope*lamda_t+b*x+error*/
 y3=ay+by*t1+40*rannor(-900)+by2;
 y4=ay+by*t1+40*rannor(-1000)+by2;
end;

end;

run;

proc corr data=sampley outp=pcorr;
var z1 y1 y2 y3 y4;
partial ay by;
run;

proc sql;
create table corrresult as
select y1,y2,y3,y4 from pcorr
where _name_= "z1";
quit;

data max; set corrresult; max=0;
if abs(y1)>max then max=abs(y1); if abs(y2)>max then max=abs(y2);
if abs(y3)>max then max=abs(y3); if abs(y4)>max then max=abs(y4);
run;

  /*use outram for parametrs, oterh options outest, outstat, outwgt */
Proc CALIS Data=sampley  UCOV AUG maxfunc =20000 maxiter=20000 outram=outputram;
  LINEQS

 /* main performance model */
 y1=F1+ 0 F2  + E1,
 y2=F1+ 1 F2  + E2,
 y3=F1+ 2 F2  + E3,
 y4=F1+ 3 F2  + E4,
```

```
 F1= a1 INTERCEPT + a11 z1   +  d1,
 F2= a2 INTERCEPT + a21 z1   + d2;

STD
    e1-e4 = s1-s4, d1-d2 = int slp;
   COV
  d1-d2 = sigma;
run;

proc sql;
create table fit as
select _name_, _ESTIM_ from outputram
where _name_ in
('N','DF','FIT','GFI','AGFI','RMR','CHISQUAR','P_CHISQ','RMSEAEST','AIC','COMPFITI','BB_NORMD','BOLRHO1','CNHOELT'
);
quit;

proc transpose data=fit out=fit1; run;
data combine; merge fit1 max rand normality; run;
data onerun; set combine; keep MAX rand1 rand2 N normal2 FIT DF GFI AGFI RMR CHISQUAR P_CHISQ RMSEAEST AIC
COMPFITI BB_NORMD BOLRHO1 CNHOELT; run;
/*proc print data=onerun; run;*/

proc datasets nolist;
append base=finalresult data=onerun ; run;

%end; /*end j*/
%mend; /*end iteration */

%iteration;
proc print data=finalresult; run;

data outfile; set finalresult; file "finalresult.txt" dlm='09'x;
put MAX rand1 rand2 N normal2 FIT DF GFI AGFI RMR CHISQUAR P_CHISQ RMSEAEST AIC COMPFITI BB_NORMD
BOLRHO1 CNHOELT;
label MAX='maxd' rand1='level' rand2='slope' N='size' normal2='normality' COMPFITI='CFI' BB_Normd='NFI';
 run;

/* the rest of the code generates the statistics */
data result2;  infile "finalresult.txt" dlm='09'x;
input dsep rand1 rand2 SIZE scenario FIT DF GFI AGFI RMR CHISQ PCHI RMSE AIC CFI NFI RHO HOTN;
sig=2;
if size=100 and dsep>0.195 then sig=1; if size=100 and dsep<=0.165 then sig=0; /*0.1 level is 0.165*/
if size=250 and dsep>0.125 then sig=1; if size=250 and dsep<=0.105 then sig=0; /*0.1 level is 0.105*/
if size=1000 and dsep>0.062 then sig=1; if size=1000 and dsep<=0.052 then sig=0; /*0.1 level is 0.052*/

 run;

proc sql;
create table stat as
select scenario, size, sig, count(size) as counts, avg(dsep) as dsepa, stderr(dsep) as dsepd, avg(fit) as fita, stderr(fit) as
fitd,
      avg(GFI) as GFIa, stderr(GFI) as GFId, avg(AGFI) as AGFIa, stderr(AGFI) as AGFId, avg(RMR) as RMRa, stderr(RMR)
as RMRd,
```

```
      avg(RMSE) as RMSEa, stderr(RMSE) as RMSEd,  avg(CHISQ) as CHISQa, stderr(CHISQ) as CHISQd, avg(PCHI) as
PCHIa, stderr(PCHI) as PCHId, avg(AIC) as AICa, stderr(AIC) as AICd,
      avg(CFI) as CFIa, stderr(CFI) as CFId, avg(NFI) as NFIa, stderr(NFI) as NFId, avg(HOTN) as HOTNa, stderr(HOTN) as
HOTNd
from result2
group by scenario, size, sig;
quit;

data stat2; set stat;
 if scenario=1 then case2='normal'; if scenario= 2 then case2='nonnormal'; if scenario=3 then case2='nonlinear'; if
scenario='4' then case2='hetero';
run;
proc print data=stat; run;

data out; set stat; file "stat.txt" dlm='09'x; put scenario size sig counts dsepa GFIa AGFIa RMRa RMSEa CHISQa  PCHIa
AICa
       CFIa NFIa HOTNa; if sig=1 or sig=0; run;

data out; set stat; file "statd.txt" dlm='09'x; put scenario size sig counts  dsepd  GFId AGFId  RMRd  RMSEd CHISQd
PCHId
       AICd  CFId  NFId HOTNd; if sig=1 or sig=0; run;
```

Online Supplementary Appendix D: Additional Results of the WOM Application

Table D1: Alternative LGM Model including a Quadratic Growth Factor

	d-seperated by Level, slope and the quadratic term of Rank							d-seperated by Level, slope and the quadratic term of Volume							
	Level_ Vol	Slope_ Vol	Quadratic _Vol	Price_ C	Price_ A	Rating	Age		Level_ RK	Slope_ RK	Quadratic _RK	Price_ C	Price_ A	Rating	Age
RK01	0.041	0.058	0.059	0.036	0.025	0.051	0.022	Vol01	0.031	0.032	0.031	0.005	0.006	0.038	0.031
RK02	0.039	0.064	0.059	0.044	0.018	0.047	0.022	Vol02	0.025	0.025	0.026	0.023	0.005	0.030	0.025
RK03	0.057	0.063	0.060	0.029	0.037	0.060	0.023	Vol03	0.031	0.032	0.033	0.021	0.001	**0.067**	0.031
RK04	0.043	0.058	0.057	0.043	0.029	**0.076**	0.024	Vol04	0.034	0.035	0.035	0.032	0.012	**0.079**	0.034
RK05	0.057	0.047	0.044	0.045	0.026	0.049	0.025	Vol05	0.021	0.021	0.022	0.020	0.005	0.058	0.021
RK06	**0.075**	0.048	0.040	0.035	0.034	0.041	0.025	Vol06	0.060	0.061	0.061	0.013	0.032	0.011	0.060
RK07	**0.070**	**0.070**	0.062	0.029	0.041	0.033	0.025	Vol07	**0.066**	**0.066**	**0.067**	0.026	0.002	0.035	**0.066**
RK08	0.050	0.049	0.045	0.040	0.033	0.031	0.025	Vol08	0.024	0.025	0.025	0.035	0.006	0.010	0.024
RK09	0.052	0.040	0.036	0.050	0.026	0.022	0.026	Vol09	0.023	0.023	0.024	0.021	0.005	0.029	0.023
RK10	0.053	0.040	0.038	0.041	0.037	0.027	0.026	Vol10	0.050	0.050	0.051	0.000	0.024	0.053	0.050
RK11	0.050	0.022	0.020	0.059	0.022	0.017	0.028	Vol11	0.022	0.022	0.023	0.024	0.000	**0.071**	0.022
RK12	0.031	0.025	0.023	**0.070**	0.008	0.013	0.026	Vol12	0.055	0.055	0.055	0.032	0.010	0.043	0.055
RK13	0.063	0.035	0.025	0.014	0.010	0.002	0.018	Vol13	0.041	0.041	0.041	0.011	0.013	0.063	0.041
RK14	0.060	0.045	0.037	0.033	0.054	0.013	0.019	Vol14	0.053	0.053	0.054	0.034	0.003	0.010	0.053
RK15	0.062	0.050	0.040	0.046	0.002	0.014	0.019	Vol15	0.032	0.032	0.033	0.033	0.004	0.006	0.032
RK16	0.052	0.039	0.031	0.038	0.011	0.047	0.018	Vol16	0.036	0.037	0.037	0.016	0.003	0.008	0.036
RK17	0.057	0.055	0.051	0.001	0.050	0.035	0.019	Vol17	0.047	0.048	0.048	0.013	0.009	0.009	0.047
RK18	0.050	0.030	0.026	0.017	0.044	0.056	0.019	Vol18	0.046	0.046	0.047	0.022	0.004	0.007	0.045
RK19	0.060	0.040	0.034	0.015	0.043	0.009	0.021	Vol19	0.025	0.025	0.025	0.018	0.008	0.051	0.025
RK20	0.063	0.058	0.054	0.001	0.054	0.007	0.021	Vol20	0.035	0.034	0.035	0.045	0.019	0.032	0.034
RK21	0.048	0.037	0.032	0.011	0.047	0.028	0.021	Vol21	0.057	0.057	0.058	0.020	0.007	0.009	0.057
RK22	0.051	0.044	0.038	0.013	0.061	0.011	0.020	Vol22	0.041	0.041	0.042	0.036	0.005	0.008	0.041
RK23	0.053	0.049	0.044	0.008	0.056	0.016	0.017	Vol23	0.041	0.041	0.042	0.009	0.023	0.005	0.041
RK24	0.050	0.018	0.063	0.010	0.041	0.023	0.020	Vol24	0.028	0.028	0.029	0.035	0.007	0.049	0.028
RK25	0.039	0.028	0.058	0.018	0.063	0.016	0.016	Vol25	0.040	0.040	0.041	0.016	0.007	0.012	0.040
RK26	0.065	0.051	0.042	0.013	0.038	0.017	0.019	Vol26	0.048	0.048	0.049	0.009	0.028	0.009	0.048
Avg	0.054	0.045	0.043	0.029	0.035	0.029	0.022	**Avg**	0.039	0.039	0.040	0.022	0.009	0.031	0.039
Max	0.075	0.070	0.063	0.070	0.063	0.076	0.028	**Max**	0.066	0.066	0.067	0.045	0.032	0.079	0.066

Figure D1: The SEM Model Results for the WOM Application

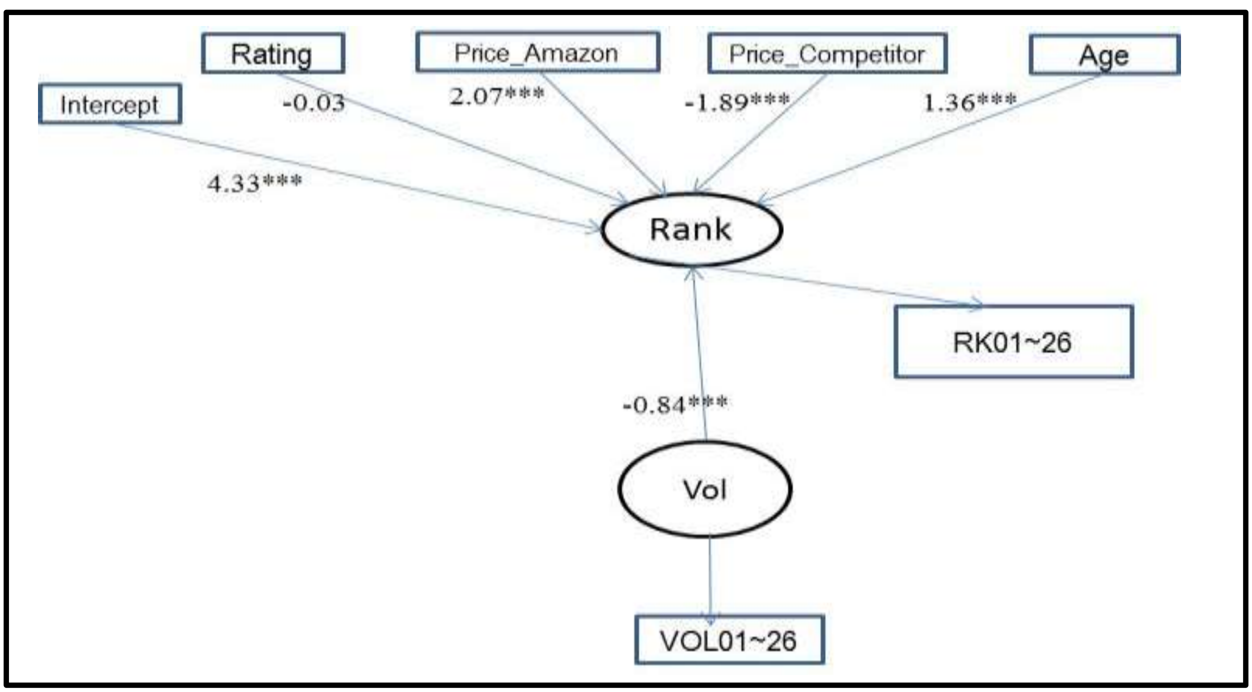


Table D2: Loadings of the Time-Varying Variables at each Time Period

	Level	Slope				Level	Slope	
	Loading	Loading	Standard Error			Loading	Loading	Standard Error
Rk01	1	0	0		Vol01	1	0	
Rk02	1	0.054	0.057		Vol02	1	0.220	0.034
Rk03	1	0.068	0.056		Vol03	1	0.440	0.024
Rk04	1	0.092	0.055		Vol04	1	0.439	0.023
Rk05	1	0.108	0.054		Vol05	1	0.352	0.026
Rk06	1	0.068	0.052		Vol06	1	0.628	0.017
Rk07	1	0.104	0.053		Vol07	1	0.760	0.013
Rk08	1	0.160	0.053		Vol08	1	0.784	0.011
Rk09	1	0.229	0.056		Vol09	1	0.791	0.030
Rk10	1	0.334	0.062		Vol10	1	0.821	0.009
Rk11	1	0.321	0.061		Vol11	1	0.777	0.010
Rk12	1	0.374	0.060		Vol12	1	0.786	0.011
Rk13	1	0.432	0.042		Vol13	1	0.787	0.010
Rk14	1	0.594	0.040		Vol14	1	0.727	0.046
Rk15	1	0.603	0.038		Vol15	1	0.683	0.014
Rk16	1	0.691	0.037		Vol16	1	0.686	0.014
Rk17	1	0.709	0.037		Vol17	1	0.840	0.009
Rk18	1	0.748	0.037		Vol18	1	0.908	0.007
Rk19	1	0.803	0.038		Vol19	1	0.890	0.008
Rk20	1	0.891	0.038		Vol20	1	0.856	0.011
Rk21	1	0.879	0.039		Vol21	1	0.919	0.008
Rk22	1	0.906	0.040		Vol22	1	0.902	0.008
Rk23	1	0.907	0.039		Vol23	1	0.951	0.008
Rk24	1	0.948	0.040		Vol24	1	0.816	0.009
Rk25	1	0.955	0.042		Vol25	1	0.951	0.006
Rk26	1	1			Vol26	1	1	

Table D3: Complete Results of the Random-Coefficients Model (Table 8 in the paper)

	Solution for Random Coefficient Effects						
Parameter	**week**	**Estimate**	**Std Err**	**t Value**	**Pr > |t|**	**Lower**	**Upper**
Intercept		10.612	0.053	201.67	<.0001		
Rating		0.048	0.010	4.85	<.0001		
Price_c		-1.687	0.039	-43.18	<.0001		
Price_a		1.767	0.044	40.43	<.0001		
Age		0.001	0.00004	29.42	<.0001		
	0	-0.445	0.053	-8.32	<.0001	-0.549	-0.343
	1	-0.446	0.053	-8.33	<.0001	-0.550	-0.340

	Solution for Random Coefficient Effects						
Parameter	**week**	**Estimate**	**Std Err**	**t Value**	**Pr > |t|**	**Lower**	**Upper**
Volume	2	-0.321	0.055	-5.80	<.0001	-0.429	-0.212
	3	-0.211	0.059	-3.58	0.0003	-0.326	-0.095
	4	-0.208	0.059	-3.52	0.0004	-0.323	-0.092
	5	-0.195	0.058	-3.37	0.0008	-0.308	-0.081
	6	-0.064	0.063	-1.03	0.305	-0.186	0.0584
	7	0.007	0.067	0.10	0.918	-0.124	0.138
	8	0.023	0.072	0.32	0.752	-0.117	0.162
	9	0.059	0.044	1.34	0.182	-0.027	0.145
	10	-0.067	0.078	-0.87	0.386	-0.218	0.084
	11	-0.073	0.074	-0.98	0.325	-0.217	0.072
	12	-0.045	0.073	-0.62	0.534	-0.187	0.097
	13	0.0198	0.075	0.26	0.794	-0.127	0.167
	14	0.070	0.040	1.74	0.082	-0.008	0.149
	15	0.089	0.067	1.34	0.179	-0.040	0.218
	16	0.092	0.067	1.37	0.172	-0.040	0.224
	17	-0.094	0.081	-1.16	0.246	-0.253	0.064
	18	-0.326	0.091	-3.58	0.0003	-0.504	-0.147
	19	-0.256	0.082	-3.14	0.002	-0.415	-0.096
	20	-0.096	0.072	-1.33	0.183	-0.238	0.045
	21	-0.285	0.083	-3.42	0.0006	-0.448	-0.121
	22	-0.313	0.083	-3.79	0.0002	-0.475	-0.151
	23	-0.219	0.086	-2.54	0.011	-0.387	-0.050
	24	-0.073	0.07	-0.94	0.347	-0.225	0.079
	25	-0.412	0.101	-4.09	<.0001	-0.609	-0.214
	26	-0.581	0.120	-4.83	<.0001	-0.816	-0.345

Note: F= 68.88, P<0.001, R^2= 0.135