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# Analysis of Product Rollover Strategies in the Presence of Strategic Customers

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Frequent product introductions emphasize the importance of product rollover strategies. With single rollover, when a new product is introduced, the old product is phased out from the market. With dual rollover, the old product remains in the market along with the new product. Anticipating the introduction of the new product and the potential markdown of the old product, strategic customers may delay their purchases. We study the interaction between product rollover strategies and strategic customer purchasing behavior and find that single rollover is more valuable when the new product's innovation is low and the number of strategic customers is high. Interestingly and counter to intuition, the firm may have to charge a lower price for the old product as well as receive a lower profit with a higher value disposal (outside) option for the old product under single rollover. Facing a market composed of both strategic and myopic customers, the firm does not necessarily reduce the stocking level as more myopic customers become strategic.

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## 1. Introduction

*To use single rollover or dual rollover, that is the question*: Is it better to phase out an old product when introducing a new product to avoid cannibalization or to continue selling it in the market for additional revenue? The answer to this question, as will be seen later, is just as complex as the dilemma faced by Shakespeare's Hamlet. Which alternative is more profitable depends critically on the innovation level of the new product over the old, the disposal value of the old product, and the proportion of customers in the market having the tendency to delay their purchases in anticipation of the introduction of the new product and a possible markdown of the old product.

Frequent product introductions are common in such industries as apparel, consumer electronics, and computers. Fashion retailers usually replenish their stock with new designs more than once per season. For example, Zara has reduced its design-to-market cycle time from a few months to a few weeks, and therefore has more frequent new clothing style introductions (Ghemawat and Nueno 2003). Many firms view frequent product introductions as important means of increasing market share and sustaining growth. This requires managing phaseouts of old products and introductions of new replacement products.

It would be ideal to introduce the new product when the old product inventory is depleted. However, this is difficult to attain in an uncertain environment. In reality, a firm usually needs to manage the leftover inventory of the old product when rolling over to the new product. This paper considers two rollover strategies: single-product (single) rollover and dual-product (dual) rollover. With dual rollover, the old product remains in the market together with the new product. With single rollover, the old product is phased out from the market and can be disposed of in ways such as fire sales, dismantling products for spare parts, recycling the material for future use, and write-offs, once the new product is introduced. Some known brand-name stores sell leftover old products at other brand-name stores, such as Neiman Marcus selling through its Last Call stores. Also, more and more U.S. companies are relying on overseas markets to clear their leftovers (Kavilanz 2008). Using different introduction dates for the new product in different regions facilitates the implementation of single rollover. Even the equipment that is obsolete in developed countries can be sold in less developed countries. Leftovers can also be sold through discount stores like TJ Maxx and Marshalls, outlet malls, and websites such as Overstock.com. Sometimes firms donate unsold items for charitable tax deductions. Tibben-Lembke (2004) lists additional approaches for disposing of leftover inventory. The main drawback of single rollover is that the revenue from disposing of leftover items is usually lower than selling them under dual rollover.

Dual rollover, despite obtaining a higher price for a leftover item by keeping it in the market than its disposal value under single rollover, has two serious drawbacks. The first is the *cannibalization effect*. With both products in the market, the old product may cannibalize sales of the new product, especially when the *innovation* (improvement of the new product over the old product) is not very high. Customers shopping in an apparel or shoe store are often attracted by the sales rack. Many discussions have taken place with regard to the cannibalization of the new iPad by the marked-down iPad 2. The second is the *postponement effect*. Strategic customers are common in the markets for durable goods with rapid innovation, such as high-tech and fashion products. These customers are forward looking and decide on what products to purchase and when to purchase them on the basis of the surplus received. According to Arends (2010), the top reasons why customers should not buy an iPad now are that it will be cheaper and better next year. Therefore, when a strategic customer is deciding on a purchase, she takes into account the future opportunity of buying the new version and the marked-down old version that is available under dual rollover. Thus, compared to single rollover, the availability of the old product in the market gives a strategic customer more incentive to delay her purchase.

By studying the performance of single rollover in mitigating cannibalization and postponement effects, we answer the following questions: Can a firm earn a higher profit by switching over to single rollover? If yes, under what conditions can single rollover increase the firm's profit and by how much? We develop a two-period model of a monopolistic firm that sells an old version in period 1 and introduces a new version in period 2 in an uncertain-sized market consisting of strategic customers. The firm decides its rollover strategy before period 1 as well as its prices and ordering quantities in both periods.

Our work contributes to both the strategic customer behavior research in operations management (OM) and product rollover strategy literature by studying the interplay between strategic waiting behavior and rollover strategies. The strategic waiting research in OM most relevant to ours is that of Su and Zhang (2008), Cachon and Swinney (2009), and Lai et al. (2010). They study mechanisms mitigating strategic waiting behavior when demand is uncertain. Su and Zhang (2008) consider quantity and price commitments, showing that these can be achieved via various supply chain contracts. Cachon and Swinney (2009) explore a quick response strategy that better matches supply and demand to reduce the likelihood of markdowns. Lai et al. (2010) show that posterior price matching can improve a firm's profit if the fraction of strategic customers is not too low, and their depreciation of the product value over time is moderate. We study how single rollover can mitigate strategic customers' waiting behavior for the first time in the OM literature. These three papers and others—Su and Zhang (2009), Su (2008), Cachon and Swinney (2011), Liu and van Ryzin (2008), Aviv and Pazgal (2008), Yin et al. (2009), Prasad et al. (2010), and Özer and Zheng (2012)—usually focus on the scenario where a firm sells a product during the regular season and then marks down the product in its final sale. However, they do not discuss a firm's willingness to sell the leftover product in the market with or without markdown or the new product introduction leading to cannibalization of sales. Erzurumlu et al. (2010) and Agrawal et al. (2013) consider product introduction strategies, but their focus is not on product rollover strategy.

Despite their importance, product rollover strategies have received attention in the literature only recently. Billington et al. (1998) and Erhun et al. (2007) provide managerial insights derived from hands-on experience. To our knowledge, only five papers consider analytical comparisons of rollover strategies (despite employing different terminologies): Levinthal and Purohit (1989), Lim and Tang (2006), Ferguson and Koenigsberg (2007), Arslan et al. (2009), and Koca et al. (2010). The last four focus on the cannibalization effect and/or product introduction and phaseout times. Furthermore, neither are the customers in these papers strategic nor their choice decisions explicitly modeled, and thus they do not consider the important interaction between rollover strategy and strategic waiting behavior. Levinthal and Purohit (1989) consider both cannibalization and postponement effects, but with a deterministic demand and without explicitly modeled customer choices. In a different setting, they show that single rollover is always better than dual rollover. However, our paper shows that dual rollover can also outperform single rollover.

We examine the performance of single rollover under three innovation cases: high, medium, and low. We show that the firm can increase its profit in all cases by adopting single rather than dual rollover under certain conditions, particularly when the proportion of strategic customers is high. We find that the innovation level strongly affects single rollover's performance in mitigating waiting behavior. In the literature for only one product version, the firm can often induce all strategic customers to buy early while extracting *all* their utility by using some mechanism (e.g., posterior price matching in Lai et al. 2010 and price commitment in Su and Zhang 2008). The same result may be expected without any old versions remaining in the market under single rollover. However, in our two-version context, this is true only when the replacement of the old version with the new version is not possible. When it is possible, which happens under a sufficiently high innovation, a strategic customer still has an incentive to delay her purchase even though no leftovers will be available later. This is because if the market is saturated (i.e., if many customers have already bought the old version), the firm may find it more profitable to lower the price of the new version to induce replacements instead of keeping it high. This creates a waiting incentive for strategic customers; that is, with a high enough innovation, the firm cannot eliminate waiting behavior even though it commits to not selling any old version leftovers in the market. In practice, especially in the consumer electronics industry with a saturated market, the price of the new version may sometimes be even less than or equal to the original price (before markdown) of the old version. This can lead to a higher customer surplus from buying the new version.

Another interesting finding is that the disposal value of the leftover old version under single rollover plays different roles under different innovation levels. With low and medium innovations, the firm benefits from a high disposal value, which is consistent with our intuition. However, with high innovation, the firm may have to charge a *lower* price for the old version in period 1 and *suffer a lower profit when the disposal value is higher*. Although this result appears to be counterintuitive, it arises because a higher disposal value leads directly to a higher inventory of the old version, which implies higher sales in period 1. When more customers carry the old version from period 1 to period 2, the market in period 2 becomes more saturated, which makes the firm more willing to price the new version lower to encourage replacements. This increases customers' likelihood of obtaining a positive surplus by waiting and therefore increases their waiting incentive. This increased waiting incentive in turn forces the firm to charge a lower price in period 1 to induce early purchases. Therefore, in addition to the direct economic benefit of a higher disposal value, there is an indirect behavioral impact as well: a higher disposal value would induce the strategic customers to wait rather than purchase. When the proportion of strategic customers is high enough, the indirect behavioral impact outweighs the direct impact, resulting in a lower profit.

We find that the firm has different product introduction policies depending on the innovation level. Specifically, when the innovation is high, the firm tends to introduce both versions in the hope that customers will purchase both. However, when the innovation is low or medium (i.e., when replacements are not possible), the firm should skip the old version to eliminate the cannibalization of the more profitable new version, as long as the innovation in the new version can compensate the firm for its loss resulting from its discounting of profit and customers' depreciation of the product value over time. Roughly speaking, a fast-innovating firm can introduce both versions since it can expect to receive payments twice from repeat customers; a moderate-innovating firm may consider skipping the first version and introducing the new version directly even though it may need to wait a little for the new version to be ready. As long as customers value the first version, a slow-innovating firm should introduce it as soon as possible to avoid the loss resulting from its profit discount and customers' value depreciation over time.

## 2. Model Description

We model a profit-maximizing firm that may introduce a product V1 in period 1 and its upgraded version V2 in period 2. Prior to period 1, the firm must choose between two product rollover strategies: *single* or *dual*. In either strategy only V1 is sold in period 1. In period 2 under single rollover, only V2 is available in the market, whereas both V2 and the leftover V1 are available under dual rollover.

We set the innovation levels of V1 and V2 as 1 and $1+\theta$, respectively, where $\theta \geq 0$ denotes the additional innovation from V1 to V2. For example, $\theta$ may represent the number of new functions introduced in V2. The higher $\theta$ is, the more customers are willing to pay. We assume that the firm makes a credible commitment for its rollover strategy prior to period 1. This is reasonable since the rollover strategy can be verified ex post, and the firm is averse to a loss of reputation resulting from reneging. Also, a rollover strategy requires preparations in advance. For example, the firm may need to plan for the required shelf space for the potential leftover V1 in period 2. In single rollover, if the phased out V1 will be sold in overseas markets, then resources such as transportation and storage space may have to be lined up ahead of time. Thus, any deviation from the committed strategy at the last moment could be prohibitively expensive. Under both rollover strategies, the firm decides the price $p_1$ and the stocking level $q_1$ for V1 in period 1, and the price $p_2$ and the stocking level $q_2$ for V2 in period 2. The firm also decides the marked-down price $p_1'$ for the leftover V1 under dual rollover, and receives the disposal value $\sigma \geq 0$ from each leftover V1 under single rollover.

The market consists of *high-end customers* and *bargain hunters*. All high-end customers have the same value for each version. For two periods together, high-end customers' value of using V1 is $v$; for only period 2, their value of using V1 is $\beta v$, where $0 < \beta < 1$. The residual value multiplier $\beta$ captures the depreciation in value due to the loss of utility in period 1 and the disutility of not being among the first adopters of the product. High-end customers' value of using V2 in period 2 is higher than their value $\beta v$ of using V1 in period 2 on account of the additional innovation $\theta$ in V2. Specifically, we assume this value to be $\beta v(1+\theta)$. The market size for high-end customers is a random variable $\mathbf{N}$ with cumulative distribution function $F(\cdot)$ and density $f(\cdot)$; $\mathbf{N}$ is not realized until at the end of period 1. A fraction $\phi$ of high-end customers in period 1 are *strategic customers*, and the rest are *myopic customers*. Without a next period to plan for, all customers are myopic in period 2. We reserve the term "myopic customers" for this kind of customers in period 1. There are unlimited bargain hunters, who arrive in the market in period 2 and consider buying only the leftover V1 if it is available under dual rollover and its marked-down price $p_1'$ is not greater than their value $\delta$ for the leftover V1, where $0 \leq \delta < \beta v$.

Strategic customers are identical, so they all reason in the same way and hold the same belief of the *waiting surplus* $W_c$, which is the surplus each of them can obtain by delaying her purchase to period 2. When making buy or not buy decisions in period 1, each strategic customer takes into account the option of waiting until period 2. Throughout this paper, we use the terms *buying now* and *waiting* for strategic customers' decisions of buying and not buying V1 in period 1, respectively. Unlike strategic customers, myopic customers decide to buy or not buy in period 1 without considering the waiting option. We assume that customers buy in the current period whenever the surpluses of buying and not buying are tied. The market size for high-end customers in period 1 is random, so it may occur that some high-end (either myopic or strategic) customers cannot get V1 because of a stockout even if they decide to buy V1 in period 1. In that case or if they decide not to buy V1, they become period 1 nonbuyers (P1NBs) in period 2. On the other hand, if they decide to buy and then find V1 in stock, they buy V1 in period 1 and become period 1 buyers (P1Bs) in period 2.

Table 1 summarizes the customers' purchase options in both periods. Under both rollover strategies, P1Bs can either continue using V1 or replace V1 with V2. P1NBs have two options (buying V2 and buying nothing) under single rollover. Under dual rollover, in addition to the two options as under single rollover, P1NBs have the third option (buying the leftover V1, if any). Bargain hunters cannot afford V2 in period 2, so they buy nothing under single rollover. But they may buy the marked-down leftover V1 under dual rollover. When both P1NBs and bargain hunters want the leftover V1 in period 2, P1NBs have purchase priority over bargain hunters. Similarly, P1NBs have purchase priority over P1Bs in purchasing V2. Customers within each group have equal priority. Similar prioritization is used in Su and Zhang (2008), Cachon and Swinney (2009), and Lai et al. (2010). In addition, under dual rollover, P1NBs, who have two versions to choose from, can switch to their second best version if they cannot get their preferred version because of a stockout; we formalize this in (13) and (14) in §2.2.

We assume that the unit production cost $c$ is the same for both versions. This is reasonable particularly for high-tech and fashion products. For example, although the price of a 16 GB iPad with both Wi-Fi and 3G was \$130 higher than that of a 16 GB iPad with Wi-Fi only, the production cost \$306.50 of the former was just \$16 higher than \$290.50 of the latter (Keizer 2010). We also assume $c \leq \beta v(1+\theta)$ so that V2 can be profitably offered. Recall that $\sigma$ is the unit disposal value for the leftover V1 under single rollover. Under dual rollover, the lowest price that the firm will charge for any leftover V1 is $\delta$ in period 2, since bargain hunters clear all V1 leftovers at this price. We assume $\sigma$, $\delta < c$ to avoid ordering an infinite amount. We also assume $\sigma < \delta$ in view of the fact that the disposal value of the leftover V1 under single rollover is usually lower than its selling price. We refer to $(\delta - \sigma)$ as the *market-disposal spread*. Inventory remaining at the end of period 2 has zero value. The firm discounts the profit in period 2 by a factor of $\alpha$, $0 < \alpha < 1$.

**Table 1 Customers' Purchase Options**

| | Period 1 | Period 2 | |
|---|---|---|---|
| | Single/dual rollover | Single rollover | Dual rollover |
| Strategic customers | Buy V1 or wait | | |
| Myopic customers | Buy or not buy V1 | | |
| P1NBs | | V2 or nothing | V1 or V2 or nothing |
| P1Bs | | V2 or continue using V1 | V2 or continue using V1 |
| Bargain hunters | N/A | Nothing | V1 or nothing |

The firm knows $v$, $\alpha$, $\beta$, $\theta$, $\phi$, $\sigma$, $\delta$, $c$, and $F(\cdot)$; high-end customers know $v$, $\beta$, and $\theta$; and bargain hunters know $\delta$. The firm holds a belief $R_f$ of the *strategic customers' reservation price* for V1 in period 1. Since it is privately formed by the firm, $R_f$ is not accessible to customers. Likewise, $W_c$, the strategic customers' belief for their waiting surplus, is not accessible to the firm. Customers observe the prices but not the stocking levels.

### 2.1. Purchase Decisions and Sequence of Events

We first analyze customers' optimal purchase decisions based on their purchase options laid out in Table 1. Myopic customers do not consider the future options, and they buy V1 in period 1 if their surplus $v - p_1$ of buying is nonnegative. So, from myopic customers' standpoint, $v$ is their reservation price in period 1. Strategic customers are forward looking and they know that if they buy V1 in period 1 (and thus become P1Bs), then in period 2 they can either continue using V1 or replace V1 with V2. If they continue using V1, then their total surplus for the two periods is $v - p_1$. Otherwise, they replace V1 with V2 and obtain a surplus of $\beta v(1+\theta) - \beta v - p_2 = \beta v\theta - p_2$, where $\beta v(1+\theta) - \beta v$ is the incremental value from using V2 instead of V1. Notice that only P1NBs and P1Bs may afford V2. P1NBs' value for V2 is $\beta v(1+\theta)$, and P1Bs' incremental value of V2 over V1 is $\beta v\theta$. The firm always sets $p_2 \geq \beta v\theta$. Otherwise, if $p_2 < \beta v\theta$, it can always increase its profit by setting $p_2 = \beta v\theta$, while selling the same number of V2. So strategic customers know that their surplus of buying now for the two periods is $v - p_1 + \max\{0, \beta v\theta - p_2\} = v - p_1$, and they buy now if this surplus $v - p_1$ is not less than their waiting surplus $W_c$. Therefore, from strategic customers' standpoint, $v - W_c$ is their reservation price in period 1.

Under single rollover, P1NBs buy V2 if the surplus $\beta v(1+\theta) - p_2$ is nonnegative; otherwise, they buy nothing. Under dual rollover, P1NBs buy V2, V1, or nothing based on their surplus rankings. For example, if $\beta v(1+\theta) - p_2 > \beta v - p_1' > 0$, then P1NBs prefer V2 to V1. Buying nothing, which provides zero surplus, is their last option. When both the leftover V1 and V2 give P1NBs the same surplus, i.e., $\beta v(1+\theta) - p_2 = \beta v - p_1'$, we assume that the firm can induce P1NBs to buy whichever version the firm prefers to sell. One explanation is that a salesperson can influence a customer's purchase decision when she is indifferent between two versions. In the remainder of this paper, we say that P1NBs prefer a particular version when they have a higher surplus from purchasing that than the other, or when they are indifferent between the two and the firm induces them to buy the particular version. Under both rollover strategies, P1Bs purchase V2 to replace V1 if $p_2 \leq \beta v\theta$; otherwise, they continue using V1. Bargain hunters want to buy the leftover V1 if $p_1' \leq \delta$.

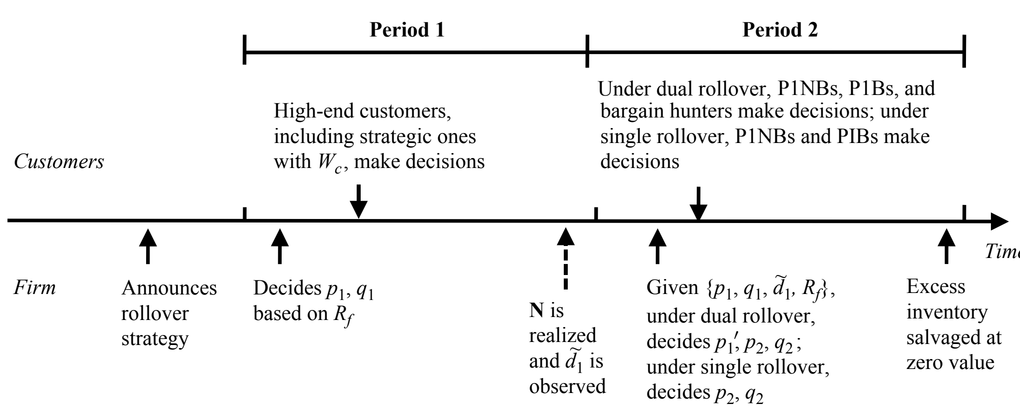
The sequence of events is specified in Figure 1. The firm announces its rollover strategy before period 1. Holding a belief $R_f$ of the strategic customers' reservation price for V1 in period 1, the firm decides price $p_1$ and stocking level $q_1$ for V1. Strategic customers consider the future options, so $R_f \leq v$. In addition, customers do not buy if the price is higher than their reservation price, so the firm should set $p_1 \leq v$. For $R_f < v$, the firm anticipates the period 1 demand to be

$$\mathbf{D}_1 = \begin{cases} \mathbf{N} & \text{if } p_1 \leq R_f, \\ (1-\phi)\mathbf{N} & \text{if } R_f < p_1 \leq v. \end{cases} \tag{1}$$

The sales in period 1 are $\mathbf{S} = \min\{q_1, \mathbf{D}_1\}$. The firm therefore sets either $p_1 = v$ or $p_1 = R_f$ to maximize its total profit. If $R_f = v$, then $\mathbf{D}_1 = \mathbf{N}$. Because $R_f = v$ occurs only in special cases, we focus on $R_f < v$ and deal with the special cases as they occur.

After the firm announces its rollover strategy and decides $p_1$ and $q_1$, all high-end customers arrive in period 1. They observe $p_1$, but not $q_1$. Strategic customers decide whether to buy now or wait and myopic customers decide whether to buy or not buy based on $p_1$ and their respective reservation prices

**Figure 1** Sequence of Events



$v-W_c$ or $v$. At the end of period 1, $\mathbf{N}$ and correspondingly $\mathbf{S}$ and $\mathbf{D}_1$ are realized as $\tilde{n}$, $\tilde{s}$, and $\tilde{d}_1$, respectively. We assume that the firm can assess the demand $\tilde{d}_1$ of V1 at the end of period 1. When there is no stockout, $\tilde{d}_1$ equals the sales $\tilde{s}$. When some high-end customers cannot get V1 because of a stockout, their visits, inquiries, and complaints help the firm to estimate $\tilde{d}_1$. Since the firm sets $p_1$ and observes $\tilde{d}_1 \in \{(1-\phi)\tilde{n}, \tilde{n}\}$, it can deduce the realized $\tilde{n}$ from $\tilde{d}_1$ at the end of period 1 by

$$\tilde{n} = \begin{cases} \tilde{d}_1 & \text{if } p_1 = R_f, \\ \tilde{d}_1/(1-\phi) & \text{if } p_1 = v. \end{cases} \tag{2}$$

In period 2, if single rollover is adopted, then the firm decides price $p_2$ and stocking level $q_2$ for V2; if dual rollover is adopted, then it decides $p_2$, $q_2$ for V2 and marked-down price $p'_1$ for the leftover V1. P1NBs, P1Bs, and bargain hunters make decisions following the discussion above.

### 2.2. Rational Expectations Equilibrium

We first describe the equilibrium under single rollover. In period 2, the firm decides $q_2$ and $p_2$ by solving the firm's optimality in period 2:

$$(q_2(y), p_2(y)) \in \arg\max_{q_2, p_2} \Pi_2^S(q_2, p_2 \mid y), \tag{3}$$

where $\Pi_2^S(q_2, p_2 \mid y)$ is the firm's profit in period 2 and the firm's information set at the end of period 1 is $y = \{q_1, p_1, \tilde{d}_1, R_f\}$. We let $Y = \{q_1, p_1, \mathbf{D}_1, R_f\}$, where $\mathbf{D}_1$ is the random demand in period 1. Because all customers are myopic in period 2 and their decisions are relatively straightforward given the prices, we implicitly incorporate their decisions through the firm's demands in period 2.

In accordance with the firm's belief $R_f$ in period 1, its expected total profit is $\mathbf{E}[\Pi^S(q_1, p_1)] = \mathbf{E}[p_1 \min\{q_1, \mathbf{D}_1\} - cq_1 + \alpha \max_{q_2, p_2} \Pi_2^S(q_2, p_2 \mid Y)]$, where the first two terms together denote the firm's profit in period 1. The firm decides $q_1$ and $p_1$ by solving its optimality problem: $(q_1(R_f), p_1(R_f)) \in \arg\max_{q_1, p_1} \mathbf{E}[\Pi^S(q_1, p_1)]$. Because we have only two possible values $v$ or $R_f$ for $p_1$, we can break down the firm's optimization problem in period 1 into the following two optimization problems:

*firm's quantity optimality in period* 1:

$$\begin{cases} \text{if } p_1 = v & q_1(R_f) \in \arg\max_{q_1} \mathbf{E}[\Pi^S(q_1, v)], \\ \text{if } p_1 = R_f & q_1(R_f) \in \arg\max_{q_1} \mathbf{E}[\Pi^S(q_1, R_f)]; \end{cases} \tag{4}$$

*firm's pricing optimality in period* 1:

$$p_1(R_f) \in \arg\max_{p_1 \in \{v, R_f\}} \mathbf{E}[\Pi^S(q_1(R_f), p_1)]. \tag{5}$$

We use $\chi \in \{0, 1\}$ to capture strategic customers' purchase decisions in period 1; $\chi = 1$ and $\chi = 0$ denote their decisions of buying now and waiting, respectively. Holding a belief of the waiting surplus $W_c$, a strategic customer makes her purchase decision after she learns the price $p_1$. So we have the following:

*strategic customer's optimality*:

$$\chi = 1 \iff v - p_1 \geq W_c. \tag{6}$$

Because myopic customers always buy early with either value of $p_1$, we have $\mathbf{D}_1 = (1-\phi)\mathbf{N} + \chi\phi\mathbf{N}$.

We adopt the concept of rational expectations equilibrium (REE). Under rational expectations, beliefs $W_c$ and $R_f$ must be consistent with their outcomes. Therefore, for any given $q_1$, $p_1$, $\chi$, $W_c$, and $R_f$, which satisfy (4)–(6), the following two conditions must hold:

*waiting surplus rational expectation*:

$$W_c = w(q_1, \chi), \tag{7}$$

*reservation price rational expectation*:

$$R_f = v - W_c, \tag{8}$$

where $w(q_1, \chi)$ is the expected waiting surplus for each waiting customer. From (7) the strategic customers' belief of the waiting surplus is consistent with the expected waiting surplus in equilibrium. According to (8), the firm's belief of the strategic customers' reservation price is consistent with the reservation price from the strategic customers' point of view.

Throughout this paper, if there are period 2 functions $(q_2(y), p_2(y))$ and period 1 numbers $(q_1, p_1, \chi, W_c, R_f)$ satisfying conditions (3)–(8), we call it a *rational expectations equilibrium under single rollover*. An *REE under dual rollover* consists of period 2 functions $(q_2(y), p_2(y), p'_1(y))$ and period 1 numbers $(q_1, p_1, \chi, W_c, R_f)$ satisfying conditions (6)–(8) and (9)–(11):

*firm's optimality in period* 2:

$$(q_2(y), p_2(y), p'_1(y)) \in \arg\max_{q_2, p_2, p'_1} \Pi_2^D(q_2, p_2, p'_1 \mid y), \tag{9}$$

*firm's quantity optimality in period* 1:

$$\begin{cases} \text{if } p_1 = v & q_1(R_f) \in \arg\max_{q_1} \mathbf{E}[\Pi^D(q_1, v)], \\ \text{if } p_1 = R_f & q_1(R_f) \in \arg\max_{q_1} \mathbf{E}[\Pi^D(q_1, R_f)], \end{cases} \tag{10}$$

*firm's pricing optimality in period* 1:

$$p_1(R_f) \in \arg\max_{p_1 \in \{v, R_f\}} \mathbf{E}[\Pi^D(q_1(R_f), p_1)], \tag{11}$$

where

$$\mathbf{E}[\Pi^D(q_1, p_1)] = \mathbf{E}\Big[p_1 \min\{q_1, \mathbf{D}_1\} - cq_1 + \alpha \max_{q_2, p_2, p_1'} \Pi_2^D(q_2, p_2, p_1' \mid Y)\Big].$$

The expected waiting surplus $w(q_1, \chi)$ for each waiting customer can be computed as

$$w(q_1, \chi) = \int_0^\infty \hat{w}(q_1, \chi, q_2, p_2, p_1' \mid \tilde{n}) f(\tilde{n})\, d\tilde{n}, \tag{12}$$

where $\hat{w}(q_1, \chi, q_2, p_2, p_1' \mid \tilde{n})$ is the average waiting surplus for each waiting customer given the market size $\tilde{n}$. We use the dependence of $q_2$, $p_2$, and $p_1'$ on $q_1$, $\chi$, and $\tilde{n}$, when performing the integration. We use the word "average" because of the rationing among customers as detailed in (13)–(15). For single rollover, we set $p_1' = \infty$ because there is no V1 available to sell in period 2.

Under dual rollover, if neither version gives P1NBs a positive surplus, then $\hat{w}(q_1, \chi, q_2, p_2, p_1' \mid \tilde{n}) = 0$. Otherwise, at least one version gives P1NBs a positive surplus, and $\hat{w}(q_1, \chi, q_2, p_2, p_1' \mid \tilde{n})$ is computed using (13) or (14). If P1NBs prefer the leftover V1 to V2 in period 2, then

$$\begin{aligned}
&\hat{w}(q_1, \chi, q_2, p_2, p_1' \mid \tilde{n}) \\
&\quad = \underbrace{\min\left\{\frac{q_1 - \tilde{s}}{\tilde{n} - \tilde{s}}, 1\right\}}_{\text{probability of getting V1}} (\beta v - p_1') \\
&\qquad + \underbrace{\left[1 - \min\left\{\frac{q_1 - \tilde{s}}{\tilde{n} - \tilde{s}}, 1\right\}\right] \min\left\{\frac{q_2}{(\tilde{n} - q_1)^+}, 1\right\}}_{\text{probability of not getting V1 but getting V2}} \\
&\qquad \cdot [\beta v(1+\theta) - p_2],
\end{aligned} \tag{13}$$

where $q_1 - \tilde{s}$ is the number of leftover V1, $\tilde{n} - \tilde{s}$ is the number of P1NBs, and $(\tilde{n} - q_1)^+ = ((\tilde{n} - \tilde{s}) - (q_1 - \tilde{s}))^+$ is the number of P1NBs who cannot get V1 because of a stockout in period 2. Recall that $\tilde{s} = \min\{q_1, \tilde{d}_1\}$ and $\tilde{d}_1 = (1 - \phi)\tilde{n} + \chi\phi\tilde{n}$. If P1NBs prefer V2 to the leftover V1 in period 2, then

$$\begin{aligned}
&\hat{w}(q_1, \chi, q_2, p_2, p_1' \mid \tilde{n}) \\
&\quad = \underbrace{\min\left\{\frac{q_2}{\tilde{n} - \tilde{s}}, 1\right\}}_{\text{probability of getting V2}} [\beta v(1+\theta) - p_2] \\
&\qquad + \underbrace{\left[1 - \min\left\{\frac{q_2}{\tilde{n} - \tilde{s}}, 1\right\}\right] \min\left\{\frac{q_1 - \tilde{s}}{(\tilde{n} - \tilde{s} - q_2)^+}, 1\right\}}_{\text{probability of not getting V2 but getting V1}} \\
&\qquad \cdot (\beta v - p_1'),
\end{aligned} \tag{14}$$

where $(\tilde{n} - \tilde{s} - q_2)^+$ is the number of P1NBs who cannot get V2 because of a stockout. Under single rollover,

$$\hat{w}(q_1, \chi, q_2, p_2, \infty \mid \tilde{n}) = \min\left\{\frac{q_2}{\tilde{n} - \tilde{s}}, 1\right\}[\beta v(1+\theta) - p_2]. \tag{15}$$

For the innovation level $\theta$, we consider three ranges: *high* $\theta \geq c/\beta v$, *medium* $(c - \delta)/\beta v \leq \theta < c/\beta v$, and *low* $\theta < (c - \delta)/(\beta v)$. Note that P1Bs purchase V2 only if $p_2 \leq \beta v\theta$, and the firm can set $p_2 = \beta v\theta$ profitably only when $\beta v\theta \geq c$, which gives the first critical value $(c/\beta v)$ for $\theta$. Under dual rollover, if V1 is priced low as $\delta$ to target bargain hunters in period 2 and V2 is priced high, P1NBs may go for V1 to get the surplus $(\beta v - \delta)$ rather than purchase V2. To attract P1NBs from V1, the highest price that the firm can charge for V2 is $(\beta v\theta + \delta)$. This price is profitable only when $\beta v\theta + \delta \geq c$, which provides the second critical value $(c - \delta)/\beta v$ for $\theta$.

Before our analysis we make some observations about possible prices in period 2. In period 2 there are only two possible prices $\beta v$ and $\delta$ for any leftover V1, and only three possible prices $\beta v\theta$, $\beta v\theta + \delta$, and $\beta v(1 + \theta)$ for V2 in equilibrium. Intuitively, $\beta v$ and $\delta$ are the respective highest prices for V1 to target P1NBs and bargain hunters in period 2. Similarly, $\beta v(1 + \theta)$, $\beta v\theta + \delta$, and $\beta v\theta$ are the highest prices for V2 in period 2 to target P1NBs, to attract P1NBs from V1 when $p_1' = \delta$, and to target P1Bs, respectively. For ease of exposition, we refer to $\beta v$ and $\delta$ as high (H) and low (L) prices for V1, respectively, and $\beta v(1+\theta)$, $\beta v\theta + \delta$, and $\beta v\theta$ as high (H), medium (M), and low (L) prices for V2, respectively, in period 2.

## 3. Low and Medium Innovations

With low and medium innovations, it is not profitable for the firm to price V2 low enough to target P1Bs. Consequently, there are only two groups of customers (P1NBs and bargain hunters) to consider in period 2. P1NBs choose from buying leftover V1 (if available under dual rollover), V2, or nothing, depending on their surplus rankings as discussed in §2.1. Bargain hunters cannot afford V2 and buy V1 only if its price is not higher than $\delta$ under dual rollover.

### 3.1. Dual Rollover

We present the low innovation case below in detail and later discuss only the difference between low and medium innovation cases for brevity. We analyze the problem in a backward manner.

The third row of Table 2 in Proposition 1 below deals with the firm's pricing and stocking level decisions in four cases in period 2 with low innovation. For any given $q_1$, $\tilde{n}$, and $R_f < v$, the four cases are characterized according to the price $p_1 \in \{R_f, v\}$ as well as the relationship between $q_1$ and the realized market size $\tilde{n}$ at the end of period 1. We also present, in the fourth row of Table 2, the expected waiting surpluses for $(p_1 = R_f, \ \chi = 1)$ and $(p_1 = v, \ \chi = 0)$ pairs, provided that the firm follows the pricing and stocking level decisions listed in the third row of the table. These two pairs are the only ones satisfying (6) and (8).

Proposition 1 (Low Innovation). *For given $q_1$, $\tilde{n}$ and $R_f < v$, the firm's period 2 optimal decisions are as shown in Table 2.*

In Table 2, L–H denotes $p'_1 = \delta, p_2 = \beta v(1+\theta)$, and $q_2 = (\tilde{n} - q_1)^+$ with the corresponding profit $\Pi_2^D = [\beta v(1+\theta) - c](\tilde{n} - q_1)^+ + \delta(q_1 - \tilde{s})$; H–H denotes $p'_1 = \beta v$, $p_2 = \beta v(1+\theta)$, and $q_2 = (\tilde{n} - q_1)^+$, with $\Pi_2^D = [\beta v(1+\theta) - c](\tilde{n} - q_1)^+ + \beta v[\min\{\tilde{n}, q_1\} - \tilde{s}]$. Note that the nomenclature for prices is consistent with our discussion at the end of §2, and similar naming conventions are used later.

According to Table 2, with low innovation the firm should always price V2 high and leave a zero surplus to the waiting customers in period 2. Notice that if the high-end market size is small relative to the stocking level of V1 (i.e., $\tilde{n} \le q_1$ when $p_1 = R_f$ and $\tilde{n} \le (\delta/(\beta v \phi + \delta(1-\phi)))q_1$ when $p_1 = v$ according to conditions of L–H strategy), then a proportionally large number of high-end customers purchase V1 in period 1. This leads to a market saturated with V1 in period 2, leaving very few P1NBs. In this case, the firm should deeply markdown the leftover V1 to target bargain hunters rather than P1NBs. Consequently, strategic customers can obtain a positive surplus from purchasing the leftover V1 if they wait. When the high-end market size is not small, the firm prices the leftover V1 high to leave the waiting customers a zero surplus. Therefore, with low innovation, the positive surplus, if any, is always from the deeply marked-down V1.

To ensure the existence of an REE, we assume that the high-end market size **N** satisfies the monotone scaled likelihood ratio (MSLR) property, i.e., for any $\kappa \in [0,1]$ and $x$ in the support of **N**, $f(\kappa x)/f(x)$ is monotone in $x$. The MSLR property is satisfied, e.g., by gamma, Weibull, uniform, exponential, power, beta, chi, and chi-squared distributions. It has also been assumed in Cachon and Swinney (2009) and Lai et al. (2010).

Period 1 price can be high $p_1 = v$ or low $p_1 = R_f$. These two cases are detailed in Lemmas 2 and 3 in the appendix. Proposition 2 below characterizes the firm's REE price and stocking level in period 1 under dual rollover by comparing its profits under high and low prices. Here the superscript $L, D$ means "low innovation and dual rollover." We use superscript $^*$ to denote the values in an REE.

Proposition 2 (Low Innovation). *Under dual rollover, there exists a unique REE. In addition:*

(i) *If $\alpha[\beta v(1+\theta) - c] < v - c$, then a $\phi^{L,D}$ exists such that*

(i.a) *if $\phi \le \phi^{L,D}$, the firm sets the high price $p_1^* = v$, and $q_1^* > 0$ is the unique solution of*

$$\big(v - \alpha[\beta v(1+\theta) - c]\big)\bar{F}\left(\frac{q_1}{1-\phi}\right) - c + \alpha\delta F\left(\frac{\delta}{\beta v\phi + \delta(1-\phi)} q_1\right) + \alpha(c - \beta v\theta)\left[F\left(\frac{q_1}{1-\phi}\right) - F(q_1)\right] = 0;$$

(i.b) *if $\phi > \phi^{L,D}$, the firm sets the low price $p_1^* = R_f$, where $c + \alpha[\beta v(1+\theta) - c] < R_f < v$, and $q_1^* > 0$; $p_1^*$ and $q_1^*$ can be uniquely determined from the two equations: $p_1 = v - (\beta v - \delta)F(q_1)$ and $F(q_1) = (p_1 - c - \alpha[\beta v(1+\theta) - c])/(p_1 - \alpha[\beta v(1+\theta) - c + \delta])$.*

(ii) *If $\alpha[\beta v(1+\theta) - c] \ge v - c$, then $q_1^* = 0$.*

The firm chooses between lowering the price in period 1 to target all high-end customers and keeping the price high to target only myopic customers. Proposition 2(i) states that when there are enough myopic customers in the market ($1 - \phi \ge 1 - \phi^{L,D}$), the firm should focus on them by setting a high price; otherwise, the firm should set a low price to induce strategic as well as myopic customers to buy early. Proposition 2(ii) provides the condition under which the firm skips V1 ($q_1^* = 0$), and we will discuss this condition after presenting the optimal rollover strategy in Proposition 5.

**Table 2** **Firm's Period 2 Optimal Decisions with Low Innovation and Dual Rollover**

| $p_1 = R_f$ | | $p_1 = v$ | |
|---|---|---|---|
| $\tilde{n} \le q_1$ | $\tilde{n} > q_1$ | $\tilde{n} \le \dfrac{\delta}{\beta v\phi + \delta(1-\phi)} q_1$ | $\tilde{n} > \dfrac{\delta}{\beta v\phi + \delta(1-\phi)} q_1$ |
| L–H | H–H | L–H | H–H |
| $w(q_1, 1) = (\beta v - \delta)F(q_1)$ | | $w(q_1, 0) = (\beta v - \delta)F\left(\dfrac{\delta}{\beta v\phi + \delta(1-\phi)} q_1\right)$ | |

The analysis and insights obtained for medium innovation are similar to those for low innovation. One difference is that with medium innovation, compared to always charging the high price $\beta v(1+\theta)$ and leaving P1NBs a zero surplus from V2 with low innovation, the firm can now price V2 less aggressively at the medium price $(\beta v\theta+\delta)$ to leave P1NBs a positive surplus. Consequently, both V1 and V2 can give waiting customers positive surpluses.

### 3.2. Single Rollover

Unlike dual rollover, leftover V1 is not in the market in period 2 under single rollover. With low and medium innovations, the firm cannot profitably sell V2 to P1Bs. Consequently, the firm always targets P1NBs in period 2. Without the cannibalization from V1, the firm has absolute pricing power for V2 and sets $p_2 = \beta v(1+\theta)$, leaving a zero surplus to P1NBs. These are detailed in Proposition 3, which is analogous to Proposition 1.

Proposition 3 (Low and Medium Innovations). *The price for V2, the stocking level for V2, and the firm's profit in period 2 are $p_2=\beta v(1+\theta)$, $q_2=\tilde{n}-\tilde{s}$, and $\Pi_2^S = [\beta v(1+\theta)-c](\tilde{n}-\tilde{s})+\sigma(q_1-\tilde{s})$, respectively.*

With $p_2=\beta v(1+\theta)$ in period 2, the expected waiting surplus is zero. Then, the reservation price for strategic customers in period 1 is $v$, the same as that for myopic customers, and thus every high-end customer (strategic or myopic) wants to buy when $p_1=v$. Therefore, we have an important result: single rollover *completely* eliminates the waiting surplus with low and medium innovations.

With low innovation and dual rollover, waiting customers can derive a positive surplus from the deeply marked-down V1. Removing the leftover V1 from the market reduces the waiting surplus to zero. The same outcome can be achieved with medium innovation, but the mechanism is more involved. With medium innovation and dual rollover, both the deeply marked-down V1 and the less aggressively priced V2 can give waiting customers positive surpluses. Single rollover completely eliminates the waiting surplus via its direct and indirect influences. Directly, waiting customers cannot purchase the deeply marked-down V1, because it is unavailable. Indirectly, without the cannibalization from V1 in period 2, the firm can set the price high at $\beta v(1+\theta)$ for V2 to capture the entire surplus of the waiting customers. Thus, the cannibalization and postponement effects are not independent. Eliminating cannibalization strengthens the firm's pricing power and thus leads to a higher price for the new version, which subsequently reduces the postponement effect. Proposition 4 provides the firm's REE decisions in period 1.

Proposition 4 (Low and Medium Innovations). *Under single rollover, there exists a unique REE, with $p_1^*=v$, and $q_1^* = F^{-1}((v-c-\alpha[\beta v(1+\theta)-c])/(v-\alpha[\beta v(1+\theta)-c+\sigma]))$ if $\alpha[\beta v(1+\theta)-c]<v-c$ and $q_1^*=0$ otherwise.*

### 3.3. Optimal Rollover Strategy

Proposition 5 presents the optimal rollover strategy with low and medium innovations.

Proposition 5 (Low and Medium Innovations). (i) *If $\alpha[\beta v(1+\theta)-c]\ge v-c$, then the firm does not introduce V1, and single rollover and dual rollover give the same profit.*

(ii) *If $\alpha[\beta v(1+\theta)-c]<v-c$, then a threshold $\Delta\ge 0$ exists such that*

(ii.a) *for market-disposal spread $\delta-\sigma\le\Delta$, a $\phi^{LM}$ exists such that single rollover is optimal iff $\phi\ge\phi^{LM}$;*

(ii.b) *for market-disposal spread $\delta-\sigma>\Delta$, dual rollover is optimal.*

The firm should offer only V2 and skip V1 (i.e., $q_1=0$) under the condition in Proposition 5(i). We see that $\alpha[\beta v(1+\theta)-c]$ is the discounted maximum unit profit by selling V2 only, whereas $v-c$ is the maximum unit profit by selling V1 only. With low and medium innovations, the two versions are so similar that it is not profitable for the firm to sell both versions to the same high-end customers, i.e., replacement is not possible. Selling one more unit of V1 to high-end customers means that the number of V2 the firm can sell will decrease by one. Thus, to eliminate cannibalization, so long as the innovation can compensate for the firm's discounted profit incorporating customers' value depreciation over time (i.e., $\alpha[\beta v(1+\theta)-c]\ge v-c$), the firm should be patient enough to wait until the new technology for V2 is ready and then just introduce V2. This result applies to a firm that dominates the market or leads in technology vis-à-vis its competitors. In contrast, in a highly competitive market, the time to market is vital, and rival firms may launch their own products to capture the market while the firm is waiting for the new technology.

Proposition 5(ii) summarizes the conditions under which single rollover can increase the firm's profit: the market-disposal spread $\delta-\sigma$ is not too large and the proportion of the strategic customers is not too low. The disadvantage of single rollover is the lower revenue from the leftover V1 in period 2, whereas its advantages include guaranteeing the high price $p_2=\beta v(1+\theta)$ for V2 in period 2 by eliminating the cannibalization from V1, and empowering the firm to set the price high at $p_1=v$ for V1 in period 1 to induce not only myopic customers, but also strategic customers to buy early. When the market-disposal spread is low, the loss from the lower revenue of the leftover V1 is small. When the proportion of strategic customers is high, their waiting behavior has a significant impact on the firm's profit. In this case, the gain from eliminating their waiting behavior may more than compensate for the loss due to the lower revenue from the leftover V1.

## 4. High Innovation

With high innovation, the incremental value gained from replacing V1 with V2 can justify the production cost $c$, and thus the firm can profitably price V2 to target P1Bs. This makes the pricing and stocking level decisions in period 2 more complicated, but at the same time more interesting, to analyze.

### 4.1. Dual Rollover

Proposition 6 provides the firm's optimal decisions in period 2 with high innovation.

Proposition 6 (High Innovation). *For given $q_1$, $\tilde{n}$, and $R_f < v$, the firm's period 2 optimal decisions are as shown in Table 3, where $A = (\beta v(1+\theta) - c)/(\beta v)$, $B = \delta/(\beta v - [\beta v(1+\theta) - \delta - c](1-\phi))$, $C = \delta/(\delta + (\beta v - 2\delta)\phi)$, and where* L-L *denotes $p_1' = \delta$, $p_2 = \beta v\theta$, and $q_2 = \tilde{n}$ with $\Pi_2^D = (\beta v\theta - c)\tilde{n} + (q_1 - \tilde{s})\delta$;* L-M *denotes $p_1' = \delta$, $p_2 = \beta v\theta + \delta$, and $q_2 = \tilde{n} - \tilde{s}$ with $\Pi_2^D = [\beta v\theta + \delta - c](\tilde{n} - \tilde{s}) + \delta(q_1 - \tilde{s})$; and* H-H *denotes $p_1' = \beta v$, $p_2 = \beta v(1+\theta)$, and $q_2 = \tilde{n} - \tilde{s}$ with $\Pi_2^D = [\beta v(1+\theta) - c](\tilde{n} - \tilde{s})$.*

According to Proposition 6, when $p_1 = v$, the strategies in period 2 depend on the proportion of strategic customers, whereas with $p_1 = R_f$, the strategies hold for every $\phi$. With high innovation, the firm can set the price of V2 as low as $\beta v\theta$ to induce P1Bs to go for a replacement. Similar to medium innovation, the positive surplus can arise from either V1 or V2, but V2 is always preferred by P1NBs, as seen from the proof of Proposition 6.

With $p_1 = R_f$ in high innovation, the firm's total profit $\mathbf{E}[\Pi^{D,I}(q_1, p_1)]$ may have one or two maximizers, unlike with low and medium innovations. Letting $q_1^-$ and $q_1^+$, respectively, be the smaller and larger maximizers of $\mathbf{E}[\Pi^{D,I}(q_1, p_1)]$ given $p_1$, we know with high innovation that the firm maximizes its profit either by producing less ($q_1 = q_1^-$) to save the market for V2 in period 2 or by producing more ($q_1 = q_1^+$) to sell more V1 in period 1. When $\arg\max_{q_1} \mathbf{E}[\Pi^{D,I}(q_1, p_1)]$ has two values, a vector of numbers $(q_1, p_1, \chi, W_c, R_f)$ satisfying REE conditions does not always exist. In such a case, we resort to a mixed strategy for the stocking level $q_1$; that is, the firm stocks $q_1^-$ with some probability and $q_1^+$ with the remaining probability. Consequently, the waiting surplus $w(q_1, \chi)$ in (7) is a weighted average of $w(q_1^-, \chi)$ and $w(q_1^+, \chi)$. Except for this, there is no change in the REE conditions. Proposition 7 summarizes the result with $p_1 = R_f$.

Proposition 7 (High Innovation). *With $p_1 = R_f$, there is either a unique $(q_1, p_1, \chi, W_c, R_f)$ or a mixture of $(q_1^-, p_1, \chi, W_c, R_f)$ and $(q_1^+, p_1, \chi, W_c, R_f)$ satisfying the REE conditions except for* (11). *In the mixture case, $q_1^- = 0$, and the firm's total profit is $\alpha[\beta v(1+\theta) - c]\mathbf{E}(\mathbf{N})$.*

Notice that by simply skipping V1 ($q_1 = 0$), the firm can earn the same profit $\alpha[\beta v(1+\theta) - c]\mathbf{E}(\mathbf{N})$ as in the mixed strategy. We break the tie by assuming that the firm skips V1 whenever the mixture case occurs. Henceforth, we use the term REE for an equilibrium not involving a mixture of stocking levels.

Surprisingly, we find that under high innovation, the firm can still (but not necessarily) introduce V1 even when $\alpha[\beta v(1+\theta) - c] \geq v - c$. This is different from low and medium innovation cases, where the firm skips V1 and introduces only V2. With low and medium innovations, the two versions are so similar that the firm cannot sell both of them to the same high-end customer. With high innovation, however, the two versions are quite different. By setting $p_1 = v$ and $p_2 = \beta v\theta$, the firm can sell both to the same customer and the maximum profit it can earn is $v - c + \alpha[\beta v\theta - c]$. This profit may be greater than $\alpha[\beta v(1+\theta) - c]$, the maximum unit profit by selling V2 only. So, with the intention of selling both versions to a high-end customer, the firm may introduce V1 even when $\alpha[\beta v(1+\theta) - c] \geq v - c$. While comparing the high and low period 1 prices, we find that the characterization of the REE price is similar to Proposition 2, except with a different threshold $\phi$.

### 4.2. Single Rollover

Under single rollover in period 2, only version V2 and two groups of customers (P1Bs and P1NBs) need to be considered. Bargain hunters are in the market, but

**Table 3 Firm's Period 2 Optimal Decisions with High Innovation and Dual Rollover**

| $p_1 = R_f$ | | $p_1 = v$ | | | | | |
|---|---|---|---|---|---|---|---|
| Every $\phi$ | | $\phi \leq \dfrac{\beta v\theta - c}{\beta v(1+\theta) - c}$ | | $\dfrac{\beta v\theta - c}{\beta v(1+\theta) - c} < \phi \leq \dfrac{\beta v\theta - c}{\beta v\theta + \delta - c}$ | | $\phi > \dfrac{\beta v\theta - c}{\beta v\theta + \delta - c}$ | |
| $\tilde{n} \leq Aq_1$ | $\tilde{n} > Aq_1$ | $\tilde{n} \leq Aq_1$ | $\tilde{n} > Aq_1$ | $\tilde{n} \leq Bq_1$ | $\tilde{n} > Bq_1$ | $\tilde{n} \leq Cq_1$ | $\tilde{n} > Cq_1$ |
| L–L | H–H | L–L | H–H | L–L | H–H | L–M | H–H |
| $w(q_1, 1) = \beta v F(Aq_1)$ | | $w(q_1, 0) = \beta v F(Aq_1)$ | | $w(q_1, 0) = \beta v F(Bq_1)$ | | $w(q_1, 0) = (\beta v - \delta)F(Cq_1)$ | |

they cannot afford V2. Therefore, there are only two possible values for $p_2$ in period 2, $\beta v\theta$ and $\beta v(1+\theta)$, which are the reservation prices for P1Bs and P1NBs, respectively. These two prices are corresponding to two values of period 2 demand, which are $\tilde{n}$ for all high-end customers (P1Bs and PINBs) and $(\tilde{n}-\tilde{s})$ for just P1NBs. This implies a trade-off between pricing low to target all high-end customers and pricing high to target only P1NBs. The firm chooses $p_2$ based on the market saturation as shown in Proposition 8.

Proposition 8 (High Innovation). *For given $q_1$, $\tilde{n}$ and $R_f < v$, the firm's period 2 optimal decisions are as shown in Table 4, where $A = (\beta v(1+\theta) - c)/(\beta v)$,* L *denotes $p_2 = \beta v\theta$ and $q_2 = \tilde{n}$ with $\Pi_2^S = (\beta v\theta - c)\tilde{n} + \sigma(q_1 - \tilde{s})$, and* H *denotes $p_2 = \beta v(1+\theta)$ and $q_2 = \tilde{n} - \tilde{s}$ with $\Pi_2^S = [\beta v(1+\theta) - c](\tilde{n} - \tilde{s}) + \sigma(q_1 - \tilde{s})$.*

Recall that with low and medium innovations, the firm has absolute pricing power for V2 by committing to single rollover and to completely eliminate strategic waiting. However, this is not so with high innovation. Comparing Proposition 8 with Proposition 6, when $p_1 = R_f$ or when $p_1 = v$ and $\phi \leq (\beta v\theta - c)/(\beta v(1+\theta) - c)$, even when V1 is not in the market under single rollover, the firm uses the same pricing strategy for V2 as with dual rollover; namely, the cannibalization between these two versions is so low with high innovation that having no V1 under single rollover *does not strengthen* the firm's pricing power for V2.

Furthermore, when $p_1 = R_f$ or when $p_1 = v$ and $\phi \leq (\beta v\theta - c)/(\beta v(1+\theta) - c)$, with the same $q_1$, strategic customers have *the same* expected waiting surpluses under both rollovers. This is different from low and medium innovation cases, where the firm can reduce the waiting surplus to zero by committing to single rollover. With medium innovation, V2 can be sold to P1NBs only, and thus the elimination of the cannibalization from V1 by committing to single rollover is equivalent to committing to the high price for V2. In contrast, the firm with high innovation has the chance to induce P1Bs to replace V1 with V2. If the market is highly saturated, which occurs when $p_1 = R_f$ and $\tilde{n} \leq Aq_1$ or when $p_1 = v$, $\phi \leq (\beta v\theta - c)/(\beta v(1+\theta) - c)$ and $\tilde{n} \leq Aq_1$, then it is more profitable to set a low price to induce replacements. This leads to a positive surplus for P1NBs. In addition, under dual rollover, although the positive surplus can be from either V2 or V1, V2 is always preferred. Therefore, despite the removal of leftover V1 under single rollover, waiting customers can still obtain the positive surplus from their preferred V2.

**Table 4** **Firm's Period 2 Optimal Decisions with High Innovation and Single Rollover**

| $p_1 = R_f$ | | $p_1 = v$ | | |
|---|---|---|---|---|
| Every $\phi$ | | $\phi \leq \dfrac{\beta v\theta - c}{\beta v(1+\theta) - c}$ | | $\phi > \dfrac{\beta v\theta - c}{\beta v(1+\theta) - c}$ |
| $\tilde{n} \leq Aq_1$ | $\tilde{n} > Aq_1$ | $\tilde{n} \leq Aq_1$ | $\tilde{n} > Aq_1$ | All $\tilde{n}$s |
| L | H | L | H | H |
| $w(q_1, 1) = \beta vF(Aq_1)$ | | $w(q_1, 0) = \beta vF(Aq_1)$ | | $w(q_1, 0) = 0$ |

When $p_1 = v$ and $\phi > (\beta v\theta - c)/(\beta v(1+\theta) - c)$, the expected waiting surplus is zero, which implies that all high-end customers want to buy in period 1 at $p_1 = v$. This seems similar to the low and medium innovation cases, when single rollover eliminates the waiting surplus completely. However, unfortunately for the firm, as shown in Lemma 1, this ideal case with zero expected waiting surplus cannot be attained in an REE. Thus, single rollover is not effective in reducing the waiting incentive even with a high proportion of strategic customers.

Lemma 1 (High Innovation). *With $\phi > (\beta v\theta - c)/(\beta v(1+\theta) - c)$, there is no REE under single rollover where $p_1 = v$ and $\chi = 0$.*

### 4.3. Optimal Rollover Strategy

Finally, we compare the performance of single rollover and dual rollover with high innovation. Let $\mathbf{E}[\Pi^{D,h}]$ and $\mathbf{E}[\Pi^{D,l}]$ be the firm's total profit under dual rollover with a period 1 high price and low price, respectively. Similarly, we define $\mathbf{E}[\Pi^{S,h}]$ and $\mathbf{E}[\Pi^{S,l}]$ under single rollover. Let

$$\mathbf{E}[\Pi^{D*}] = \max\{\mathbf{E}[\Pi^{D,h}], \mathbf{E}[\Pi^{D,l}]\} \quad \text{and}$$
$$\mathbf{E}[\Pi^{S*}] = \max\{\mathbf{E}[\Pi^{S,h}], \mathbf{E}[\Pi^{S,l}]\}.$$

Consequently, $\mathbf{E}[\Pi^{S*}]$ and $\mathbf{E}[\Pi^{D*}]$ are the firm's REE profits under single and dual rollover, respectively. Let $\phi^H = \inf\{\phi\colon \mathbf{E}[\Pi^{S*}] = \mathbf{E}[\Pi^{D*}]$, where $0 \leq \phi < 1\}$.

Proposition 9 (High Innovation). (i) *If there is an REE under single rollover, then we have the following*:

(i.a) *If a $\phi^H$ does not exist, then dual rollover is always better than single rollover.*

(i.b) *If a $\phi^H$ exists, then single rollover is optimal iff $\phi \geq \phi^H$.*

(ii) *Otherwise, the firm skips V1 under single rollover, and dual rollover is always better than single rollover.*

From §4.2, single rollover does not strengthen the firm's pricing power for V2. In addition, with high innovation, P1NBs prefer V2 to V1. Thus, even without V1 under single rollover, P1NBs can still get their preferred option V2. Hence, it is not clear how the firm can improve its profit by phasing out V1. Proposition 10 helps us to understand how single rollover gains an advantage over dual rollover.

Proposition 10. *Suppose that there is an REE under single rollover. Then we have the following*:

(i) *If $p_1^* = v$ under single rollover, then dual rollover is always better than single rollover.*

(ii) *The necessary and sufficient condition for the existence of a $\phi^H$ is that $\mathbf{E}[\Pi^{S,l}] \geq \mathbf{E}[\Pi^{D,l}]$.*

Proposition 10(i) states that single rollover cannot outperform dual rollover if a high period 1 price is in REE, i.e., $p_1^* = v$ under single rollover. Equivalently, $p_1^* = R_f$ under single rollover is a necessary condition for single rollover to be optimal. Proposition 10(ii) further states that if single rollover can beat dual rollover when both rollovers use low period 1 prices (i.e., $\mathbf{E}[\Pi^{S,l}] \geq \mathbf{E}[\Pi^{D,l}]$), then there always exists a $\phi^H$ such that single rollover outperforms dual rollover when $\phi \geq \phi^H$.

To gain additional insights, we explain how $\mathbf{E}[\Pi^{S,l}] \geq \mathbf{E}[\Pi^{D,l}]$ can occur. When $p_1 = R_f$, according to Propositions 6 and 8, the structure of the expected waiting surplus $w(q_1, 1)$ is the same under both rollover strategies. In addition, using $\tilde{s} = \min\{\tilde{n}, q_1\} = q_1$ when $\tilde{n} > ((\beta v(1+\theta) - c)/(\beta v))q_1 \geq q_1$, we have

$$\begin{aligned}
&\mathbf{E}[\Pi^{S,l}(q_1, R_f)]\\
&= \mathbf{E}[R_f \min\{\mathbf{N}, q_1\} - cq_1]\\
&\quad + \alpha \int_0^{((\beta v(1+\theta)-c)/(\beta v))q_1} \big[(\beta v\theta - c)x\\
&\qquad\qquad + \sigma(q_1 - \min\{x, q_1\})\big] f(x)\,dx\\
&\quad + \alpha \int_{((\beta v(1+\theta)-c)/(\beta v))q_1}^{\infty} \big[(\beta v(1+\theta) - c)(x - q_1)\big] f(x)\,dx,
\end{aligned} \tag{16}$$

$$\begin{aligned}
&\mathbf{E}[\Pi^{D,l}(q_1, R_f)]\\
&= \mathbf{E}[R_f \min\{\mathbf{N}, q_1\} - cq_1]\\
&\quad + \alpha \int_0^{((\beta v(1+\theta)-c)/(\beta v))q_1} \big[(\beta v\theta - c)x\\
&\qquad\qquad + \delta(q_1 - \min\{x, q_1\})\big] f(x)\,dx\\
&\quad + \alpha \int_{((\beta v(1+\theta)-c)/(\beta v))q_1}^{\infty} [(\beta v(1+\theta) - c)(x - q_1)] f(x)\,dx.
\end{aligned} \tag{17}$$

Profits (16) and (17) are structurally the same except that each leftover V1 is disposed of at value $\sigma$ under single rollover and is sold at the price $\delta$ under dual rollover. This can be further shown to be the only difference between the equilibrium conditions under single and dual rollovers for a low $p_1$. Since $\sigma < \delta$, it seems that dual rollover outperforms single rollover. Counter to this intuition, the opposite can interestingly occur.

The revenue loss from disposing of the leftovers is usually believed to be the main drawback of single rollover as discussed in the introduction. However, this *direct economic* loss is only a part of the story. With a potential revenue loss in period 2 compared to dual rollover, the firm tends to order less V1 at the beginning of period 1. This leads to lower expected sales, and, in turn, to a market less saturated with V1 in period 2. Recall that with high innovation, the firm has the option to induce P1Bs to make replacement purchase by setting a low price for V2, which leaves P1Bs with zero surplus and waiting customers with positive surplus. In a less saturated market, however, instead of a low price for V2, the firm tends to price V2 high, which gives waiting customers zero surplus. Strategic customers recognize that a potential revenue loss under single rollover leads to a higher possibility of zero waiting surplus, so they are more willing to purchase in period 1 unless $p_1$ is very high. This is the *indirect behavioral* impact of the potential revenue loss from the leftovers under single rollover, which favorably affects the firm's profit. When the behavioral gain in period 1 more than offsets the economic loss in period 2, we have $\mathbf{E}[\Pi^{S,l}] \geq \mathbf{E}[\Pi^{D,l}]$. In this regard, with high innovation, the firm can actually benefit from the revenue loss from the leftovers—the so-called "drawback" of single rollover. Thus, if the proportion of strategic customers is so high that their behavior has a significant impact on the firm's profit and the low period 1 price is in REE under both rollovers (i.e., $\mathbf{E}[\Pi^{S*}] = \mathbf{E}[\Pi^{S,l}]$ and $\mathbf{E}[\Pi^{S*}] = \mathbf{E}[\Pi^{S,l}]$), then the firm can earn a higher profit under single rollover.

If we consider a higher disposal value $\sigma' > \sigma$ under single rollover, then similar to the above analysis, we can argue that the firm may suffer from the higher disposal value when the proportion of strategic customers in the market is high. In other words, with many strategic customers in the market, when facing more than one disposal options under single rollover, the firm may want to choose a lower value disposal option rather than a higher value one. This is because a lower stocking level of V1 resulting from a lower value disposal option enables the firm to charge a higher price for V1, and thus possibly to earn a higher profit. We will explore this interesting and counterintuitive finding further in §5.

# 5. Numerical Analysis and Discussion

In this section, we first quantify the value of single rollover over dual rollover. Specifically, we study how the innovation level $\theta$ and the proportion $\phi$ of strategic customers affect the value of single rollover. We then investigate how other model parameters impact the optimal rollover strategy. After that, we use numerical examples to illustrate the extent to which a firm can suffer from a high disposal value option. Finally, we introduce a flexible rollover strategy and discuss its additional value, if any, with respect to single and dual rollovers.

## 5.1. Value of Single Rollover

Our numerical study starts with a base case with $v = 10$, $c = 5$, $\alpha = 0.6$, $\beta = 0.6$, $\delta = 3$, $\sigma = 1$, and a gamma-distributed **N** with mean 50 and standard deviation 25. Additional instances are generated by varying one of the model parameters, which will be explained as needed. For each test instance generated, we obtain the REE and compute the expected equilibrium profit using Matlab. Generalizing the notations in §4.3 for low, medium, and high innovations, we use $\mathbf{E}[\Pi^{S*}]$ and $\mathbf{E}[\Pi^{D*}]$ to denote the firm's respective equilibrium profits under single and dual rollovers. We define $(\mathbf{E}[\Pi^{S*}] - \mathbf{E}[\Pi^{D*}])/\mathbf{E}[\Pi^{D*}]$ as the (relative) value of single rollover compared to dual rollover.

Figures 2(a)–2(c) are the respective examples for low, medium, and high innovations with the parameter values in the base case. From these figures, as the innovation increases, the value of single rollover decreases. In low and medium innovation cases, the profit increase with single rollover can be as much as 29% and 14%, respectively. With high innovation, the profit increase is less than 1%, which supports the observation from our analysis that single rollover is more valuable when the innovation is low or medium. With high innovation, single rollover neither ensures the firm higher pricing power for V2, because the cannibalization is already low, nor completely eliminates the waiting incentive.

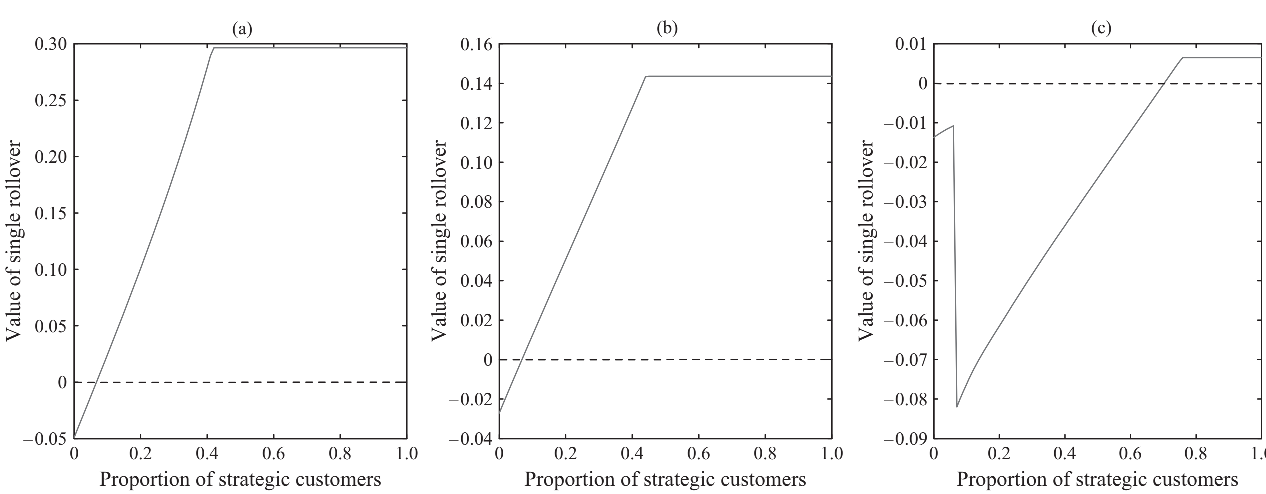
In Propositions 5 and 9, single rollover has an advantage over dual rollover when the proportion of strategic customers is greater than a critical value. With low and medium innovations in Figures 2(a) and 2(b), single rollover outperforms dual rollover as long as more than 10% of the customers are strategic, which is common these days (McWilliams 2004); with high innovation in Figure 2(c), single rollover is optimal when the proportion of strategic customer is more than 70%, which is foreseeable in the future considering that customers are better aided by technical fora (e.g., Computerworld.com) and online deal fora (e.g., DealSea.com). We obtain similar critical values for the proportion of strategic customers with other parameters.

From Proposition 5, with low and medium innovations, a necessary condition for single rollover to outperform dual rollover is that the market-disposal spread $\delta - \sigma$ be less than the threshold $\Delta$. Our numerical study shows that $\Delta$ can be very large, which implies that single rollover can outperform dual rollover in a wide parameter range. For example, in addition to the base case, we run numerical experiments with $\delta = 3$ and $\sigma = 0$ that may correspond to donation of the leftovers. These experiments show that even with such a low value of $\sigma$, single rollover does much better than dual rollover. Without reporting detailed results here, it suffices to mention only that the profit increase can be as much as 26% and 13%, respectively, for $\theta = 0.2$ and $\theta = 0.6$, as long as the proportion of strategic customers is more than 10%. This means that single rollover can be widely adopted in practice to increase a firm's profit, especially when the innovation is not very high.

### 5.2. Sensitivity of Rollover Strategies to Parameters

Starting from the base case, we investigate how the optimal rollover strategy changes as one of the model parameters changes. Specifically, we vary the standard deviation of market size over the range $[15, 35]$, the production cost $c$ over the range $[3.5, 6.5]$, the residual value multiplier $\beta$ over the range $[0.6, 0.8]$, and the proportion $\phi$ of strategic customers over the range $[0, 1]$ in steps of 5, 0.1, 0.05, and 0.1, respectively. For brevity, we do not report detailed results here and instead focus our discussion on low ($\theta = 0.2$) and high ($\theta = 0.9$) innovations, with a note that, consistent with our earlier analysis, the numerical results for medium innovation are similar to those for low innovation.

**Figure 2** **Value of Single Rollover $(\mathbf{E}[\Pi^{S*}] - \mathbf{E}[\Pi^{D*}])/\mathbf{E}[\Pi^{D*}]$ vs. Proportion of Strategic Customers ($\phi$)**




*Note.* (a) Low innovation, $\theta = 0.2$; (b) medium innovation, $\theta = 0.6$; (c) high innovation, $\theta = 0.9$.

Regardless of the values of parameters, the optimal rollover strategy switches from dual rollover to single rollover as the proportion of strategic customers increases, except when the firm skips V1, in which case there is no difference between the two rollover strategies. As the market size uncertainty increases, intuitively, by keeping the leftovers in the market and then setting their price after the demand realization under dual rollover, the firm has more leverage to deal with the mismatch of supply and demand. Our study supports this intuition by showing that higher uncertainties point to dual rollover with low innovation. When the innovation level is high, the choice between single and dual rollovers is not affected by the market size uncertainty, but mainly by the proportion of strategic customers.

As the production cost increases, one can reason that the firm is less likely to adopt single rollover. This is because with a higher production cost, the firm would reduce the stocking level of V1, which results in a lower waiting incentive, whose mitigation would not be critical enough to warrant single rollover. Surprisingly, the opposite happens in our study with high innovation. A possible explanation is that with a higher production cost and then a lower stocking level, the number of leftovers decreases. This may reduce the revenue loss from disposing of the leftovers if single rollover is adopted, and hence softens the primary disadvantage of single rollover compared to dual rollover.

The impact of the residual value multiplier $\beta$ on the optimal rollover strategy is complicated. With high innovation, on one hand, a larger $\beta$ tends to increase the waiting surplus by increasing the utility surplus $\beta v$ (Propositions 6 and 8). On the other hand, with a larger $\beta$, the firm can charge higher prices for products in period 2. So to have more demand in period 2, it prefers to stock less V1 in period 1, which tends to reduce the waiting surplus. Therefore, it is not clear which rollover strategy becomes more profitable as $\beta$ increases. According to our study, a larger $\beta$ favors dual rollover with high innovation. When the innovation level is low, the impact of waiting behavior is so strong that, regardless of the residual value multiplier, the firm should adopt single rollover as long as more than 10% of the customers are strategic.

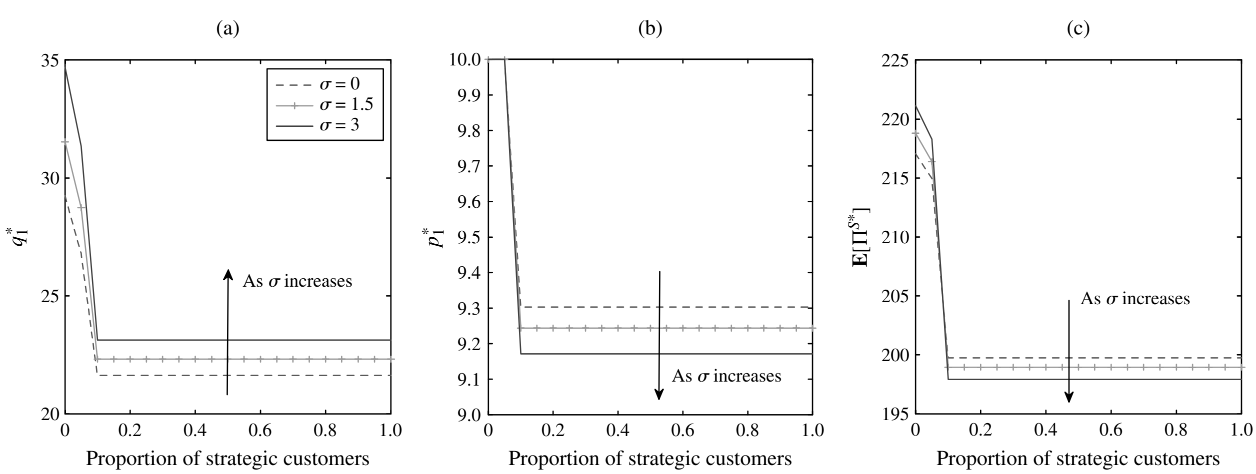
In addition to the optimal rollover strategy, we also examine the firm's stocking decision in period 1. It is commonly accepted that a firm should order less when selling to strategic customers compared to myopic customers; that is, strategic waiting behavior leads to a lower stocking level. In contrast, our paper finds that the firm may stock more V1 when more customers become strategic (i.e., as $\phi$ increases). This is because as the proportion of strategic customers increases, instead of setting a high period 1 price to sell V1 to only myopic customers, the firm may find it more profitable to set a low price and stock more to serve both myopic and strategic customers.

### 5.3. Impact of Disposal Value

Under single rollover, we have two counterintuitive observations from our earlier discussion with high innovation: customers can be charged a *lower* price for V1 when the firm has a *higher* disposal value option, and the firm can *suffer from* its higher disposal value option. We use Figure 3 to illustrate how the equilibrium values of $q_1^*$, $p_1^*$, and $\mathbf{E}[\Pi^{S*}]$ change as the disposal value $\sigma$ increases. In Figure 3, the parameter values follow the base case and $\theta = 0.9$.

According to Figure 3(a), as the disposal value $\sigma$ increases, the firm orders more V1, which is consistent with the traditional newsvendor results. However, *in contrast with the conventional wisdom* that the firm should charge a higher price when having a better alternative/external option (a higher $\sigma$), the

**Figure 3** **Impact of Disposal Value $\sigma$ with High Innovation**




*Note.* (a) $q_1^*$; (b) $p_1^*$; (c) $\mathbf{E}[\Pi^{S*}]$.

firm sets instead a lower price for V1 in Figure 3(b). This is because a higher $q_1$ (resulting from a higher $\sigma$) leads to a higher waiting surplus (see the expression of the expected waiting surplus in Proposition 8), which in turn forces the firm to charge a lower price for V1 to induce strategic customers to buy early. From Figures 3(a) and 3(b), there is a substitute relationship between $p_1^*$ and $q_1^*$. This interplay also underscores the importance of joint pricing and stocking level consideration. With a lower disposal value $\sigma$, although the firm receives less per-unit revenue from the leftovers in period 2, a higher $p_1$ in period 1 can help the firm to overcome the revenue loss and then to earn a higher profit when there are enough strategic customers, such as when $\phi > 10\%$ in Figure 3(c). In our extensive numerical study, we find that with high innovation and high proportion of strategic customers, the firm always charges a lower price for V1 and suffers from a lower profit when it has a higher disposal value option, as long as V1 is introduced. Therefore, a firm does not necessarily benefit from a high value disposal option. Possibly the lowest value disposal option is that of donating the leftovers, and it can give the highest profit. Thus, a profit-making objective can lead to socially responsible outcomes.

### 5.4. Value of Flexible Rollover

In our model the firm decides the rollover strategy prior to period 1. Instead, if the firm can postpone its rollover strategy decision until the demand is realized at the end of period 1, then it can choose the strategy according to the known numbers of P1Bs, P1NBs, and leftovers. Although the firm benefits from this flexible rollover in period 2, its two-period total profit $\mathbf{E}[\Pi^{F*}]$ may suffer. This is because with the possibility of purchasing the marked-down V1 under flexible rollover, strategic customers have more incentive to wait in period 1 compared to under single rollover. Our numerical study shows that, consistent with our analysis, when the proportion $\phi$ of strategic customers is high, and thus their waiting behavior has a great impact on the firm's profit, the firm actually suffers from being flexible. Specifically, using the same parameters as in Figure 2, the (relative) value of flexibility ($\mathbf{E}[\Pi^{F*}]/\max\{\mathbf{E}[\Pi^{S*}], \mathbf{E}[\Pi^{D*}]\} - 1$) can be as low as $-22\%$, $-12\%$ and $-0.6\%$, respectively, for $\theta = 0.2$, $\theta = 0.6$ and $\theta = 0.9$. Also, we find that the value of flexibility, if positive, is negligible except with medium innovation ($\theta = 0.6$) and $\phi$ less than 15%, where the highest value of flexibility is around 3%. So the firm can improve its profit through the flexible strategy when the cannibalization between the two versions is neither very high nor very low and not many customers are strategic.

## 6. Conclusion

This paper analyzes a firm's single and dual (product) rollover strategies under different levels of innovation in the presence of strategic customers, who may postpone their purchases of an existing old product (V1) in anticipation of the introduction of an improved new product (V2) and a markdown of V1. When the new improved product is introduced, the old product leftovers are kept in the market under dual rollover, whereas the leftovers are disposed of under single rollover. By removing the leftovers from the market, single rollover helps to reduce not only the cannibalization of the new product sales by the old product, but also the waiting incentive of strategic customers. A two-period model is studied in which the firm decides its rollover strategy prior to period 1, and the product prices and ordering quantities in each period. The market size is uncertain and is realized at the end of period 1.

We find that single rollover can improve a firm's profit in all (low, medium, and high) innovation cases under conditions obtained in this paper. The most noteworthy among these is that a firm should adopt single rollover when the innovation is not high and there are many strategic customers in the market. When using single rollover, a firm is better off with a higher disposal value option under low and medium innovations. However and contrary to the intuition, the firm under high innovation can suffer a decreasing profit with increasing disposal value. This is because an increase in the disposal value raises strategic customers' waiting incentive and thus force the firm to charge a lower price for V1 to induce strategic customers to buy early. So the firm may seek a low value disposal option such as donation of leftovers, which interestingly makes a profit-maximizing firm also a socially responsible one. We also find that a firm having the flexibility to choose its rollover strategy at the end of period 1 can surprisingly have a lower profit (than the best of single rollover and dual rollover chosen prior to period 1) in some situations, especially when there are many strategic customers in the market.

Although our model can be applied to many industries, the insights are sharper for fashion-like industries or industries with frequent new-version introductions such as consumer electronics, computers, software, and apparel. For each industry under consideration, our analysis also yields testable propositions. First, dual rollover is more likely to be adopted in relatively more innovative industries. This supports the finding of Erhun et al. (2007) that the high-tech industry commonly tends to use dual rollover. Second, single rollover is more popular in the industries facing more strategic customers. The number of such industries tends to increase as more and more customers become better informed and strategic.

Following Dhebar (1994), Lee and Lee (1998), and Kornish (2001), we have assumed that there is no secondhand market. So period 1 buyers scrap V1 after buying V2. This assumption holds for instance when a firm designs its product in a way so as to make it unsuitable for alternative purposes and, therefore, not salable in the secondhand markets. Familiar examples are those of telecom service providers locking cell phones, and software firms inserting computer codes into their products to eliminate duplication and transfer; also see Oraiopoulos et al. (2012) for other examples of information products with no secondhand markets. Nevertheless, if there is a secondhand market and period 1 buyers can salvage their used V1 at its resale value, then period 1 buyers are more willing to replace V1 with V2. This enables the firm to price V2 higher. If the resale value is exogenous and period 1 nonbuyers do not purchase the used V1 due to concerns of, for example, cleanliness (e.g., apparel industry) or condition uncertainty (e.g., consumer electronics industry), then our analysis can be repeated to find that the current qualitative results obtained without resale would also hold with resale.

We have also assumed that all high-end customers arrive at the market in period 1. Instead, if a new stream of high-end customers arrive in period 2 and they are interested in buying only V2, the following two changes can be expected. On the one hand, the firm has more incentive to price V2 high to target period 1 nonbuyers, which reduces the waiting incentive and hence the effectiveness of single rollover. On the other, the larger the number of high-end customers in period 2 is, the more additional profit can be made with single rollover by reducing the cannibalization from V1 and then keeping the price of V2 high. Despite these two changes, the trade-off between these two rollover strategies remains essentially the same.

Several possible directions for future research follow from our model. Although we consider consumer heterogeneity (strategic versus myopic and high-end versus low-end), it would be interesting to allow further heterogeneity within each segment. For example, if high-end customers have different valuations for the product, the firm may want to mark down the current version in advance of introducing the new version. Also, our model can be extended to incorporate competition, which can force the firm to introduce both versions to avoid leaving the market to its competitors, even though it may be advantageous for the firm to skip V1 when there is no competition.

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## Appendix. Proofs

Proof of Proposition 1. With more than one version and customer segment in period 2, different versions can be targeted to different segments. We first list all possible targeting strategies, and then list the highest prices that can be used to execute each targeting strategy and consequential stocking levels. After that, we investigate the conditions under which a certain strategy (involving both prices and stocking levels) in period 2 is most profitable in two cases $[p_1 = R_f]$ and $[p_1 = v]$. Finally, we compute the expected waiting surpluses in each of the two cases. We use B to denote bargain hunters and use a vector notation to denote a targeting strategy; the first (respectively, second) entry in the vector denotes the customer segments targeted by version V1 (respectively, V2) in period 2. For example, [{B}, {P1B, P1NB}] means targeting B with V1 and both P1Bs and P1NBs with V2. Under each targeting strategy, we also discuss P1NBs' preference between V1 and V2. With low innovation, all of the firm's possibly optimal targeting strategies are [{P1NB}, {P1NB}], [{B}, {P1NB}], [{B,P1NB}, {P1NB}].

*The highest prices to execute* [{P1NB}, {P1NB}] *are high–high prices,* $p'_1 = \beta v$ and $p_2 = \beta v(1+\theta)$, which are the P1NB's respective reservation prices for V1 and V2. With $\beta v(1+\theta) - c < \beta v$, the firm induces P1NBs to buy V1 rather than V2. Because $\beta v(1+\theta) - c \geq 0$ and there is no uncertainty in period 2, the firm produces exactly $[(\tilde{n} - \tilde{s}) - (q_1 - \tilde{s})]^+ = (\tilde{n} - q_1)^+$ of V2 to meet all unsatisfied P1NB demand overflowed from V1. So, the firm's profit in period 2 is $\Pi_2^{D(\text{H–H})} = [\beta v(1+\theta) - c](\tilde{n} - q_1)^+ + \beta v \min\{\tilde{n} - \tilde{s}, q_1 - \tilde{s}\}$; $\Pi_2^{D(\text{H–H})}$ denotes the firm's profit with H–H strategy in period 2. Similar naming conventions are used throughout this appendix.

*The highest prices to execute* [{B,P1NB}, {P1NB}] *are low–high prices,* $p'_1 = \delta$ and $p_2 = \beta v(1+\theta)$, which are the B's reservation price for V1 and the P1NB's reservation price for V2, respectively. With the positive surplus $(\beta v - \delta)$ from V1 and zero surplus from V2, P1NBs prefer V1 and switch to V2 only when V1 is out of stock. The remaining V1s (if any) after the P1NB's purchase are sold to B. The profit is $\Pi_2^{D(\text{L–H})} = [\beta v(1+\theta) - c](\tilde{n} - q_1)^+ + \delta(q_1 - \tilde{s})$.

To execute [{B}, {P1NB}], the firm needs to make V2 the P1NB's first option, which requires $\beta v(1+\theta) - p_2 \geq \beta v - \delta$, i.e., $p_2 \leq \beta v\theta + \delta$. However, $p_2 \leq \beta v\theta + \delta$ implies a negative margin from V2 with low innovation $\theta < (c-\delta)/(\beta v)$. So, [{B}, {P1NB}] cannot be successfully executed.

$[p_1 = R_f]$: $\tilde{s} = \min\{q_1, \tilde{n}\}$. If $\tilde{n} \leq q_1$, then $\tilde{s} = \tilde{n}$. The L–H strategy is better as $\Pi_2^{D(\text{H–H})} = 0 < \delta(q_1 - \tilde{n}) = \Pi_2^{D(\text{L–H})}$. If $\tilde{n} > q_1$, then $\tilde{s} = q_1$. So $\Pi_2^{D(\text{H–H})} = [\beta v(1+\theta) - c](\tilde{n} - q_1) = \Pi_2^{D(\text{L–H})}$. Without loss of generality, we take H–H as optimal. Finally, we compute $w(q_1, 1)$. When $\tilde{n} > q_1$, the firm uses the H–H strategy and leaves zero surplus to waiting customers. When $\tilde{n} \leq q_1$, with $p_2 = \beta v(1+\theta)$, the surplus from V2 is also zero. Because P1NBs prefer V1 to V2 when $\tilde{n} \leq q_1$, according to (12) and (13), we have

$$
\begin{aligned}
w(q_1, 1) &= \int_0^{q_1} \left\{ \min\left\{ \frac{q_1 - x}{x - x}, 1 \right\} (\beta v - \delta) + 0 \right\} f(x)\, dx + 0 \\
&= (\beta v - \delta) F(q_1).
\end{aligned}
$$

$[p_1 = v]$: $\tilde{s} = \min\{q_1, (1-\phi)\tilde{n}\}$. If $\tilde{n} \ge q_1/(1-\phi)$, then $\tilde{s} = q_1$. So $\Pi_2^{D(\text{H–H})} = \Pi_2^{D(\text{L–H})}$ as in the case $[p_1 = R_f]$. If $q_1 < \tilde{n} < q_1/(1-\phi)$, then $\tilde{s} = (1-\phi)\tilde{n}$. Because

$$\begin{aligned}\Pi_2^{D(\text{H–H})} &= [\beta v(1+\theta) - c](\tilde{n} - q_1) + \beta v[q_1 - (1-\phi)\tilde{n}]\\ &> [\beta v(1+\theta) - c](\tilde{n} - q_1) + \delta[q_1 - (1-\phi)\tilde{n}] = \Pi_2^{D(\text{L–H})},\end{aligned}$$

the H–H strategy is optimal. If $\tilde{n} \le q_1$, then $\tilde{s} = (1-\phi)\tilde{n}$, $\Pi_2^{D(\text{H–H})} = \beta v[1-(1-\phi)]\tilde{n}$, and $\Pi_2^{D(\text{L–H})} = \delta[q_1 - (1-\phi)\tilde{n}]$.

$$\Pi_2^{D(\text{H–H})} \le \Pi_2^{D(\text{L–H})} \Leftrightarrow \tilde{n} \le \frac{\delta}{\beta v\phi + \delta(1-\phi)} q_1.$$

Since $\delta/(\beta v\phi + \delta(1-\phi)) < \delta/(\delta\phi + \delta(1-\phi)) = 1$, we have $(\delta/(\beta v\phi + \delta(1-\phi)))q_1 < q_1$. So the L–H strategy is optimal only for $\tilde{n} \le (\delta q_1)/(\beta v\phi + \delta(1-\phi))$. We can compute

$$\begin{aligned}w(q_1, 0) &= \int_0^{(\delta/(\beta v\phi+\delta(1-\phi)))q_1} \left\{\min\left\{\frac{q_1-(1-\phi)x}{x-(1-\phi)x}, 1\right\}(\beta v - \delta) + 0\right\}\\ &\quad \cdot f(x)\,dx + 0 = (\beta v - \delta)F\left(\frac{\delta}{\beta v\phi + \delta(1-\phi)} q_1\right).\end{aligned}$$

In the first equality, when $x > (\delta q_1)/(\beta v\phi + \delta(1-\phi))$, the firm uses the H–H strategy, which leads to zero surplus to waiting customers. The second equality follows from $\min\{(q_1 - (1-\phi)x)/(x-(1-\phi)x), 1\} = 1$ when $x < (\delta/(\beta v\phi + \delta(1-\phi)))q_1 < q_1$. $\square$

Lemma 2 (Low Innovation). *With $p_1 = v$, the unique solution to the REE conditions except for the firm's pricing optimality* (11) *is $\chi = 0$, $W_c = (\beta v - \delta)F((\delta/(\beta v\phi + \delta(1-\phi)))q_1)$, $R_f = v - (\beta v - \delta)F((\delta/(\beta v\phi + \delta(1-\phi)))q_1)$, and $q_1$ as defined in* (i) *and* (ii) *as follows*:

(i) *If $\alpha[\beta v(1+\theta) - c] < v - c$, then $q_1 > 0$ and it is the unique solution of $\partial\mathbf{E}[\Pi^{D,h}(q_1, v)]/\partial q_1 = 0$.*

(ii) *If $\alpha[\beta v(1+\theta) - c] \ge v - c$, then $q_1 = 0$.*

Proof of Lemma 2. From (6)–(8) and Proposition 1, with $p_1 = v$ and $q_1$, we have the values for $\chi$, $W_c$, and $R_f$. So we focus on finding $q_1$. Note that

$$\begin{aligned}&\mathbf{E}[\Pi^{D,h}(q_1, v)]\\ &\quad = \mathbf{E}[v\min\{(1-\phi)\mathbf{N}, q_1\} - cq_1]\\ &\qquad + \alpha\int_0^{(\delta/(\beta v\phi+\delta(1-\phi)))q_1} \{[\beta v(1+\theta) - c](x - q_1)^+\\ &\qquad\qquad + \delta[q_1 - \min\{(1-\phi)x, q_1\}]\} f(x)\,dx\\ &\qquad + \alpha\int_{(\delta/(\beta v\phi+\delta(1-\phi)))q_1}^{\infty} \{[\beta v(1+\theta) - c](x - q_1)^+\\ &\qquad\qquad + \beta v[\min\{x, q_1\} - \min\{(1-\phi)x, q_1\}]\} f(x)\,dx,\end{aligned}$$

where the first term $\mathbf{E}[v\min\{(1-\phi)\mathbf{N}, q_1\} - cq_1]$ is the expected profit in period 1, and the last two terms together are the expected profit in period 2 by using the pricing and stocking level values from Proposition 1. We have

$$\begin{aligned}&\frac{\partial\mathbf{E}[\Pi^{D,h}(q_1, v)]}{\partial q_1}\\ &\quad = (v - \alpha[\beta v(1+\theta) - c])\bar{F}\left(\frac{q_1}{1-\phi}\right) - c\\ &\qquad + \alpha\delta F\left(\frac{\delta}{\beta v\phi + \delta(1-\phi)} q_1\right) + \alpha(c - \beta v\theta)\left[F\left(\frac{q_1}{1-\phi}\right) - F(q_1)\right],\end{aligned}$$

$$\left(\frac{\partial\mathbf{E}[\Pi^{D,h}(q_1, v)]}{\partial q_1}\right)\bigg|_{q_1=0} = v - c - \alpha[\beta v(1+\theta) - c],$$

$$\left(\frac{\partial\mathbf{E}[\Pi^{D,h}(q_1, v)]}{\partial q_1}\right)\bigg|_{q_1=\infty} = \alpha\delta - c < 0,$$

and

$$\begin{aligned}&\frac{\partial^2\mathbf{E}[\Pi^{D,h}(q_1, v)]}{\partial q_1^2}\\ &\quad = f(q_1)\left[\frac{\alpha\delta^2}{\beta v\phi + \delta(1-\phi)} \frac{f((\delta/(\beta v\phi + \delta(1-\phi)))q_1)}{f(q_1)}\right.\\ &\qquad \left. - \frac{v(1-\alpha\beta)}{1-\phi}\frac{1}{f(q_1)/f(q_1/(1-\phi))} - \alpha(c - \beta v\theta)\right].\end{aligned}$$

From the MSLR property, $f((\delta/(\beta v\phi + \delta(1-\phi)))q_1)/f(q_1)$ and $f(q_1)/f(q_1/(1-\phi))$ are monotone in $q_1$ in the same direction. So

$$\text{if } \frac{\partial^2\mathbf{E}[\Pi^{D,h}(q_1, v)]}{\partial q_1^2} \text{ crosses 0, it crosses at most once.} \tag{18}$$

By combining (18) and $(\partial\mathbf{E}[\Pi^{D,h}(q_1, v)]/\partial q_1)|_{q_1=\infty} < 0$, we know that if $(\partial\mathbf{E}[\Pi^{D,h}(q_1, v)]/\partial q_1)|_{q_1=0} = v - c - \alpha[\beta v(1+\theta) - c] > 0$, then $\mathbf{E}[\Pi^{D,h}(q_1, v)]$ must be unimodal. Moreover, there exists a unique positive root achieving $\partial\mathbf{E}[\Pi^{D,h}(q_1, v)]/\partial q_1 = 0$, and this unique positive root $q_1$ maximizes $\mathbf{E}[\Pi^{D,h}(q_1, v)]$. This proves Claim (i).

*Claim* (ii). Rewrite the derivative as

$$\begin{aligned}\frac{\partial\mathbf{E}[\Pi^{D,h}(q_1, v)]}{\partial q_1} &= v - c - \alpha[\beta v(1+\theta) - c]\\ &\quad - v(1-\alpha\beta)F\left(\frac{q_1}{1-\phi}\right)\\ &\quad - \alpha(c - \beta v\theta)F(q_1)\\ &\quad + \alpha\delta F\left(\frac{\delta}{\beta v\phi + \delta(1-\phi)} q_1\right).\end{aligned}$$

Because $(\delta/(\beta v\phi + \delta(1-\phi)))q_1 < q_1 < q_1/(1-\phi)$, $\beta v\theta < c$, $\alpha < 1$, and $\beta < 1$, we have

$$\begin{aligned}&\frac{\partial\mathbf{E}[\Pi^{D,h}(q_1, v)]}{\partial q_1}\\ &< v - c - \alpha[\beta v(1+\theta) - c] - v(1-\alpha\beta)F\left(\frac{\delta}{\beta v\phi + \delta(1-\phi)} q_1\right)\\ &\quad - \alpha(c - \beta v\theta)F\left(\frac{\delta}{\beta v\phi + \delta(1-\phi)} q_1\right) + \alpha\delta F\left(\frac{\delta}{\beta v\phi + \delta(1-\phi)} q_1\right)\\ &= v - c - \alpha[\beta v(1+\theta) - c] - \{v - \alpha[\beta v(1+\theta) - c + \delta]\}\\ &\quad \cdot F\left(\frac{\delta}{\beta v\phi + \delta(1-\phi)} q_1\right) < 0.\end{aligned}$$

The last inequality follows from $v - c - \alpha[\beta v(1+\theta) - c] \le 0$ and $v - \alpha[\beta v(1+\theta) - c + \delta] > v - c - \alpha[\beta v(1+\theta) - c]$. So $q_1 = 0$. $\square$

Lemma 3 (Low Innovation). *With $p_1 = R_f$, the unique solution to the REE conditions except for* (11) *is $\chi = 1$, $W_c = (\beta v - \delta)F(q_1)$, and $p_1$, $q_1$ as defined in* (i) *and* (ii) *as follows*:

(i) *If $\alpha[\beta v(1+\theta) - c] < v - c$, then $p_1 > c + \alpha[\beta v(1+\theta) - c]$ and $q_1 > 0$, and they can be uniquely determined from two equations: $p_1 = v - (\beta v - \delta)F(q_1)$ and $F(q_1) = (p_1 - c - \alpha[\beta v(1+\theta) - c])/(p_1 - \alpha[\beta v(1+\theta) - c + \delta])$.*

(ii) *If $\alpha[\beta v(1+\theta) - c] \ge v - c$, then $q_1 = 0$.*

Proof of Lemma 3. From (6)–(8) and Proposition 1, with $p_1 = R_f$ and $q_1$, we have the values for $\chi$ and $W_c$. From Proposition 1, we have $w(q_1, 1) = (\beta v - \delta)F(q_1)$, which together with (7) and (8) yields $R_f = v - (\beta v - \delta)F(q_1)$. So, according to (6), (10), and $R_f = v - (\beta v - \delta)F(q_1)$, we can obtain $q_1$ and $p_1$ by solving the simultaneous equations

$$q_1 = \arg\max_{q_1} \mathbf{E}[\Pi^{D,l}(q_1, R_f)], \quad p_1 = R_f = v - (\beta v - \delta)F(q_1),$$

where

$$\begin{aligned}\mathbf{E}[\Pi^{D,l}(q_1, R_f)] = \mathbf{E}[&R_f \min\{\mathbf{N}, q_1\} - cq_1] \\ &+ \alpha\delta \int_0^{q_1} (q_1 - x) f(x)\,dx \\ &+ \alpha[\beta v(1+\theta) - c] \int_{q_1}^{\infty} (x - q_1) f(x)\,dx\end{aligned}$$

is the firm's expected total profit. By treating the right-hand sides of the simultaneous equations as functions of $p_1$ and $q_1$, we have

$$q_1(p_1) = \arg\max_{q_1} \mathbf{E}[\Pi^{D,l}(q_1, p_1)], \tag{19}$$

$$p_1(q_1) = v - (\beta v - \delta)F(q_1). \tag{20}$$

We prove Claims (i) and (ii) in three steps.

*Step* 1. Optimal $q_1$ in (19) is continuous and nondecreasing in $p_1$ as in Lemma 4, which is stated and proved below.

*Step* 2. From (20), we see that $p_1$ decreases in $q_1$. Also, $p_1 = v$ when $q_1 = 0$; $p_1 = v(1-\beta) + \delta$ when $q_1 = \infty$; and $v(1-\beta) + \delta \le p_1 \le v$.

*Step* 3. From Steps 1 and 2, there must be a crossing point of (19) and (20). In addition, this crossing point is unique due to the nondecreasing property of $q_1(p_1)$ in (19) and the decreasing property of $p_1(q_1)$ in (20). □

Lemma 4 (Low Innovation). *For a given $p_1$, there exists a maximizer $q_1^o(p_1) \ge 0$ of $\mathbf{E}[\Pi^{D,l}(q_1, p_1)]$, satisfying the following conditions*:

$$\text{(i)}\quad q_1^o(p_1) = \begin{cases} 0 & \text{if } 0 < p_1 \le c + \alpha[\beta v(1+\theta) - c], \\ F^{-1}\left(\dfrac{p_1 - c - \alpha(\beta v(1+\theta) - c)}{p_1 - \alpha[\beta v(1+\theta) - c + \delta]}\right) & \\ & \text{if } p_1 > c + \alpha[\beta v(1+\theta) - c]; \end{cases}$$

(ii) *$q_1^o(p_1)$ is continuous and nondecreasing in $p_1$ for all $p_1 \ge 0$.*

Proof of Lemma 4. Similar to the case $p_1 = v$, we have

$$\begin{aligned}\frac{\partial \mathbf{E}[\Pi^{D,l}(q_1, p_1)]}{\partial q_1} &= p_1 \bar{F}(q_1) - c - \alpha[\beta v(1+\theta) - c]\bar{F}(q_1) \\ &\quad + \alpha\delta F(q_1), \\ \frac{\partial^2 \mathbf{E}[\Pi^{D,l}(q_1, p_1)}{\partial q_1^2} &= f(q_1)\{\alpha[\beta v(1+\theta) + \delta - c] - p_1\}, \\ \left.\frac{\partial \mathbf{E}[\Pi^{D,l}(q_1, p_1)]}{\partial q_1}\right|_{q_1=0} &= p_1 - c - \alpha[\beta v(1+\theta) - c], \quad \text{and} \\ \left.\frac{\partial \mathbf{E}[\Pi^{D,l}(q_1, p_1)}{\partial q_1}\right|_{q_1=\infty} &= \alpha\delta - c < 0.\end{aligned}$$

(i) We investigate the maximizer $q_1^o(p_1)$ of $\mathbf{E}[\Pi^{D,l}(q_1, p_1)]$ in three cases described by increasing $p_1$. For $p_1 < \alpha[\beta v(1+\theta) + \delta - c]$, since $c + \alpha[\beta v(1+\theta) - c] > \alpha[\beta v(1+\theta) + \delta - c]$, we have

$$\frac{\partial^2 \mathbf{E}[\Pi^{D,l}(q_1, p_1)]}{\partial q_1^2} > 0 \quad \text{and} \quad \left.\frac{\partial \mathbf{E}[\Pi^{D,l}(q_1, p_1)]}{\partial q_1}\right|_{q_1=0} < 0.$$

Considering

$$\left.\frac{\partial \mathbf{E}[\Pi^{D,l}(q_1, p_1)]}{\partial q_1}\right|_{q_1=\infty} < 0,$$

we have $q_1^o(p_1) = 0$. For $\alpha[\beta v(1+\theta) + \delta - c] \le p_1 \le c + \alpha[\beta v(1+\theta) - c]$, $q_1^o(p_1) = 0$ due to

$$\frac{\partial^2 \mathbf{E}[\Pi^{D,l}(q_1, p_1)]}{\partial q_1^2} \le 0 \quad \text{and} \quad \left.\frac{\partial \mathbf{E}[\Pi^{D,l}(q_1, p_1)}{\partial q_1}\right|_{q_1=0} \le 0.$$

For $p_1 > c + \alpha[\beta v(1+\theta) - c]$, since $\partial^2 \mathbf{E}[\Pi^{D,l}(q_1, p_1)]/\partial q_1^2 < 0$,

$$\left.\frac{\partial \mathbf{E}[\Pi^{D,l}(q_1, p_1)]}{\partial q_1}\right|_{q_1=0} > 0 \quad \text{and} \quad \left.\frac{\partial \mathbf{E}[\Pi^{D,l}(q_1, p_1)]}{\partial q_1}\right|_{q_1=\infty} < 0,$$

$q_1^o(p_1)$ is the unique solution for $\partial \mathbf{E}[\Pi^{D,l}(q_1, p_1)]/\partial q_1 = 0$, i.e.,

$$F(q_1^o) = \frac{p_1 - c - \alpha(\beta v(1+\theta) - c)}{p_1 - \alpha[\beta v(1+\theta) - c + \delta]}.$$

Together, these three cases prove (i).

(ii) For $p_1 = c + \alpha[\beta v(1+\theta) - c]$, we have

$$F^{-1}\left(\frac{p_1 - c - \alpha(\beta v(1+\theta) - c)}{p_1 - \alpha[\beta v(1+\theta) - c + \delta]}\right) = F^{-1}(0) = 0,$$

so $q_1^o(p_1)$ is continuous in $p_1 \ge 0$. Since

$$\frac{p_1 - c - \alpha(\beta v(1+\theta) - c)}{p_1 - \alpha[\beta v(1+\theta) - c + \delta]}$$

increases in $p_1 > c + \alpha[\beta v(1+\theta) - c]$, $q_1^o(p_1)$ is nondecreasing in $p_1 \ge 0$. □

In this appendix, we use superscripts $D$, $h$ (respectively, $D$, $l$) on order quantity $q_1$ and/or price $p_1$ to denote "dual rollover high price $p_1 = v$" (respectively, "dual rollover low price $p_1 = R_f$").

Proof of Proposition 2. Denote the stocking level defined in Lemma 2(i) as $q_1^{D,h}(\phi)$ to emphasize its dependence on $\phi$. From Lemma 3(i), with $p_1 = R_f$, the price and stocking level are independent of $\phi$, and thus are denoted by $(p_1^{D,l}, q_1^{D,l})$. According to $\mathbf{E}[\Pi^{D,h}(q_1, v)]$ in the proof of Lemma 2 and $\mathbf{E}[\Pi^{D,l}(q_1, R_f)]$ in the proof of Lemma 3, we have

$$\begin{aligned}&\mathbf{E}[\Pi^{D,h}(q_1^{D,h}(\phi), v; \phi)] \\ &= \mathbf{E}\big[v \min\{(1-\phi)\mathbf{N}, q_1^{D,h}(\phi)\} - cq_1^{D,h}(\phi)\big] \\ &\quad + \alpha \int_0^{(\delta/(\beta v\phi + \delta(1-\phi)))q_1^{D,h}(\phi)} \big\{[\beta v(1+\theta) - c](x - q_1^{D,h}(\phi))^+ \\ &\qquad\qquad + \delta\big[q_1^{D,h}(\phi) - \min\{(1-\phi)x, q_1^{D,h}(\phi)\}\big]\big\} f(x)\,dx \\ &\quad + \alpha \int_{(\delta/(\beta v\phi + \delta(1-\phi)))q_1^{D,h}(\phi)}^{\infty} \big\{[\beta v(1+\theta) - c](x - q_1^{D,h}(\phi))^+ \\ &\qquad\qquad + \beta v\big[\min\{x, q_1^{D,h}(\phi)\} \\ &\qquad\qquad - \min\{(1-\phi)x, q_1^{D,h}(\phi)\}\big]\big\} f(x)\,dx,\end{aligned}$$

$$\begin{aligned}
&\mathbf{E}[\Pi^{D,l}(q_1^{D,l},p_1^{D,l})]\\
&=\mathbf{E}\big[p_1^{D,l}\min\{\mathbf{N},q_1^{D,l}\}-cq_1^{D,l}\big]+\alpha\int_0^{q_1^{D,l}}\delta(q_1^{D,l}-x)f(x)\,dx\\
&\quad+\alpha\int_{q_1^{D,l}}^{\infty}[\beta v(1+\theta)-c](x-q_1^{D,l})f(x)\,dx.
\end{aligned}$$

Then,

$$\begin{aligned}
\lim_{\phi\to 0}\mathbf{E}[\Pi^{D,h}(q_1^{D,h}(\phi),v;\phi)]
&=\mathbf{E}[v\min\{\mathbf{N},q_1^{D,h}(0)\}-cq_1^{D,h}(0)]\\
&\quad+\alpha\int_0^{q_1^{D,h}(0)}\delta[q_1^{D,h}(0)-x]f(x)\,dx\\
&\quad+\alpha\int_{q_1^{D,h}(0)}^{\infty}[\beta v(1+\theta)-c][x-q_1^{D,h}(0)]f(x)\,dx.
\end{aligned}$$

We have

$$\begin{aligned}
\lim_{\phi\to 0}\mathbf{E}[\Pi^{D,h}(q_1^{D,h}(\phi),v;\phi)]&\ge\lim_{\phi\to 0}\mathbf{E}[\Pi^{D,h}(q_1^{D,l},v;\phi)]\\
&\ge\mathbf{E}[\Pi^{D,l}(q_1^{D,l},p_1^{D,l})],
\end{aligned}$$

where the last inequality is from $v\ge p_1^{D,l}$.

From Lemma 2, we know that when $q_1^{D,h}(\phi)>0$, it solves $\partial\mathbf{E}[\Pi^{D,h}(q_1,v)]/\partial q_1=0$. So for $q_1^{D,h}(\phi)>0$, according to the envelope theorem (Mas-Colell et al. 1995), we have

$$\begin{aligned}
&\frac{d\mathbf{E}[\Pi^{D,h}(q_1^{D,h}(\phi),v;\phi)]}{d\phi}\\
&=\frac{\partial\mathbf{E}[\Pi^{D,h}(q_1,v;\phi)]}{\partial\phi}\bigg|_{q_1=q_1^{D,h}(\phi)}\\
&=-v\int_0^{(1/(1-\phi))q_1^{D,h}(\phi)}xf(x)\,dx\\
&\quad+\alpha\int_0^{(\delta/(\beta v\phi+\delta(1-\phi)))q_1^{D,h}(\phi)}\delta xf(x)\,dx\\
&\quad+\alpha\int_{(\delta/(\beta v\phi+\delta(1-\phi)))q_1^{D,h}(\phi)}^{q_1^{D,h}(\phi)}\beta vxf(x)\,dx\\
&\quad+\alpha\int_{q_1^{D,h}(\phi)}^{(1/(1-\phi))q_1^{D,h}(\phi)}\beta vxf(x)\,dx+0\\
&=(\alpha\delta-v)\int_0^{(\delta/(\beta v\phi+\delta(1-\phi)))q_1^{D,h}(\phi)}xf(x)\,dx\\
&\quad+(\alpha\beta v-v)\int_{(\delta/(\beta v\phi+\delta(1-\phi)))q_1^{D,h}(\phi)}^{(1/(1-\phi))q_1^{D,h}(\phi)}xf(x)\,dx<0.
\end{aligned}$$

The last inequality is because $\alpha\delta-v<0$ and $\alpha\beta v-v<0$. So $\mathbf{E}[\Pi^{D,h}(q_1^{D,h}(\phi),v;\phi)]$ is decreasing in $\phi$. Note that $\mathbf{E}[\Pi^{D,l}(q_1^{D,l},p_1^{D,l})]$ is constant in $\phi$. Let $\phi^{L,D}=\inf\{\phi\colon \mathbf{E}[\Pi^{D,h}(q_1^{D,h}(\phi),v;\phi)]\le\mathbf{E}[\Pi^{D,l}(q_1^{D,l},p_1^{D,l})]$, where $0\le\phi<1\}$. Then, $\mathbf{E}[\Pi^{D,h}(q_1^{D,h}(\phi),v;\phi)]\ge\mathbf{E}[\Pi^{D,l}(q_1^{D,l},p_1^{D,l})]$, if and only if $\phi\le\phi^{L,D}$. We have now proved the existence of a unique REE, the existence of $\phi^{L,D}$, and the firm's pricing strategy in Claim (i). The remaining results in (i.a) and (i.b) are from Claim (i) in Lemma 2 and Claim (i) in Lemma 3, respectively. Similarly, Claim (ii) in Proposition 2 is from Lemmas 2 and 3. □

Proof of Proposition 3. With sales $\tilde{s}$ and market size $\tilde{n}$, the firm orders $q_2=\tilde{n}-\tilde{s}$ to sell V2 to every P1NB. The firm receives the revenue $\sigma$ from each leftover V1. So, $\Pi_2^S=[\beta v(1+\theta)-c](\tilde{n}-\tilde{s})+\sigma(q_1-\tilde{s})$. □

Proof of Proposition 4. The firm can successfully induce all high-end customers to buy in period 1 by setting $p_1=v$. So the firm finds the stocking level $q_1$ by maximizing $\mathbf{E}[\Pi^S(q_1,v)]=\mathbf{E}[v\min\{\mathbf{N},q_1\}-cq_1]+\alpha\int_0^\infty[\beta v(1+\theta)-c](x-\min\{x,q_1\})f(x)\,dx+\alpha\sigma[q_1-\min\{\mathbf{N},q_1\}]$, with respect to $q_1$. Note that

$$\begin{aligned}
\frac{\partial\mathbf{E}[\Pi^S(q_1,v)]}{\partial q_1}&=[v-\alpha(\beta v(1+\theta)-c+\sigma)]\bar{F}(q_1)+\alpha\sigma-c,\\
\frac{\partial^2\mathbf{E}[\Pi^S(q_1,v)]}{\partial q_1^2}&=-[v-\alpha(\beta v(1+\theta)-c+\sigma)]f(q_1),\\
\frac{\partial\mathbf{E}[\Pi^S(q_1,v)]}{\partial q_1}\bigg|_{q_1=0}&=v-c-\alpha[\beta v(1+\theta)-c],\quad\text{and}\\
\frac{\partial\mathbf{E}[\Pi^S(q_1,v)]}{\partial q_1}\bigg|_{q_1=\infty}&=\alpha\sigma-c<0.
\end{aligned}$$

If $v-c-\alpha[\beta v(1+\theta)-c]>0$, then $(\partial\mathbf{E}[\Pi^S(q_1,v)]/\partial q_1)|_{q_1=0}>0$. As $v-\alpha[\beta v(1+\theta)-c+\sigma]>v-c-\alpha[\beta v(1+\theta)-c]>0$, $\mathbf{E}[\Pi^S(q_1,v)]$ is concave in $q_1$. So we obtain the REE solution $q_1^*=F^{-1}((v-c-\alpha(\beta v(1+\theta)-c))/(v-\alpha[\beta v(1+\theta)-c+\sigma]))$ by solving

$$\frac{\partial\mathbf{E}[\Pi^S(q_1,v)]}{\partial q_1}=0.$$

If $v-c-\alpha[\beta v(1+\theta)-c]\le 0$, then $(\partial\mathbf{E}[\Pi^S(q_1,v)]/\partial q_1)|_{q_1=0}\le 0$. Since $(\partial\mathbf{E}[\Pi^S(q_1,v)]/\partial q_1)|_{q_1=\infty}<0$, no matter whether $\mathbf{E}[\Pi^S(q_1,v)]$ is concave or convex in $q_1$, $q_1^*=0$. □

Proof of Proposition 5. The proof for low innovation is as follows, and the proof for medium innovation is obtained similarly. Claim (i) is the direct result from Lemmas 2 and 3 and Proposition 4. With $q_1^*=0$, there is no leftover V1, and thus there is no difference between single rollover and dual rollover.

*Claim* (ii). Let $\mathbf{E}[\Pi^{S*}]$ be the firm's REE profit under single rollover, and let $q_1^{S*}$ be the stocking level defined in Proposition 4. Let $\mathbf{E}[\Pi^{D,h}]$ (respectively, $\mathbf{E}[\Pi^{D,l}]$) be the profit associated with solutions defined in Lemma 2 (respectively, Lemma 3). Let $\mathbf{E}[\Pi^{D*}]=\max\{\mathbf{E}[\Pi^{D,h}],\mathbf{E}[\Pi^{D,l}]\}$ be the firm's REE profit under dual rollover. From $\mathbf{E}[\Pi^{D,l}(q_1^{D,l},p_1^{D,l})]$, $\lim_{\phi\to 0}\mathbf{E}[\Pi^{D,h}(q_1^{D,h}(\phi),v;\phi)]$ in the proof of Proposition 2 and $\mathbf{E}[\Pi^S(q_1,v)]$ in the proof of Proposition 4, we have

$$\begin{aligned}
\mathbf{E}[\Pi^{D,l}]&=\mathbf{E}[\Pi^{D,l}(q_1^{D,l},p_1^{D,l})]\\
&=\mathbf{E}[p_1^{D,l}\min\{\mathbf{N},q_1^{D,l}\}-cq_1^{D,l}]\\
&\quad+\alpha[\beta v(1+\theta)-c]\int_{q_1^{D,l}}^{\infty}(x-q_1^{D,l})f(x)\,dx\\
&\quad+\alpha\delta\int_0^{q_1^{D,l}}(q_1^{D,l}-x)f(x)\,dx,\\
\lim_{\phi\to 0}\mathbf{E}[\Pi^{D,h};\phi]&=\lim_{\phi\to 0}\mathbf{E}[\Pi^{D,h}(q_1^{D,h}(\phi),v;\phi)]\\
&=\mathbf{E}[v\min\{\mathbf{N},q_1^{D,h}(0)\}-cq_1^{D,h}(0)]\\
&\quad+\alpha[\beta v(1+\theta)-c]\int_{q_1^{D,h}(0)}^{\infty}[x-q_1^{D,h}(0)]f(x)\,dx\\
&\quad+\alpha\delta\int_0^{q_1^{D,h}(0)}[q_1^{D,h}(0)-x]f(x)\,dx,\quad\text{and}
\end{aligned}$$

$$\begin{aligned}\mathbf{E}[\Pi^{S*}] &= \mathbf{E}[\Pi^{S}(q_1^{S*}, v)]\\ &= \mathbf{E}[v\min\{\mathbf{N}, q_1^{S*}\} - cq_1^{S*}]\\ &\quad + \alpha[\beta v(1+\theta)-c]\int_{q_1^{S*}}^{\infty}(x-q_1^{S*})f(x)\,dx\\ &\quad + \alpha\sigma\int_0^{q_1^{S*}}(q_1^{S*}-x)f(x)\,dx.\end{aligned}$$

From Lemmas 2 and 3 and Proposition 4, $q_1 > 0$ when $c+\alpha[\beta v(1+\theta)-c] < v$. From the proof of Proposition 2, with $q_1 > 0$, $\mathbf{E}[\Pi^{D*}] = \mathbf{E}[\Pi^{D,h}]$ decreases in $\phi$ when $\phi \le \phi^{L,D}$, and $\mathbf{E}[\Pi^{D*}] = \mathbf{E}[\Pi^{D,l}]$ is constant in $\phi$ when $\phi > \phi^{L,D}$. Clearly, $\mathbf{E}[\Pi^{S*}]$ is also constant in $\phi$. Comparing $\lim_{\phi\to 0}\mathbf{E}[\Pi^{D,h};\phi]$ and $\mathbf{E}[\Pi^{S*}]$ above, we have

$$\begin{aligned}\lim_{\phi\to 0}\mathbf{E}[\Pi^{D,h};\phi] &= \lim_{\phi\to 0}\mathbf{E}[\Pi^{D,h}(q_1^{D,h}(\phi), v;\phi)]\\ &\ge \lim_{\phi\to 0}\mathbf{E}[\Pi^{D,h}(q_1^{S*}, v;\phi)] \ge \mathbf{E}[\Pi^{S}(q_1^{S*}, v)]\\ &= \mathbf{E}[\Pi^{S*}],\end{aligned}$$

where the last inequality is due to $\delta > \sigma$. Note that in the extreme case $\sigma = \delta$, $\lim_{\phi\to 0}\mathbf{E}[\Pi^{D,h}(q_1, v;\phi)] = \mathbf{E}[\Pi^{S}(q_1, v)]$. So in this extreme case, $\lim_{\phi\to 0}\mathbf{E}[\Pi^{D,h};\phi] = \mathbf{E}[\Pi^{S*}]$. Because $\mathbf{E}[\Pi^{S*}]$ is increasing in $\sigma$, if we decrease $\sigma$ by starting with $\sigma=\delta$, there exists a threshold $\Delta \ge 0$ such that for $\delta-\sigma\le\Delta$, there exists a $\phi^L$, $\phi^L < \phi^{L,D}$, at which $\mathbf{E}[\Pi^{D*}] = \mathbf{E}[\Pi^{D,h}] = \mathbf{E}[\Pi^{S*}]$. Furthermore, single rollover is optimal iff $\phi \ge \phi^L$. This proves Claim (ii.a). When $\delta-\sigma > \Delta$, no $\phi$ can set $\mathbf{E}[\Pi^{D*}] = \mathbf{E}[\Pi^{S*}]$. So dual rollover is optimal. Claim (ii.b) follows. □

Proof of Proposition 6. With high innovation, the firm can profitably target P1Bs with V2 by setting $p_2 = \beta v\theta$. This increases the number of targeting strategies from three in the low innovation cases to seven: [{B}, {P1NB}], [{B}, {P1B, P1NB}], [{P1NB}, {P1NB}], [{P1NB}, {P1B, P1NB}], [{B, P1NB}, {P1NB}], [{B, P1NB}, {P1B}], [{B, P1NB}, {P1B, P1NB}]. Our analysis show that the strategies used are L–L, L–M, and H–H as listed in Proposition 6, which result in [{B}, {P1B, P1NB}] , [{B}, {P1NB}], and [{P1NB}, {P1NB}]. The proof is similar to that of Proposition 1, but requires more comparisons. We omit the details for brevity. □

Proof of Proposition 7. With $p_1 = R_f$, we have $\tilde{s} = \min\{\tilde{n}, q_1\}$. The firm's profit is

$$\begin{aligned}&\mathbf{E}[\Pi^{D,l}(q_1, R_f)]\\ &= \mathbf{E}[R_f\min\{\mathbf{N}, q_1\} - cq_1]\\ &\quad + \alpha\int_0^{((\beta v(1+\theta)-c)/(\beta v))q_1}[(\beta v\theta - c)x + \delta(q_1 - \min\{x, q_1\})]f(x)\,dx\\ &\quad + \alpha\int_{((\beta v(1+\theta)-c)/(\beta v))q_1}^{\infty}[(\beta v(1+\theta)-c)(x-\min\{x,q_1\})]f(x)\,dx.\end{aligned}$$

We have $R_f = v - \beta v F(((\beta v(1+\theta)-c)/(\beta v))q_1)$ from (7), (8), and Proposition 6. Similar to (19) and (20), we have (21) and (22). We need to investigate if and how many times $q_1(p_1)$ and $p_1(q_1)$ cross each other in three steps: Step 1, for (21), how the optimal $q_1$ changes as $p_1$ increases; Step 2, for (22), how $p_1$ changes as $q_1$ increases; and Step 3, under which conditions (21) and (22) have crossing point(s):

$$q_1(p_1) = \arg\max_{q_1}\mathbf{E}[\Pi^{D,l}(q_1, p_1)], \tag{21}$$

$$p_1(q_1) = v\left(1 - \beta F\left(\frac{\beta v(1+\theta)-c}{\beta v}q_1\right)\right). \tag{22}$$

*Step* 1.

$$\begin{aligned}&\frac{\partial \mathbf{E}[\Pi^{D,l}(q_1,p_1)]}{\partial q_1}\\ &= p_1\bar{F}(q_1) - c - \alpha[\beta v(1+\theta)-c]\bar{F}\left(\frac{\beta v(1+\theta)-c}{\beta v}q_1\right) + \alpha\delta F(q_1),\\ &\left.\frac{\partial \mathbf{E}[\Pi^{D,l}(q_1,p_1)]}{\partial q_1}\right|_{q_1=0} = p_1 - c - \alpha[\beta v(1+\theta)-c],\\ &\left.\frac{\partial \mathbf{E}[\Pi^{D,l}(q_1,p_1)]}{\partial q_1}\right|_{q_1=\infty} = \alpha\delta - c < 0, \quad \text{and}\\ &\frac{\partial^2 \mathbf{E}[\Pi^{D,l}(q_1,p_1)]}{\partial q_1^2}\\ &= f(q_1)\left[\alpha\delta - p_1 + \frac{\alpha[\beta v(1+\theta)-c]^2}{\beta v}\frac{f([1+\theta-c/(\beta v)]q_1)}{f(q_1)}\right].\end{aligned}$$

Since $f(\cdot)$ has the MSLR property, if $\partial^2\mathbf{E}[\Pi^{D,l}(q_1,p_1)]/\partial q_1^2$ crosses 0, then it crosses at most once (cross-once property). We also know $\partial^2\mathbf{E}[\Pi^{D,l}(q_1,p_1)]/(\partial p_1\partial q_1) = \bar{F}(q_1) > 0$.

From $(\partial\mathbf{E}[\Pi^{D,l}(q_1,p_1)]/\partial q_1)|_{q_1=0} = p_1 - c - \alpha[\beta v(1+\theta)-c]$, the shape of $\mathbf{E}[\Pi^{D,l}(q_1,p_1)]$ around $q_1 = 0$ depends on $p_1$. Particularly, $(\partial\mathbf{E}[\Pi^{D,l}(q_1,p_1)]/\partial q_1)|_{q_1=0} < 0$ if and only if $p_1 < c+\alpha[\beta v(1+\theta)-c]$. So $\arg\max_{q_1}\mathbf{E}[\Pi^{D,l}(q_1,p_1)]$ depends on the value of $p_1$, and we investigate it in three cases described by increasing $p_1$:

*Case* (I). $p_1 = 0$.

*Case* (II). $0 < p_1 \le c+\alpha[\beta v(1+\theta)-c]$. This case has two subcases:

*Case* (II.a). $\partial\mathbf{E}[\Pi^{D,l}(q_1,p_1)]/\partial q_1 < 0$ for all $q_1 > 0$ and all $p_1 \in (0, c+\alpha[\beta v(1+\theta)-c])$.

*Case* (II.b). $\partial\mathbf{E}[\Pi^{D,l}(q_1,p_1)]/\partial q_1 \ge 0$ for some $q_1 > 0$ and some $p_1 \in (0, c+\alpha[\beta v(1+\theta)-c])$.

*Case* (III). $p_1 > c+\alpha[\beta v(1+\theta)-c]$.

We analyze the optimal $q_1$ in all these cases in Lemmas 5 and 6, which are stated below but proved later.

Lemma 5. *Under* Case (II.b):

(i) *As $p_1$ increases, the number of roots of $\partial\mathbf{E}[\Pi^{D,l}(q_1,p_1)]/\partial q_1 = 0$ increases from zero to one, and reaches finally two.*

(ii) *If there is no or one root for $\partial\mathbf{E}[\Pi^{D,l}(q_1,p_1)]/\partial q_1 = 0$, then the optimal stocking level is zero.*

(iii) *If there are two roots for $\partial\mathbf{E}[\Pi^{D,l}(q_1,p_1)]/\partial q_1 = 0$, then $\mathbf{E}[\Pi^{D,l}(q_1^-(p_1),p_1)] < \mathbf{E}[\Pi^{D,l}(q_1^+(p_1),p_1)]$, where $q_1^-(p_1)$ and $q_1^+(p_1)$ are the smaller and larger roots, respectively. Moreover, the smaller root $q_1^-(p_1)$ decreases in $p_1$ and the larger root $q_1^+(p_1)$ increases in $p_1$.*

(iv) *If $p_1 = c+\alpha[\beta v(1+\theta)-c]$, then $\mathbf{E}[\Pi^{D,l}(q_1,p_1)]$ first increases and then decreases. Moreover, $\mathbf{E}[\Pi^{D,l}(q_1,p_1)]$ has a unique positive maximizer $q_1^+(p_1)$, which is the larger root for $\partial\mathbf{E}[\Pi^{D,l}(q_1,p_1)]/\partial q_1 = 0$.*

(v) *If there are two roots of* $\partial \mathbf{E}[\Pi^{D,I}(q_1, p_1)]/\partial q_1 = 0$, *then there exists a critical unique price* $p_1^J$ *such that*

$$\mathbf{E}[\Pi^{D,I}(0, p_1^J)] = \mathbf{E}[\Pi^{D,I}(q_1^+(p_1^J), p_1^J)] \quad \textit{and}$$
$$p_1^J < c + \alpha[\beta v(1+\theta) - c].$$

*In addition,* $\mathbf{E}[\Pi^{D,I}(q_1^+(p_1), p_1)] > (\textit{respectively}, <) \mathbf{E}[\Pi^{D,I}(0, p_1)]$ *when* $p_1 > (\textit{respectively}, <)\ p_1^J$.

Lemma 6. *For a given* $p_1$, *there exists a maximizer* $q_1^o(p_1) \geq 0$ *of* $\mathbf{E}[\Pi^{D,I}(q_1, p_1)]$, *satisfying the following conditions*:

(i) *In Cases* (I), (II.a), *and* (III),

$$q_1^o(p_1) = \begin{cases} 0 & \textit{if Case (I) for } p_1 = 0, \\ 0 & \textit{if Case (II.a) for } 0 < p_1 \leq c + \alpha[\beta v(1+\theta) - c], \\ \multicolumn{2}{l}{\textit{the unique positive root of } \partial \mathbf{E}[\Pi^{D,I}(q_1, p_1)]/\partial q_1 = 0} \\ & \textit{if Case (III) for } p_1 > c + \alpha[\beta v(1+\theta) - c]. \end{cases}$$

(ii) *In Case* (II.b) *for* $0 < p_1 \leq c + \alpha[\beta v(1+\theta) - c]$,

$$q_1^o(p_1) = \begin{cases} 0 & \textit{if } 0 < p_1 \leq p_1^J, \\ q_1^+(p_1) & \textit{if } p_1^J \leq p_1 \leq c + \alpha[\beta v(1+\theta) - c], \end{cases}$$

*where* $p_1^J$ *and* $q_1^+(p_1)$ *are defined in Lemma* 5.

(iii) *In all cases,* $q_1^o(p_1)$ *is nondecreasing in* $p_1$ *for all* $p_1 \geq 0$.

(iv) *If Case* (II.a) *occurs when* $0 < p_1 \leq c + \alpha[\beta v(1+\theta) - c]$, *then, combining Cases* (I), (II.a), *and* (III), $q_1^o(p_1)$ *is continuous in* $p_1$ *for all* $p_1 \geq 0$. *If Case* (II.b) *occurs when* $0 < p_1 \leq c + \alpha[\beta v(1+\theta) - c]$, *then, combining Cases* (I), (II.b), *and* (III), $q_1^o(p_1)$ *is continuous everywhere except at* $p_1 = p_1^J$.

*Step* 2. Solving $q_1$ from (22), we have

$$q_1^{-1}(p_1) = \frac{\beta v}{\beta v(1+\theta) - c} F^{-1}\left(\frac{1}{\beta}(1 - p_1/v)\right).$$

Clearly, $q_1$ decreases in $p_1$. In addition, $q_1 = 0$ when $p_1 = v$; $q_1 = \infty$ when $p_1 = v(1-\beta)$; and it is impossible that $p_1 > v$ or $p_1 < v(1-\beta)$. For ease of exposition, we define the $q_1^{-1}(p_1)$ as

$$q_1^{-1}(p_1) = \begin{cases} \dfrac{\beta v}{\beta v(1+\theta) - c} F^{-1}\left(\dfrac{1}{\beta}\left(1 - \dfrac{p_1}{v}\right)\right) & \\ \qquad\qquad \text{if } v(1-\beta) \leq p_1 \leq v, \\ \infty \qquad\qquad \text{if } p_1 < v(1-\beta). \end{cases}$$

This extended function $q_1^{-1}(p_1)$ does not change the existence or not of the crossing point.

*Step* 3. Steps 1 and 2 show that if there is a crossing point between (21) and (22), then the crossing point must be unique due to the nondecreasing property of $q_1^o(p_1)$ in (21) and the decreasing property of $q_1^{-1}(p_1)$ in (22). In addition, a crossing point corresponds to a unique vector $(q_1, p_1, \chi, W_c, R_f)$ satisfying the REE conditions except (11). So, to show the existence of the vector $(q_1, p_1, \chi, W_c, R_f)$, we just need to show that there is a crossing point between (21) and (22). We denote the crossing point (if any) as $(p_1, q_1)$.

If Case (II.a) occurs, then $q_1^o(p_1)$ is continuous and nondecreasing in $p_1$ for all $p_1$'s, and there must be a crossing point. If Case (II.b) occurs, then a crossing point does not exist if and only if all of the following conditions are satisfied: $v(1-\beta) < p_1^J < v$ and $q_1^+(p_1^J) > (\beta v/(\beta v(1+\theta) - c))F^{-1}((1/\beta) \cdot (1 - p_1^J/v))$. Under these conditions, if we can show that there exists a unique combination of $(q_1^-, p_1, \chi, W_c, R_f)$ and $(q_1^+, p_1, \chi, W_c, R_f)$ satisfying the REE conditions except (11), then we complete the proof of Proposition 7. From Lemma 5, at the jump point $p_1 = p_1^J$, $\mathbf{E}[\Pi^{D,I}(q_1^+(p_1^J), p_1^J)] = \mathbf{E}[\Pi^{D,I}(0, p_1^J)] = \alpha[\beta v(1+\theta) - c]\mathbf{E}(\mathbf{N})$, which means both $q_1 = q_1^+(p_1^J)$ and $q_1 = 0$ maximize $\mathbf{E}[\Pi^{D,I}(q_1, p_1^J)]$. So, if the firm orders $q_1 = 0$ with probability $\lambda$ $(0 < \lambda < 1)$ and $q_1 = q_1^+(p_1^J)$ with probability $1 - \lambda$, it can still get the maximum profit. The resulting expected waiting surplus is

$$\lambda \beta v F\left(\frac{\beta v(1+\theta) - c}{\beta v} \times 0\right) + (1-\lambda)\beta v F\left(\frac{\beta v(1+\theta) - c}{\beta v} q_1^+(p_1^J)\right)$$
$$= (1-\lambda)\beta v F\left(\frac{\beta v(1+\theta) - c}{\beta v} q_1^+(p_1^J)\right),$$

and thus we must have $\chi = 1$, $W_c = (1-\lambda)\beta v F(((\beta v(1+\theta) - c)/(\beta v))q_1^+(p_1^J))$, and $p_1^J = R_f = v - W_c$. This means we need to show that there exists a $\lambda$ satisfying $p_1^J = v(1 - \beta(1-\lambda) \cdot F(((\beta v(1+\theta) - c)/(\beta v))q_1^+(p_1^J)))$. Since

$$q_1^+(p_1^J) > \frac{\beta v}{\beta v(1+\theta) - c} F^{-1}\left(\frac{1}{\beta}\left(1 - \frac{p_1^J}{v}\right)\right),$$

we have $p_1^J > v(1 - \beta F((\beta v(1+\theta) - c)/\beta v)q_1^+(p_1^J)))$. Moreover, we know $p_1^J < v$, so we can always find a $\lambda \in (0, 1)$ satisfying $p_1^J = v(1 - \beta(1-\lambda)F(((\beta v(1+\theta) - c)/(\beta v))q_1^+(p_1^J)))$. Letting $q_1^+ = q_1^+(p_1^J)$ and $q_1^- = 0$, we finish the proof. $\square$

Proof of Lemma 5. (i) When $p_1 = 0$,

$$\frac{\partial \mathbf{E}[\Pi^{D,I}(q_1, p_1)]}{\partial q_1} < 0 \quad \text{for all } q_1.$$

From $\partial^2 \mathbf{E}[\Pi^{D,I}(q_1, p_1)]/\partial p_1 \partial q_1 > 0$, $\partial \mathbf{E}[\Pi^{D,I}(q_1, p_1)]/\partial q_1$ increases as $p_1$ increases. From the definition of Case (II.b), $\partial \mathbf{E}[\Pi^{D,I}(q_1, p_1)]/\partial q_1 \geq 0$ for some $q_1 > 0$ and some $p_1 \in (0, c + \alpha[\beta v(1+\theta) - c])$. Because $\partial \mathbf{E}[\Pi^{D,I}(q_1, p_1)]/\partial q_1$ is continuous in $p_1$, if we increase $p_1$ by starting with $p_1 = 0$, then we can always find a $\hat{p}_1 \in (0, c + \alpha[\beta v(1+\theta) - c])$ such that $\partial \mathbf{E}[\Pi^{D,I}(q_1, \hat{p}_1)]/\partial q_1 = 0$ for some $q_1 > 0$ and

$$\frac{\partial \mathbf{E}[\Pi^{D,I}(q_1, \hat{p}_1)]}{\partial q_1} < 0 \quad \text{for other } q_1\text{'s}.$$

Because $\partial \mathbf{E}[\Pi^{D,I}(q_1, p_1)]/\partial q_1 < 0$ for all $q_1 > 0$ when $p_1 < \hat{p}_1$, and $\partial \mathbf{E}[\Pi^{D,I}(q_1, p_1)]/\partial q_1 > 0$ for some $q_1 > 0$ when $p_1 > \hat{p}_1$, we know that $\hat{p}_1$ is unique. Next, we prove by contradiction that when $p_1 = \hat{p}_1$, there is only one $q_1 > 0$ at which $\partial \mathbf{E}[\Pi^{D,I}(q_1, p_1)]/\partial q_1 = 0$. Suppose that there are two $q_1$'s at which $\partial \mathbf{E}[\Pi^{D,I}(q_1, p_1)]/\partial q_1 = 0$. Because $\partial \mathbf{E}[\Pi^{D,I}(q_1, p_1)]/\partial q_1 < 0$ for other $q_1$'s, the sign of $\partial^2 \mathbf{E}[\Pi^{D,I}(q_1, p_1)]/\partial q_1^2$ changes more than once, which contradicts with the cross-once property. Similarly, we can prove that it is impossible to have more than two $q_1$'s at which $\partial \mathbf{E}[\Pi^{D,I}(q_1, p_1)]/\partial q_1 = 0$. So there is only one root for $\partial \mathbf{E}[\Pi^{D,I}(q_1, p_1)]/\partial q_1 = 0$ when $p_1 = \hat{p}_1$, and there is no root for $\partial \mathbf{E}[\Pi^{D,I}(q_1, p_1)]/\partial q_1 = 0$ when $p_1 < \hat{p}_1$. Now we prove that when $p_1 > \hat{p}_1$, there are two roots for $\partial \mathbf{E}[\Pi^{D,I}(q_1, p_1)]/\partial q_1 = 0$. From analysis above, when $p_1 > \hat{p}_1$, $\partial \mathbf{E}[\Pi^{D,I}(q_1, p_1)]/\partial q_1 > 0$ for some $q_1 > 0$. Because

$$\left.\frac{\partial \mathbf{E}[\Pi^{D,I}(q_1, p_1)]}{\partial q_1}\right|_{q_1=0} < 0 \quad \text{and} \quad \left.\frac{\partial \mathbf{E}[\Pi^{D,I}(q_1, p_1)]}{\partial q_1}\right|_{q_1=\infty} < 0,$$

there are at least two roots for $\partial\mathbf{E}[\Pi^{D,l}(q_1,p_1)]/\partial q_1 = 0$. If there are more than two roots, then the sign of $\partial^2\mathbf{E}[\Pi^{D,l}(q_1,p_1)]/\partial q_1^2$ must change more than once, which contradicts with the cross-once property. So there are two roots for $\partial\mathbf{E}[\Pi^{D,l}(q_1,p_1)]/\partial q_1 = 0$ when $p_1 > \hat{p}_1$. Combining the analysis above, we get Claim (i).

(ii) Because $\partial\mathbf{E}[\Pi^{D,l}(q_1,p_1)]/\partial q_1 < 0$ for all $q_1 > 0$ when $p_1 < \hat{p}_1$, the optimal stocking level is zero. Similarly, as when $p_1 = \hat{p}_1$, $\partial\mathbf{E}[\Pi^{D,l}(q_1,p_1)]/\partial q_1 = 0$ for only one $q_1$ and $\partial\mathbf{E}[\Pi^{D,l}(q_1,p_1)]/\partial q_1 < 0$ for other $q_1$'s, we know that the optimal stocking level is zero.

(iii) When $p_1 > \hat{p}_1$, there are two roots for

$$\frac{\partial\mathbf{E}[\Pi^{D,l}(q_1,p_1)]}{\partial q_1} = 0.$$

Together with

$$\left.\frac{\partial\mathbf{E}[\Pi^{D,l}(q_1,p_1)]}{\partial q_1}\right|_{q_1=0} < 0 \quad\text{and}\quad \left.\frac{\partial\mathbf{E}[\Pi^{D,l}(q_1,p_1)]}{\partial q_1}\right|_{q_1=\infty} < 0,$$

we know that as $q_1$ increases, $\mathbf{E}[\Pi^{D,l}(q_1,p_1)]$ is decreasing for $q_1 \le q_1^-(p_1)$, increases for $q_1^-(p_1) \le q_1 \le q_1^+(p_1)$, and decreases for $q_1 \ge q_1^+(p_1)$. This implies $\mathbf{E}[\Pi^{D,l}(q_1^-(p_1),p_1)] < \mathbf{E}[\Pi^{D,l}(q_1^+(p_1),p_1)]$. For $p_1 > \hat{p}_1$, $\partial\mathbf{E}[\Pi^{D,l}(q_1,p_1)]/\partial q_1$ is positive only for $q_1 \in [q_1^-(p_1), q_1^+(p_1)]$. Notice that

$$\frac{\partial\mathbf{E}[\Pi^{D,l}(q_1,p_1)]}{\partial q_1}$$

increases in $p_1$ for all $q_1$'s.

$$\frac{\partial\mathbf{E}[\Pi^{D,l}(q_1,p_1+\varepsilon)]}{\partial q_1}$$

is positive over a larger interval for $q_1 \in [q_1^-(p_1+\varepsilon), q_1^+(p_1+\varepsilon)]$ that includes the original interval. So $q_1^-(p_1+\varepsilon) \le q_1^-(p_1)$ and $q_1^+(p_1+\varepsilon) \ge q_1^+(p_1)$.

(iv) When $p_1 = c+\alpha[\beta v(1+\theta)-c]$, we have

$$\left.\frac{\partial\mathbf{E}[\Pi^{D,l}(q_1,p_1)]}{\partial q_1}\right|_{q_1=0} = 0.$$

With

$$\left.\frac{\partial\mathbf{E}[\Pi^{D,l}(q_1,p_1)]}{\partial q_1}\right|_{q_1=\infty} < 0$$

and the cross-once property, the only two possibilities for $\mathbf{E}[\Pi^{D,l}(q_1,p_1)]$ are that (a) $\partial\mathbf{E}[\Pi^{D,l}(q_1,p_1)]/\partial q_1 < 0$ for all $q_1 > 0$ and (b) $\mathbf{E}[\Pi^{D,l}(q_1,p_1)]$ first increases, reaches a root, and then decreases. From $\partial^2\mathbf{E}[\Pi^{D,l}(q_1,p_1)]/\partial p_1\partial q_1 > 0$, $\partial\mathbf{E}[\Pi^{D,l}(q_1,p_1)]/\partial q_1$ increases as $p_1$ increases, so (a) implies that $\partial\mathbf{E}[\Pi^{D,l}(q_1,p_1)]/\partial q_1 < 0$ for all $q_1 > 0$ for all $p_1 < c + \alpha[\beta v(1+\theta)-c]$, which violates the definition of Case (II.b). Thus, (b) is the only possible case. In addition, $q_1^-(p_1) = 0$, and $q_1^+(p_1)$ is the unique positive maximizer. Claim (iv) follows.

(v) We first show that there is a unique $p_1^J$. Note that (a) $\mathbf{E}[\Pi^{D,l}(0,p_1)] = \alpha[\beta v(1+\theta)-c]\mathbf{E}(\mathbf{N})$ regardless of the value of $p_1$, (b) $\mathbf{E}[\Pi^{D,l}(q_1,p_1)]$ is continuous and increasing in $p_1$ for each fixed $q_1 > 0$, (c) $\mathbf{E}[\Pi^{D,l}(q_1,\hat{p}_1)] < \mathbf{E}[\Pi^{D,l}(0,\hat{p}_1)]$ for all $q_1$'s, and (d) when $p_1 > \hat{p}_1$, there are two roots for $\partial\mathbf{E}[\Pi^{D,l}(q_1,p_1)]/\partial q_1 = 0$. Starting from $p_1 = \hat{p}_1$ and increasing $p_1$, a unique $p_1^J$ always exists at which

$$\mathbf{E}[\Pi^{D,l}(0,p_1^J)] = \mathbf{E}[\Pi^{D,l}(q_1^+(p_1^J),p_1^J)].$$

Furthermore, if $p_1 < p_1^J$,

$$\begin{aligned}\mathbf{E}[\Pi^{D,l}(q_1^+(p_1),p_1)] &< \mathbf{E}[\Pi^{D,l}(q_1^+(p_1),p_1^J)]\\ &\le \mathbf{E}[\Pi^{D,l}(q_1^+(p_1^J),p_1^J)] = \mathbf{E}[\Pi^{D,l}(0,p_1^J)].\end{aligned}$$

If $p_1 > p_1^J$,

$$\begin{aligned}\mathbf{E}[\Pi^{D,l}(q_1^+(p_1),p_1)] &\ge \mathbf{E}[\Pi^{D,l}(q_1^+(p_1^J),p_1)]\\ &> \mathbf{E}[\Pi^{D,l}(q_1^+(p_1^J),p_1^J)] = \mathbf{E}[\Pi^{D,l}(0,p_1^J)].\end{aligned}$$

Next we show that $p_1^J < c+\alpha[\beta v(1+\theta)-c]$ by contradiction. If $p_1^J = c+\alpha[\beta v(1+\theta)-c]$, $\mathbf{E}[\Pi^{D,l}(0,p_1^J)] < \mathbf{E}[\Pi^{D,l}(q_1^+(p_1^J),p_1^J)]$ from (iv), which violates the definition of $p_1^J$. Because $\mathbf{E}[\Pi^{D,l}(q_1^+(p_1),p_1)]$ increases in $p_1$ and $\mathbf{E}[\Pi^{D,l}(0,p_1)]$ is constant in $p_1$, if $p_1^J > c+\alpha[\beta v(1+\theta)-c]$, again $\mathbf{E}[\Pi^{D,l}(0,p_1^J)] < \mathbf{E}[\Pi^{D,l}(q_1^+(p_1^J),p_1^J)]$. So $p_1^J < c+\alpha[\beta v(1+\theta)-c]$. □

Proof of Lemma 6. (i) Case (I). When $p_1 = 0$,

$$\begin{aligned}\frac{\partial\mathbf{E}[\Pi^{D,l}(q_1,p_1)]}{\partial q_1} &= -c-\alpha[\beta v(1+\theta)-c]\bar{F}\left(\frac{\beta v(1+\theta)-c}{\beta v}q_1\right)\\ &\quad+\alpha\sigma F(q_1) < 0\end{aligned}$$

for all $q_1$. So $q_1 = 0$ is optimal.

Case (II.a). When $0 < p_1 < c+\alpha[\beta v(1+\theta)-c]$,

$$\frac{\partial\mathbf{E}[\Pi^{D,l}(q_1,p_1)]}{\partial q_1} < 0$$

for all $q_1 \ge 0$. So $q_1 = 0$ is optimal. By a limit argument, when $p_1 = c+\alpha[\beta v(1+\theta)-c]$, we have $\partial\mathbf{E}[\Pi^{D,l}(q_1,p_1)]/\partial q_1 \le 0$ for all $q_1 \ge 0$. We know that $\mathbf{E}[\Pi^{D,l}(q_1,p_1)]$ cannot be constant over an interval of $q_1$; otherwise, $\partial^2\mathbf{E}[\Pi^{D,l}(q_1,p_1)]/\partial q_1^2$ is zero over the interval, which violates the cross-once property. So $q_1^o(p_1) = 0$ when $p_1 = c+\alpha[\beta v(1+\theta)-c]$.

Case (III). When $p_1 > c+\alpha[\beta v(1+\theta)-c]$, by combining

$$\left.\frac{\partial\mathbf{E}[\Pi^{D,l}(q_1,p_1)]}{\partial q_1}\right|_{q_1=0} > 0, \quad \left.\frac{\partial\mathbf{E}[\Pi^{D,l}(q_1,p_1)]}{\partial q_1}\right|_{q_1=\infty} < 0$$

and the cross-once property, we know that $\mathbf{E}[\Pi^{D,l}(q_1,p_1)]$ is unimodal. Furthermore, there exists a unique positive root for $\partial\mathbf{E}[\Pi^{D,l}(q_1,p_1)]/\partial q_1 = 0$, and this unique positive root maximizes $\mathbf{E}[\Pi^{D,l}(q_1,p_1)]$.

(ii) According to Lemma 5, when $0 < p_1 \le p_1^J$, $q_1^o(p_1) = 0$ for the scenarios in which there is zero or one root, or there are two roots but $\mathbf{E}[\Pi^{D,l}(0,p_1)] \ge \mathbf{E}[\Pi^{D,l}(q_1^+(p_1),p_1)]$. When $p_1^J \le p_1 \le c+\alpha[\beta v(1+\theta)-c]$, there are two roots and $\mathbf{E}[\Pi^{D,l}(0,p_1)] \le \mathbf{E}[\Pi^{D,l}(q_1^+(p_1),p_1)]$.

(iii) For Cases (I) and (II.a), $q_1^o(p_1)$ is constant in $p_1$. For Case (II.b), we have increasing $q_1^+(p_1)$ from Lemma 5(iii). We can prove the increasing property of $q_1^o(p_1)$ for Case (III) in the same way as the proof of Lemma 5(iii) except that there is a single root here. This single root satisfies the properties that $q_1^+(p_1)$ satisfies in Lemma 5(iii).

(iv) Cases (I), (II.a), and (III). The unique root of $\partial\mathbf{E}[\Pi^{D,l}(q_1,p_1)]/\partial q_1 = 0$ is continuous in $p_1$ in Case (III). From the analysis for Case (II.a) above, $q_1 = 0$ is maximizer for $\mathbf{E}[\Pi^{D,l}(q_1,p_1)]$ as well as the unique root for $\partial\mathbf{E}[\Pi^{D,l}(q_1,p_1)]/\partial q_1 = 0$ when $p_1 = c+\alpha[\beta v(1+\theta)-c]$. So $q_1^o(p_1)$ is continuous in $p_1$ for all $p_1 \ge 0$.

Cases (I), (II.b), and (III). For $p_1 > c + \alpha[\beta v(1+\theta) - c]$, $q_1^o(p_1)$ is the unique root of $\partial \mathbf{E}[\Pi^{D,l}(q_1, p_1)]/\partial q_1 = 0$ and thus is continuous. For $p_1^J \le p_1 \le c + \alpha[\beta v(1+\theta) - c]$, $q_1^o(p_1) = q_1^+(p_1)$ is always the larger root of $\partial \mathbf{E}[\Pi^{D,l}(q_1, p_1)]/\partial q_1 = 0$ and is also continuous. Because there is a unique positive root of $\partial \mathbf{E}[\Pi^{D,l}(q_1, p_1)]/\partial q_1 = 0$ when $p_1 > c + \alpha[\beta v(1+\theta) - c]$, this unique positive root corresponds to $q_1^+(p_1)$ rather than $q_1^-(p_1)$ for $p_1^J \le p_1 \le c + \alpha[\beta v(1+\theta) - c]$; that is, the unique positive root of $\partial \mathbf{E}[\Pi^{D,l}(q_1, p_1)]/\partial q_1 = 0$ when $p_1 > c + \alpha[\beta v(1+\theta) - c]$ evolves from $q_1^+(p_1)$ for $p_1^J \le p_1 \le c + \alpha[\beta v(1+\theta) - c]$. Otherwise, we would need another root corresponding to $q_1^+(p_1)$, which violates the fact that there is a unique positive root of $\partial \mathbf{E}[\Pi^{D,l}(q_1, p_1)]/\partial q_1 = 0$ when $p_1 > c + \alpha[\beta v(1+\theta) - c]$. So, $q_1^o(p_1)$ is continuous at $p_1 = c + \alpha[\beta v(1+\theta) - c]$. Finally, $q_1^o(p_1)$ jumps from 0 to $q_1^+(p_1^J)$ only at $p_1 = p_1^J$. □

Proof of Proposition 8. We have argued for the strategies L and H in the main body of this paper. Similar to the proof of Proposition 1, we can investigate the conditions under which a certain strategy is optimal. In case $[p_1 = R_f]$, following (12) and (15),

$$\begin{aligned} w(q_1, 1) &= \int_0^{((\beta v(1+\theta)-c)/(\beta v))q_1} 1 \times [\beta v(1+\theta) - \beta v\theta] f(x)\,dx + 0 \\ &= \beta v F\left(\frac{\beta v(1+\theta) - c}{\beta v} q_1\right). \end{aligned}$$

In case $[p_1 = v]$, if $\phi \le (\beta v\theta - c)/(\beta v(1+\theta) - c)$, we can compute $w(q_1, 0)$ in the same way as in $[p_1 = R_f]$; if $\phi > (\beta v\theta - c)/(\beta v(1+\theta) - c)$, $w(q_1, 0) = 0$ since the firm always uses the H strategy. □

Proof of Lemma 1. If there is an REE with $p_1 = v$ and $\chi = 0$, then we have $R_f < v$. Otherwise, if $R_f = v$, then $\chi = 1$. However, in case $[p_1 = v]$, with $\chi = 0$, we have $w(q_1, 0) = 0$ from Proposition 8, which leads to $R_f = v$, a contradiction with $R_f < v$. Thus, such an REE does not exist. □

Proving Proposition 9 requires two results stated and proved below as Lemmas 7 and 8.

Lemma 7 (High Innovation). *Under dual rollover, there exists a unique REE. In addition, a $\phi^{H,D}$ exists such that if $\phi \le \phi^{H,D}$, then the firm sets the high price $p_1^* = v$; otherwise, the firm sets the low price $p_1^* = R_f$.*

Proof of Lemma 7. If the firm needs to use the mixed strategy with $p_1 = R_f$, then its profit is $\alpha[\beta v(1+\theta) - c]\mathbf{E}(N)$. The firm can achieve the same profit by setting $p_1 = v$ and $q_1 = 0$. So an equilibrium with $p_1 = R_f$ using the mixed strategy is always dominated by an equilibrium with $p_1 = v$. Define $\mathbf{E}[\Pi^{D,h\text{-All}}(q_1^{D,h}(\phi), v; \phi)]$ as the firm's total profit with $p_1 = v$ across the three cases defined by $\phi$ in Proposition 6. We can show that $\mathbf{E}[\Pi^{D,h\text{-All}}(q_1^{D,h}(\phi), v; \phi)]$ is continuous and nonincreasing in $\phi$. If a unique $(q_1^{D,l}, p_1^{D,l}, \chi, W_c, R_f)$ satisfying the REE conditions except for (11) exists, $\mathbf{E}[\Pi^{D,l}(q_1^{D,l}, p_1^{D,l})]$ (see the proof of Proposition 7 for its expression) is constant in $\phi$. Otherwise, from Proposition 7, $\mathbf{E}[\Pi^{D,l}(q_1^{D,l}, p_1^{D,l})] = \alpha[\beta v(1+\theta) - c]\mathbf{E}(\mathbf{N})$ is also constant in $\phi$. Defining $\phi^{H,D}$ as $\phi^{H,D} = \inf\{\phi\colon \mathbf{E}[\Pi^{D,h\text{-All}}(q_1^{D,h}(\phi), v; \phi)] \le \mathbf{E}[\Pi^{D,l}(q_1^{D,l}, p_1^{D,l})]$, where $0 \le \phi < 1\}$, the result follows. □

Lemma 8 (High Innovation). *Under single rollover, either there exists a unique REE or the firm skips V1. In addition, there exists a $\phi^{H,S}$, $0 \le \phi^{H,S} \le (\beta v\theta - c)/(\beta v(1+\theta) - c)$, such that if $\phi \le \phi^{H,S}$, then the firm sets the high price $p_1^* = v$; otherwise, the firm sets the low price $p_1^* = R_f$.*

Proof of Lemma 8. From Lemma 1, the only possible REE is with $p_1 = R_f$ when $\phi > (\beta v\theta - c)/(\beta v(1+\theta) - c)$. So with $\phi > (\beta v\theta - c)/(\beta v(1+\theta) - c)$, if the firm needs to mix the stocking levels with $p_1 = R_f$, then the associated equilibrium cannot be dominated by an equilibrium with $p_1 = v$, unlike in the dual rollover case. Define

$$\begin{aligned} \phi^{H,S} &= \min\{\bar{\phi}, (\beta v\theta - c)/(\beta v(1+\theta) - c)\}, \quad \text{where} \\ \bar{\phi} &= \inf\{\phi\colon \mathbf{E}[\Pi^{S,h}(q_1^{S,h}(\phi), v; \phi)] \\ &\qquad \le \mathbf{E}[\Pi^{S,l}(q_1^{S,l}, p_1^{S,l})],\ \text{where } 0 \le \phi < 1\}, \end{aligned}$$

$\mathbf{E}[\Pi^{S,h}(q_1^{S,h}(\phi), v; \phi)]$ is the profit with $p_1 = v$ for $\phi \le (\beta v\theta - c)/(\beta v(1+\theta) - c)$, and $\mathbf{E}[\Pi^{S,l}(q_1^{S,l}, p_1^{S,l})]$ is the profit with $p_1 = R_f$. The result follows. □

Proof of Proposition 9. (i) When $\phi \le \phi^{H,S}$,

$$\begin{aligned} \mathbf{E}[\Pi^{D*}] &\ge \mathbf{E}[\Pi^{D,h}] = \mathbf{E}[\Pi^{D,h}(q_1^{D,h}, v)] \\ &> \mathbf{E}[\Pi^{D,h}(q_1^{S,h}, v)] \\ &> \mathbf{E}[\Pi^{S,h}(q_1^{S,h}, v)] = \mathbf{E}[\Pi^{S,h}] = \mathbf{E}[\Pi^{S*}]. \end{aligned}$$

The last inequality is because when $\phi \le \phi^{H,S} \le (\beta v\theta - c)/(\beta v(1+\theta) - c)$, the expressions of $\mathbf{E}[\Pi^{D,h}(q_1, v)]$ and $\mathbf{E}[\Pi^{S,h}(q_1, v)]$ are structurally the same, and the only difference between them lies in $\delta$ versus $\sigma$, and $\delta > \sigma$. So $\mathbf{E}[\Pi^{D*}] > \mathbf{E}[\Pi^{S*}]$ when $\phi \le \phi^{H,S}$. This implies $\phi^H > \phi^{H,S}$. From the proofs of Lemmas 7 and 8, $\mathbf{E}[\Pi^{D*}]$ is continuous in $\phi$, and $\mathbf{E}[\Pi^{S*}]$ is continuous in $\phi$ except that there may be a drop at $\phi = \phi^{H,S}$ if $\phi^{H,S} = (\beta v\theta - c)/(\beta v(1+\theta) - c)$. Furthermore, $\mathbf{E}[\Pi^{D*}]$ (respectively, $\mathbf{E}[\Pi^{S*}]$) is nonincreasing in $\phi$ when $\phi < \phi^{H,D}$ (respectively, $\phi < \phi^{H,S}$), and constant in $\phi$ when $\phi \ge \phi^{H,D}$ (respectively, $\phi \ge \phi^{H,S}$). As $\mathbf{E}[\Pi^{D*}] > \mathbf{E}[\Pi^{S*}]$ when $\phi \le \phi^{H,S}$, if there does not exist such a $\phi^H$, dual rollover is always better than single rollover. This proves Claim (i.a).

Next, we prove Claim (i.b). First, we can prove that if $\phi^{H,S} \ge \phi^{H,D}$, then $\phi^H$ does not exist. This is because when $\phi^{H,S} \ge \phi^{H,D}$ we have (a) for $\phi < \phi^{H,D} \le \phi^{H,S}$, $\mathbf{E}[\Pi^{D*}] > \mathbf{E}[\Pi^{S*}]$, and, (b) for $\phi \ge \phi^{H,D}$, $\mathbf{E}[\Pi^{D*}]$ is continuous and constant in $\phi$ and $\mathbf{E}[\Pi^{S*}]$ is nonincreasing in $\phi$. Facts (a) and (b) together show that $\mathbf{E}[\Pi^{D*}] > \mathbf{E}[\Pi^{S*}]$ for all $\phi$'s, and thus $\phi^H$ does not exist. So, if $\phi^H$ exists, we must have $\phi^{H,S} < \phi^{H,D}$. In addition, when $\phi > \phi^{H,D} > \phi^{H,S}$, both $\mathbf{E}[\Pi^{S*}]$ and $\mathbf{E}[\Pi^{D*}]$ are continuous and constant. So, if $\phi^H$ exists, we must have $\phi^H \le \phi^{H,D}$. Together with the fact $\phi^H > \phi^{H,S}$ from analysis above, we know $\phi^{H,S} < \phi^H \le \phi^{H,D}$.

When $\phi > \phi^H$, because $\mathbf{E}[\Pi^{S*}]$ is continuous and constant and $\mathbf{E}[\Pi^{D*}]$ is continuous and nonincreasing, we have $\mathbf{E}[\Pi^{S*}] \ge \mathbf{E}[\Pi^{D*}]$. When $\phi \le \phi^{H,S} < \phi^H$, we have $\mathbf{E}[\Pi^{D*}] > \mathbf{E}[\Pi^{S*}]$ from analysis above. Furthermore, as $\mathbf{E}[\Pi^{D*}]$ is nonincreasing while $\mathbf{E}[\Pi^{S*}]$ is constant when $\phi^{H,S} < \phi < \phi^H$ and $\mathbf{E}[\Pi^{D*}] = \mathbf{E}[\Pi^{S*}]$ when $\phi = \phi^H$, we have again $\mathbf{E}[\Pi^{D*}] > \mathbf{E}[\Pi^{S*}]$. So $\mathbf{E}[\Pi^{D*}] > \mathbf{E}[\Pi^{S*}]$ when $\phi < \phi^H$. Combining the cases above, we get Claim (i.b).

(ii) Under single rollover, if there is no REE, then the firm skips V1 (Lemma 8) and its total profit is $\alpha[\beta v(1+\theta)-c]\mathbf{E}(N)$. Since

$$\begin{aligned}\mathbf{E}[\Pi^{D*}] &\geq \mathbf{E}[\Pi^{D,h}] = \mathbf{E}[\Pi^{D,h}(q_1^{D,h}, v)]\\ &\geq \mathbf{E}[\Pi^{D,h}(0,v)] = \alpha[\beta v(1+\theta)-c]\mathbf{E}(\mathbf{N}),\end{aligned}$$

dual rollover is always better than single rollover. □

Proof of Proposition 10. Claim (i) is from the proof of Proposition 9: When $\phi \leq \phi^{H,S}$, that is, when $\mathbf{E}[\Pi^{S*}] = \mathbf{E}[\Pi^{S,h}]$, we have $\mathbf{E}[\Pi^{D*}] > \mathbf{E}[\Pi^{S*}]$. This shows that single rollover can outperform dual rollover only when $\mathbf{E}[\Pi^{S*}] = \mathbf{E}[\Pi^{S,l}]$. If $\mathbf{E}[\Pi^{S,l}] < \mathbf{E}[\Pi^{D,l}]$, then $\mathbf{E}[\Pi^{D*}] \geq \mathbf{E}[\Pi^{D,l}] > \mathbf{E}[\Pi^{S,l}]$, which means that $\phi^H$ does not exist. So $\mathbf{E}[\Pi^{S,l}] \geq \mathbf{E}[\Pi^{D,l}]$ is a necessary condition for $\phi^H$ to exist. Note that $\mathbf{E}[\Pi^{D*}] > \mathbf{E}[\Pi^{S*}]$ when $\phi \leq \phi^{H,S}$, and $\mathbf{E}[\Pi^{D*}]$ is continuous and constant in $\phi$ when $\phi \geq \phi^{H,D}$, but $\mathbf{E}[\Pi^{S*}]$ is nonincreasing in $\phi$ and $\mathbf{E}[\Pi^{S*}] = \mathbf{E}[\Pi^{S,l}]$ when $\phi \geq \phi^{H,S}$. So, if $\mathbf{E}[\Pi^{S,l}] \geq \mathbf{E}[\Pi^{D,l}]$, then such a $\phi^H$ must exist. □

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