THE EFFECTS OF SUPPLY UNCERTAINTY ON VOLUNTARY DISCLOSURE OF DEMAND

by

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This work is dedicated to God and family, particularly my mother, Barbara, and my father, Jeff, for sustaining me throughout this journey.

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Supply disruptions are increasing in magnitude and frequency imbuing our global economy with greater levels of supply uncertainty. In Cournot duopoly, I examine how supply uncertainty affects the voluntary disclosure of a firm's private demand information. In addition to supply uncertainty, I allow for variation in product substitutability (substitutability effect) and in the degree to which private demand information affects the rival firm's demand (spillover effect). I show that the partial disclosure equilibrium depends on a firm's competitive advantage characterized by the relative importance of the spillover effect, substitutability effect, and the firms' supply uncertainty. In particular, I show that absent supply uncertainty the firm with private information about product demand discloses good news (bad news) when the substitutability effect (spillover effect) is most pronounced. By incorporating the role of supply uncertainty in a firm's disclosure decision, I demonstrate that supply uncertainty diminishes the substitutability effect and enhances the spillover effect. I show that with supply uncertainty the firm possessing private information about product demand discloses good news (bad news) when

the supply uncertainty does not (does) mute the relative dominance of the substitutability effect over the spillover effect. I also show that the disclosure region for both good news and bad news, in general, decreases with increases in supply uncertainty of both the firm and its rival.

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CURRICULUM VITAE

CHAPTER 1

INTRODUCTION

Supply uncertainty – the possibility that a firm's order quantity may not be fulfilled by its supplier in time to meet customer demand – has reached a critical mass in recent years causing firms to regard their exposure to such uncertainty as an increasingly important factor in their strategic decision making. My study examines the effects of supply uncertainty on a firm's (strategic) decision to voluntarily disclose its private demand information to a rival firm. Prior literature has demonstrated that a firm's decision to disclose its private demand information to its rival is multifaceted and considers various elements of the competitive environment. Clinch & Verrecchia (1997) establish a link between product substitutability (used to model competitive intensity) and a firm's decision to voluntarily disclose its private demand information to its rival. Bagnoli & Watts (2015b) also establish a link between product substitutability and a firm's decision to voluntarily disclose its private demand information to its rival while considering the added feature that the private demand information may also be informative of the rival's own demand (i.e., there is meaningful information spillover). In my study, I examine how supply uncertainty in addition to product substitutability and information spillover affects a firm's decision to voluntarily disclose private demand information to its rival. That is, I consider a firm's decision to voluntarily disclose its private demand information in the traditional competitive environment consisting of product substitutability and information spillover with the added feature of supply uncertainty, i.e., the potential that a firm's order quantity is not entirely fulfilled.

The motivation for my study stems from the increased attention being given to supply uncertainty caused by recent disruptions in the supply chains of firms worldwide. For example, in 2017, Hurricane Harvey hit Louisiana and Texas, disrupting some of the largest US oil refineries and petrochemical plants. This disruption reduced the available supply of key plastics and resins for a range of industries creating shortages in the market and greater supply uncertainty (Contessa, 2022). More recently, the COVID-19 pandemic forced factories around the world that were integral to the development of integrated circuits to shut down increasing product lead times and supply uncertainty in industries such as phones, laptops, cars, watches and refrigerators. The McKinsey Consulting Group, in their August 6, 2020 report, noted that the frequency and magnitude of events causing supply uncertainty is increasing and that this is causing firms to re-examine the impact of supply uncertainty on various aspects of their operations (Lund et al., 2020).

I build on the prior literature to examine the consequences of supply uncertainty on voluntary disclosure of private demand information. Specifically, I examine a traditional one-period Cournot duopoly where a firm and its rival face demand uncertainty with a linear inverse demand function. The firm privately observes a signal that is informative about its demand. I allow the degree to which the firm's private demand information is relevant to its rival's demand, i.e., the degree of information spillover inherent in the signal, to vary. In addition, I allow for the products being sold by the firm and its rival to be either perfect or imperfect substitutes. After privately observing the signal on its demand, the firm chooses to voluntarily disclose or conceal its private demand information. If the firm discloses its private demand information, it incurs a fixed cost from issuing the disclosure. Similar to Verrecchia (1983, 1987) and Bagnoli & Watts

(2013), this cost prevents what is referred to in the literature as the unraveling result, which is full disclosure of the firm's private demand information (Grossman, 1981; Milgrom, 1981).

To product substitutability, information spillover, and private (demand) information, I add the key issue of supply uncertainty. Specifically, the firm and its rival place orders with suppliers that may not fulfill said orders. This is because the firms' suppliers face uncertain input availability or uncertain yields. Thus, when the firm and its rival choose their order quantities, they do so knowing that their suppliers may fall short in fulfilling the entire order quantity, and this results in the number of units the firm and its rival sell to their customers potentially being lower than their original orders. This characterization of supply uncertainty is similar to Fang and Shou's (2015) model. I assume that the supply market is competitive such that the unit input costs are normalized to zero.

The sequence of events unfold as follows. First, the firm learns its private demand information. Second, the firm chooses to voluntarily disclose or conceal its demand information. Third, the firm and its rival choose their order quantities. Last, the suppliers deliver a proportion of the order quantities, that is sold by the firm and its rival to their customers and the profits are realized.

Before discussing the voluntary disclosure results, I provide an overview of how supply uncertainty interacts with the firm's private demand information via information spillover and product substitutability to affect the firm's competitive advantage. Note that the extent to which the firm's private demand information gives it a competitive advantage depends on the extent to which products are substitutes and the degree to which spillover is inherent in the information. To see this, suppose the firm's private demand information indicates above-average customer demand; the firm increases its order quantity and, because the firms' products are substitutes, this causes the rival to back down – I refer to this as the substitutability effect. Conversely, because information spillover implies the firm's private demand information also has relevance to the rival's demand, when the firm's private demand information indicates above-average demand, information spillover implies the rival will increase its order quantity causing the firm to back down – I refer to this as the spillover effect. The combined effect of product substitutability and information spillover on the firm's order quantity determines its competitive advantage. When the substitutability effect dominates the spillover effect, the firm's order quantity is increasing in its private demand information and the firm's competitive advantage is therefore high. The converse is true when the substitutability effect is dominated by the spillover effect (see Bagnoli & Watts, 2015b). Importantly, supply uncertainty diminishes the substitutability effect and enhances the spillover effect resulting in a lower order quantity, and thus, chips away at the firm's competitive advantage. The reason for this outcome stems from the underlying payoff structure of the firm. Specifically, the firm's linear inverse demand function implies concavity in the firm's payoff function, and this, in turn, implies an aversion to (supply) risk. Given the firm's aversion to supply risk, when the firm's supply uncertainty is high, rather than increasing its order quantity at the risk of creating a glut of product in the market which would reduce the price the firm is able to charge for its products, the firm decreases its order quantity. A lower order quantity weakens the firm's ability to exploit its private demand information, and this gives the rival firm opportunity to step in and increase its own order quantity. Thus, when the firm's supply uncertainty is high the competitive advantage arising from the substitutability and spillover effects is muted.

Given the firm's disclosure choice is made after privately observing the signal on its demand, I show that a partial disclosure equilibrium arises where the firm either discloses good news (and conceals bad news) or discloses bad news (and conceals good news) information, voluntarily, about its demand – here, good news and bad news refers to signals of high and low private demand information, respectively. Specifically, I first show that there is a threshold of private demand information above which the firm will voluntarily disclose its private demand information to its rival, and below which the firm will conceal its private demand information. To develop the intuition behind my partial (good news) disclosure equilibrium, consider first the case when there is no supply uncertainty: the firm discloses good news and conceals bad news when its competitive advantage is high, i.e., when the substitutability effect dominates the spillover effect. This is because when the substitutability effect is sufficiently large relative to the spillover effect, the firm can use disclosure to increase its order quantity and stave-off its rival's order quantity over the region where its private demand information is high. In contrast, the firm discloses bad news and conceals good news when its competitive advantage is low, i.e., when the substitutability effect is dominated by the spillover effect. In this case, I show that there is a threshold of private demand information below which the firm will voluntarily disclose its private demand information to its rival, and above which the firm will conceal its private demand information. This is because when the substitutability effect is sufficiently small relative to the spillover effect, the firm can use disclosure to increase its order quantity and stave-off its rival's order quantity over the region where its private demand information is low. This tradeoff between the substitutability and spillover effects in my model is similar to how product substitutability and information spillover have been shown in the prior literature to affect voluntary disclosures (e.g., Bagnoli & Watts, 2015b; Arya, Mittendorf, & Yoon, 2019). However, the prior literature does not consider voluntary disclosure after observing the private demand information. As such, I extend the insights from the prior literature to a setting with partial disclosure equilibriums.

As has been discussed, supply uncertainty decreases the firm's competitive advantage. Given this, there are, essentially, two cases to consider in determining how supply uncertainty affects the firm's equilibrium disclosure outcomes; in each case I assume that the substitutability effect, absent supply uncertainty, is large relative to the spillover effect. First, when supply uncertainty is low, the decrease in the firm's competitive advantage is minimal and the substitutability effect dominates the spillover effect. It follows that the firm's competitive advantage is high in this circumstance and, for reasons aforementioned, the firm chooses to disclose good news to its rival (conceal bad news). Second, when supply uncertainty is high, the decrease in the firm's competitive advantage is significant and the substitutability effect is now dominated by the spillover effect. In this circumstance, the firm's competitive advantage is effectively low and, for reasons aforementioned, the firm chooses to disclose bad news to its rival (conceal good news). Overall, I show that depending on whether supply uncertainty is low or high relative to the combined effects of product substitutability and information spillover, either bad news only or good news only is voluntarily disclosed.[1]

I also examine the effects of supply uncertainty on the equilibrium disclosure thresholds. In general, the firm's supply uncertainty has two effects. In the setting where the firm's supply

[1] Noticeably absent from this analysis is the rival's supply uncertainty. In my setting where only the firm has private demand information, the rival's supply uncertainty does not determine whether the firm's competitive advantage is high or low.

uncertainty is low enough such that the competitive advantage of the substitutability effect dominates the spillover effect, the threshold for voluntarily disclosing good news increases with increases in the firm's supply uncertainty. Specifically, as the firm's supply uncertainty increases its competitive advantage diminishes and for this reason the good news disclosure threshold increases. In the setting where the supply uncertainty is high enough such that the competitive advantage of the substitutability effect is dominated by the spillover effect, the threshold for voluntarily disclosing bad news decreases with increases in the firm's supply uncertainty when the spillover effect is large. As the firm's supply uncertainty increases it further diminishes the firm's competitive advantage, which leads to the bad news threshold to decrease. Overall, in each of these settings, increasing the firm's supply uncertainty decreases the region over which the firm finds it advantageous to disclose its good news or bad news private demand information and this, in turn, decreases the likelihood of voluntary disclosure.

Collectively, these results provide useful testable insights. First, I demonstrate that examining the relationship between supply uncertainty and incidences of disclosure, in general, may not yield statistically significant results because in the presence of supply uncertainty both good news and bad news disclosures are possible. Second, my results suggest that it is important to distinguish between good news and bad news disclosures when examining the relationship between supply uncertainty and voluntary disclosures. Furthermore, it is also important to consider the product substitutability and information spillover effects. Specifically, good (bad) news disclosures occur in markets characterized by high (low) product substitutability, low (high) information spillover and low (high) supply uncertainty. Third, the comparative static results on the disclosure threshold suggest that the likelihood of good news and bad news disclosures are negatively related to supply uncertainty.

I contribute to the extant literature on voluntary disclosures by building on Clinch and Verrecchia (1997). In their study, Clinch and Verrecchia (1997) examine a Cournot duopoly with imperfect substitutes and a private signal on demand that is of pure common value (meaning there is perfect information spillover and the informed firm's competitive advantage is always low) and show that no voluntary disclosure occurs for extremely good news or extremely bad news; voluntary disclosure occurs only in the intermediate news region. While the private signal on demand in their study may indicate additional positive or negative demand, the private signal on demand in my study considers only additional positive demand. Because of this, what they characterize as intermediate good news disclosure is, in effect, equivalent to what I characterize as bad news disclosure. In stark contrast to their result, I find that "extremely" good news disclosure can also occur when the common value of the firm's private demand information is sufficiently low, i.e., when the spillover effect is sufficiently small. However, supply uncertainty effectively increases the common value of the firm's private demand information, which weakens the firm's incentive to disclose good news and strengthens its incentive to disclose bad news demand information. Thus, when the informed firm's supply uncertainty is sufficiently high only bad news disclosure occurs.

In an extension, I consider also the case in which the firm commits to a disclosure strategy prior to observing its private demand information. In this setting, I extend Bagnoli & Watts (2015b) whose information structure also considers imperfect substitutes and intermediate values of information spillover. In the absence of supply uncertainty, my disclosure rule mirrors theirs:

fully disclose when the substitutability effect is sufficiently large relative to the spillover effect, otherwise do not disclose. For reasons similar to those aforementioned in the previous case, layering in supply uncertainty puts pressure on the firm to conceal its private demand information. In particular, when the substitutability effect is relatively high and the firm’s supply uncertainty is low, the firm’s competitive advantage is high and full disclosure is optimal. Increasing the firm’s supply uncertainty diminishes the substitutability effect and enhances the spillover effect tipping the scales in favor of concealing the firm’s private demand information. Hence, when supply uncertainty is sufficiently high, the firm’s competitive advantage is effectively low and it conceals it's private demand information.

CHAPTER 2

LITERATURE REVIEW

The subject of voluntary disclosures in product markets has been studied across multiple disciplines beginning with Gal-Or (1985, 1986) and Raith (1996) in economics and Darrough (1993) and Sankar (1995) in accounting. An important and distinguishing feature of studies on voluntary disclosures is the juncture of a firm's disclosure decision relative to when it obtains its private information (A. Beyer et al., 2010). To be specific, firms may choose what to do with their privately known information – disclose or conceal – either after or before the information is obtained. Studies that examine ex post disclosure incentives consider settings in which a firm chooses to disclose or conceal its private information after obtaining it. The standard disclosure equilibrium in studies on ex post disclosure incentives is full disclosure per the unraveling result (Grossman and Hart, 1980; Grossman, 1981; Milgrom, 1981; and Milgrom and Roberts, 1986). When the unraveling result fails to hold, such as when there are costs of disclosure or uncertainty about whether the firm actually has any information at all, a partial disclosure equilibrium may arise. For example, Bagnoli & Watts (2013) examine ex post disclosure incentives when there is a fixed cost of disclosure. Their study is the first to consider a setting in which two Cournot competitors selling imperfect substitutes possess private demand information exclusively about a rival. That is, their information structure departs from the pure private value and pure common value structures typical in the prior literature. They find that when markets are sufficiently similar each firm discloses bad news about its rival and withholds good news.

Jansen (2008) examines ex post disclosure incentives when there is uncertainty about whether a rival firm has private information. Specifically, Jansen (2008) considers a setting in which two Cournot competitors selling imperfect substitutes can invest in acquiring information about a common value demand intercept. Because firms may fail to acquire information even when they invest in information acquisition, a partial disclosure equilibrium may arise. When the rival is uncertain about whether the firm managed to acquire private demand information the disclosure equilibrium is one in which bad news (a low demand intercept) is disclosed while good news (a high demand intercept) is withheld.

Other more recent studies to feature a partial disclosure equilibrium include Arya & Ramanan (2022) who demonstrate that by judiciously managing its disclosure about cost information in one market, a firm can manage information about common-value demand in another market that it has a financial interest in via. a licensee. They find that when the licensee's incumbency advantage is sufficiently high, disclosing low-cost information and concealing high-cost information is the optimal disclosure strategy. Additionally, Cheynel & Ziv (2021) develop a micro-foundation in a Cournot game to support the proprietary cost hypothesis introduced by Verrecchia (1983, 1990). In particular, Cheynel & Ziv (2021) explicitly derive the (proprietary) cost as a function of market structure and find that when the market is sufficiently competitive good news disclosure occurs. However, as the market increases in competitive intensity, withholding disclosure in high common-value demand states preserves the informed firm's competitive advantage making non-disclosure more attractive, thus, leading to less disclosure.

Closest to my own study is Clinch & Verrecchia (1997) who examine a Cournot duopoly with imperfect substitutes and a private signal on demand that is of common-value. They show that each firm withholds disclosure of its private demand information for extremely good news and extremely bad news; voluntary disclosure occurs only in the intermediate news region.

Furthermore, Clinch & Verrecchia (1997) develop a link between competitive intensity (modeled by product differentiation) and disclosure practices. They find that as the competitive intensity increases both the range of information disclosed and the (ex ante) likelihood of information being disclosed, decrease.

In each of the aforementioned studies various facets of competition are considered within the context of a firm's disclosure choice, e.g., product substitutability, information acquisition costs, uncertainty about the existence of private information, private demand information that is of common-value, private demand information about a rival, having multiple rivals etc. I contribute to the ex post voluntary disclosure literature by extending Clinch & Verrecchia (1997) in two ways. First, I differentiate myself by considering an additional facet of competition not yet considered in the voluntary disclosure literature, namely, supply uncertainty, and second, I allow for information spillover effects.

Studies that examine ex ante disclosure incentives consider settings in which a firm chooses to disclose or conceal its private information before obtaining it. The literature that examines ex ante disclosure incentives for firms competing in product markets is extensive with recent work focusing on disclosure-related subtleties, including supply market effects (Arya, Mittendorf, & Yoon, 2019); capital market effects (Xiong & Yang, 2021); bias in disclosure (Bagnoli and Watts 2010; Friedman, Hughes, and Saouma 2016); dual-purpose firms (Arya, Mittendorf, & Ramanan, 2019); third-party analysts (Arya & Mittendorf, 2007); delegation of disclosure (Theilen, 2007; Bagnoli & Watts, 2015a); investment considerations (Arya, Frimor, Mittendorf, & Pfeiffer, 2023); and, disclosure of competitive intelligence (Bagnoli & Watts, 2015b), to name a few. The generalized setup of Bagnoli & Watts (2015b) delineates the key trade-offs of the substitutability and spillover effects salient in my study. In an extension, my study builds on Bagnoli & Watts (2015b) by jointly considering the effects of product substitutability, information spillover, and supply uncertainty on a firm's ex ante disclosure decision.

Finally, there is a vast literature in operations management (OM) that examines supply uncertainty. The extant literature on supply uncertainty in OM can be grouped into three categories: (i) the proportional random-yield model, which assumes supply uncertainty is a random function of the input level (e.g., Deo & Corbett, 2009; Fang and Shou, 2015), (ii) the stochastic lead-time model, which models the lead-time as a random variable (Bagchi, Hayya, & Chu, 1986; Song, 1994; Zipkin, 2000; Wang & Tomlin, 2009), and (iii) the supply disruption model, which typically models supply uncertainty as the supplier being in one of two states: "up" or "down" (e.g., Arreola-Risa & DeCroix, 1998; Snyder & Shen, 2006).[2] My study utilizes a variation of the proportional random-yield model. The proportional random-yield model has been used in a variety of contexts in the OM literature to examine the competitive effects of supply uncertainty; particularly relevant to this study are those studies that examine the effects of supply uncertainty when firms compete in quantity, i.e., Cournot competition. For example, Deo & Corbett (2009) examine in a Cournot oligopoly of symmetric firms the impact of supply uncertainty on a firm's production quantity, total industry output, and the incentive for additional firms to enter the (influenza vaccine) market. They find that supply uncertainty leads to a diminution of firm and industry output and can contribute to a high degree of firm concentration in the industry. Jansen & Ozaltin (2016), also motivated by the US influenza vaccine market,

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[2] Additional examples of studies utilizing the stochastic lead-time model and the supply disruption model are given in the literature review section of Fang & Shou (2015).

examine a Cournot oligopoly of asymmetric firms facing capacity constraints and supply uncertainty. Each firm is assigned a score that ranks it among other firms according to its capacity, unit production cost, random yield mean, and variance. An equilibrium threshold is derived based on this score where firms with scores below the threshold produce below or equal to their capacity and firms with scores above the threshold produce at full capacity. Jansen & Ozaltin (2016) argue that their result provides a decision aid tool for modeling endogenous market response to interventions such as government subsidies aimed at eliminating supply side inefficiencies.

Tang & Kouvelis (2011) investigate the benefits of supplier diversification from dual-sourcing, symmetric (Cournot) duopolists. Their results indicate that when the suppliers' supply uncertainty is strongly negatively correlated, dual sourcing increases the expected market output and improves the firms' expected profits over sole sourcing. In essence, dual sourcing adds value by reducing a firm's exposure to supply uncertainty thereby reducing the market output inefficiencies caused by it – this is referred to by the authors as the diversification effect.

Jung (2020) study firms' sourcing strategies at a global level – offshore and onshore supply bases – under supply and demand uncertainty. Their work is related to Tang & Kouvelis (2011), though differing in several aspects. First, Tang & Kouvelis (2011) analyze symmetric suppliers whereas Jung (2020) consider asymmetric suppliers in terms of market responsiveness (onshore supply bases being more responsive than offshore supply bases); this causes new issues in sourcing decisions such as information asymmetry/acquisition and sequential order quantity decisions. Second, the focus of Tang & Kouvelis (2011) is on the value of dual-sourcing under competition, whereas Jung (2020) analyzes the entire sourcing game between rivals and highlights a potential risk of dual-sourcing in a competitive environment. Specifically, Jung (2020) observe that a firm and its rival always choose dual sourcing in equilibrium, which is optimal, except when demand uncertainty is low and supply uncertainty is highly correlated between onshore (offshore) suppliers. In this scenario, a prisoner's dilemma arises, and the firms are better off single-sourcing their key inputs.

Zhang et al. (2019) likewise investigate the benefits from supplier diversification for dual-sourcing (Cournot) duopolists except they differentiate themselves by allowing the a firm's competitor to also (potentially) be one of its suppliers. The firm may, in addition to or in lieu of the competitor-supplier, source a key input from a non-competitor supplier that faces supply uncertainty. In equilibrium, when supply uncertainty is high, the firm chooses a dual-sourcing strategy; this creates price competition between suppliers driving down the cost of the key input for the firm. When supply uncertainty is low, competition between suppliers increases and the competitor-supplier quits supplying the firm with its key input. In this scenario, the firm relies solely on the non-competitor supplier and is better off than if it were to rely solely on the competitor-supplier, but still prefers a dual-sourcing strategy.

Fang and Shou (2015) in a Cournot duopoly between supply chains subject to supply uncertainty examine how the levels of supply uncertainty and competitive intensity effect the equilibrium decisions of a retailer's order quantity, contract offering (between a firm and its supplier), and a firm's supply chain centralization choice. They find that a retailer should increase its order quantity if the rival's supply becomes less reliable or if its own supply becomes more reliable; and, they find that supply chain centralization is a dominant strategy, the desirability of which is enhanced by high supply uncertainty or low supply chain competition.

Chen et al. (2020) study the implications of supply uncertainty in firms' horizontal mergers and acquisitions decisions in a Cournot oligopolistic market. In particular, Chen et al. (2020) provide a cogent explanation for the Cournot merger paradox given in Salant et al. (1983) which is that even without a marginal cost reduction, firms facing high supply uncertainty can obtain a competitive advantage and increased profits by merging when doing so results in a statistical reduction of supply uncertainty. For this reason, Chen et al. (2020) conclude that firms facing high supply uncertainty are likely to engage in mergers even when the marginal cost reduction is not significant.

Cheng et al. (2021) consider a two-echelon supply chain network where the downstream retailers compete and quantity and place orders with the upstream suppliers who compete in (wholesale) price and face supply uncertainty. The equilibrium is such that suppliers facing lower levels of supply uncertainty command higher wholesale prices, receive larger orders from their retailers, and are more profitable, in expectation.

Lee & Lu (2015) study the inventory competition model (see Parlar, 1988) under supply uncertainty to obtain insights into how supply uncertainty affects firms' ordering decisions and expected profits in equilibrium. Lee & Lu (2015) consider the case when firms can invest in decreasing their supply uncertainty prior to competing in quantity. Their results demonstrate that quantity and reliability serve as complimentary instruments in inventory competition – that is, firms facing lower levels of supply uncertainty increase their production quantity, and this prevents their competitors from doing so. Lee & Lu (2015) also find that the inventory competition game is submodular under some assumptions. Their results further demonstrate that competition in quantity can discourage firms from seeking improvements in their supply uncertainty.

To the best of my knowledge, the operations management literature has not explored the effects of supply uncertainty on voluntary disclosures of private demand information between firms competing in quantity, i.e., Cournot competition.

CHAPTER 3

MODEL

Consider a one-period, Cournot competition among two firms, indexed $i = F, R$. Firm i faces a linear inverse demand function $p_i = A_i - \acute{q}_i - k\acute{q}_j$ for $i, j = F, R$, $i \neq j$, where p_i denotes firm i's retail price, A_i is it's demand intercept, $k \in (0,1]$ is the degree of product substitutability, and $\acute{q}_i$ is the quantity firm i makes available for sale.

Each firm faces underlying uncertainty in demand that is reflected in its own demand intercept by $A_i = a + \gamma_i d_F$. The demand uncertainty in a firm's demand intercept stems from uncertain demand information tied to firm F, denoted d_F, and is assumed to be distributed uniformly between zero and $\bar{d}_F$.[3] The term γ_i, $\gamma_i \in [0,1]$, measures the sensitivity of firm i's demand to the uncertain demand information tied to firm F, that is, $\gamma_i = \frac{\partial A_i}{\partial d_F}$. I normalize $\gamma_F = 1$ and allow γ_R to vary to reflect the fact that a firm's demand is (weakly) more impacted by its own demand information than by information obtained from a competitor. Intuitively, as in Bagnoli and Watts (2015), the parameter γ_F can be thought of as representing the usefulness of firm F's demand information in determining its own demand and the parameter γ_R can be thought of as the extent to which there is information spillover. Thus, at one end of the spectrum, when $\gamma_R = 0$, firm F's uncertain demand information d_F is a private-value shock to firm F's demand and does not affect the demand of firm R. By way of example, consider competition

[3] I consider the case when only firm F is endowed with private information on demand uncertainty; and, I assume that demand uncertainty is distributed uniformly for analytical simplicity, i.e., to obtain closed-form solutions for the equilibrium order quantities.

between FORLOH and KUIU in the market for hunting jackets. If FORLOH's market research indicates that consumers are willing to pay more for Made in America branded gear, then this information (positively) affects FORLOH's demand, but not KUIU's demand, whose products are made overseas. At the other end of the spectrum, when $\gamma_R = 1$, firm F's demand uncertainty is a common-value shock to demand affecting both firm F's and firm R's demand symmetrically. In keeping with the example of competition between FORLOH and KUIU, if FORLOH's market research instead indicates that camouflage is en vogue and consumers are therefore willing to pay more for hunting jackets, in general, then the market research information affects KUIU's demand with the same sensitivity as it does its own demand. In summary, the parameter γ_R represents the effect information spillover has on firm R's demand and, thus, captures the usefulness of firm F's demand information to firm R in determining its own demand.

Prior to choosing its order quantity, firm F privately observes a signal, denoted d_F, resolving the uncertainty in its demand. After privately observing signal d_F, and before choosing its order quantity, firm F faces the decision to disclose its private demand information to its rival, firm R, or to conceal it. If firm F discloses its private demand information it incurs a fixed cost, denoted c, from issuing the disclosure. The upper bound on c is given by Assumption (IS), below.

$$c < \frac{k\Delta_F \bar{d}_F}{[4\Delta_F\Delta_R - k^2]^2}\left\{a[2\Delta_R - k]\left|\gamma_R - \frac{k}{2\Delta_F}\right| - \frac{1}{4}k\bar{d}_F\left[\gamma_R - \frac{k}{2\Delta_F}\right]^2\right\} \tag{IS}$$

In my analysis, the fixed cost of disclosure can be thought of as the cost incurred to have an outside auditor verify the disclosure.[4] Knowing that disclosures will be scrutinized by an outside auditor and assuming the penalty for falsifying disclosures is prohibitively costly, if firm F discloses its private demand information it does so truthfully (see Verrecchia, 1983; 1997; Bagnoli and Watts, 2013; Arya et al., 2022).

Each firm procures products from a competitive supply market to sell in the final (retail) product market. Unit costs paid to the supplier are normalized to zero. The supply market is characterized by uncertain input availability or uncertain yields, resulting in the number of $\acute{q}_i$ sold being uncertain. Following Fang and Shou (2015), I let the quantity sold $\acute{q}_i$ equal $\alpha_i Q_i$ where $\alpha_i \in [0,1]$ represents the supply yield rate and is a random variable with mean μ_i and variance σ_i^2 and Q_i is the quantity that is ordered by firm i. While neither the firm nor its rival is aware of the quantities that they will get from their suppliers, the distribution of each firm's supply uncertainty is common knowledge. This formulation of supply uncertainty is similar to what is referred to in the operations management literature as the proportional random yield model and has been considered in a variety of settings to reflect potential differences in target production quantities and actual production quantities (e.g., Deo & Corbett, 2009). In these settings, the production yield is uncertain and thus may not meet the target production quantities. It follows in this study that the quantity delivered by a supplier equals the quantity sold by the retailer and is a random variable. Thus, my representation of supply uncertainty as an uncertain

[4] Alternatively, the disclosure-related cost (c) may be interpreted as disclosing demand information that may be used by regulators, labor unions, or employees in a way that is harmful to firm F's profits. For example, employees may demand higher wages when they believe the firm is doing well. In any case, assuming a fixed cost of disclosure is a modeling convenience for sustaining a partial disclosure equilibrium similar to Bagnoli & Watts (2013).

proportion of the order quantity reflects the possibility that not all of the order quantity is procured and sold.[5] To focus on supply uncertainty in an analytically tractable fashion, I assume that firms F and R do not have backup suppliers to make good on any shortfall in supplies.

The firms have common priors about the information structure presented in the model and I assume that supply uncertainty across firms F and R as well as the demand uncertainty of firm F are independent. As is standard, I assume the intercept of the inverse demand function, a, is sufficiently large to ensure positive order quantities and expected profits; specifically, I assume that $a > 2\bar{d}_F$.[6]

The sequence of events unfold as follows:

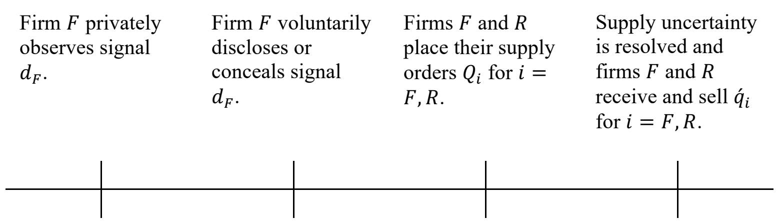

Figure 1. Timeline.

[5] Normalizing unit costs of inputs to zero makes it tempting to consider the firm and its rival ordering infinitely many units from their suppliers. However, in such a case, the linear inverse demand function will depress the retail product price, such that it is not a viable strategy.

[6] In the appendix, I refer to this as Assumption (PQ).

CHAPTER 4

ANALYSIS & RESULTS

In the Figure 1 Timeline, note that firm F first privately observes the signal about its demand and chooses to disclose or conceal it; then both firms choose their equilibrium order quantities. Accordingly, I work backwards to determine firm F's disclosure choice by first characterizing the equilibrium order quantities for a given disclosure choice and then I compare expected profits under each to determine the conditions under which a particular disclosure choice arises in equilibrium. When characterizing a firm's equilibrium order quantity and deriving its expected profit, each firm has rational expectations about the other firm. This implies that the act of concealing firm F's private demand information, given it has observed the signal d_F, in and of itself can be telling about the value of firm F's private demand information. Hence, a Perfect Bayesian Equilibrium is determined based on identifying a set of strategies for the firm and its rival, each of which are best responses given beliefs, and beliefs are consistent with Bayesian updating given disclosure choices.

4.1 Equilibrium Order Quantity

Firms F and R choose their order quantities to maximize their (conditional) expected profits. A firm's expected profit is conditional on firm F's disclosure choice $\Omega \in \{D, \phi\}$, where D indicates firm F discloses its private demand information and ϕ indicates firm F conceals it. For a given realization of signal d_F and for firm R's (posterior) belief, denoted b_F, about the value of signal d_F, firm F's conditional expected profit is given by Equation (1'). Note that firm R's belief is defined as the expectation of firm F's private demand information conditional on firm

F's disclosure choice, i.e., $b_F = E[d_F|\Omega]$. Here, the notation $\Omega = D$ is used to denote the range of d_F-values implied by firm F's disclosure choice over which the true value of signal d_F is expected to be disclosed, and $\Omega = \phi$ is used to denote the range of d_F-values implied by firm F's disclosure choice over which the true value of signal d_F is expected to be concealed. It is without loss of generality to restrict attention to the posterior mean because, as will be seen, the firms' best response functions are linear in firm F's private demand information, that is, signal d_F.

$$\Pi_F^{\Omega}(d_F, b_F) = E_{\alpha_F,\alpha_R}\{p_F \acute{q}_F - 1^{\Omega} c\} \quad (1')$$

Firm F's conditional expected profit given by Equation (1') is the revenue it generates, made up of its retail price (p_F) and quantity sold ($\acute{q}_F$), less the fixed cost of issuing a disclosure (c), where 1^{Ω} is an indicator variable that takes on the value of 1 (0) when firm F discloses (conceals) its private demand information. The expectation operator is over each firms' uncertain supply yield rate (α_F, α_R). Substituting for the retail price (p_F) and quantity sold ($\acute{q}_F$), taking expectation over each firms' uncertain supply yield rate (α_F, α_R), and using the coefficient of variation, i.e., $\Delta_i = \left[1 + \frac{\sigma_i^2}{\mu_i^2}\right]$ for $i = F, R$, the conditional expected profit for firm F is given by Equation (1).[7] In writing firm F's conditional expected profit, $\hat{Q}_R(b_F)$ is used to denote firm F's Cournot conjecture of firm R's quantity choice recognizing that firm R can condition its quantity choice on its belief about the value of firm F's private demand information. Thus, firm F's expected profit is affected by its private demand information, i.e., signal d_F, in two ways, the first being a direct effect in which the demand intercept a is augmented by a realization of signal

[7] Since I focus on supply uncertainty, I discuss changes in Δ_i as reflecting changes in σ_i^2. As such, in the discussions, I hold μ_i constant and vary σ_i^2 (see Chen et al., 2020).

d_F and the second being an indirect effect in which firm R's order quantity is shaped by its belief about the value of signal d_F.

$$\Pi_F^{\Omega}(d_F, b_F) = E_{\alpha_F,\alpha_R}\{p_F \acute{q}_F - 1^{\Omega}c\}$$

$$= E_{\alpha_F,\alpha_R}\{[a + d_F - \alpha_F, Q_F - k\alpha_R \hat{Q}_R(b_F)]\alpha_F Q_F - 1^{\Omega}c\}$$

$$= [(a + d_F)\mu_F - (\mu_F^2 + \sigma_F^2)Q_F - k\mu_F\mu_R \hat{Q}_R(b_F)]Q_F - 1^{\Omega}c$$

$$= \left[a + d_F - \left(1 + \frac{\sigma_F^2}{\mu_F^2}\right)\mu_F Q_F - k\mu_R \hat{Q}_R(b_F)\right]\mu_F Q_F - 1^{\Omega}c$$

$$= [a + d_F - \Delta_F \mu_F Q_F - k\mu_R \hat{Q}_R(b_F)]\mu_F Q_F - 1^{\Omega}c \qquad (1)$$

Similarly, firm R's conditional expected profit is given by Equation (2). Notice that the expectation operator in Equation (2) is likewise over each firms' uncertain supply yield rate (α_F, α_R) but, additionally, also over firm F's private demand information d_F since firm F may choose to conceal this information from its rival, firm R. In writing firm R's conditional expected profit, $\hat{Q}_F(d_F, b_F)$ is used to denote firm R's Cournot conjecture of firm F's quantity choice recognizing that firm F can condition its quantity choice on both its private demand information and its rival's belief. Thus, here, too, firm F's private demand information has a direct and indirect effect on (firm R's) conditional expected profits. The direct effect on firm R's conditional expected profit comes from augmenting the demand intercept a by $\gamma_R b_F$, the portion of firm F's private demand information relevant to firm R's demand, and the indirect effect comes from the way in which firm R's belief b_F shapes firm F's order quantity.

$$\Pi_R^{\Omega}(d_F, b_F) = E_{\alpha_F,\alpha_R,d_F}\{p_R \acute{q}_R\}$$

$$= E_{\alpha_F,\alpha_R,d_F}\{[a + \gamma_R \tilde{d}_F - \alpha_R Q_R - k\alpha_F \hat{Q}_F(d_F, b_F)]\alpha_R Q_R\}$$

$$= \left[(a + \gamma_R b_F)\mu_R - (\mu_R^2 + \sigma_R^2)Q_R - k\mu_R\mu_F E_{d_F}\{\hat{Q}_F(d_F, b_F)\}\right]Q_R$$

$$= \left[a + \gamma_R b_F - \left(1 + \frac{\sigma_R^2}{\mu_R^2}\right)\mu_R Q_R - k\mu_F E_{d_F}\{\hat{Q}_F(d_F, b_F)\}\right]\mu_R Q_R$$

$$= \left[a + \gamma_R b_F - \Delta_R\mu_R Q_R - k\mu_F E_{d_F}\{\hat{Q}_F(d_F, b_F)\}\right]\mu_R Q_R \quad (2)$$

Note that firm F's private demand information in Equation (2) is reflected in firm R's belief $b_F = E[d_F|\Omega]$. The extent to which firm F's private demand information directly affects firm R's conditional expected profit depends on the usefulness of the information in determining firm R's demand, which is characterized in Equation (2) by the amount of spillover, denoted γ_R, inherent in the information. It follows that firm R faces uncertainty in its demand only when firm F conceals information that is useful to firm R in determining its own demand, i.e., when $\gamma_R > 0$.

Firms F and R maximize their conditional expected profits by choosing order quantities Q_F and Q_R, respectively. Each firms' maximization problem is given by Equation (3) for $i = F, R$.

$$\underset{Q_i}{Max}\ \Pi_i^\Omega(\cdot) \quad (3)$$

The first-order conditions of Equation (3), when rearranged, produce the best-response functions of each firm. Jointly solving the two best-response functions, and noting that Cournot conjectures hold in equilibrium, i.e., $\hat{Q}_R(b_F) = Q_R(b_F)$ and $\hat{Q}_F(d_F, b_F) = Q_F(d_F, b_F)$, yields the equilibrium order quantities $Q_i^\Omega(\cdot)$ and expected profits $\Pi_i^\Omega(\cdot)$ for $i = F, R$ conditional on firm F's disclosure choice provided in Lemma 1. The details of all derivations/proofs are provided in the appendix.

Lemma 1. Conditional on firm F's disclosure choice Ω, for a given signal d_F and firm R's belief b_F:

i. The equilibrium order quantities for firms F and R are given by:

$$Q_F^{\Omega}(d_F, b_F) = \frac{1}{\mu_F}\left(\frac{[2\Delta_R - k]a}{4\Delta_F\Delta_R - k^2} + \frac{d_F}{2\Delta_F} - \frac{b_F k[2\Delta_F\gamma_R - k]}{2\Delta_F[4\Delta_F\Delta_R - k^2]}\right), \text{ and}$$

$$Q_R^{\Omega}(b_F) = \frac{1}{\mu_R}\left(\frac{[2\Delta_F - k]a}{4\Delta_F\Delta_R - k^2} + \frac{b_F[2\Delta_F\gamma_R - k]}{4\Delta_F\Delta_R - k^2}\right).$$

ii. The equilibrium expected profits for firms F and R are given by:

$$\Pi_F^{\Omega}(d_F, b_F) = \Delta_F[\mu_F Q_F^{\Omega}(d_F, b_F)]^2 - 1^{\Omega}c, \text{ and}$$

$$\Pi_R^{\Omega}(b_F) = \Delta_R[\mu_R Q_R^{\Omega}(b_F)]^2.$$

Lemma 1(i) provides the equilibrium order quantities for firms F and R. To highlight the effect of supply uncertainty, consider as a baseline the case in which neither firm faces supply uncertainty ($\Delta_R = \Delta_F = 1$). The equilibrium order quantities for firms F and R are given by $Q_F^{\Omega}(\Delta_R = \Delta_F = 1, \tilde{d}_F, b_F) = \frac{1}{\mu_F}\left(\frac{a}{2+k} + \frac{d_F}{2} - \frac{b_F k[2\gamma_R - k]}{2[4-k^2]}\right)$ and $Q_R^{\Omega}(\Delta_R = \Delta_F = 1, d_F, b_F) = \frac{1}{\mu_R}\left(\frac{a}{2+k} + \frac{b_F[2\gamma_R - k]}{[4-k^2]}\right)$. I use this benchmark to demonstrate that supply uncertainty is the only additional feature in my model to affect disclosure decisions through a firm's equilibrium order quantity.

Lemma 1(ii) provides the equilibrium expected profits for firms F and R. Notice that when firm F conceals its private demand, as is standard in Cournot models, each firms' expected profit is proportional to the square of its expected order quantity (for example, see Bagnoli & Watts, 2015b; Arya et al., 2019, 2021) – disclosure results in firm F incurring a fixed cost of disclosure. This shows that the effects of supply uncertainty on expected profits work primarily through the equilibrium order quantities. As such, it is critical to understand how supply uncertainty affects each firms' equilibrium order quantity.

To see how supply uncertainty affects each firms' equilibrium order quantity, consider the case when firm R alone faces supply uncertainty ($\Delta_R > \Delta_F = 1$). Additionally, consider the case when firm F's and firm R's products are not substitutes ($k = 0$). In this scenario, the equilibrium order quantities are given by $Q_F^{\Omega}(\Delta_R > \Delta_F = 1, k = 0, d_F) = \frac{1}{\mu_F}\left(\frac{a}{2} + \frac{d_F}{2}\right)$ and $Q_R^{\Omega}(\Delta_R > \Delta_F = 1, k = 0, b_F) = \frac{1}{\mu_R}\left(\frac{a}{2\Delta_R} + \frac{b_F[\gamma_R]}{2\Delta_R}\right)$. Notice that while firm F's order quantity does not depend on firm R's supply uncertainty, firm R's order quantity decreases. In essence, the heightened risk of being unable to fulfill customer demand causes firm R to order less from its suppliers making supply uncertainty (Δ_R) akin to an additional cost faced by firm R. Homing in on the second term in firm R's order quantity, observe that the increase in firm R's order quantity is amplified by information spillover (γ_R). Hence, the greater the effect the information spillover has on order quantities (hereafter the spillover effect) the greater the cost supply uncertainty imposes on firm R. This shows how firm R's supply uncertainty interacts with firm F's private demand information to affect order quantities. Specifically, firm R's supply uncertainty has the potential to increase firm F's competitive advantage, however, since the firms' products are not substitutes, firm F cannot capitalize on its rival's supply uncertainty.

At the other end of the spectrum, consider the case when firm R alone faces supply uncertainty and the firms' products are perfect substitutes ($k = 1$). In this scenario, the equilibrium order quantities are given by $Q_F^{\Omega}(\Delta_R > \Delta_F = 1, k = 1, d_F, b_F) = \frac{1}{\mu_F}\left(\frac{[2\Delta_R - 1]a}{4\Delta_R - 1} + \frac{d_F}{2} - \frac{b_F[2\gamma_R - 1]}{2[4\Delta_R - 1]}\right)$ and $Q_R^{\Omega}(\Delta_R > \Delta_F = 1, k = 1, b_F) = \frac{1}{\mu_R}\left(\frac{a}{4\Delta_R - 1} + \frac{b_F[2\gamma_R - 1]}{4\Delta_R - 1}\right)$. Firm R's supply uncertainty causes firm R's order quantity to decrease. Since the products are (perfect) substitutes, firm F can potentially capitalize on firm R's supply uncertainty in two ways: one is directly through the inverse demand function's intercept parameter a; and the other is (potentially) through firm F's private demand information. The latter depends on the magnitude of the spillover effect relative to that of the substitutability effect, or the effect product substitutability has on a firm's order quantity. The net effect is made precise by the condition $[2\gamma_R - 1]$ nestled in the last term of each firms' equilibrium order quantity. Specifically, firm R's supply uncertainty causes firm F's order quantity to increase (decrease) when the spillover effect is large (small) relative to the substitutability effect; notice that this is the opposite outcome information spillover and supply uncertainty have on firm R's order quantity. In essence, whenever the spillover effect is relatively large (small), firm *R* increases (decreases) its order quantity and firm *F* correspondingly decreases (increases) its order quantity; accordingly, firm R's supply uncertainty has the effect of increasing (decreasing) firm F's competitive advantage. The underlying theme in the foregoing discussion is that a firm's competitive advantage is increasing in its equilibrium order quantity, and the extent to which firm F's private demand information gives firm F a competitive advantage depends on the combined effect of three things: product substitutability (k), information spillover (γ_R), and supply uncertainty (Δ_i for $i = F, R$).

With the equilibria conditional on firm F's disclosure choice Ω detailed in Lemma 1, I now compare equilibrium expected profits under each disclosure choice to determine the nature and extent of firm F's disclosures.

4.2 Equilibrium Disclosure Choice

Since firm F's expected profit put forth in Lemma 1(ii) is monotonic in its equilibrium order quantity, the question of whether disclosure is value-increasing given the decision is made after observing signal d_F depends on whether disclosure will increase or decrease firm F's equilibrium order quantity. In this section, I show that one of two partial disclosure equilibriums may arise – either partial good news disclosure or partial bad news disclosure – depending on firm F's competitive advantage. I show in Proposition 1 that a partial good news disclosure equilibrium obtains in which firm F discloses (conceals) its private demand information when demand is high (low); specifically, there exists d_F^* such that firm F discloses its private demand information when $d_F \in \left[d_F^*, \bar{d}_F\right]$ and conceals its private demand information when $d_F \in [0, d_F^*)$. Additionally, I show in Proposition 2 that a partial bad news disclosure equilibrium obtains in which firm F discloses (conceals) its private demand information when demand is low (high); here, there exists d_F^{**} such that firm F discloses its private demand information when $d_F \in [0, d_F^{**}]$ and conceals its private demand information when $d_F \in \left(d_F^{**}, \bar{d}_F\right]$.

Proposition 1. Firm F discloses its private demand information, i.e., signal d_F, for d_F-values greater than d_F^* and conceals its private demand information for d_F-values less than or equal to d_F^*, if the condition in Equation (4) is satisfied.

$$\Delta_F < \frac{k}{2\gamma_R} \tag{4}$$

Specifically, d_F^* is the unique, interior solution to the equation $Z^* = A^*(d_F^*)^2 + B^* d_F^* + C^* = 0$, where $A^* = \frac{1}{2}k\left[4\Delta_R - \frac{3}{2}\gamma_R k - \frac{k^2}{4\Delta_F}\right]\left[\frac{k}{2\Delta_F} - \gamma_R\right]$, $B^* = ak[2\Delta_R - k]\left[\frac{k}{2\Delta_F} - \gamma_R\right]$, and $C^* = -\frac{\left[4\Delta_F\Delta_R - k^2\right]^2 c}{\Delta_F}$.

Proposition 1 establishes that a unique, interior equilibrium disclosure cutoff denoted d_F^* obtains when the condition in Equation (4) is satisfied. Intuitively, the disclosure cutoff d_F^* is the point at which firm F is indifferent between disclosing and concealing its good news private demand information. In this setting, for the range of d_F-values greater than the disclosure cutoff d_F^*, firm F discloses its (good news) private demand information and firm R's posterior belief is $b_F = d_F$. For the range of d_F-values less than or equal to the disclosure cutoff d_F^*, firm F conceals its (bad news) private demand information and, from this, firm R draws a negative inference implying a posterior belief of $b_F = E[d_F | d_F \leq d_F^*] = \frac{1}{2} d_F^*$.

The primary innovation in Proposition 1 is the consideration of supply uncertainty in a firm's decision to disclose its good news private demand information. To develop the intuition for Proposition 1, consider the baseline case when firm F does not face supply uncertainty ($\Delta_F = 1$). Additionally, consider the case when firm F's and firm R's products are not substitutes ($k = 0$). In this scenario, disclosure is unprofitable for firm F and firm F conceals its private demand information. In essence, when the firms' products are not substitutes the firms are not competitors and firm F withholds disclosure because it is unable to exploit its private demand information (see the third term in firm F's equilibrium order quantity). Hence, product substitutability is a necessary condition for partial good news disclosure.

Continuing with the baseline case when firm F does not face supply uncertainty ($\Delta_F = 1$), consider the other end of the spectrum when firm F's and firm R's products are perfect substitutes ($k = 1$). In this scenario, the partial good news disclosure equilibrium obtains when the spillover effect is small relative to the substitutability effect, i.e., when the spillover effect is $\gamma_R < 0.5$. The reason for this is best understood by considering the special case in which firm

F's private demand information is of pure private-value ($\gamma_R = 0$). With the spillover effect suppressed, the substitutability effect is the singular effect through which firm F's private demand information impacts firm R's order quantity, which implies the condition in Equation (4) is always satisfied. The dominance of the substitutability effect causes firm R to decrease its order quantity when firm F discloses that demand is high (i.e., good news). This is because by disclosing that demand is high, firm F is conveying to firm R that it intends to increase its order quantity to fulfill the increase in demand and, because the firms' products are (perfect) substitutes, this causes firm R to back down on its order quantity. It is worth noting that when the condition in Equation (4) is satisfied, disclosure always results in firm R backing down, which benefits firm F. However, for disclosure to be profitable, the benefit from getting firm R to back down on its order quantity must exceed the fixed cost associated with disclosure. Hence, Proposition 1 posits a disclosure strategy for firm F when the private demand information is high enough so that firm R revises its belief upwards and cuts its order quantity such that firm F increases its expected profits enough to (at least) cover the fixed cost of disclosure.

Consider the general case presented in Proposition 1 when firm F's and firm R's products may be perfect or imperfect substitutes, i.e., $k \in (0,1]$, and when the degree of information spillover varies, i.e., $\gamma_R \in [0,1]$. In this scenario, absent supply uncertainty, the competitive advantage firm F derives from its private demand information is captured by the right-hand side of the condition in Equation (4), i.e., $\frac{k}{2\gamma_R}$, and can be either high or low. Firm F's competitive advantage is high when its private demand information implies higher order quantities for firm F; in the baseline case ($\Delta_F = 1$) this is achieved when $1 < \frac{k}{2\gamma_R}$. Notice that firm F's competitive advantage is increasing in product substitutability and decreasing in information spillover. The novelty in Proposition 1 is that it demonstrates how firm F's supply uncertainty affects firm F's competitive advantage thereby affecting its incentive to disclose its good news private demand information. By examining the condition in Equation (4) it is apparent that supply uncertainty takes away from firm F's competitive advantage by diminishing the substitutability effect and by enhancing the spillover effect. To develop the intuition, note that supply uncertainty (Δ_F) calls into question firm F's ability to fulfill customer demand due to unreliable suppliers, and firm R considers this in its response to good news disclosures. When firm F's supply uncertainty is low, the risk is low of not being able to fulfill customer demand and firm F retains its (high) competitive advantage causing firm R to back down when good news is disclosed. Conversely, when firm F's supply uncertainty is high, the risk is high of not being able to fulfill customer demand and firm R will not back down when good news is disclosed; rather, firm R increases its order quantity to fulfill demand where firm F cannot. In this way, supply uncertainty diminishes the substitutability effect (i.e., attenuates firm F's ability to use good news disclosure to get firm R to back down) and enhances the spillover effect (i.e., signals to firm R to increase its order quantity). Thus, when firm F's supply uncertainty is sufficiently high its competitive advantage is effectively low quashing firm F's incentive to disclose its good news private demand information.[8] This gives rise to Proposition 2.

[8] Technically, since $\Delta_F \in [1,2]$, it is when $1 < k/2\gamma_R < 2$ that firm F's supply uncertainty may alter firm F's competitive advantage from being high (i.e., $\Delta_F < k/2\gamma_R$) to low (i.e., $\Delta_F > k/2\gamma_R$).

Proposition 2. Firm F discloses its private demand information, i.e., signal d_F, for d_F-values less than d_F^{**} and conceals its private demand information for d_F-values greater than or equal to d_F^{**}, if Equation (4) is not satisfied with strict inequality, i.e., $\Delta_F > \frac{k}{2\gamma_R}$. Specifically, d_F^{**} is the unique, interior solution to the equation $Z^{**} = A^{**}(d_F^{**})^2 + B^{**}d_F^{**} + C^{**} = 0$, where $A^{**} = \frac{1}{2}k\left[4\Delta_R - \frac{3}{2}\gamma_R k - \frac{k^2}{4\Delta_F}\right]\left[\gamma_R - \frac{k}{2\Delta_F}\right]$, $B^{**} = ak[2\Delta_R - k]\left[\gamma_R - \frac{k}{2\Delta_F}\right] - k\left[2\Delta_R - \frac{1}{2}\gamma_R k - \frac{k^2}{4\Delta_F}\right]\left[\gamma_R - \frac{k}{2\Delta_F}\right]\bar{d}_F$, and $C^{**} = -ak[2\Delta_R - k]\left[\gamma_R - \frac{k}{2\Delta_F}\right]\bar{d}_F + \left\{\frac{1}{2}k\left[\gamma_R - \frac{k}{2\Delta_F}\right]\bar{d}_F\right\}^2 + \frac{\left[4\Delta_F\Delta_R - k^2\right]^2 c}{\Delta_F}$.

Proposition 2 establishes a unique, interior equilibrium disclosure cutoff denoted d_F^{**} and puts forth the condition under which bad news disclosure occurs. Intuitively, the disclosure cutoff d_F^{**} is the point at which firm F is indifferent between disclosing and concealing its bad news private demand information. In this setting, for the range of d_F-values less than the disclosure cutoff d_F^{**}, firm F discloses its bad news private demand information and firm R's posterior belief is $b_F = d_F$. For the range of d_F-values greater than or equal to the disclosure cutoff d_F^{**}, firm F conceals its good news private demand information and, from this, firm R draws a negative inference implying a posterior belief of $b_F = E[d_F | d_F \geq d_F^{**}] = \frac{1}{2}\left[d_F^{**} + \bar{d}_F\right]$.

The primary innovation in Proposition 2 is the consideration of supply uncertainty in a firm's decision to disclose its bad news private demand information. To develop the intuition for Proposition 2, consider the baseline case when firm F does not face supply uncertainty ($\Delta_F = 1$). Additionally, consider the case when firm F's and firm R's products are perfect substitutes ($k = 1$). In this scenario, bad news disclosure occurs when the spillover effect is large relative to the substitutability effect. The reason for this is best understood by considering the special case in which firm F's private demand information is of pure common-value ($\gamma_R = 1$), which is similar to Clinch and Verrecchia (1997), implying the condition in Equation (4) is never satisfied. First note that when the condition in Equation (4) is not satisfied with strict inequality, the third term in firm F's order quantity is negative (i.e., decreasing in firm R's belief), and correspondingly, the second term in firm R's order quantity is positive (i.e., increasing in firm R's belief). This follows from firm R treating disclosures about firm F's demand as being disclosures about its own demand. As such, when F discloses its private demand information, firm R will increase its order quantity and firm F will decrease its order quantity. When firm F's private demand information is low, if firm F conceals its private demand information, then firm R's belief is the posterior mean of the private demand information, which will result in firm R increasing its order quantity by more than if it were to disclose. Thus, firm F finds it beneficial to disclose (conceal) its private demand information when bad news (good news) is observed, i.e., when demand is low (high), to forestall excessive increases in firm R's order quantity. Importantly, this benefit from disclosure is large enough to cover the fixed costs of disclosure.

Continuing with the baseline case when firm F does not face supply uncertainty ($\Delta_F = 1$), consider the case when firm F's and firm R's products may be perfect or imperfect substitutes, i.e., $k \in (0,1]$. Additionally, consider the case when the degree of information spillover varies, i.e., $\gamma_R \in [0,1]$. Notice that in this scenario when $1 > \frac{k}{2\gamma_R}$ firm F's competitive advantage is low implying bad news is disclosed while good news is concealed. The lower bound on firm F's supply uncertainty, i.e., $\underline{\Delta}_F = 1$, means layering in supply uncertainty has the mere effect of perpetuating the status quo by strengthening firm F's incentive to disclose (conceal) its bad news (good news) private demand information. This follows from supply uncertainty diminishing the substitutability effect and enhancing the spillover effect. The interesting case is when $1 < \frac{k}{2\gamma_R}$ and supply uncertainty is high (note that Proposition 1 considers the case when supply uncertainty is low). In this circumstance, firm F's competitive advantage is effectively low due to its supply uncertainty and bad news is disclosed (good news is concealed). Thus, taken together, Propositions 1 and 2 establish the role supply uncertainty plays in firm F's decision to disclose either its good news or bad news private demand information. In summary, when the substitutability effect dominates the spillover effect, firm F finds it advantageous to disclose good news when supply uncertainty is low and to disclose bad news when supply uncertainty is high.

The conditions in Propositions 1 and 2 are stated as strict inequalities. In the following Proposition I complete the analysis by considering the case when the condition in equation (4) is satisfied at equality.

Proposition 3. Firm F conceals its private demand information, i.e., signal $\tilde{d}_F$, if Equation (4) is satisfied at equality, i.e., $\Delta_F = \frac{k}{2\gamma_R}$.

Proposition 3 considers the case when firm F's supply uncertainty exactly balances the spillover and substitutability effects. In this case, the second term of firm R's order quantity exactly equals zero (see Lemma 1) implying firm R's order quantity is independent of firm F's private demand information. Accordingly, firm F cannot influence firm R's order quantity by disclosing its private demand information and obtain benefits from doing so. It follows that since there are no benefits to disclosure, but only the fixed cost of disclosure, in this knife-edged case firm F finds it optimal to conceal its private demand information.

4.3 Equilibrium Disclosure Cutoff

Propositions 1, 2 and 3 focus on the role supply uncertainty plays in determining the equilibrium disclosure. I next examine how the disclosure cutoff thresholds demarcating good news and bad news in each disclosure equilibrium characterized in Propositions 1 and 2 are impacted by supply uncertainty.

Proposition 4. The disclosure cutoff d_F^* is increasing in the firms' supply uncertainty (Δ_F, Δ_R), i.e., $[dd_F^*/d\Delta_i] > 0$ for $i = F, R$.

Proposition 4 shows how the partial good news disclosure equilibrium cutoff d_F^* changes with respect to the firms' supply uncertainty. Specifically, the disclosure cutoff d_F^* is increasing in the firms' supply uncertainty resulting in a contraction of the good news disclosure region; because firm F's private demand information is uniformly distributed over 0 and $\bar{d}_F$, the implication is that supply uncertainty decreases the likelihood firm F discloses good news, i.e., less good news is disclosed, on average. Undergirding this result is the rationale that firm F discloses its (good news) private demand information when doing so gets firm R to back down on its order quantity, and conceals it, otherwise. Proposition 1 showed that the efficacy of good news disclosure in getting firm R to back down is curtailed by firm F's supply uncertainty. Specifically, by diminishing the substitutability effect and by enhancing the spillover effect, firm F's supply uncertainty hampers firm F's ability to exploit its good news private demand information; and, for this reason, less good news is disclosed, on average. When firm R faces supply uncertainty, however, it is the reduced need (rather than ability) to use disclosure to get firm R to back down on its order quantity that results in fewer good news disclosures. Intuitively, by causing firm R to decrease its order quantity, firm R's supply uncertainty makes good news disclosures less impactful/important.

This demonstrates that whether it is firm F or firm R that faces supply uncertainty, the efficacy of good news disclosure in getting firm R to back down is curtailed and firm F therefore discloses less good news.

Proposition 5. The disclosure cutoff d_F^{**} is decreasing in firm R's supply uncertainty, Δ_R, i.e., $[dd_F^{**}/d\Delta_R] < 0$; and decreasing (increasing) in firm F's supply uncertainty, Δ_F, if the spillover effect, γ_R, is sufficiently high (low), i.e., there exists $\gamma_R^{**} \in \left(\frac{k}{2\Delta_F}, 1\right)$ such that $[dd_F^{**}/d\Delta_F] < 0$ for $\gamma_R \in (\gamma_R^{**}, 1]$ and $[dd_F^{**}/d\Delta_F] > 0$ for $\gamma_R \in \left(\frac{k}{2\Delta_F}, \gamma_R^{**}\right)$.

Proposition 5 shows how the partial bad news disclosure equilibrium cutoff d_F^{**} changes with respect to the firms' supply uncertainty. Notice that the effect firm F's supply uncertainty has on the disclosure cutoff d_F^{**} depends on the spillover effect. Specifically, there exists a cutoff denoted γ_R^{**} on the interior of the region $\left(\frac{k}{2\Delta_F}, 1\right)$ such that the disclosure cutoff d_F^{**} is increasing in firm F's supply uncertainty when the spillover effect is sufficiently low, i.e., when $\gamma_R \in \left(\frac{k}{2\Delta_F}, \gamma_R^{**}\right)$, causing the bad news disclosure region to expand and, thus, increasing the likelihood of bad news disclosure.[9] This result reflects the fact that firm F's supply uncertainty, by enhancing the spillover effect, decreases firm F's incentive to disclose good news

[9] Technically, the cutoff γ_R^{**} is the γ_R that solves the equation $0 = -8\Delta_F^2\Delta_R\gamma_R + 8\Delta_F\Delta_R k - k^3$.

(complementing Proposition 4) and increases firm F's incentive to disclose bad news. Conversely, the disclosure cutoff d_F^{**} is decreasing in firm F's supply uncertainty when the spillover effect is sufficiently high, i.e., when $\gamma_R \in (\gamma_R^{**}, 1]$ causing the bad news disclosure region to contract and, thus, decreasing the likelihood of bad news disclosure. The intuition for this result comports with the discussion following Proposition 4, specifically, by diminishing the substitutability effect, firm F's supply uncertainty dulls the impact of (bad news) disclosure on the firms' order quantities weakening the incentive for firm F to disclose its bad news private demand information. Finally, the disclosure cutoff d_F^{**} is decreasing in firm R's supply uncertainty. When firm R faces supply uncertainty firm R decreases its order quantity and, consistent with previously established findings, the need for firm F to use bad news disclosure as a way of getting firm R to back down is abated.

With these results in hand, Propositions 4 and 5 taken together imply that firm F's supply uncertainty, by enhancing the spillover effect, makes good news disclosure less likely and bad news disclosure more likely; and, by diminishing the substitutability effect, makes disclosure, overall, less efficacious and, therefore, less likely. Similarly, Propositions 4 and 5 taken together imply that firm R's supply uncertainty, by causing firm R's order quantity to decrease, abates the need for firm F to use disclosure, generally, as a means of getting firm R to back down; here, too, the effect is less disclosure, on average.

CHAPTER 5

EX ANTE DISCLOSURE CASE

Complementing the analysis of supply uncertainty's effect on a firm's decision to voluntarily disclose its private demand information, I turn to the case when firm F commits prior to privately observing signal d_F to disclose its private demand information when obtained. To allow for easier comparisons between the analyses of ex post and ex ante disclosure, I maintain the assumptions given heretofore in the model with one exception, specifically, I set $c = 0$.[10] In essence, because firm F pre-commits to a disclosure policy, the unraveling result does not apply obviating the need for a fixed cost of disclosure (see Bagnoli & Watts, 2013). This change to the model is minor but useful because it allows me to directly compare my results to the results of other studies in the prior literature on voluntary disclosures where ex ante disclosure incentives are examined, most notably Bagnoli & Watts (2015b).

Given this setup, I now turn to solving the model. The solution concept is one of Perfect Bayesian Equilibrium. Thus, I work backwards and begin in the second stage of the game by solving for the firms' equilibrium order quantities and expected profits conditional on firm F's disclosure policy $\Omega \in \{D, \phi\}$. Here, in the ex ante case, D denotes a policy a full disclosure and ϕ denotes a policy of no disclosure. Lemma 1(i) gives the equilibrium order quantities with one

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[10] Setting the fixed cost of disclosure equal to zero is not critical, but it does simplify the analysis and make it easier to compare the insights with prior literature. Specifically, when $c = 0$ the disclosure condition implied by Equation (4), i.e., $\Delta_F < k/2\gamma_R$, is both necessary and sufficient for full disclosure in equilibrium; however, when $c > 0$ the disclosure condition is only necessary. In the latter case, coordinating quantities through disclosure must result in a net benefit sufficiently large to overcome the additional hurdle c represents. In either case, $\Delta_F < k/2\gamma_R$ drives the disclosure decision, which is why I let $c = 0$.

proviso. That is, when firm F conceals its private demand information, firm R's belief (b_F) nestled in the last term of each firms' equilibrium order quantity is the unconditional mean of signal d_F, i.e., $b_F = E[d_F]$. This follows from the fact that in the ex ante disclosure case firm R cannot draw a negative inference when firm F conceals its private demand information. Alternatively, when firm F discloses its private demand information firm R's belief is $b_F = d_F$. The (ex ante) equilibrium expected profit for firm F conditional on its disclosure policy is given by Equation (5).

$$E_{d_F}\{\Pi_F^{\Omega}\} = E_{d_F}\{\Delta_F[\mu_F Q_F^{\Omega}]^2\} \tag{5}$$

Note that Equation (5) is obtained by taking the expectation of firm F's (ex post) equilibrium expected profit given by Lemma 1(ii) over firm F's private demand information d_F. Comparing firm F's equilibrium expected profit under each disclosure policy yields Proposition 6.

Proposition 6. In the case when firm F commits to a disclosure policy prior to privately observing information on its demand, i.e., signal d_F, firm F chooses full disclosure if and only if Equation (4) is satisfied, i.e., $\Delta_F < \frac{k}{2\gamma_R}$, and no disclosure otherwise.

Proposition 6 posits the condition under which firm F pre-commits to a policy of full disclosure, in equilibrium. To develop the intuition for Proposition 6, consider the baseline case when firm F does not face supply uncertainty ($\Delta_F = 1$). Additionally, consider the case when firm F's and firm R's products are not substitutes ($k = 0$). In this scenario, firm F and firm R are not competitors and, for that reason, there is no benefit from coordinating order quantities through disclosure. Hence, firm F pre-commits to a policy of no disclosure. This shows that product substitutability is a necessary condition for disclosure to occur.

Continuing with the baseline case, consider the other end of the spectrum when firm F's and firm R's products are perfect substitutes ($k = 1$). In this scenario, firm F's disclosure policy depends on the magnitude of the substitutability effect relative to that of the spillover effect. Consider the two extreme cases when firm F's private demand information is either of 1) pure common-value ($\gamma_R = 1$) or 2) pure private-value ($\gamma_R = 0$). The results from these special cases are consistent with the findings established in the prior literature. That is, when firm F's private demand information is of pure common-value firm F's competitive advantage is low, and concealing its private demand information is optimal. Conversely, when firm F's private demand information is of pure private-value firm F's competitive advantage is high and full disclosure is optimal.

The case first examined by Bagnoli & Watts (2015) that I am extending permits competitors' products to be imperfect substitutes, i.e., $k \in (0,1]$, and considers intermediate values of information spillover, i.e., $\gamma_R \in [0,1]$. In this scenario, it is apparent from the disclosure condition $1 < \frac{k}{2\gamma_R}$ that firm F pre-commits to a policy of full disclosure when the substitutability effect is sufficiently large relative to the spillover effect, i.e., when firm F's competitive advantage is high. The innovation in Proposition 6 demonstrates how firm F's supply uncertainty affects firm F's decision to fully disclose or conceal its private demand information. As in the ex post case, by diminishing the substitutability effect and by enhancing the spillover effect, supply uncertainty has the combined effect of corroding firm F's incentive to disclose its private demand information. Thus, per Equation (4), when the substitutability effect dominates the spillover effect and firm F's supply uncertainty is sufficiently low, firm F retains its competitive advantage and therefore finds it advantageous to pre-commit to a policy a full disclosure.

However, as supply uncertainty increases, firm F's incentive to disclose its private demand information wanes, and when supply uncertainty is sufficiently high firm F loses its competitive advantage and the scales tip in favor of firm F concealing its private demand information.

Overall, this is an interesting result because it runs counter to the notion that firms by virtue of having an information advantage also have a competitive advantage. Specifically, Proposition 6 puts forth a plausible circumstance in which sufficiently high levels of supply uncertainty may altogether take away the competitive advantage a firm's private demand information might otherwise give it. The crux of the result is that when firm F faces sufficiently high levels of supply uncertainty, firm F's ability to exploit its private demand information is diminished causing firm F to pre-commit to a policy of no disclosure.

CHAPTER 6

CONCLUSION

This study acknowledges the increasing importance of supply uncertainty in a firm's paradigm for strategic decision-making by examining the effects of supply uncertainty on a firm's decision to voluntarily disclose (or conceal) its private demand information to a rival firm. The philosophical framework employed in this study takes the approach of characterizing a firm's competitive advantage, be it high or low, and correlating that with a firm's ex post disclosure choice. The key forces in this study that combine to characterize a firm's competitive advantage are 1) product substitutability, 2) information spillover, and 3) supply uncertainty. Prior literature has shown that two facets of competition, in particular, are influential in determining a firm's competitive advantage thereby determining its disclosure choice/policy, namely product substitutability and information spillover. A firm's competitive advantage is increasing in product substitutability and decreasing in information spillover implying a firm's competitive advantage is high (low) when information spillover is sufficiently low (high) relative to product substitutability. When a firm's competitive advantage is high, a partial good news disclosure equilibrium arises, and the informed firm finds it advantageous to disclose good news (i.e., high demand) and withhold bad news (i.e., low demand). Alternatively, when a firm's competitive advantage is low, a bad news disclosure equilibrium arises, and the informed firm finds it advantageous to disclose bad news (i.e., low demand) and withhold good news (i.e., high demand). The novelty in this study is the joint consideration of product substitutability, information spillover, and supply uncertainty. By diminishing the effect of product substitutability and by enhancing the effect of information spillover, supply uncertainty decreases a firm's incentive to disclose good news and increases a firm's incentive to disclose bad news. Thus, one of the key findings in this study is that when a firm's competitive advantage is high absent supply uncertainty, the informed firm may experience (sufficiently) low levels of supply uncertainty and a partial good news disclosure equilibrium be sustained. However, when the informed firm experiences (sufficiently) high levels of supply uncertainty its competitive advantage is effectively low and the incentive to disclose good news is reversed resulting in a partial bad news disclosure equilibrium. In this way, the informed firm's supply uncertainty affects the types of disclosures that arise in equilibrium.

Additionally, the effects of supply uncertainty on the equilibrium disclosure cutoff(s) are also examined. In general, because supply uncertainty curtails the efficacy of disclosure in getting the rival firm to back down, disclosure occurs with less frequency, on average. More specifically, when firm F faces supply uncertainty good news disclosure is less likely and bad news disclosure is more likely; this follows from supply uncertainty diminishing the substitutability effect and by enhancing the spillover effect.

To complete the analysis, this study also examines the ex ante disclosure case. Here, the forces remain the same as in the ex post disclosure case, however the disclosure equilibrium is one of full disclosure or no disclosure, i.e., bang-bang. When the firm's competitive advantage is high there is full disclosure and when the firm's competitive advantage is low there is no disclosure. The key finding in this extension is that when a firm's competitive advantage is high absent supply uncertainty, the informed firm may experience (sufficiently) low levels of supply uncertainty and a policy of full disclosure be sustained in equilibrium. However, when the informed firm experiences (sufficiently) high levels of supply uncertainty its competitive advantage is effectively low and the incentive to fully disclose its private demand information is reversed resulting in a policy of no disclosure, in equilibrium.

APPENDIX

PROOFS OF LEMMAS AND PROPOSITIONS

Deriving Firm *F*'s Expected Profit Function

Firm F's expected profits $\Pi_F^{\Omega}(d_F, b_F)$, conditional on its disclosure choice Ω at the time of placing the order quantity, Q_F is derived below.

$$\Pi_F^{\Omega}(\cdot) = E_{\alpha_F,\alpha_R}\{p_F \acute{q}_F - 1^{\Omega}c\}$$

$$= E_{\alpha_F,\alpha_R}\left\{\left[a + d_F - \alpha_F Q_F - k\alpha_R \hat{Q}_R(b_F)\right]\alpha_F Q_F - 1^{\Omega}c\right\}$$

Substituting $p_F = a + d_F - \alpha_F Q_F - k\alpha_R \hat{Q}_R(b_F)$ and $\acute{q}_F = \alpha_F Q_F$;

$$= E_{\alpha_F,\alpha_R}\left\{(a + d_F)\alpha_F Q_F - \alpha_F^2 Q_F^2 - k\alpha_R \alpha_F \hat{Q}_R(b_F) Q_F - 1^{\Omega}c\right\}$$

$$= (a + d_F)\mu_F Q_F - (\mu_F^2 + \sigma_F^2)Q_F^2 - k\mu_F \mu_R \hat{Q}_R(b_F) Q_F - 1^{\Omega}c$$

Using $E[\alpha_F^2] = (\mu_F^2 + \sigma_F^2)$, and $E[\tilde{\alpha}_i] = \mu_i$ for $i = F, R$, and $Cov(\alpha_F, \alpha_R) = 0$;

$$= \left[a + d_F - \Delta_F \mu_F Q_F - k\mu_R \hat{Q}_R(b_F)\right]\mu_F Q_F - 1^{\Omega}c \quad \text{(A1)}$$

Using $\Delta_F = \left(1 + \frac{\sigma_F^2}{\mu_F^2}\right)$ and rearranging.

In (A1), $\hat{Q}_R(b_F)$ denotes firm F's conjecture of firm R's quantity choice, where b_F represents firm R's belief about the value of firm F's private demand information, d_F.

Deriving Firm *R*'s Expected Profit Function

Firm R's expected profit function $\Pi_R^{\Omega}(b_F)$, conditional on firm F's disclosure choice Ω, is given by Equation (A2):

$\Pi_R^{\Omega}(\cdot)$	$=$	$E_{\alpha_F,\alpha_R}\{p_R \acute{q}_R\}$	
	$=$	$E_{\alpha_F,\alpha_R,d_F}\{[a+\gamma_R d_F-\alpha_R Q_R-k\alpha_F \hat{Q}_F(d_F,b_F)]\alpha_R Q_R\}$	Substituting $p_R = a+\gamma_R d_F - \alpha_R Q_R - k\alpha_F \hat{Q}_F(d_F,b_F)$ and $\acute{q}_R = \alpha_R Q_R$;
	$=$	$E_{\alpha_F,\alpha_R,d_F}\{(a+\gamma_R d_F)\alpha_R Q_R - \alpha_R^2 Q_R^2 - k\alpha_F\alpha_R \hat{Q}_F(d_F,b_F)Q_R\}$	
	$=$	$(a+\gamma_R b_F)\mu_R Q_R - (\mu_R^2+\sigma_R^2)Q_R^2 - k\mu_R\mu_F E_{d_F}[\hat{Q}_F(d_F,b_F)]Q_R$	Using $E[\alpha_R^2] = (\mu_R^2+\sigma_R^2)$ and $E[\tilde{\alpha}_i] = \mu_i$ for $i = F,R$ and $Cov(\alpha_F,\alpha_R) = 0$;
	$=$	$\left[a+\gamma_R b_F - \Delta_R\mu_R Q_R - k\mu_F E_{d_F}[\hat{Q}_F(d_F,b_F)]\right]\mu_R Q_R$	Using $\Delta_R = \left(1+\frac{\sigma_R^2}{\mu_R^2}\right)$ and rearranging. (A2)

In (A2), $\hat{Q}_F(d_F,b_F)$ denotes firm R's conjecture of firm F's quantity choice, where b_F represents firm R's belief about the value of firm F's private demand information, d_F.

Proof of Lemma 1(i).

Differentiate $\Pi_F^{\Omega}(d_F,b_F)$ in Equation (A1) with respect to Q_F and set equal to zero to obtain firm F's first-order condition given by Equation (A3). The superscript Ω is suppresed from Q_i^{Ω} for $i = F,R$ for brevity.

$$\frac{\partial \Pi_F^{\Omega}(d_F,b_F)}{\partial Q_F} = a + d_F - 2\mu_F\Delta_F Q_F - k\mu_R\hat{Q}_R(b_F) = 0 \Rightarrow Q_F = \frac{[a+d_F]-k\mu_R\hat{Q}_R(b_F)}{2\mu_F\Delta_F} \quad \text{(A3)}$$

Equation (A3) is firm F's best-response function.

Differentiate $\Pi_R^{\Omega}(d_F,b_F)$ in Equation (A2) with respect to Q_R and set equal to zero to obtain firm R's first-order condition given by Equation (A4).

$$\frac{\partial \Pi_R^{\Omega}(d_F,b_F)}{\partial Q_R} = a + \gamma_R b_F - 2\mu_R \Delta_R Q_R - k\mu_F E_{\tilde{d}_F}\left[\hat{Q}_F(d_F,b_F)\right] = 0 \Rightarrow Q_R = \tag{A4}$$

$$\frac{[a+\gamma_R b_F] - k\mu_F E_{\tilde{d}_F}[\hat{Q}_F(d_F,b_F)]}{2\mu_R \Delta_R} = \left(\frac{a+\gamma_R b_F}{2\mu_R \Delta_R}\right) - \left\{\frac{k\mu_F}{2\mu_R \Delta_R}\right\}\left[E_{\tilde{d}_F}\left[\hat{Q}_F(d_F,b_F)\right]\right]$$

Using the in-equilibrium identity $\hat{Q}_i(\cdot) = Q_i$ for $i = F, R$ in (A3) we get $E_{d_F}[Q_F] = \frac{a+b_F-k\mu_R Q_R}{2\mu_F \Delta_F}$.

Substituting $E_{d_F}[Q_F] = \frac{a+b_F-k\mu_R Q_R}{2\mu_F \Delta_F}$ in for $E_{\tilde{d}_F}\left[\hat{Q}_F(d_F,b_F)\right]$ in (A4) we get

$$Q_R = \left(\frac{a+\gamma_R b_F}{2\mu_R \Delta_R}\right) - \left\{\frac{k\mu_F}{2\mu_R \Delta_R}\right\}\left[\frac{a+b_F-k\mu_R Q_R}{2\mu_F \Delta_F}\right] = \left(\frac{a+\gamma_R b_F}{2\mu_R \Delta_R}\right) - \left[\frac{ak+b_F k-k^2\mu_R Q_R}{4\Delta_F \Delta_R \mu_R}\right] =$$

$$\frac{2a\Delta_F + 2b_F \Delta_F \gamma_R - ak - b_F k + k^2 \mu_R Q_R}{4\Delta_F \Delta_R \mu_R}$$

Collecting Q_R to the LHS and rearranging we get

$$Q_R = \frac{1}{\mu_R}\left(\frac{a[2\Delta_F - k] - b_F[k - 2\Delta_F \gamma_R]}{4\Delta_F \Delta_R - k^2}\right) = \frac{1}{\mu_R}\left(\frac{a[T1] + b_F[DC1]}{QDEN}\right) = Q_R^{\Omega}(b_F) > 0, \tag{A5}$$

where $T1 = [2\Delta_F - k]$, $DC1 = [k - 2\Delta_F \gamma_R]$ and $QDEN = 4\Delta_F \Delta_R - k^2$. $QDEN > 0$ because the minimum value it can take is positive; to see this, evaluate $QDEN$ using $\Delta_i \geq 1$ for $i = F, R$, and $k \leq 1$. $T1 > DC1 \Rightarrow [2\Delta_F - k] > [k - 2\Delta_F \gamma_R] \Rightarrow 2\Delta_F[1 + \gamma_R] > k$, where the last inequality follows because $\Delta_i \geq 1$ for $i = F, R$, $\gamma_R \leq 1$ and $k \leq 1$. $a > b_F$ by assumption (PQ). Putting these together establishes the last inequality in Equation (A5).

To obtain firm F's equilibrium order quantity $Q_F(d_F, b_F)$, substitute $Q_R(b_F)$ from Equation (A5) in Equation (A3), noting that $Q_R(b_F) = \hat{Q}_R(b_F) = Q_R$ in equilibrium to get

$$\begin{aligned} Q_F &= \left(\frac{a+d_F}{2\mu_F \Delta_F}\right) - \left\{\frac{k\mu_R}{2\mu_F \Delta_F}\right\}\left[\frac{1}{\mu_R}\left(\frac{a[2\Delta_F - k] - b_F[k - 2\Delta_F \gamma_R]}{4\Delta_F \Delta_R - k^2}\right)\right] \\ &= \frac{(a+d_F)(4\Delta_F \Delta_R - k^2) - ak[2\Delta_F - k] + b_F k[k - 2\Delta_F \gamma_R]}{2\mu_F \Delta_F (4\Delta_F \Delta_R - k^2)} \\ &= \frac{a(4\Delta_F \Delta_R - k^2 - 2\Delta_F k + k^2) + d_F(4\Delta_F \Delta_R - k^2) - b_F k[2\Delta_F \gamma_R - k]}{2\mu_F \Delta_F (4\Delta_F \Delta_R - k^2)} \end{aligned}$$

$$= \frac{2\Delta_F a(2\Delta_R - k) + d_F(4\Delta_F\Delta_R - k^2) - b_F k[2\Delta_F\gamma_R - k]}{2\mu_F\Delta_F(4\Delta_F\Delta_R - k^2)}$$

$$= \frac{2\Delta_F a(2\Delta_R - k)}{2\mu_F\Delta_F(4\Delta_F\Delta_R - k^2)} + \frac{d_F(4\Delta_F\Delta_R - k^2)}{2\mu_F\Delta_F(4\Delta_F\Delta_R - k^2)} - \frac{b_F k[2\Delta_F\gamma_R - k]}{2\mu_F\Delta_F(4\Delta_F\Delta_R - k^2)}$$

$$= \frac{1}{\mu_F}\left(\frac{a(2\Delta_R - k)}{(4\Delta_F\Delta_R - k^2)} + \frac{d_F}{2\Delta_F} - \frac{b_F k[2\Delta_F\gamma_R - k]}{2\Delta_F(4\Delta_F\Delta_R - k^2)}\right) = \frac{1}{\mu_F}\left(\frac{aT2}{QDEN} + \frac{d_F}{2\Delta_F} - \frac{b_F kDC2}{QDEN}\right) > 0, \qquad \text{(A6)}$$

where $T2 = (2\Delta_R - k)$, $DC2 = \frac{k[2\Delta_F\gamma_R - k]}{2\Delta_F}$ and $QDEN = (4\Delta_F\Delta_R - k^2)$. $QDEN > 0$ because the minimum value it can take is positive; to see this, evaluate $QDEN$ using $\Delta_i \geq 1$ for $i = F, R$, and $k \leq 1$. $T2 > DC2 \Rightarrow [2\Delta_R - k] > \{[2\Delta_F\gamma_R - k]k/2\Delta_F\} \Rightarrow 2\Delta_F[2\Delta_R - k] > [2\Delta_F\gamma_R - k] \Rightarrow 4\Delta_F\Delta_R > 2\Delta_F(\gamma_R + 1)k - k^2 = DC3$. Differentiate $DC3$ with respect to k and set it to zero and rearrange to get $k^* = \Delta_F(\gamma_R + 1) > 1$ because $\Delta_F \geq 1$ and $\gamma_R \leq 1$. Since the second order condition is negative $DC3$ is concave in k. Thus, $DC3$ achieves a maximum for $k = 1$. For $T2 > DC2 \Rightarrow 4\Delta_F\Delta_R > DC3(k = 1) = 2\Delta_F(\gamma_R + 1) - 1 > 2\Delta_F(\gamma_R + 1) \Rightarrow 2\Delta_R > (\gamma_R + 1)$, where is last inequality holds because $\Delta_F \geq 1$ and $\gamma_R \leq 1$. $a > b_F$ by assumption (PQ). Putting these together establishes the last inequality in Equation (A6). QED.

Proof of Lemma 1(ii)

Expressing the first order condition for firm *F* in Equation (A3) with equilibrium quantities and multiplying by $\mu_F Q_F^\Omega$ we get

$$\mu_F Q_F^\Omega[a + d_F - 2\mu_F\Delta_F Q_F^\Omega - k\mu_R Q_R^\Omega] = 0 \Rightarrow \mu_F Q_F^\Omega[a + d_F - \mu_F\Delta_F Q_F^\Omega - k\mu_R Q_R^\Omega] =$$

$$\Pi_F^\Omega(d_F, b_F) + 1^\Omega c = \{\mu_F Q_F^\Omega\}^2$$

Rearranging the last equality, we get $\Pi_F^\Omega(d_F, b_F) = \{\mu_F Q_F^\Omega\}^2 - 1^\Omega c$.

Expressing the first order condition for firm *R* in Equation (A4) with equilibrium quantities and multiplying by $\mu_R Q_R^\Omega$ we get

$\mu_R Q_R^{\Omega}\left[a+\gamma_R b_F - 2\Delta_R\mu_R Q_R^{\Omega} - k\mu_F E_{\tilde{d}_F}[Q_F^{\Omega}(d_F,b_F)]\right] = 0 \Rightarrow \mu_R Q_R^{\Omega}\left[a+\gamma_R b_F - \right.$

$\left.\Delta_R\mu_R Q_R^{\Omega} - k\mu_F E_{\tilde{d}_F}[Q_F^{\Omega}(d_F,b_F)]\right] = \Pi_R^{\Omega}(b_F) = \{\mu_R Q_R^{\Omega}\}^2$. QED.

Proof of Proposition 1.

When equation (4) is satisfied, firm F does not disclose its private demand information and firm R rationally conjectures that firm F is withholding bad news for $\tilde{d}_F \in [0, d_F^*]$ implying firm R's posterior belief is $b_F = 0.5d_F^*$ for $\tilde{d}_F \in [0, d_F^*]$; firm F discloses its private demand information for $\tilde{d}_F \in [d_F^*, \bar{d}_F]$ implying firm R's posterior belief is $b_F = \tilde{d}_F$ for $\tilde{d}_F \in [d_F^*, \bar{d}_F]$. To derive d_F^*, note that d_F^* is the point at which firm F is indifferent between disclosing and concealing its private demand information and, thus, $\Pi_F^D(\tilde{d}_F = d_F^*, b_F = d_F^*) = \Pi_F^{\phi}(\tilde{d}_F = d_F^*, b_F = 0.5d_F^*)$, i.e., firm F's expected profit evaluated at $\tilde{d}_F = d_F^*$ is equal for disclosure and non-disclosure. Using Lemma 1(ii) we get the following, where Ω denotes disclosure of $\tilde{d}_F = d_F^*$ and thus $b_F = d_F^*$, and ϕ denotes non-disclosure at $\tilde{d}_F = d_F^*$ and thus $b_F = 0.5d_F^*$.

Let $X^* = [\tilde{d}_F = d_F^*, b_F = d_F^*]$ and $Y^* = [\tilde{d}_F = d_F^*, b_F = 0.5d_F^*]$.

$$\Pi_F^D(X^*) = \Delta_F[\mu_F Q_F^D(X^*)]^2 - c = \Delta_F\left[\mu_F Q_F^{\phi}(Y^*)\right]^2 = \Pi_F^{\phi}(Y^*)$$

$$\Rightarrow \Delta_F[\mu_F Q_F^D(X^*)]^2 - c - \Delta_F\left[\mu_F Q_F^{\phi}(Y^*)\right]^2 = 0$$

$$\Rightarrow \Delta_F\mu_F^2\left[Q_F^D(X^*) - Q_F^{\phi}(Y^*)\right]\left[Q_F^D(X^*) + Q_F^{\phi}(Y^*)\right] - c = 0 \qquad \text{(A7)}$$

From Lemma 1(i), using $Q_F^D(X^*) = \frac{2\Delta_F a[2\Delta_R - k] + d_F^*[4\Delta_F\Delta_R - k^2] - d_F^* k[2\Delta_F\gamma_R - k]}{\mu_F 2\Delta_F[4\Delta_F\Delta_R - k^2]}$ and

$Q_F^{\phi}(Y^*) = \frac{2\Delta_F a[2\Delta_R - k] + d_F^*[4\Delta_F\Delta_R - k^2] - 0.5d_F^* k[2\Delta_F\gamma_R - k]}{\mu_F 2\Delta_F[4\Delta_F\Delta_R - k^2]}$ we get

$$\left[Q_F^D(X^*) - Q_F^{\phi}(Y^*)\right] = \frac{-0.5d_F^* k[2\Delta_F\gamma_R - k]}{\mu_F 2\Delta_F[4\Delta_F\Delta_R - k^2]} \qquad \text{(A8)}$$

$$\left[Q_F^D(X^*) + Q_F^{\phi}(Y^*)\right] = \frac{4\Delta_F a[2\Delta_R - k] + 2d_F^*[4\Delta_F\Delta_R - k^2] - 1.5d_F^* k[2\Delta_F\gamma_R - k]}{\mu_F 2\Delta_F[4\Delta_F\Delta_R - k^2]} \quad \text{(A9)}$$

Using the above $\left[Q_F^D(X^*) - Q_F^{\phi}(Y^*)\right]$ and $\left[Q_F^D(X^*) + Q_F^{\phi}(Y^*)\right]$ in (A7)

$$\begin{aligned}
0 &= \Delta_F\mu_F^2\left[Q_F^D(X^*) - Q_F^{\phi}(Y^*)\right]\left[Q_F^D(X^*) + Q_F^{\phi}(Y^*)\right] - c \\
&= \left\{\frac{\Delta_F\mu_F^2}{4\mu_F^2\Delta_F^2[4\Delta_F\Delta_R - k^2]^2}\right\}[\{-0.5d_F^* k[2\Delta_F\gamma_R - k]\}\{[4\Delta_F a[2\Delta_R - k] + 2d_F^*[4\Delta_F\Delta_R - k^2] - \\
&\quad 1.5d_F^* k[2\Delta_F\gamma_R - k]\} - c] \\
&= -\left[\frac{4c\Delta_F^2[4\Delta_F\Delta_R - k^2]^2}{\Delta_F}\right] - 0.5k[2\Delta_F\gamma_R - k]4\Delta_F a[2\Delta_R - k]d_F^* + \{0.75k[2\Delta_F\gamma_R - \\
&\quad k]k[2\Delta_F\gamma_R - k] - k[2\Delta_F\gamma_R - k][4\Delta_F\Delta_R - k^2]\}(d_F^*)^2 \\
&= -\left[\frac{4c\Delta_F^2[4\Delta_F\Delta_R - k^2]^2}{\Delta_F}\right] - 4\Delta_F^2 ak\left[\frac{k}{2\Delta_F} - \gamma_R\right][2\Delta_R - k]d_F^* + \{k[2\Delta_F\gamma_R - k][1.5k\Delta_F\gamma_R - \\
&\quad 0.75k^2 - 4\Delta_F\Delta_R + k^2]\}(d_F^*)^2 \\
&= -\left[\frac{4c\Delta_F^2[4\Delta_F\Delta_R - k^2]^2}{\Delta_F}\right] - 4\Delta_F^2 ak\left[\frac{k}{2\Delta_F} - \gamma_R\right][2\Delta_R - k]d_F^* + \{4\Delta_F^2\left(\frac{k}{2}\right)\left[\left(\frac{k}{2\Delta_F}\right) - \right. \\
&\quad \left.\gamma_R\right]\left[4\Delta_R - 1.5\gamma_R k - \frac{k^2}{4\Delta_F}\right](d_F^*)^2 \\
&\Rightarrow \frac{k}{2}\left[4\Delta_R - 1.5\gamma_R k - \frac{k^2}{4\Delta_F}\right]\left[\frac{k}{2\Delta_F} - \gamma_R\right](d_F^*)^2 + ak\left[\frac{k}{2\Delta_F} - \gamma_R\right][2\Delta_R - k]d_F^* - \\
&\quad \frac{c[4\Delta_F\Delta_R - k^2]^2}{\Delta_F} = Z^*, \text{ with } A^*, B^*, \text{ and } C^* \text{ as defined in Proposition 1.}
\end{aligned}$$

To show that d_F^* is a unique, interior solution, begin by establishing the signs of A^*, B^*, and C^*. $A^* = \frac{k}{2}\left[4\Delta_R - \frac{3}{2}\gamma_R k - \frac{k^2}{4\Delta_F}\right]\left[\frac{k}{2\Delta_F} - \gamma_R\right] = (A_1^*)(A_2^*)(A_3^*)$, where $A_1^* = \frac{k}{2}$, $A_2^* = \left[4\Delta_R - \frac{3}{2}\gamma_R k - \frac{k^2}{4\Delta_F}\right]$, and $A_3^* = \left[\frac{k}{2\Delta_F} - \gamma_R\right]$. $A_1^* > 0$ because $k \in (0,1]$. To see that $A_2^* > 0$, note that A_2^* is increasing in Δ_i for $i = F, R$, and decreasing in k and γ_R. Accordingly, evaluating A_2^* at $\Delta_i = 1$

(the lowest value of parameter Δ_i) and $k = \gamma_R = 1$ (the highest values of parameters k and γ_R) yields the lower bound of $A_2^*(\Delta_i = 1,\ k = 1,\ \gamma_R = 1) = \frac{9}{4}$, thus implying $A_2^* > 0$. $A_3^* > 0$ by the condition in equation (4). Thus, $A^* > 0$.

$B^* = ak[2\Delta_R - k]\left[\frac{k}{2\Delta_F} - \gamma_R\right] = (B_1^*)(B_2^*)(B_3^*)$, where $B_1^* = ak$, $B_2^* = [2\Delta_R - k]$, and $DC^* = \left[\frac{k}{2\Delta_F} - \gamma_R\right]$. $B_1^* > 0$ because $a > 0$ and $k \in (0,1]$. To see that B_2^* is positive, note that B_2^* is increasing in Δ_R and decreasing k. Accordingly, evaluating B_2^* at $\Delta_R = 1$ (the lowest value of parameter Δ_R) and $k = 1$ (the highest value of parameter k) yields the lower bound of $B_2^*(\Delta_R = 1,\ k = 1) = 1$, thus implying $B_2^* > 0$. $B_3^* > 0$ by the condition in equation (4). Thus, $B^* > 0$. $C^* < 0$ because $c > 0$ and $\Delta_F \in [1,2]$, and $QDEN = 4\Delta_F\Delta_R - k^2 > 0$ as shown in Lemma 1(i).

To show that $d_F^* \in \left(0, \overline{d}_F\right)$. The closed form solution to d_F^* is given by $d_F^* = \frac{-B^* \pm \sqrt{(B^*)^2 - 4(A^*)(C^*)}}{2A^*}$. Since $A^* > 0$, $B^* > 0$, and $C^* < 0$ implies $\sqrt{(B^*)^2 - 4(A^*)(C^*)}$ has two real valued roots with $d_F^* = \frac{-B^* - \sqrt{(B^*)^2 - 4(A^*)(C^*)}}{2A^*} < 0$, which is not in the domain of $\left(0, \overline{d}_F\right)$.

Consider the solution, $d_F^* = \frac{-B^* + \sqrt{(B^*)^2 - 4(A^*)(C^*)}}{2A^*}$. The numerator, i.e., $-B^* + \sqrt{(B^*)^2 - 4(A^*)(C^*)} > -B^* + \sqrt{(B^*)^2} = 0$, where the inequality follows because $A^* > 0$ and $C^* < 0$. The denominator is positive because $A^* > 0$. Thus, d_F^* must be greater than zero.

To show that $d_F^* < \overline{d}_F$, note that $Z^*(d_F^* = 0) = C^* < 0$. Also, $Z^*(\cdot)$ is continuous and monotonically increasing with respect to d_F^*. It follows that $d_F^* < \overline{d}_F$, if and only if $Z^*\left(d_F^* = \overline{d}_F\right) > 0$. Assume that $Z^*\left(d_F^* = \overline{d}_F\right) < 0$.

$$Z^*\left(d_F^* = \overline{d}_F\right) = \frac{k}{2}\left[4\Delta_R - \frac{3}{2}\gamma_R k - \frac{k^2}{4\Delta_F}\right]\left[\frac{k}{2\Delta_F} - \gamma_R\right]\left(\overline{d}_F\right)^2 + ak[2\Delta_R - k]\left[\frac{k}{2\Delta_F} - \gamma_R\right]\overline{d}_F - \frac{\left[4\Delta_F\Delta_R - k^2\right]^2 c}{\Delta_F} < 0$$

$$\Rightarrow \frac{k}{2}\left[4\Delta_R - \frac{3}{2}\gamma_R k - \frac{k^2}{4\Delta_F}\right]\left[\frac{k}{2\Delta_F} - \gamma_R\right](\overline{d}_F)^2 + ak[2\Delta_R - k]\left[\frac{k}{2\Delta_F} - \gamma_R\right]\overline{d}_F < \frac{\left[4\Delta_F\Delta_R - k^2\right]^2 c}{\Delta_F}$$

$$\Rightarrow \frac{\Delta_F}{[4\Delta_F\Delta_R - k^2]^2}\left\{\frac{k}{2}\left[4\Delta_R - \frac{3}{2}\gamma_R k - \frac{k^2}{4\Delta_F}\right]\left[\frac{k}{2\Delta_F} - \gamma_R\right](\overline{d}_F)^2 + ak[2\Delta_R - k]\left[\frac{k}{2\Delta_F} - \gamma_R\right]\overline{d}_F\right\} < c$$

$$\Rightarrow \frac{k\Delta_F\overline{d}_F}{[4\Delta_F\Delta_R - k^2]^2}\left\{\frac{k}{2}\left[4\Delta_R - \frac{3}{2}\gamma_R k - \frac{k^2}{4\Delta_F}\right]\left[\frac{k}{2\Delta_F} - \gamma_R\right]\overline{d}_F + ak[2\Delta_R - k]\left[\frac{k}{2\Delta_F} - \gamma_R\right]\right\} < c$$

$$\Rightarrow \frac{k\Delta_F\overline{d}_F}{[4\Delta_F\Delta_R - k^2]^2}\left\{ak[2\Delta_R - k]\left[\frac{k}{2\Delta_F} - \gamma_R\right]\right\} < c, \text{ because } \left[4\Delta_R - \frac{3}{2}\gamma_R k - \frac{k^2}{4\Delta_F}\right] = A_2^* > 0 \text{ and } A_3^* = \left[\frac{k}{2\Delta_F} - \gamma_R\right] > 0$$

$$\Rightarrow \frac{k\Delta_F\overline{d}_F}{[4\Delta_F\Delta_R - k^2]^2}\left\{ak[2\Delta_R - k]\left[\frac{k}{2\Delta_F} - \gamma_R\right] - \frac{1}{4}k\bar{d}_F\left[\gamma_R - \frac{k}{2\Delta_F}\right]^2\right\} < c \qquad \text{(A10)}$$

because $\frac{1}{4}k\bar{d}_F\left[\gamma_R - \frac{k}{2\Delta_F}\right]^2 > 0$

Equation (A10) is a contradiction with assumption (IS). Thus, $Z^*\left(d_F^* = \overline{d}_F\right) > 0$, and $d_F^* \in \left(0, \overline{d}_F\right)$. QED.

Proof of Proposition 2.

When equation (4) is not satisfied with strict inequality, firm F does not disclose its private demand information and firm R rationally conjectures that firm F is withholding good news for

$\tilde{d}_F \in [d_F^{**}, \bar{d}_F]$ implying firm R's posterior belief is $b_F = 0.5[d_F^{**} + \bar{d}_F]$ for $\tilde{d}_F \in [d_F^{**}, \bar{d}_F]$; firm F discloses its private demand information for $d_F \in [0, d_F^{**}]$ implying firm R's posterior belief is $b_F = \tilde{d}_F$ for $\tilde{d}_F \in [0, d_F^{**}]$. To derive d_F^{**}, note that d_F^{**} is the point at which firm F is indifferent between disclosing and concealing its private demand information and, thus, $\Pi_F^D(\tilde{d}_F = d_F^{**}, b_F = d_F^{**}) = \Pi_F^\phi(d_F = d_F^{**}, b_F = 0.5[d_F^{**} + \bar{d}_F])$, i.e., firm F's expected profit evaluated at $\tilde{d}_F = d_F^{**}$ is equal for disclosure and non-disclosure. Using Lemma 1(ii) we get the following, where Ω denotes disclosure of $\tilde{d}_F = d_F^{**}$ and thus $b_F = d_F^{**}$, and ϕ denotes non-disclosure at $\tilde{d}_F = d_F^{**}$ and thus $b_F = 0.5[d_F^{**} + \bar{d}_F]$.

Let $X^{**} = [\tilde{d}_F = d_F^{**}, b_F = d_F^{**}]$ and $Y^{**} = [\tilde{d}_F = d_F^{**}, b_F = 0.5[d_F^{**} + \bar{d}_F]]$.

$$\Pi_F^D(X^{**}) = \Delta_F[\mu_F Q_F^D(X^{**})]^2 - c = \Delta_F[\mu_F Q_F^\phi(Y^{**})]^2 = \Pi_F^\phi(Y^{**})$$

$$\Rightarrow \Delta_F[\mu_F Q_F^D(X^{**})]^2 - c - \Delta_F[\mu_F Q_F^\phi(Y^{**})]^2 = 0$$

$$\Rightarrow \Delta_F \mu_F^2 [Q_F^D(X^{**}) - Q_F^\phi(Y^{**})][Q_F^D(X^{**}) + Q_F^\phi(Y^{**})] - c = 0 \quad \text{(A11)}$$

From Lemma 1(i) using $Q_F^D(X^{**}) = \frac{2\Delta_F a[2\Delta_R - k] + d_F^{**}[4\Delta_F\Delta_R - k^2] - d_F^{**}k[2\Delta_F\gamma_R - k]}{\mu_F 2\Delta_F[4\Delta_F\Delta_R - k^2]}$

and $Q_F^\phi(Y^{**}) = \frac{2\Delta_F a[2\Delta_R - k] + d_F^{**}[4\Delta_F\Delta_R - k^2] - 0.5[d_F^{**} + \bar{d}_F]k[2\Delta_F\gamma_R - k]}{\mu_F 2\Delta_F[4\Delta_F\Delta_R - k^2]}$ we get

$$[Q_F^D(X^{**}) - Q_F^\phi(Y^{**})] = \frac{0.5[\bar{d}_F - d_F^{**}]k[2\Delta_F\gamma_R - k]}{\mu_F 2\Delta_F[4\Delta_F\Delta_R - k^2]} \quad \text{(A12)}$$

$$[Q_F^D(X^{**}) + Q_F^\phi(Y^{**})] = \frac{4\Delta_F a[2\Delta_R - k] + 2d_F^{**}[4\Delta_F\Delta_R - k^2] - [1.5d_F^{**} + 0.5\bar{d}_F]k[2\Delta_F\gamma_R - k]}{\mu_F 2\Delta_F[4\Delta_F\Delta_R - k^2]} \quad \text{(A13)}$$

Using the above $[Q_F^D(X^{**}) - Q_F^\phi(Y^{**})]$ and $[Q_F^D(X^{**}) + Q_F^\phi(Y^{**})]$ in (A11)

$$0 \quad = \quad -\Delta_F \mu_F^2 [Q_F^D(X^{**}) - Q_F^\phi(Y^{**})][Q_F^D(X^{**}) + Q_F^\phi(Y^{**})] + c$$

$$= -\left\{\frac{\Delta_F\mu_F^2}{4\mu_F^2\Delta_F^2[4\Delta_F\Delta_R-k^2]^2}\right\}\{0.5[\bar{d}_F - d_F^{**}]k[2\Delta_F\gamma_R - k]\}\{4\Delta_F a[2\Delta_R - k] + 2d_F^{**}[4\Delta_F\Delta_R - k^2] - [1.5d_F^{**} + 0.5\bar{d}_F]k[2\Delta_F\gamma_R - k]\} + c$$

$$= -\left\{\frac{\Delta_F}{4\Delta_F^2[4\Delta_F\Delta_R-k^2]^2}\right\}\{0.5\bar{d}_F k[2\Delta_F\gamma_R - k] - 0.5d_F^{**}k[2\Delta_F\gamma_R - k]\}\{4\Delta_F a[2\Delta_R - k] + d_F^{**}[8\Delta_F\Delta_R - 2k^2 - 3\Delta_F\gamma_R k + 1.5k^2] - 0.5\bar{d}_F k[2\Delta_F\gamma_R - k]\} + c$$

$$= 0.5k[2\Delta_F\gamma_R - k][8\Delta_F\Delta_R - 3\Delta_F\gamma_R k - 0.5k^2](d_F^{**})^2 + \{0.5k[2\Delta_F\gamma_R - k]4\Delta_F a[2\Delta_R - k] - 0.25\bar{d}_F k^2[2\Delta_F\gamma_R - k]^2 - 0.5\bar{d}_F k[2\Delta_F\gamma_R - k][8\Delta_F\Delta_R - 2k^2 - 3\Delta_F\gamma_R k + 1.5k^2]\}d_F^{**} + 4c\Delta_F[4\Delta_F\Delta_R - k^2]^2 - 2\bar{d}_F k[2\Delta_F\gamma_R - k]\Delta_F a[2\Delta_R - k] + 0.25(\bar{d}_F)^2 k^2[2\Delta_F\gamma_R - k]^2 \quad \text{(A13')}$$

$$= -0.5k2\Delta_F\left[\gamma_R - \frac{k}{2\Delta_F}\right]2\Delta_F\left[4\Delta_R - \frac{3}{2}\gamma_R k - \frac{k^2}{4\Delta_F}\right](d_F^{**})^2 + \left\{-4\Delta_F^2 ak[2\Delta_R - k]\left[\gamma_R - \frac{k}{2\Delta_F}\right] + 0.5\bar{d}_F k2\Delta_F\left[\gamma_R - \frac{k}{2\Delta_F}\right]4\Delta_F\left[2\Delta_R - \frac{1}{2}\gamma_R k - \frac{k^2}{4\Delta_F}\right]\right\}d_F^{**} - \left[\frac{4c\Delta_F^2[4\Delta_F\Delta_R - k^2]^2}{\Delta_F}\right] + 4\Delta_F^2\bar{d}_F ak\left[\gamma_R - \frac{k}{2\Delta_F}\right][2\Delta_R - k] - 4\Delta_F^2(\bar{d}_F)^2\frac{1}{4}k^2\left[\gamma_R - \frac{k}{2\Delta_F}\right]^2$$

$$= -0.5k4\Delta_F^2\left[\gamma_R - \frac{k}{2\Delta_F}\right]\left[4\Delta_R - \frac{3}{2}\gamma_R k - \frac{k^2}{4\Delta_F}\right](d_F^{**})^2 + \left\{-ak[2\Delta_R - k]\left[\gamma_R - \frac{k}{2\Delta_F}\right] + \bar{d}_F k\left[\gamma_R - \frac{k}{2\Delta_F}\right]\left[2\Delta_R - \frac{1}{2}\gamma_R k - \frac{k^2}{4\Delta_F}\right]\right\}4\Delta_F^2 d_F^{**} - \left[\frac{4\Delta_F^2 c[4\Delta_F\Delta_R - k^2]^2}{\Delta_F}\right] + 4\Delta_F^2\bar{d}_F ak\left[\gamma_R - \frac{k}{2\Delta_F}\right][2\Delta_R - k] - 4\Delta_F^2(\bar{d}_F)^2\frac{1}{4}k^2\left[\gamma_R - \frac{k}{2\Delta_F}\right]^2$$

$$= \quad -0.5k\left[\gamma_R - \frac{k}{2\Delta_F}\right]\left[4\Delta_R - \frac{3}{2}\gamma_R k - \frac{k^2}{4\Delta_F}\right](d_F^{**})^2 + \left\{-ak[2\Delta_R - k]\left[\gamma_R - \frac{k}{2\Delta_F}\right] + \bar{d}_F k\left[\gamma_R - \frac{k}{2\Delta_F}\right]\left[2\Delta_R - \frac{1}{2}\gamma_R k - \frac{k^2}{4\Delta_F}\right]\right\} d_F^{**} - \left[\frac{c\left[4\Delta_F\Delta_R - k^2\right]^2}{\Delta_F}\right] + \bar{d}_F ak\left[\gamma_R - \frac{k}{2\Delta_F}\right][2\Delta_R - k] - \left(\bar{d}_F\right)^2 \frac{1}{4}k^2\left[\gamma_R - \frac{k}{2\Delta_F}\right]^2$$

$$= \quad 0.5k\left[\gamma_R - \frac{k}{2\Delta_F}\right]\left[4\Delta_R - \frac{3}{2}\gamma_R k - \frac{k^2}{4\Delta_F}\right](d_F^{**})^2 + \left\{ak[2\Delta_R - k]\left[\gamma_R - \frac{k}{2\Delta_F}\right] - \bar{d}_F k\left[\gamma_R - \frac{k}{2\Delta_F}\right]\left[2\Delta_R - \frac{1}{2}\gamma_R k - \frac{k^2}{4\Delta_F}\right]\right\} d_F^{**} + \left[\frac{c\left[4\Delta_F\Delta_R - k^2\right]^2}{\Delta_F}\right] - \bar{d}_F ak\left[\gamma_R - \frac{k}{2\Delta_F}\right][2\Delta_R - k] + \left(\bar{d}_F\right)^2 \frac{1}{4}k^2\left[\gamma_R - \frac{k}{2\Delta_F}\right]^2$$

$$\Rightarrow \quad \frac{k}{2}\left[\gamma_R - \frac{k}{2\Delta_F}\right]\left[4\Delta_R - \frac{3}{2}\gamma_R k + \frac{k^2}{4\Delta_F}\right](d_F^{**})^2 + \left\{ak[2\Delta_R - k]\left[\gamma_R - \frac{k}{2\Delta_F}\right] - \bar{d}_F k\left[\gamma_R - \frac{k}{2\Delta_F}\right]\left[2\Delta_R - \frac{1}{2}\gamma_R k - \frac{k^2}{4\Delta_F}\right]\right\} d_F^{**} + \left[\frac{c\left[4\Delta_F\Delta_R - k^2\right]^2}{\Delta_F}\right] - \bar{d}_F ak\left[\gamma_R - \frac{k}{2\Delta_F}\right][2\Delta_R - k] + \left(\bar{d}_F\right)^2 \frac{1}{4}k^2\left[\gamma_R - \frac{k}{2\Delta_F}\right]^2 = Z^{**} = 0$$

with A^{**}, B^{**}, C^{**} as defined in Proposition 2.

To show that d_F^{**} is a unique, interior solution, begin by establishing the signs of A^{**}, B^{**}, and C^{**}.

$A^{**} = \frac{k}{2}\left[4\Delta_R - \frac{3}{2}\gamma_R k - \frac{k^2}{4\Delta_F}\right]\left[\gamma_R - \frac{k}{2\Delta_F}\right] = (A_1^{**})(A_2^{**})(A_3^{**})$, where $A_1^{**} = \frac{k}{2}$, $A_2^{**} = \left[4\Delta_R - \frac{3}{2}\gamma_R k - \frac{k^2}{4\Delta_F}\right]$, $A_3^{**} = \left[\gamma_R - \frac{k}{2\Delta_F}\right]$. $A_1^{**} > 0$ because $k \in (0,1]$. To see that $A_2^{**} > 0$, note that A_2^{**} is increasing in Δ_i for $i = F, R$, and decreasing in k and γ_R. Accordingly, evaluating A_2^{**} at $\Delta_i = 1$ (the lowest value of parameter Δ_i) and $k = \gamma_R = 1$ (the highest values of parameters k

and γ_R) yields the lower bound of $A_2^{**}(\Delta_i = 1,\ k = 1,\ \gamma_R = 1) = \frac{9}{4}$, thus implying $A_2^{**} > 0$.

$A_3^{**} > 0$ by the condition in equation (4) not being satisfied. Thus, $A^{**} > 0$.

$B^{**} = ak[2\Delta_R - k]\left[\gamma_R - \frac{k}{2\Delta_F}\right] - k\left[2\Delta_R - \frac{1}{2}\gamma_R k - \frac{k^2}{4\Delta_F}\right]\left[\gamma_R - \frac{k}{2\Delta_F}\right]\bar{d}_F = k\left\{-\left[2\Delta_R - \frac{1}{2}\gamma_R k - \frac{k^2}{4\Delta_F}\right]\bar{d}_F + a[2\Delta_R - k]\right\}\left[\gamma_R - \frac{k}{2\Delta_F}\right] = (B_1^{**})(B_2^{**})(B_3^{**})$, where $B_1^{**} = k$, $B_2^{**} = a[2\Delta_R - k] - \bar{d}_F\left[2\Delta_R - \frac{1}{2}\gamma_R k - \frac{k^2}{4\Delta_F}\right]$ and $B_3^{**} = \left[\gamma_R - \frac{k}{2\Delta_F}\right]$. $B_1^{**} > 0$ because $k \in (0,1]$. $B_2^{**} = a[2\Delta_R - k] - \bar{d}_F\left[2\Delta_R - \frac{1}{2}\gamma_R k - \frac{k^2}{4\Delta_F}\right] > 2\bar{d}_F[2\Delta_R - k] - \bar{d}_F\left[2\Delta_R - \frac{1}{2}\gamma_R k - \frac{k^2}{4\Delta_F}\right] = \bar{d}_F\left\{2(\Delta_R - k) + \frac{1}{2}\gamma_R k + \frac{k^2}{4\Delta_F}\right\} > 0$; where the first inequality is obtained by using assumption (PQ), i.e. $a > 2\bar{d}_F$, and the second inequality is obtained using $\Delta_R \geq 1$ and $k \leq 1$. $B_3^{**} > 0$ by the condition in equation (4) not being satisfied. Thus, $B^{**} > 0$.

$C^{**} = -\,ak[2\Delta_R - k]\left[\gamma_R - \frac{k}{2\Delta_F}\right] + \left\{\frac{1}{2}k\left[\gamma_R - \frac{k}{2\Delta_F}\right]\bar{d}_F\right\}^2 + \frac{[4\Delta_F\Delta_R - k^2]^2 c}{\Delta_F} = \frac{k\Delta_F\bar{d}_F}{[4\Delta_F\Delta_R - k^2]^2}\left\{-a[2\Delta_R - k]\left[\gamma_R - \frac{k}{2\Delta_F}\right] + \frac{1}{4}k\bar{d}_F\left(\gamma_R - \frac{k}{2\Delta_F}\right)^2\right\} + c < 0$, where the last inequality follows from assumption (IS).

To show that $d_F^{**} \in (0, \bar{d}_F)$. The closed form solution to d_F^{**} is given by $d_F^{**} = \frac{-B^{**} \pm \sqrt{(B^{**})^2 - 4(A^{**})(C^{**})}}{2A^{*}}$. Since $A^{**} > 0$, $B^{**} > 0$, and $C^{**} < 0$ implies $\sqrt{(B^{**})^2 - 4(A^{**})(C^{**})}$ has two real valued roots with $d_F^{**} = \frac{-B^{*} - \sqrt{(B^{*})^2 - 4(A^{*})(C^{*})}}{2A^{*}} < 0$, which is not in the domain of $(0, \bar{d}_F)$. Consider the solution, $d_F^{**} = \frac{-B^{*} + \sqrt{(B^{*})^2 - 4(A^{*})(C^{*})}}{2A^{*}}$. The numerator, i.e., $-B^{**} + \sqrt{(B^{**})^2 - 4(A^{**})(C^{**})} > -B^{**} + \sqrt{(B^{**})^2} = 0$, where the inequality follows because $A^{*} > 0$, and $C^{*} < 0$. The denominator is positive because $A^{*} > 0$. Thus, d_F^{**} must be greater than zero.

To show that $d_F^{**} < \overline{d}_F$, note that $Z^{**}(d_F^{**} = 0) = C^* < 0$. Also, $Z^{**}(\cdot)$ is continuous and monotonically increasing with respect to d_F^{**}. It follows that $d_F^{**} < \overline{d}_F$, if and only if $Z^{**}(d_F^{**} = \overline{d}_F) > 0$. Assume that $Z^{**}(d_F^{**} = \overline{d}_F) < 0$.

$$\begin{aligned}
Z^{**}\left(d_F^{**} = \overline{d}_F\right) &= \tfrac{k}{2}\left[\gamma_R - \tfrac{k}{2\Delta_F}\right]\left[4\Delta_R - \tfrac{3}{2}\gamma_R k + \tfrac{k^2}{4\Delta_F}\right]\left(\overline{d}_F\right)^2 + \Big\{ak[2\Delta_R - k]\left[\gamma_R - \tfrac{k}{2\Delta_F}\right] - \bar{d}_F k\left[\gamma_R - \tfrac{k}{2\Delta_F}\right]\left[2\Delta_R - \tfrac{1}{2}\gamma_R k - \tfrac{k^2}{4\Delta_F}\right]\Big\}\overline{d}_F + \\
&\quad \left[\tfrac{c\left[4\Delta_F\Delta_R - k^2\right]^2}{\Delta_F}\right] - \bar{d}_F ak\left[\gamma_R - \tfrac{k}{2\Delta_F}\right][2\Delta_R - k] + \left(\bar{d}_F\right)^2 \tfrac{1}{4}k^2\left[\gamma_R - \tfrac{k}{2\Delta_F}\right]^2 < 0 \\
&= \left(\bar{d}_F\right)^2 k\left[\gamma_R - \tfrac{k}{2\Delta_F}\right]\left[2\Delta_R - \tfrac{3}{4}\gamma_R k - \tfrac{k^2}{8\Delta_F} - 2\Delta_R + \tfrac{1}{2}\gamma_R k + \tfrac{k^2}{4\Delta_F}\right] + \left(\bar{d}_F\right)^2 \tfrac{1}{4}k^2\left[\gamma_R - \tfrac{k}{2\Delta_F}\right]^2 + \bar{d}_F ak\left[\gamma_R - \tfrac{k}{2\Delta_F}\right][2\Delta_R - k][1-1] + \left[\tfrac{c\left[4\Delta_F\Delta_R - k^2\right]^2}{\Delta_F}\right] < 0 \\
&= -\left(\bar{d}_F\right)^2 \tfrac{1}{4}k^2\left[\gamma_R - \tfrac{k}{2\Delta_F}\right]^2 + \left(\bar{d}_F\right)^2 \tfrac{1}{4}k^2\left[\gamma_R - \tfrac{k}{2\Delta_F}\right]^2 + \left[\tfrac{c\left[4\Delta_F\Delta_R - k^2\right]^2}{\Delta_F}\right] < 0 \\
&= \left[\tfrac{c\left[4\Delta_F\Delta_R - k^2\right]^2}{\Delta_F}\right] < 0,
\end{aligned}$$

a contradiction because $[4\Delta_F\Delta_R - k^2] = QDEN > 0$, see Lemma 1; $\Delta_F > 1$ and $c > 0$.

Proof of Proposition 3.

When equation (4) is satisfied at equality from Lemma 1(i) using $\gamma_R = \frac{k}{2\Delta_F}$, we get $Q_F^D = Q_F^\phi = \frac{2\Delta_F a[2\Delta_R - k] + d_F[4\Delta_F\Delta_R - k^2]}{\mu_F 2\Delta_F[4\Delta_F\Delta_R - k^2]}$. Thus, $\Pi_F^D(\cdot) = \Delta_F[\mu_F Q_F^D]^2 - c < \Delta_F\left[\mu_F Q_F^\phi\right]^2 = \Pi_F^\phi(\cdot)$, where the inequality follows from using $Q_F^D = Q_F^\phi$ and $c > 0$.

Proof of Proposition 4.

From equation (A7) d_F^* is solution to the following equation.

$$Z^* \quad = \quad \Delta_F\mu_F^2\left[Q_F^D(X^*) - Q_F^\phi(Y^*)\right]\left[Q_F^D(X^*) + Q_F^\phi(Y^*)\right] - c = 0.$$

Define Z_1^* and Z_2^* and use (A8) and (A9) to get

$$Z_1^* \quad = \quad \Delta_F\mu_F\left[Q_F^D(X^*) - Q_F^\phi(Y^*)\right] = \frac{d_F^* k(k - 2\Delta_F\gamma_R)}{4[4\Delta_F\Delta_R - k^2]} > 0 \qquad \text{(A14)}$$

$$Z_2^* \quad = \quad \mu_F\left[Q_F^D(X^*) + Q_F^\phi(Y^*)\right] = \frac{4\Delta_F a[2\Delta_R - k] + 2d_F^*[4\Delta_F\Delta_R - k^2] - 1.5d_F^* k[2\Delta_F\gamma_R - k]}{2\Delta_F[4\Delta_F\Delta_R - k^2]} > 0 \qquad \text{(A15)}$$

Where the inequality for Z_1^* follows from equation (4) being satisfied, and the inequality for Z_2^* follows from Lemma 1. Thus,

$$Z^* \quad = \quad Z_1^* Z_2^* - c = 0$$

$$[\partial Z^*/\partial\Delta_i] \quad = \quad [\partial Z_1^*/\partial\Delta_i]Z_2^* + [\partial Z_2^*/\partial\Delta_i]Z_1^* = 0 \qquad \text{(A16)}$$

Differentiating Z_1^* with respect to Δ_F we get

$$[\partial Z_1^*/\partial\Delta_F] \quad = \quad \left[\frac{d_F^* k}{4[4\Delta_F\Delta_R - k^2]^2}\right][(4\Delta_F\Delta_R - k^2)(-2\gamma_R) - 4\Delta_R(k - 2\Delta_F\gamma_R)]$$

$$= \quad \left[\frac{d_F^* k}{4[4\Delta_F\Delta_R - k^2]^2}\right][-8\Delta_F\Delta_R\gamma_R + 2\gamma_R k^2 - 4\Delta_R k + 8\Delta_F\Delta_R\gamma_R]$$

$$= \quad \left[\frac{d_F^* k^2}{2[4\Delta_F\Delta_R - k^2]^2}\right][\gamma_R k - 2\Delta_R] < 0, \qquad \text{(A17)}$$

where the last inequality follows because $\gamma_R \leq 1$, $k \leq 1$ and $\Delta_R \geq 1$.

Differentiating Z_2^* with respect to Δ_F we get

$$
\begin{aligned}
[\partial Z_2^*/\partial \Delta_F] &= \left[\frac{1}{2\Delta_F^2[4\Delta_F\Delta_R - k^2]^2}\right][(4\Delta_F\Delta_R - k^2)\{4\Delta_F a[2\Delta_R - k] + 8d_F^*\Delta_F\Delta_R - \\
&\quad 3d_F^* k\Delta_F\gamma_R\} - (8\Delta_F\Delta_R - k^2)\{4\Delta_F a[2\Delta_R - k] + 2d_F^*[4\Delta_F\Delta_R - \\
&\quad k^2] - 1.5d_F^* k[2\Delta_F\gamma_R - k]\}] \\
&= \left[\frac{1}{2\Delta_F^2[4\Delta_F\Delta_R - k^2]^2}\right][(4\Delta_F\Delta_R - k^2)\{4\Delta_F a[2\Delta_R - k] + 8d_F^*\Delta_F\Delta_R - \\
&\quad 3d_F^* k\Delta_F\gamma_R\} - (4\Delta_F\Delta_R - k^2)\{4\Delta_F a[2\Delta_R - k] + 8d_F^*\Delta_F\Delta_R - \\
&\quad 2d_F^* k^2 - 3d_F^* k\Delta_F\gamma_R + 1.5d_F^* k^2\} - 4\Delta_F\Delta_R\{4\Delta_F a[2\Delta_R - k] + \\
&\quad 2d_F^*[4\Delta_F\Delta_R - k^2] - 1.5d_F^* k[2\Delta_F\gamma_R - k]\}] \\
&= \left[\frac{1}{2\Delta_F^2[4\Delta_F\Delta_R - k^2]^2}\right][-(4\Delta_F\Delta_R - k^2)\{-0.5d_F^* k^2\} - \\
&\quad 4\Delta_F\Delta_R\{4\Delta_F a[2\Delta_R - k] + 2d_F^*[4\Delta_F\Delta_R - k^2] - 1.5d_F^* k[2\Delta_F\gamma_R - \\
&\quad k]\}] \\
&= \left[\frac{1}{2\Delta_F^2[4\Delta_F\Delta_R - k^2]^2}\right][-(4\Delta_F\Delta_R - k^2)\{-0.5d_F^* k^2 + 8d_F^* 4\Delta_F\Delta_R\} - \\
&\quad 4\Delta_F\Delta_R\{4\Delta_F a[2\Delta_R - k] - 1.5d_F^* k[2\Delta_F\gamma_R - k]\}] \\
&= \left[\frac{1}{2\Delta_F^2[4\Delta_F\Delta_R - k^2]^2}\right][-(4\Delta_F\Delta_R - k^2)0.5d_F^*\{16\Delta_F\Delta_R - k^2\} - \qquad \text{(A18)} \\
&\quad 4\Delta_F\Delta_R\{4\Delta_F a[2\Delta_R - k] + 1.5d_F^* k[k - 2\Delta_F\gamma_R]\}] < 0,
\end{aligned}
$$

where the last inequality follows because $(4\Delta_F\Delta_R - k^2) > 0$ (see proof of Lemma 1), $[2\Delta_R - k]$, (see proof of Lemma 1), $[k - 2\Delta_F\gamma_R] > 0$ (equation (4) is satisfied), and $\{16\Delta_F\Delta_R - k^2\} > 0$ from $(4\Delta_F\Delta_R - k^2) > 0$. Thus, $[\partial Z^*/\partial \Delta_F] < 0$ using (A14), (A15), (A17), (A18) in (A16).

Differentiate $Z^* = A^*(d_F^*)^2 + B^* d_F^* + C^* = 0$ from Proposition 1 to get $[\partial Z^*/\partial d_F^*] = 2A^* d_F^* + B^* > 0$, where the last inequality follows because $A^* > 0$, $B^* > 0$, and $d_F^* > 0$ (see proof of Proposition 1).

Totally differentiate Z^* with respect to Δ_i to get

$$\left[\frac{\partial Z^*}{\partial d_F^*}\right]\left\{\frac{d d_F^*}{d\Delta_i}\right\} + \left[\frac{\partial Z^*}{\partial \Delta_i}\right] = 0$$

$$\Rightarrow \frac{d d_F^*}{d\Delta_i} = -\left\{\left[\frac{\partial Z^*}{\partial \Delta_i}\right] \Big/ \left[\frac{\partial Z^*}{\partial d_F^*}\right]\right\} \qquad \text{(A19)}$$

Using $[\partial Z^*/\partial d_F^*] > 0$ and $[\partial Z^*/\partial \Delta_F] < 0$ in (A21) we get $\frac{d d_F^*}{d\Delta_F} > 0$.

Differentiating Z_1^* with respect to Δ_R we get

$$\begin{aligned}[\partial Z_1^*/\partial \Delta_R] &= -\left[\frac{d_F^* k(k - 2\Delta_F\gamma_R)}{4[4\Delta_F\Delta_R - k^2]^2}\right][4\Delta_F] < 0 \\ &= -\left[\frac{d_F^* \Delta_F k(k - 2\Delta_F\gamma_R)}{[4\Delta_F\Delta_R - k^2]^2}\right] \qquad \text{(A20)}\end{aligned}$$

where the last inequality follows because equation (4) is satisfied and $(4\Delta_F\Delta_R - k^2) > 0$.

Differentiating Z_2^* with respect to Δ_R we get

$$\begin{aligned}[\partial Z_2^*/\partial \Delta_R] &= \left[\frac{1}{2\Delta_F[4\Delta_F\Delta_R - k^2]^2}\right][(4\Delta_F\Delta_R - k^2)\{8\Delta_F a + 8d_F^*\Delta_F\} - \\ &\quad (4\Delta_F)\{4\Delta_F a[2\Delta_R - k] + 2d_F^*[4\Delta_F\Delta_R - k^2] - 1.5d_F^* k[2\Delta_F\gamma_R - \\ &\quad k]\}] \\ &= \left[\frac{1}{2\Delta_F[4\Delta_F\Delta_R - k^2]^2}\right]\big[4\Delta_F a\{2(4\Delta_F\Delta_R - k^2) - 4\Delta_F[2\Delta_R - k]\} - \\ &\quad (4\Delta_F\Delta_R - k^2)\{8d_F^*\Delta_F - 8d_F^*\Delta_F\} - (4\Delta_F)1.5d_F^* k[k - 2\Delta_F\gamma_R]\big] \\ &= \left[\frac{1}{2\Delta_F[4\Delta_F\Delta_R - k^2]^2}\right]\big[4\Delta_F a\{4\Delta_F k - 2k^2\} - 6d_F^* k\Delta_F[k - 2\Delta_F\gamma_R]\big]\end{aligned}$$

$$= \left[\frac{1}{2\Delta_F[4\Delta_F\Delta_R-k^2]^2}\right]\left[8\Delta_F ak\{2\Delta_F-k\}-6d_F^* k\Delta_F[k-2\Delta_F\gamma_R]\right]$$

$$= \left[\frac{2k\Delta_F}{2\Delta_F[4\Delta_F\Delta_R-k^2]^2}\right]\left[4a\{2\Delta_F-k\}-3d_F^*[k-2\Delta_F\gamma_R]\right]$$

$$= \left[\frac{k\left[4a\{2\Delta_F-k\}-3d_F^*[k-2\Delta_F\gamma_R]\right]}{[4\Delta_F\Delta_R-k^2]^2}\right] \quad \text{(A21)}$$

Using (A14), (A15), (A20) and (A21) in (A16) we get

$$[\partial Z^*/\partial\Delta_R] = -\left[\frac{d_F^*\Delta_F k(k-2\Delta_F\gamma_R)}{[4\Delta_F\Delta_R-k^2]^2}\right]\left\{\frac{4\Delta_F a[2\Delta_R-k]+2d_F^*[4\Delta_F\Delta_R-k^2]-1.5d_F^*k[2\Delta_F\gamma_R-k]}{2\Delta_F[4\Delta_F\Delta_R-k^2]}\right\}+$$

$$\left[\frac{k\left[4a\{2\Delta_F-k\}-3d_F^*[k-2\Delta_F\gamma_R]\right]}{[4\Delta_F\Delta_R-k^2]^2}\right]\left\{\frac{d_F^*k(k-2\Delta_F\gamma_R)}{4[4\Delta_F\Delta_R-k^2]}\right\}$$

$$= -\left[\frac{d_F^*k(k-2\Delta_F\gamma_R)}{4[4\Delta_F\Delta_R-k^2]^3}\right]\{2(4\Delta_F a[2\Delta_R-k]+2d_F^*[4\Delta_F\Delta_R-k^2]-$$

$$1.5d_F^*k[2\Delta_F\gamma_R-k])-k\left[4a\{2\Delta_F-k\}-3d_F^*[k-2\Delta_F\gamma_R]\right]\}$$

$$= -\left[\frac{d_F^*k(k-2\Delta_F\gamma_R)}{4[4\Delta_F\Delta_R-k^2]^3}\right]\{4\Delta_F a[4\Delta_R-2k-2k+k^2]+2d_F^*[4\Delta_F\Delta_R-$$

$$k^2]+4.5d_F^*k[k-2\Delta_F\gamma_R]]\}$$

$$= -\left[\frac{d_F^*k(k-2\Delta_F\gamma_R)}{4[4\Delta_F\Delta_R-k^2]^3}\right]\{4\Delta_F a[4(\Delta_R-k)+k^2]+2d_F^*[4\Delta_F\Delta_R-k^2]+$$

$$4.5d_F^*k[k-2\Delta_F\gamma_R]\}<0,$$

where the last inequality follows from equation (4), $[4\Delta_F\Delta_R-k^2]>0$, $\Delta_R\geq 1$, $k\leq 1$ and $d_F^*>0$.

Using $[\partial Z^*/\partial d_F^*]>0$ and $[\partial Z^*/\partial\Delta_R]<0$ in (A19) we get $\frac{dd_F^*}{d\Delta_R}>0$. QED.

Proof of Proposition 5.

From equation (A11), d_F^{**} is the solution to the following equation.

$$Z^{**} = -\Delta_F \mu_F^2 \left[Q_F^D(X^{**}) - Q_F^{\phi}(Y^{**})\right]\left[Q_F^D(X^{**}) + Q_F^{\phi}(Y^{**})\right] + c = 0$$

Define Z_1^{**} and Z_2^{**} and use (A12) and (A13) to get

$$Z_1^{**} = -\mu_F\left[Q_F^D(X^{**}) - Q_F^{\phi}(Y^{**})\right] = -\left[\frac{[\bar{d}_F - d_F^{**}]k[2\Delta_F\gamma_R - k]}{4\Delta_F[4\Delta_F\Delta_R - k^2]}\right] < 0 \quad \text{(A22)}$$

$$Z_2^{**} = \Delta_F\mu_F\left[Q_F^D(X^{**}) + Q_F^{\phi}(Y^{**})\right] = \frac{4\Delta_F a[2\Delta_R - k] + 2d_F^{**}[4\Delta_F\Delta_R - k^2]}{2[4\Delta_F\Delta_R - k^2]} - \frac{[1.5d_F^{**} + 0.5\bar{d}_F]k[2\Delta_F\gamma_R - k]}{2[4\Delta_F\Delta_R - k^2]} > 0 \quad \text{(A23)}$$

where the inequality for Z_1^{**} follows from equation (4) not being satisfied, and the inequality for Z_2^{**} follows from Lemma 1. Thus,

$$Z^{**} = Z_1^{**}Z_2^{**} - c = 0$$

$$[\partial Z^{**}/\partial\Delta_i] = [\partial Z_1^{**}/\partial\Delta_i]Z_2^{**} + [\partial Z_2^{**}/\partial\Delta_i]Z_1^{**} = 0 \quad \text{(A24)}$$

Differentiating Z_1^{**} with respect to Δ_F we get

$$[\partial Z_1^{**}/\partial\Delta_F] = -\left[\frac{[\bar{d}_F - d_F^{**}]k}{4\Delta_F^2[4\Delta_F\Delta_R - k^2]^2}\right][(4\Delta_F^2\Delta_R - \Delta_F k^2)(2\gamma_R) - [2\Delta_F\gamma_R - k](8\Delta_F\Delta_R - k^2)]$$

$$= -\left[\frac{[\bar{d}_F - d_F^{**}]k}{4\Delta_F^2[4\Delta_F\Delta_R - k^2]^2}\right][8\Delta_F^2\Delta_R\gamma_R - 2\Delta_F\gamma_R k^2 - 16\Delta_F^2\Delta_R\gamma_R + 2\Delta_F\gamma_R k^2 + 8\Delta_F\Delta_R k - k^3]$$

$$= -\left[\frac{[\bar{d}_F - d_F^{**}]k}{4\Delta_F^2[4\Delta_F\Delta_R - k^2]^2}\right][-8\Delta_F^2\Delta_R\gamma_R + 8\Delta_F\Delta_R k - k^3]. \quad \text{(A25)}$$

Note that $-\left[\frac{[\bar{d}_F - d_F^{**}]k}{4\Delta_F^2[4\Delta_F\Delta_R - k^2]^2}\right] < 0$ and $[-8\Delta_F^2\Delta_R\gamma_R + 8\Delta_F\Delta_R k - k^3]$ is decreasing in γ_R in (A25), so (A25) is positive if and only if γ_R is sufficiently large. Evaluating (A25) at $\gamma_R = (k/2\Delta_F)$ we get

$$[\partial Z_1^{**}/\partial \Delta_F]_{\gamma_R=(k/2\Delta_F)} = -\left[\frac{[\bar{d}_F - d_F^{**}]k}{4\Delta_F^2[4\Delta_F\Delta_R - k^2]^2}\right][-8\Delta_F^2\Delta_R(k/2\Delta_F) + 8\Delta_F\Delta_R k - k^3]$$

$$= -\left[\frac{[\bar{d}_F - d_F^{**}]k}{4\Delta_F^2[4\Delta_F\Delta_R - k^2]^2}\right][-4\Delta_F\Delta_R k + 8\Delta_F\Delta_R k - k^3]$$

$$= -\left[\frac{[\bar{d}_F - d_F^{**}]k^2[4\Delta_F\Delta_R - k^2]}{4\Delta_F^2[4\Delta_F\Delta_R - k^2]^2}\right] < 0, \quad \text{(A26)}$$

where the last inequality follows because $d_F^{**} < \bar{d}_F$ (see proof of Proposition 2) and $[4\Delta_F\Delta_R - k^2] > 0$ (see proof of Lemma 1). Using $[Z_1^{**}]_{\gamma_R=(k/2\Delta_F)} = 0$ in (A24) we get

$$[\partial Z^{**}/\partial \Delta_i]_{\gamma_R=(k/2\Delta_F)} = [\partial Z_1^{**}/\partial \Delta_F]_{\gamma_R=(k/2\Delta_F)}[Z_2^{**}]_{\gamma_R=(k/2\Delta_F)} < 0 \quad \text{(A27)}$$

Differentiate $Z^{**} = A^{**}(d_F^{**})^2 + B^{**}d_F^{**} + C^{**} = 0$ from Proposition 2 to get $[\partial Z^{**}/\partial d_F^{**}] = 2A^{**}d_F^{**} + B^{**} > 0$, where the last inequality follows because $A^{**} > 0$, $B^{**} > 0$, and $d_F^{**} > 0$ (see proof of Proposition 2).

Totally differentiate Z^{**} with respect to Δ_i to get

$$\left[\frac{\partial Z^{**}}{\partial d_F^{**}}\right]\left\{\frac{d d_F^{**}}{d\Delta_i}\right\} + \left[\frac{\partial Z^{**}}{\partial \Delta_i}\right] = 0$$

$$\Rightarrow \frac{d d_F^{**}}{d\Delta_i} = -\left\{\left[\frac{\partial Z^{**}}{\partial \Delta_i}\right] \Big/ \left[\frac{\partial Z^{**}}{\partial d_F^{*}}\right]\right\} \quad \text{(A28)}$$

Using $[\partial Z^{**}/\partial d_F^{**}] > 0$, (A25) is positive if and only if γ_R is sufficiently large, and $[\partial Z^{**}/\partial \Delta_i]_{\gamma_R=(k/2\Delta_F)} < 0$ in (A28), we get $\left[\frac{d d_F^{**}}{d\Delta_F}\right]_{\gamma_R=(k/2\Delta_F)} > 0$ if and only if $\gamma_R < \gamma_R^{**}$, where γ_R^{**} is the unique γ_R-value that solves (A25) = 0.

Differentiating Z_2^{**} with respect to Δ_F we get

$$[\partial Z_2^{**}/\partial \Delta_F] = \left[\frac{1}{2\Delta_F^2[4\Delta_F\Delta_R-k^2]^2}\right]\big[(4\Delta_F\Delta_R-k^2)\{4a[2\Delta_R-k]+8d_F^{**}\Delta_R-$$

$$(1.5d_F^{**}+0.5\bar{d}_F)2\gamma_R k\}-4\Delta_R\{4\Delta_F a[2\Delta_R-k]+2d_F^{**}[4\Delta_F\Delta_R-$$

$$k^2]-[1.5d_F^{**}+0.5\bar{d}_F]k[2\Delta_F\gamma_R-k]\}\big]$$

$$= \left[\frac{1}{2\Delta_F^2[4\Delta_F\Delta_R-k^2]^2}\right]\big[4a[2\Delta_R-k]\{(4\Delta_F\Delta_R-k^2)-4\Delta_F\Delta_R\}+$$

$$(4\Delta_F\Delta_R-k^2)(8d_F^{**}\Delta_R-8d_F^{**}\Delta_R)+(1.5d_F^{**}+$$

$$0.5\bar{d}_F)k\{[2\Delta_F\gamma_R-k]4\Delta_R-2\gamma_R(4\Delta_F\Delta_R-k^2)\}\big]$$

$$= \left[\frac{1}{2\Delta_F^2[4\Delta_F\Delta_R-k^2]^2}\right]\big[-4ak^2[2\Delta_R-k]-2k^2(1.5d_F^{**}+0.5\bar{d}_F)[2\Delta_R- \tag{A29}$$

$$\gamma_R k]\big]<0,$$

where the last inequality follows because $(4\Delta_F\Delta_R-k^2)>0$ (see proof of Lemma 1), $[2\Delta_R-k]>0$, (see proof of Lemma 1), $[2\Delta_R-\gamma_R k]>0$, $\Delta_R\geq 1$, $\gamma_R\leq 1$ and $k\leq 1$).

Evaluating (A25) at $\gamma_R=1$ we get

$$[\partial Z_1^{**}/\partial \Delta_F]_{\gamma_R=1} = -\left[\frac{[\bar{d}_F-d_F^{**}]k}{4\Delta_F^2[4\Delta_F\Delta_R-k^2]^2}\right][-8\Delta_F^2\Delta_R+8\Delta_F\Delta_R k-k^3]$$

$$= -\left[\frac{[\bar{d}_F-d_F^{**}]k8\Delta_F\Delta_R}{4\Delta_F^2[4\Delta_F\Delta_R-k^2]^2}\right][k-\Delta_F-k^3]$$

$$= -\left[\frac{2[\bar{d}_F-d_F^{**}]k\Delta_R}{\Delta_F[4\Delta_F\Delta_R-k^2]^2}\right][k-\Delta_F-k^3]>0, \tag{A30}$$

where the last inequality follows because $\Delta_R\geq 1$ and $k\leq 1$. Using the signs of (A22), (A23), (A29) and (A30) we get $[\partial Z^{**}/\partial \Delta_i]_{\gamma_R=1}>0$. Using $[\partial Z^{**}/\partial d_F^{**}]>0$, (A25) is positive if and only if γ_R is sufficiently large, and $[\partial Z^{**}/\partial \Delta_i]_{\gamma_R=1}>0$ in (A28) we get $\left[\frac{dd_F^{**}}{d\Delta_F}\right]_{\gamma_R=1}<0$ if and only if $\gamma_R>\gamma_R^{**}$, where γ_R^{**} is the unique γ_R-value that solves (A25) = 0.

Differentiating Z_1^{**} with respect to Δ_R we get

$$[\partial Z_1^{**}/\partial \Delta_R] \quad = \quad \left[\frac{[\bar{d}_F - d_F^{**}]k[2\Delta_F\gamma_R - k]}{[4\Delta_F\Delta_R - k^2]^2}\right] > 0, \qquad \text{(A31)}$$

Differentiating Z_2^{**} with respect to Δ_R we get

$$[\partial Z_2^{**}/\partial \Delta_R] \quad = \quad \left[\frac{1}{2[4\Delta_F\Delta_R - k^2]^2}\right]\Big[(4\Delta_F\Delta_R - k^2)\{8\Delta_F a + 8d_F^{**}\Delta_F\} -$$

$$4\Delta_F\big\{4\Delta_F a[2\Delta_R - k] + 2d_F^{**}[4\Delta_F\Delta_R - k^2] - [1.5d_F^{**} +$$

$$0.5\bar{d}_F]k[2\Delta_F\gamma_R - k]\big\}\Big]$$

$$= \quad \left[\frac{1}{2[4\Delta_F\Delta_R - k^2]^2}\right]\Big[4\Delta_F a\{2(4\Delta_F\Delta_R - k^2) - 4\Delta_F[2\Delta_R - k]\} -$$

$$(4\Delta_F\Delta_R - k^2)\{8d_F^{**}\Delta_F - 8d_F^{**}\Delta_F\} + (4\Delta_F)[1.5d_F^{**} +$$

$$0.5\bar{d}_F]k[2\Delta_F\gamma_R - k]\Big]$$

$$= \quad \left[\frac{1}{2[4\Delta_F\Delta_R - k^2]^2}\right]\Big[4\Delta_F a\{4\Delta_F k - 2k^2\} + 4\Delta_F[1.5d_F^{**} +$$

$$0.5\bar{d}_F]k[2\Delta_F\gamma_R - k]\Big]$$

$$= \quad \left[\frac{2\Delta_F k\{2a\{2\Delta_F - k\} + [1.5d_F^{**} + 0.5\bar{d}_F][2\Delta_F\gamma_R - k]\}}{[4\Delta_F\Delta_R - k^2]^2}\right] \qquad \text{(A32)}$$

Using (A22), (A23), (A31) and (A32) in (A28) we get

$$[\partial Z^{**}/\partial \Delta_R] \quad = \quad -\left[\frac{[\bar{d}_F - d_F^{**}]k[2\Delta_F\gamma_R - k]}{4\Delta_F[4\Delta_F\Delta_R - k^2]}\right]\left[\frac{2\Delta_F k\{2a\{2\Delta_F - k\} + [1.5d_F^{**} + 0.5\bar{d}_F][2\Delta_F\gamma_R - k]\}}{[4\Delta_F\Delta_R - k^2]^2}\right] + \left[\frac{[\bar{d}_F - d_F^{**}]k[2\Delta_F\gamma_R - k]}{[4\Delta_F\Delta_R - k^2]^2}\right]\left\{\frac{4\Delta_F a[2\Delta_R - k] + 2d_F^{**}\big[4\Delta_F\Delta_R - k^2\big] - [1.5d_F^{**} + 0.5\bar{d}_F]k[2\Delta_F\gamma_R - k]}{2[4\Delta_F\Delta_R - k^2]}\right\}$$

$$= \quad -\left[\frac{[\bar{d}_F - d_F^{**}]k[2\Delta_F\gamma_R - k]}{2[4\Delta_F\Delta_R - k^2]}\right]\left[\frac{k\{2a\{2\Delta_F - k\} + [1.5d_F^{**} + 0.5\bar{d}_F][2\Delta_F\gamma_R - k]\}}{[4\Delta_F\Delta_R - k^2]^2}\right] + \left[\frac{[\bar{d}_F - d_F^{**}]k[2\Delta_F\gamma_R - k]}{[4\Delta_F\Delta_R - k^2]^2}\right]\left\{\frac{4\Delta_F a[2\Delta_R - k] + 2d_F^{**}\big[4\Delta_F\Delta_R - k^2\big] - [1.5d_F^{**} + 0.5\bar{d}_F]k[2\Delta_F\gamma_R - k]}{2[4\Delta_F\Delta_R - k^2]}\right\}$$

$$= -\left[\frac{[\bar{d}_F - d_F^{**}]k[2\Delta_F\gamma_R - k]}{2[4\Delta_F\Delta_R - k^2]^3}\right]\Big[2ak\{2\Delta_F - k\} + [1.5d_F^{**} + 0.5\bar{d}_F]k[2\Delta_F\gamma_R - k] -$$

$$4\Delta_F a[2\Delta_R - k] - 2d_F^{**}[4\Delta_F\Delta_R - k^2] + [1.5d_F^{**} + 0.5\bar{d}_F]k[2\Delta_F\gamma_R - k]\Big]$$

$$= -\left[\frac{d_F^{**}k(k - 2\Delta_F\gamma_R)}{4[4\Delta_F\Delta_R - k^2]^3}\right]\{4\Delta_F a[4(\Delta_R - k) + k^2] + 2d_F^{**}[4\Delta_F\Delta_R - k^2] +$$

$$4.5d_F^{**}k[k - 2\Delta_F\gamma_R]\} < 0,$$

where the last inequality follows from equation (4), $[4\Delta_F\Delta_R - k^2] > 0$, $\Delta_R \geq 1$, $k \leq 1$ and $d_F^{**} > 0$.

Using $[\partial Z^{**}/\partial d_F^{**}] > 0$ and $[\partial Z^{**}/\partial \Delta_R] < 0$ in (A28) we get $\frac{dd_F^{**}}{d\Delta_R} > 0$. QED.

Proof of Proposition 6.

The expected profit for firm F under full disclosure, noting that firm R's belief is $b_F = d_F$, is given by $E_{d_F}\{\Pi_F^D(d_F, b_F = d_F)\}$ defined in Equation (A33).

$$E_{d_F}\{\Pi_F^D(\cdot)\} = \Delta_F E_{d_F}\{[\mu_F Q_F^{\Omega}(d_F, b_F = d_F)]^2\}$$

$$= \left\{\frac{\Delta_F\mu_F^2}{4\mu_F^2\Delta_F^2[4\Delta_F\Delta_R - k^2]^2}\right\}E_{d_F}\{2\Delta_F a[2\Delta_R - k] + d_F[4\Delta_F\Delta_R - k^2] -$$

$$d_F k[2\Delta_F\gamma_R - k]\}^2$$

$$= \left\{\frac{1}{4\Delta_F[4\Delta_F\Delta_R - k^2]^2}\right\}E_{d_F}\{2\Delta_F a[2\Delta_R - k] + 2\Delta_F d_F[2\Delta_R - k\gamma_R]\}^2$$

$$= \left\{\frac{4\Delta_F^2}{4\Delta_F[4\Delta_F\Delta_R - k^2]^2}\right\}E_{d_F}\{a[2\Delta_R - k] + d_F[2\Delta_R - k\gamma_R]\}^2$$

$$= \left\{\frac{\Delta_F}{[4\Delta_F\Delta_R - k^2]^2}\right\}E_{d_F}[\{a[2\Delta_R - k]\}^2 + \{2a[2\Delta_R - k]d_F[2\Delta_R -$$

$$k\gamma_R]\} + \{d_F[2\Delta_R - k\gamma_R]\}^2]$$

$$= \left\{\frac{\Delta_F[a^2[2\Delta_R-k]^2+2a[2\Delta_R-k][2\Delta_R-k\gamma_R]\tau_F+[\tau_F^2+\sigma_F^2][2\Delta_R-k\gamma_R]^2]}{[4\Delta_F\Delta_R-k^2]^2}\right\} \text{ using} \tag{A33}$$

$E_{d_F}[d_F] = \tau_F$ and $E_{d_F}[\{d_F\}^2] = [\tau_F^2 + \sigma_F^2]$

Note that $\tau_F = 0.5\bar{d}_F$ and $\sigma_F^2 = \left(\bar{d}_F\right)^2/12$.

The expected profit for firm F under no-disclosure, noting that firm R's belief is $b_F = \delta_F$, is given by $E_{d_F}\left\{\Pi_F^{\phi}(d_F, b_F = \delta_F)\right\}$ defined in equation (A34).

$$E_{d_F}\left\{\Pi_F^{\phi}(\cdot)\right\} = \Delta_F E_{d_F}\left\{\left[\mu_F Q_F^{\phi}(d_F, b_F = \delta_F)\right]^2\right\}$$

$$= \left\{\frac{\Delta_F\mu_F^2}{4\mu_F^2\Delta_F^2[4\Delta_F\Delta_R-k^2]^2}\right\} E_{d_F}\{2\Delta_F a[2\Delta_R - k] + d_F[4\Delta_F\Delta_R - k^2] - \delta_F k[2\Delta_F\gamma_R - k]\}^2,$$ using $Q_F^{\phi}(\cdot)$ from Lemma 1.

$$= \left\{\frac{1}{4\Delta_F[4\Delta_F\Delta_R-k^2]^2}\right\} E_{d_F}[\{2\Delta_F a[2\Delta_R - k] - \delta_F k[2\Delta_F\gamma_R - k]\}^2 + 2\{2\Delta_F a[2\Delta_R - k] - \delta_F k[2\Delta_F\gamma_R - k]\}\{d_F[4\Delta_F\Delta_R - k^2]\} + \{d_F[4\Delta_F\Delta_R - k^2]\}^2]$$

$$= \left\{\frac{1}{4\Delta_F[4\Delta_F\Delta_R-k^2]^2}\right\} [\{2\Delta_F a[2\Delta_R - k] - \tau_F k[2\Delta_F\gamma_R - k]\}^2 + 2\{2\Delta_F a[2\Delta_R - k] - \tau_F k[2\Delta_F\gamma_R - k]\}\{\delta_F[4\Delta_F\Delta_R - k^2]\} + \{[\tau_F^2 + \sigma_F^2][4\Delta_F\Delta_R - k^2]^2\}]$$

$$= \left\{\frac{1}{4\Delta_F[4\Delta_F\Delta_R-k^2]^2}\right\} \{4\Delta_F^2 a^2[2\Delta_R - k]^2 - 4\Delta_F a[2\Delta_R - k]\tau_F k[2\Delta_F\gamma_R - k] + \tau_F^2 k^2[2\Delta_F\gamma_R - k]^2 + 4\Delta_F a[2\Delta_R - k]\delta_F[4\Delta_F\Delta_R - k^2] - 2\tau_F^2 k[2\Delta_F\gamma_R - k][4\Delta_F\Delta_R - k^2] + \tau_F^2[4\Delta_F\Delta_R - k^2]^2 + \sigma_F^2[4\Delta_F\Delta_R - k^2]^2\}$$

$$= \left\{\frac{1}{4\Delta_F[4\Delta_F\Delta_R-k^2]^2}\right\}\{4\Delta_F^2a^2[2\Delta_R-k]^2+4\Delta_F a[2\Delta_R-k]\tau_F[4\Delta_F\Delta_R-2\Delta_F\gamma_R k]+\tau_F^2\{k^2[2\Delta_F\gamma_R-k]^2-2k[2\Delta_F\gamma_R-k][4\Delta_F\Delta_R-k^2]+[4\Delta_F\Delta_R-k^2]^2\}+\sigma_F^2[4\Delta_F\Delta_R-k^2]^2\}$$

$$= \left\{\frac{1}{4\Delta_F[4\Delta_F\Delta_R-k^2]^2}\right\}\{4\Delta_F^2a^2[2\Delta_R-k]^2+8\Delta_F^2a[2\Delta_R-k]\tau_F[2\Delta_R-\gamma_R k]+\tau_F^2([4\Delta_F\Delta_R-k^2]-k[2\Delta_F\gamma_R-k])^2+\sigma_F^2[4\Delta_F\Delta_R-k^2]^2\}$$

$$= \left\{\frac{1}{4\Delta_F[4\Delta_F\Delta_R-k^2]^2}\right\}\{4\Delta_F^2a^2[2\Delta_R-k]^2+8\Delta_F^2a[2\Delta_R-k]\tau_F[2\Delta_R-\gamma_R k]+\tau_F^2(4\Delta_F\Delta_R-2\Delta_F\gamma_R k)^2+\sigma_F^2[4\Delta_F\Delta_R-k^2]^2\}$$

$$= \left\{\frac{1}{4\Delta_F[4\Delta_F\Delta_R-k^2]^2}\right\}\{4\Delta_F^2a^2[2\Delta_R-k]^2+8\Delta_F^2a[2\Delta_R-k]\tau_F[2\Delta_R-\gamma_R k]+4\Delta_F^2\tau_F^2(2\Delta_R-\gamma_R k)^2+\sigma_F^2[4\Delta_F\Delta_R-k^2]^2\}$$

$$= \left\{\frac{4\Delta_F^2}{4\Delta_F[4\Delta_F\Delta_R-k^2]^2}\right\}\{a^2[2\Delta_R-k]^2+2a[2\Delta_R-k]\tau_F[2\Delta_R-\gamma_R k]+\tau_F^2(2\Delta_R-\gamma_R k)^2\}+\left\{\frac{\sigma_F^2[4\Delta_F\Delta_R-k^2]^2}{4\Delta_F[4\Delta_F\Delta_R-k^2]^2}\right\}$$

$$= \left\{\frac{\Delta_F}{[4\Delta_F\Delta_R-k^2]^2}\right\}\{a^2[2\Delta_R-k]^2+2a[2\Delta_R-k]\tau_F[2\Delta_R-\gamma_R k]+\tau_F^2(2\Delta_R-\gamma_R k)^2\}+\left\{\frac{\sigma_F^2}{4\Delta_F}\right\} \qquad \text{(A34)}$$

Firm F will commit to full disclosure if and only if $Z^D = E_{d_F}\{\Pi_F^D(d_F, b_F = d_F)\} - E_{d_F}\{\Pi_F^\phi(d_F, b_F = \delta_F)\} > 0$. Using $E_{d_F}\{\Pi_F^D(d_F, b_F = d_F)\}$ and $E_{d_F}\{\Pi_F^\phi(d_F, b_F = \delta_F)\}$ from equations (A33) and (A34), we get

$$Z^D = \left\{\frac{\Delta_F\sigma_F^2[[2\Delta_R-k\gamma_R]^2]}{[4\Delta_F\Delta_R-k^2]^2}\right\} - \left\{\frac{\sigma_F^2}{4\Delta_F}\right\}$$

$$= \left\{\frac{\sigma_F^2}{4\Delta_F[4\Delta_F\Delta_R-k^2]^2}\right\}\{4\Delta_F^2[[2\Delta_R-k\gamma_R]^2]-[4\Delta_F\Delta_R-k^2]^2\}$$

$$= \left\{\frac{\sigma_F^2(2\Delta_F[2\Delta_R-k\gamma_R]+[4\Delta_F\Delta_R-k^2])}{4\Delta_F[4\Delta_F\Delta_R-k^2]^2}\right\}\{2\Delta_F[2\Delta_R-k\gamma_R]-[4\Delta_F\Delta_R-k^2]\}$$

$$= k\left\{\frac{\sigma_F^2(2\Delta_F[2\Delta_R-k\gamma_R]+[4\Delta_F\Delta_R-k^2])}{4\Delta_F[4\Delta_F\Delta_R-k^2]^2}\right\}\{2\Delta_F\gamma_R-k\} \tag{A35}$$

Equation (A35) is positive when $\{2\Delta_F\gamma_R - k\} > 0$, i.e., $\Delta_F < \frac{k}{2\gamma_R}$; this follows because $\sigma_F^2 > 0$, $[2\Delta_R - k\gamma_R] > 0$ (see Lemma 1 proof), $[4\Delta_F\Delta_R - k^2] > 0$ (see Lemma 1 proof), and $k > 0$.

QED.

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BIOGRAPHICAL SKETCH

Ammon Butcher is a professor in accounting at Brigham Young University – Idaho, where he also obtained a bachelor's degree in accounting. In addition to a bachelor's degree, Ammon has a master's degree in accounting from The Ohio State University and a PhD in management science with an accounting concentration from The University of Texas at Dallas. Ammon is a Certified Public Accountant with experience in both the public and corporate sectors. When Ammon is not working, he can be found volunteering at his church, playing fetch with his golden doodle, or dining out with friends.

CURRICULUM VITAE

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Education

- University of Texas at Dallas — Richardson, TX

  PhD Candidate in Management Science, Accounting Concentration — Aug. 2018 – Present

  Eugene McDermott Graduate Fellow — Expected Grad.: Spring 2023

- The Ohio State University — Columbus, OH

  Master's in accounting — Aug. 2014 – Jul. 2015

  Fisher College of Business Graduate Fellow

- Brigham Young University – Idaho — Rexburg, ID

  BS in Accounting — Sep. 2008 – Dec. 2013

  The Accounting Department Outstanding Graduate

Teaching Experience

- Financial Accounting 2301, University of Texas at Dallas — Richardson, TX

  Student Evaluation: 4.65/5.00 — Aug. 2021 – Dec. 2021

Professional Experience

- Senior Accountant, Cardinal Health — Dublin, OH

  Nov. 2015 – Mar. 2017

- Tax Intern, Deloitte & Touche — Dallas, TX

  Jan. 2014 – Mar. 2014

Licenses & Certifications

- CPA, State of Ohio

Awards

- Eagle Scout

Research

- "The Effects of Supply Uncertainty on Voluntary Disclosure of Demand" (Dissertation)
  - Dissertation Committee: Suresh Radhakrishnan (chair), Brian Mittendorf (co-chair), Ashiq Ali & Ram Natarajan (committee members).

ABSTRACT: Supply disruptions are increasing in magnitude and frequency imbuing our global economy with greater levels of supply uncertainty. In Cournot duopoly, I examine how supply uncertainty affects the voluntary disclosure of a firm's private demand information. In addition to supply uncertainty, I allow for variation in product substitutability (substitutability effect) and in the degree to which private demand information affects the rival firm's demand (spillover effect). I show that the partial disclosure equilibrium depends on a firm's competitive advantage characterized by the relative importance of the spillover effect, substitutability effect, and the firms' supply uncertainty. In particular, I show that absent supply uncertainty the firm with private information about product demand discloses good news (bad news) when the substitutability effect (spillover effect) is most pronounced. By incorporating the role of supply uncertainty in a firm's disclosure decision, I demonstrate that supply uncertainty diminishes the substitutability effect and enhances the spillover effect. I show that with supply uncertainty the firm possessing private information about product demand discloses good news (bad news) when the supply uncertainty does not (does) mute the relative dominance of the substitutability effect over the spillover effect. I also show that the disclosure region for both good news and bad news, in general, decreases with increases in supply uncertainty of both the firm and its rival.

- "Managerial conservatism and Risky Project Choice" with Suresh Radhakrishnan; work-in-progress (writing stage)

ABSTRACT: We examine the effect of conservatism on the investor's problem of inducing the manager to select the risky project. The manager assesses the business environment and the private signal he observes can be conservative. Specifically, managerial conservatism represents that the manager assessment is more likely to flag the true bad business environment as bad as well as the true good business environment as bad. We find that when the agency cost is sufficiently large, the investor prefers the manager to choose the more (less) risky project when the business environment is good (bad), and also prefers more conservative managers. Managerial conservatism, i.e., the bias in the manager's assessment of the business environment, is informative about the good business environment, which in turn reduces the expected bonus. This benefit of managerial conservatism is balanced with the opportunity cost of not choosing the high-return, high-risk project in the good state. Overall, when the agency cost is sufficiently large, we show that the benefit dominates the opportunity cost, i.e., the investor prefers managerial conservatism. We also show that as the difference in the project risk increases, the effect of managerial conservatism is reduced. This occurs because differential returns across the projects are themselves informative to mitigate the agency problem. There are two important implications of these findings. First, our finding puts to question the notion that managers have to be aggressive/optimistic to select risky projects. Second, while conservatism helps to mitigate agency problems with the choice of risky projects, the degree of usefulness of conservatism decreases as the differential riskiness increases.

- "Intermediaries with Biased Information in a Not-For-Profit Environment"; work-in-progress (analysis stage).

  I model an intermediary who helps to provide information to donors about a not-for-profit enterprise's effectiveness to attain the social cause. I plan to examine whether donors prefer an information intermediary that provides biased or unbiased information.

Workshops/Conferences Attended

- AES-UCSD Summer School in Theory and Structural Estimation — San Diego, CA, Aug. 2019
- American Accounting Association annual meeting — San Diego, CA, Aug. 2022

Memberships

- American Accounting Association (AAA)