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In-Season Transshipments Among Competitive Retailers

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A decentralized system of competing retailers that order and sell the same product in a sales season is studied. When a customer demand occurs at a stocked-out retailer, that retailer requests a unit to be transshipped from another retailer who charges a transshipment price. If this request is rejected, the unsatisfied customer may go to another retailer with a customer overflow probability. Each retailer decides on the initial order quantity from a manufacturer and on the acceptance/rejection of *each* transshipment request. For two retailers, we show that retailers' optimal transshipment policies are dynamic and characterized by chronologically nonincreasing inventory holdback levels. We analytically study the sensitivity of holdback levels to explain interesting findings, such as smaller retailers and geographically distant retailers benefit more from transshipments. Numerical experiments show that retailers substantially benefit from using optimal transshipment policies compared to no sharing. The expected sales increase in all but a handful of over 3,000 problem instances. Building on the two-retailer optimal policies, we suggest an effective heuristic transshipment policy for a multiretailer system.

Key words: dynamic transshipment policy; demand overflow; decentralized distribution system

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1. Introduction

A common method of inventory sharing among independent retailers is retailer-to-retailer trade, called *transshipment*. In transshipment-based inventory sharing, a retailer with sufficient inventory may be willing to sell her inventory to a stocked-out retailer. This allows a retailer to satisfy demand through other retailers without frequent shipments from the generally distant manufacturer.

Transshipment applications are reported in many retail industries such as automotive, apparel, sporting goods, toys, furniture, information technology products, and shoes, among production facilities, and in after-sale services for aircraft and automotive spare parts (Kukreja et al. 2001, Özdemir et al. 2006, Rudi et al. 2001, Hu et al. 2008). Time-based competition and higher demand variability have the potential to encourage adoption of transshipments (Harris 2006). In addition, the use of less-than-truckload carriers and enterprise resource planning software facilitate transshipments. For example, car manufacturers provide Intranet systems connecting their retailers for information exchange (Zhao and Atkins 2009).

Independent retailers may decline to send transshipments because they tend to see each other as competitors. The belief that an unsatisfied customer at a stocked-out retailer may buy from another retailer

fuels competition among retailers. In a study of 71,000 customers, Corsten and Gruen (2004) found that customers lose patience with stockouts. Customer overflow to another retailer may happen when a stocked-out retailer, whose transshipment request is denied, cannot satisfy a customer demand. While the unwillingness of customers to wait motivates a stocked-out retailer to request a transshipment, the possibility of the customer overflow motivates retailers with on-hand inventory to reject the request. Transshipments provide a retailer with the option of accessing other retailers' inventories and markets. They are real options with which retailers hedge against risks of both stockouts and leftover inventory. This paper provides a flexible sharing mechanism that is regulated by inventory *holdback levels*. It is an attractive alternative to pure competition with no inventory sharing and pure cooperation with complete sharing.

Our study provides easily implementable transshipment and ordering policies for a decentralized retailer system where transshipments can potentially be made immediately after each demand arrival and unsatisfied customers may visit another retailer to satisfy their demands. Allowing a transshipment after each demand arrival is an important aspect of modeling reality as each customer wants to know the availability of the product, either directly from stock or

by transshipment, upon his or her visit to the store. We do not consider in-advance transshipments to avoid potential future stockouts or delaying a transshipment request, say to potentially increase the profit from it or its chance of acceptance. In this paper, as in practice, a transshipment is requested to meet a demand immediately after the realization of that demand. This is called an *in-season* transshipment.

Although transshipments are reported among centralized and decentralized retailers, studies tend to focus on centralized ones (e.g., Çömez et al. 2012). We review studies concerning decentralized systems. Rudi et al. (2001) study a system of retailers who use transshipments for demand and inventory matching at the end of a sales season. Extending this, Shao et al. (2011) analyze the manufacturer's benefit from retailers' transshipments. Anupindi et al. (2001) consider both inventory sharing through transshipment and physical pooling by using common inventories. Sošić (2006) extends Anupindi et al. (2001) by introducing a partial pooling policy. In these studies, transshipments occur after all demands are realized at all retailers.

When a transshipment can happen after each demand, Grahovac and Chakravarty (2001) use a one-for-one replenishment policy for both requesting and accepting a transshipment. Zhao et al. (2005) assume an (S, K) policy for transshipments and replenishments. There S is the order-up-to level and K is the threshold sharing level. They adopt ideas from rationing policies of multiple demand classes (Desphande et al. 2003) as customer demand and transshipment requests are different demand classes. Zhao et al. (2006) extend both Grahovac and Chakravarty (2001) and Zhao et al. (2005) by considering a base-stock replenishment policy, a threshold level for sending transshipment requests, and another threshold for filling requests. Considering inventory sharing and customer overflow, Anupindi and Bassok (1999) and Zhao and Atkins (2009) compare two extreme models: no inventory sharing and complete sharing. In a nontransshipment context, Chen et al. (2011) obtain a dynamic rationing policy for demand fulfillment of an e-retailer that carries no inventory and attempts to meet his demand first from inventory at a primary retailer and then from a secondary retailer. Inventory is sequentially rationed from these two retailers that do not share inventory with each other. We differ from these studies of decentralized systems by obtaining an *optimal transshipment policy* characterized by *dynamic holdback levels* for inventory-sharing retailers.

We first study a system of two independent retailers who maximize their own profits. Each retailer places a manufacturer order at the beginning of a sales season. During the season, if a retailer stocks out when a

customer demand occurs, *he* (*requesting retailer*) places a transshipment request to the *other* (*requested*) retailer. If *she* (*requested retailer*) accepts the request, the unit is transshipped after charging a transshipment price to him. Otherwise, the unsatisfied customer leaves the requesting retailer and may visit the requested retailer with a customer overflow probability. Therefore, a requested retailer may be willing to transship depending on the transshipment price, the expected revenue from a possible customer overflow, and the likelihood of selling the requested unit before the end of the season. We show that retailers' optimal transshipment policies are characterized by dynamic inventory holdback levels that change during the season.

Each retailer's transshipment price is assumed to be exogenously set to a constant value during the season. Such prices arise in practice as "[transshipment] prices are set by an external agency, such as a common supplier," according to Rudi et al. (2001, p. 1674). When the manufacturer is much larger than the retailers and has competitive power, it may dictate transshipment prices to retailers. According to the authors' private communication with an automobile dealer (Schunck 2009), dealers are dictated to use a transshipment price that is equal to the cost of the car to the requested dealer. Our framework yields the retailer profits under constant transshipment prices. So it is useful to set these prices before the season, possibly with a game-theoretic model that takes profits as inputs under different prices. Studying the negotiation of *dynamic* transshipment prices, Çakanyıldırım et al. (2012) show that certain negotiation power structures lead to constant transshipment prices.

With more than two retailers, a transshipment policy should specify where a stocked-out retailer requests transshipments from when two or more retailers have inventory. For a given requested retailer, a holdback policy based only on the inventory level at that retailer ceases to be optimal. Optimal policies for multiple retailers are significantly more demanding in information requirements and computations. To address this, we convert the optimal solution of a two-retailer system to a heuristic for multiple retailers. The total expected profit is computed with this heuristic and compared with the total expected centralized profit, which is a natural profit upper bound on the decentralized system. This comparison reveals that the total retailer profits under heuristic transshipments differ slightly from the upper bound, so the heuristic performs well.

Supplementing existing literature, this paper simultaneously captures several aspects that arise in practice in decentralized retailer systems. Each transshipment is requested immediately after the associated demand occurs. An unsatisfied demand at

a retailer can overflow to another retailer. For this practical setting, the optimal transshipment policy and its sensitivity to system parameters are obtained in §2. We numerically study the effect of optimal transshipments on retailers' and manufacturers' profits in §3 and the performance of our multiretailer heuristic in §4. All proofs and counterexamples can be found in the e-companion appendix (available at <http://msom.journal.informs.org/>).

2. Transshipment Model

A decentralized system of two retailers who receive inventory from a manufacturer once at the beginning of a sales season is studied. During the season, if the retailers have available on-hand inventory, they immediately satisfy their customers' demands. If one of the retailers has no inventory to satisfy his demand, he sends a transshipment request to the other retailer. The requested retailer, while determining how to maximize her profit, either accepts or rejects the request. If the request is accepted, the cost of transportation τ is paid by the requesting retailer. A unit at retailer i is sold at sale price r_i . Expecting a visit from an unsatisfied customer with overflow probability θ_i , requested retailer i may reject the request. With probability $1 - \theta_i$, the customer leaves the system of two retailers and neither retailer earns the revenue.

For a transshipment from retailer i , retailer j pays transshipment price t_i . Then retailer j obtains $r_j - t_i - \tau$ by selling the transshipped unit, while retailer i forgoes at least the salvage value s_i to earn t_i . If $r_j - t_i - \tau < 0$, receiving a transshipment causes a loss for retailer j . If $t_i < s_i$, no transshipment request is accepted by retailer i . Besides, $r_j - \tau \leq r_i$ should hold to avoid an arbitrage opportunity to send units to the high-priced market and sell there. In summary,

$$s_i \leq t_i \leq r_j - \tau \leq r_i \quad \text{for } i, j \in \{1, 2\} \quad \text{and} \quad i \neq j. \quad (1)$$

To capture the dynamics of in-season transshipments, a model is developed by dividing the sales season into N short decision periods. Periods are short enough so that there can be at most one unit demand in each period, i.e., at retailer 1 with probability p_1 , at retailer 2 with probability p_2 , or at neither with probability $1 - p_1 - p_2$, where $p_1 + p_2 \leq 1$. As N increases by a factor and p_1 and p_2 decrease by the same factor, the demands converge to independent Poisson processes with means Np_1 and Np_2 . For similar demand models, see Lee and Hersh (1993) and Talluri and van Ryzin (2004). For correlated demand models, see Çömez et al. (2010) and Wee and Dada (2005).

Without loss of generality, the profit is formulated only for retailer 1, as that for retailer 2 is analogous.

The number of decision periods remaining until the end of the sales season is denoted by $n \leq N$. $\pi_n^i(x_1, x_2)$ is the maximum expected profit of retailer i in the remaining n periods with current inventory levels x_1 and x_2 .

When both retailers have positive inventory levels, each customer demand can be satisfied by the receiving retailer. Receiving a demand, retailer 1 sells a unit to earn r_1 . Otherwise, retailer 1 has no cost or revenue.

$$\begin{aligned} \pi_n^1(x_1, x_2) &= (1 - p_1 - p_2)\pi_{n-1}^1(x_1, x_2) \\ &\quad + p_1[r_1 + \pi_{n-1}^1(x_1 - 1, x_2)] \\ &\quad + p_2\pi_{n-1}^1(x_1, x_2 - 1), \end{aligned} \quad (2)$$

for $x_1, x_2 \in \mathcal{N} := \{1, 2, \dots\}$. When both retailers are stocked out, demand is lost. With zero inventory in stock, there is no change in a retailer's profit from one period to the next.

$$\pi_n^1(0, 0) = \pi_{n-1}^1(0, 0) = \pi_0^1(0, 0) = 0. \quad (3)$$

If retailer 2 stocks out before retailer 1, retailer 1 replies to a transshipment request comparing profits when accepting or rejecting it. By transshipping a unit to retailer 2, retailer 1 earns t_1 . Rejecting the transshipment request may cause the unsatisfied customer to visit retailer 1 with probability θ_1 . In this case, retailer 1 earns r_1 from the customer. With probability $(1 - \theta_1)$ the customer leaves the system and no revenue is obtained. Thus,

$$\begin{aligned} \pi_n^1(x_1, 0) &= (1 - p_1 - p_2)\pi_{n-1}^1(x_1, 0) + p_1[r_1 + \pi_{n-1}^1(x_1 - 1, 0)] \\ &\quad + p_2 \max\{t_1 + \pi_{n-1}^1(x_1 - 1, 0), \theta_1(r_1 + \pi_{n-1}^1(x_1 - 1, 0)) \\ &\quad \quad + (1 - \theta_1)\pi_{n-1}^1(x_1, 0)\}. \end{aligned} \quad (4)$$

If retailer 1 stocks out before retailer 2, retailer 1 asks for transshipments from retailer 2 to meet his demand. Retailer 1 expects retailer 2 to behave rationally to maximize her profit while responding to retailer 1's transshipment request. Let \mathbb{I}_{21}^{n, x_2} be the indicator associated with the accept/reject decision of retailer 2 in response to the transshipment request of retailer 1 in period n when the inventory level at retailer 2 is x_2 .

$$\mathbb{I}_{21}^{n, x_2} = \begin{cases} 1 & \text{if } t_2 + \pi_{n-1}^2(0, x_2 - 1) \geq \theta_2(r_2 + \pi_{n-1}^2(0, x_2 - 1)) \\ & \quad + (1 - \theta_2)\pi_{n-1}^2(0, x_2), \\ 0 & \text{otherwise.} \end{cases}$$

If the transshipment request is accepted ($\mathbb{I}_{21}^{n, x_2} = 1$), retailer 1 pays the transshipment price t_2 and the transportation cost τ to receive the unit, which is then sold to the customer for r_1 . Otherwise $\mathbb{I}_{21}^{n, x_2} = 0$, and

retailer 1 loses the customer demand. The resulting expected profit of retailer 1 is

$$\begin{aligned} \pi_n^1(0, x_2) &= (1 - p_1 - p_2)\pi_{n-1}^1(0, x_2) + p_2\pi_{n-1}^1(0, x_2 - 1) \\ &\quad + p_1\mathbb{I}_{21}^{n, x_2}(r_1 - t_2 - \tau + \pi_{n-1}^1(0, x_2 - 1)) \\ &\quad + p_1(1 - \mathbb{I}_{21}^{n, x_2})(\theta_2\pi_{n-1}^1(0, x_2 - 1) \\ &\quad \quad \quad + (1 - \theta_2)\pi_{n-1}^1(0, x_2)). \end{aligned} \quad (5)$$

A retailer asks for a transshipment in (5) when he stocks out. If the market price, transshipment price, or transportation cost are dynamic during the season, the stocked-out retailer may delay a transshipment request to save competitor's inventory for future periods, which is outside the scope of this paper. At the end of the season, the remaining inventory at retailer i is sold at s_i :

$$\pi_0^i(x_1, x_2) = s_i x_i. \quad (6)$$

The objective of each retailer i is to maximize total expected profit, which is the expected profit π_N^i minus the cost of inventory S_i purchased at the purchase cost c_i per unit, paid by retailer i to the manufacturer at the beginning of the season. Then the profit of retailer i is

$$J_i(S_1, S_2) = \pi_N^i(S_1, S_2) - c_i S_i. \quad (7)$$

2.1. Optimal Holdback Level-Based Policy

When a transshipment request is received, retailer 1 can determine the trade-off between accepting and rejecting a request, which is represented by the maximum in (4). To better understand this trade-off, we define $\delta_n^1(x) := \pi_n^1(x, 0) - \pi_n^1(x - 1, 0)$ as the *marginal benefit* (of keeping an extra unit of inventory at retailer 1) when $x \in \mathcal{N}$ and $x_2 = 0$ in period n . From (4) and (6), extra inventory can only increase profit in the remaining periods, so $\delta_n^1(x) \geq 0$. The marginal benefit function can be written by using (4) for $x \geq 2$ and $x = 1$ separately, as the expression for $x = 1$ includes the profit function $\pi_n^1(0, 0)$, which is zero from (3):

$$\begin{aligned} \delta_n^1(x) &= (p_1 + p_2)\delta_{n-1}^1(x-1) + (1 - p_1 - p_2)\delta_{n-1}^1(x) \\ &\quad + p_2[\max\{t_1, \theta_1 r_1 + (1 - \theta_1)\delta_{n-1}^1(x)\} \\ &\quad \quad - \max\{t_1, \theta_1 r_1 + (1 - \theta_1)\delta_{n-1}^1(x-1)\}], \\ &\quad \quad \quad x \geq 2. \end{aligned} \quad (8)$$

$$\begin{aligned} \delta_n^1(1) &= p_1 r_1 + (1 - p_1 - p_2)\delta_{n-1}^1(1) \\ &\quad + p_2 \max\{t_1, \theta_1 r_1 + (1 - \theta_1)\delta_{n-1}^1(1)\}. \end{aligned} \quad (9)$$

At the end of the season, $\delta_0^1(x_1) = s_1$. The maximum in (4) can be rewritten as $\pi_{n-1}^1(x - 1, 0) + \max\{t_1, \theta_1 r_1 + (1 - \theta_1)\delta_{n-1}^1(x)\}$. Then the request of retailer 2 is

accepted if and only if $\theta_1 r_1 + (1 - \theta_1)\delta_{n-1}^1(x) \leq t_1$. Putting parameters on the right-hand side, we get

$$\delta_{n-1}^1(x) \leq (t_1 - \theta_1 r_1)/(1 - \theta_1). \quad (10)$$

We refer to $(t_1 - \theta_1 r_1)/(1 - \theta_1)$ as retailer 1's marginal cost of rejecting a request. Toward the characterization of the optimal transshipment policy, it suffices to examine the monotonicity of the marginal benefit δ_n^i because the marginal cost is constant.

LEMMA 1. For $x \in \mathcal{N}$ and $n \in \mathcal{N} \cup \{0\}$,

- (i) the marginal benefit of keeping an extra unit of inventory is nonincreasing in inventory level: $\delta_n^i(x+1) \leq \delta_n^i(x)$;
- (ii) the marginal benefit cannot be more than the unit selling price: $\delta_n^i(x) \leq r_i$;
- (iii) the marginal benefit is nondecreasing in the number of remaining periods: $\delta_n^i(x) \leq \delta_{n+1}^i(x)$;
- (iv) the marginal benefit cannot be less than the salvage value: $\delta_n^i(x) \geq s_i$.

Recalling the transshipment acceptance condition (10), Lemma 1(i) leads to the existence of an optimal transshipment policy based on holdback levels. Lemma 1(iii) implies that retailers have a higher marginal benefit of rejecting a request earlier in a sales season. Knowing that the marginal cost function is constant, retailers should be more willing to accept transshipment requests when there are fewer periods remaining in the sales season. Lemma 1 leads to the optimal transshipment policy stated in the following theorem.

THEOREM 1. There exist inventory holdback levels \tilde{x}_n^i for retailer i such that it is optimal to reject (respectively, accept) the transshipment request when $x_i \leq \tilde{x}_n^i$ (respectively, $x_i > \tilde{x}_n^i$). The holdback levels are nondecreasing in the remaining number of periods: $\tilde{x}_1^i \leq \tilde{x}_2^i \leq \dots \leq \tilde{x}_n^i \dots \leq \tilde{x}_N^i$.

The holdback level \tilde{x}_n^1 can be obtained as $\tilde{x}_n^1 := \max\{x \in \mathcal{N}: \delta_{n-1}^1(x) > (t_1 - \theta_1 r_1)/(1 - \theta_1)\}$. From Lemma 1(i), if $\delta_{n-1}^1(1) \leq (t_1 - \theta_1 r_1)/(1 - \theta_1)$, then $\tilde{x}_n^1 = 0$; i.e., complete sharing is optimal when the expected benefit of keeping one unit of inventory for retailer 1 is sufficiently low. On the contrary, if $\delta_0^1(x) = s_1 > (t_1 - \theta_1 r_1)/(1 - \theta_1)$, then the holdback level of retailer 1 in period 1 is infinite. An infinite holdback level at a retailer in period n indicates that the retailer is not willing to send a transshipment regardless of her inventory level. By Theorem 1, an infinite holdback level in period 1 indicates infinite holdback levels in all periods and so a no sharing policy is optimal. In other words, if the customer overflow probability is sufficiently large, i.e., $\theta_i > (t_i - s_i)/(r_i - s_i)$, then retailer i never sends any transshipments. To study both retailers' transshipment policies, we assume that $\theta_i \leq (t_i - s_i)/(r_i - s_i)$ for $i \in \{1, 2\}$.

THEOREM 2. If $\theta_i \leq (t_i - s_i)/(r_i - s_i)$ for $i \in \{1, 2\}$,
(i) in period 1, retailer i has a zero holdback level: $\tilde{x}_1^i = 0$;
(ii) the holdback level of retailer i cannot decrease by more than one over a period: $\tilde{x}_{n+1}^i - \tilde{x}_n^i \leq 1$.

Theorem 2 states that if retailer i has any inventory in the last period, she is willing to send a transshipment regardless of her inventory level. The highest decrease in holdback levels between two consecutive periods is one, which is the maximum demand in a period. The holdback level-based transshipment policy is similar to the threshold inventory rationing policy used to model demand satisfaction in the multiple customer demand class literature. A transshipment accept/reject decision is based on the trade-off between selling a unit inventory for a low margin current transshipment request and keeping it for a high margin, but possible, future direct customer sale. The corresponding trade-off in a multiple demand class problem is between using a unit for a low-class immediate demand and saving the unit for a future high-class demand. Different from the rationing policies of the multiple demand class studies, we show the optimality of a dynamic holdback level-based transshipment policy for two independent retailers.

2.2. Sensitivity of Holdback Levels

Holdback levels depend on the marginal benefit and marginal cost of rejecting a transshipment request. Because the sensitivity of the marginal cost is fairly straightforward, we focus on the sensitivity of the marginal benefit at retailer i with respect to (wrt) parameters p_1 , p_2 , t_i , r_i , s_i , and θ_i . This sensitivity is analyzed by appropriately bounding the changes in the benefit of rejecting a request. The results are reported in the next theorem. Neither the benefit nor the cost of rejecting a request depends on the transportation cost τ , which is paid by the requesting retailer, as long as (1) is satisfied. Then the holdback levels are insensitive to τ .

THEOREM 3. The holdback levels at retailer i are nondecreasing in demand probabilities p_1 and p_2 , sale price r_i , salvage value s_i , and customer overflow probability θ_i . These levels are nonincreasing in the transshipment price t_i .

Strong competition between retailers in the same geographic district can be modeled by increasing the customer overflow probability θ_i . An increase in θ_i leads to higher holdback levels at retailer i by Theorem 3, which indicates less willingness to transship among nearby retailers. Zhao and Atkins (2009) report a similar effect of competition on transshipments. In their analysis of complete sharing and no sharing policies, it is suggested that retailers cooperate with holdback level $\tilde{x}_n^i = 0$ when θ_i is low, and do not cooperate with $\tilde{x}_n^i = \infty$ when θ_i is high. This principle is implemented, for example, by some

automobile dealers who compete with nearby dealers while cooperating with dealers farther away. Our model, with the additional flexibility of $0 < \tilde{x}_n^i < \infty$, smoothes the effect of competition on inventory sharing. It suggests that competing retailers use transshipments selectively with high, but still finite, holdback levels.

Theorem 3 also states that an increase in the expected market size through an increase in either p_1 or p_2 increases holdback levels at both retailers. Note that this result is valid for constant N , so an increase in p_i means an increase in retailer i 's total expected demand. This is consistent with the wide application of transshipments in industries with slow-moving products where Np_i is low (Grahovac and Chakravarty 2001). On the other hand, the relative sensitivity of a retailer's holdback levels to the demand probabilities p_1 and p_2 is an interesting question that is not answered by Theorem 3. For a fixed expected market size $N(p_1 + p_2)$, demand probabilities affect holdback levels as stated by the following theorem.

THEOREM 4. The holdback levels at retailer i are nondecreasing in her own expected market size Np_i when the total expected market size is constant.

By Theorem 4, if some retailer j customers migrate to retailer i 's market, the holdback levels at retailer i cannot decrease and those at retailer j cannot increase. In other words, retailer i 's demand is dominant over retailer j 's demand in determining retailer i 's transshipment policy. For example, when p_1 increases and p_2 decreases by the same amount, holdback levels at retailer 1 either remain the same or increase.

2.3. Inventory Ordering Game

Because retailers have similar delivery lead times when buying from the same manufacturer, they usually order at about the same time without knowing the other's order quantity, which leads to a Cournot game. The optimal order quantity of a retailer can be defined as a best response to the other retailer's quantity choice:

$$S_1^*(S_2) = \arg \max_{S_1} J_1(S_1, S_2) \quad \text{and}$$

$$S_2^*(S_1) = \arg \max_{S_2} J_2(S_1, S_2).$$

A pure strategy equilibrium (S_1^e, S_2^e) satisfies $S_1^e = S_1^*(S_2^e)$ and $S_2^e = S_2^*(S_1^e)$.

To establish the existence of an equilibrium in the space of integers, submodularity of profits is a property that is commonly used (Zhao et al. 2005). To show the submodularity of J_i in our model, the profit function π_n^i should be submodular for all $n \in \{0, 1, \dots, N\}$. This strong condition does not hold in

our context as established by a counterexample in the e-companion appendix.

Although submodularity does not hold in general, the existence of an equilibrium for ordering noninteger amounts can be shown by extending the definition of profit functions. Noninteger orders are present in many inventory studies, including those of inventory sharing (Anupindi et al. 2001, Dong and Rudi 2004). Using interpolation (Phillips 2003), the profit function J_i can be extended over nonintegers S_1 and S_2 . The existence of a pure strategy equilibrium follows in view of Theorem 1.2 of Fudenberg and Tirole (1991), as we can prove that the extended profit $J_i(S_1, S_2)$ is continuous and concave in S_i .

3. Performance of Optimal Transshipment Policies

3.1. Retailers' Benefits from Transshipments

To quantify retailer benefits, we compare the expected retailer profits when there is *no sharing* of inventories to those when there is *optimal sharing* via transshipments. Numerical experiments are run with instances P0–P22 in Table 1. These instances all have $c_1 = c_2 = c$ and $r_1 = r_2 = r$. In each setting, $p_2 = 0.15$, $s_2 = 2$, $\theta_2 = 0.2$, $t_2 = 7$ and only one of the parameters in $(p_1, s_1, c, r, \tau, \theta_1, t_1) = (0.15, 2, 5, 11, 1, 0.2, 7)$ is altered at a time to see the effect of the altered parameter. P0 denotes the base problem setting with no alteration.

The percent increase in the expected profit of retailer i is denoted by ΔJ_i . Formally,

$$\Delta J_i = \frac{J_i(S_1^e(t_1, t_2), S_2^e(t_1, t_2)) - J_i^{NS}(S_1^{NS}, S_2^{NS})}{J_i^{NS}(S_1^{NS}, S_2^{NS})} \cdot 100.$$

Above, J_i^{NS} and S_i^{NS} , respectively, are the expected profit and equilibrium order quantity of retailer i with no sharing. The percent change in the total order quantities of retailers with optimal sharing is denoted by ΔS . The percent change in retailers' total safety stocks is denoted by ΔSS , where safety stock of retailer i with ordering level S_i is calculated as $S_i - Np_i$. A positive (negative) change indicates an increase (decrease) in stock amounts. In some problems, there are multiple equilibria, but all have the same total order quantity. In these problems, the average values of ΔJ_1 and ΔJ_2 are reported.

When p_1 increases in Table 1, ΔJ_1 becomes smaller as retailer 1 focuses on sales in her own larger market with higher holdback levels by Theorem 3. On the other hand, retailer 2 can access customers in larger retailer 1's market via transshipments sent to retailer 1. However, retailer 2 can lose access to inventory at retailer 1 who increases her holdback levels. The effect of access to the larger retailer 1's market

Table 1 Optimal Sharing vs. No Sharing for $N = 60$

Changing parameter		(S_1^e, S_2^e)	ΔJ_1	ΔJ_2	ΔS	ΔSS
$p_1 = 0.10$	P0	(10, 10)	4.10	4.10	0	0
	P1	(7, 10)	5.48	3.56	0	0
$p_1 = 0.25$	P2	(16, 10)	2.81	5.79	-3.70	-33.3
$p_1 = 0.35$	P3	(23, 10)	2.13	5.41	0	0
$s_1 = 1$	P4	(9, 11)	4.16	5.33	0	0
$s_1 = 3$	P5	(11, 10)	3.13	3.96	0	0
$s_1 = 4$	P6	(12, 10)	2.12	3.96	0	0
$c = 3$	P7	(12, 12)	1.57	1.57	0	0
$c = 7$	P8	(9, 9)	6.67	6.67	0	0
$c = 9$	P9	(7, 8) ^a	7.87 ^a	7.87 ^a	7.14	-25
$r = 8$	P10	(9, 10) ^a	4.73 ^a	4.73 ^a	5.56	^b
$r = 9$	P11	(10, 10)	4.98	4.98	0	0
$r = 13$	P12	(10, 11) ^a	3.77 ^a	3.77 ^a	-4.55	-25
$\tau = 2$	P13	(10, 10)	3.37	3.37	0	0
$\tau = 3$	P14	(10, 10)	2.67	2.67	0	0
$\tau = 4$	P15	(10, 11) ^a	1.22 ^a	1.22 ^a	5.00	50
$\theta_1 = 0$	P16	(10, 10)	5.77	4.40	0	0
$\theta_1 = 0.3$	P17	(10, 10)	3.40	3.89	0	0
$\theta_1 = 0.5$	P18	(10, 10)	2.32	3.21	0	0
$t_1 = 4$	P19	(10, 10)	2.27	4.38	0	0
$t_1 = 5$	P20	(10, 10)	2.78	4.71	0	0
$t_1 = 9$	P21	(10, 10)	5.68	2.75	0	0
$t_1 = 10$	P22	(10, 11)	4.90	1.91	5	50

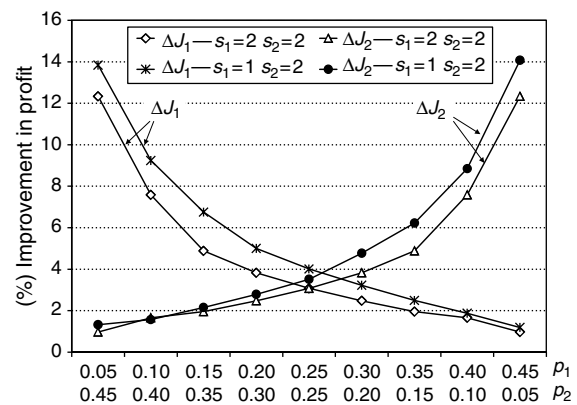
^a Indicates multiple equilibria.

^b ΔSS in P10 is undefined because the total safety stock is zero with no sharing.

on ΔJ_2 mostly dominates the effect of losing access to retailer 1's inventory, so ΔJ_2 generally increases in p_1 in Table 1. Similarly, when market size $N(p_1 + p_2)$ is constant, the benefit of transshipment is higher for the smaller retailer; see Figure 1. Because smaller retailers can expect relatively more benefits from transshipments, larger retailers should demand higher transshipment prices and/or be more reluctant to share inventory.

As the salvage value s_i increases or the purchase cost c decreases, the cost of leftover inventory drops

Figure 1 Effect of Relative Demand Probabilities on the Improvement in Expected Retailer Profits for $r = 10$, $t_1 = t_2 = 6$, $c = 4$, $\tau = 1$, $\theta_1 = \theta_2 = 0.1$, $s_1 \in \{1, 2\}$, and $s_2 = 2$



and retailers can stock more. Therefore, the transshipment option is not executed frequently by retailers. Besides, with increasing s_i , retailer i 's inventory becomes more valuable to her according to Theorem 3. Both cases lead to drops in ΔJ_1 and ΔJ_2 in Table 1. The same outcome occurs when competition is intensified with a higher value of θ_i or when the profit margin $r - t_i - \tau$ per transshipped unit is decreased with a higher value of τ . The effects of sale price r and transshipment price t_1 on ΔJ_1 and ΔJ_2 are not monotone. As r increases, retailers stock more and increase their holdback levels. Therefore, we cannot clearly say whether more or fewer transshipments happen with higher r . Increasing t_1 decreases holdback levels at retailer 1 and can lead to more transshipments. However, it decreases the profit margin per transshipped unit for the requesting retailer 2, which can lead him to order more from the manufacturer.

The total orders with optimal sharing can be greater ($\Delta S > 0$) or less ($\Delta S < 0$) than the total equilibrium orders with no sharing. While a decrease in orders may be expected as transshipment is a type of inventory pooling, Yang and Schrage (2009) show that inventory pooling can lead to a rise in inventory levels. Dong and Rudi (2004) show, for a single-period centralized system, that this inventory anomaly can be observed when the purchasing cost is high with respect to sales price. In Table 1, P9, P10, P15, and P22 demonstrate the inventory anomaly in our context.

To further substantiate the conclusions drawn from Table 1, 3,000 randomly generated problem instances are solved. Each parameter in these instances is sampled from a uniform distribution over the following ranges: $p_1, p_2 \in (0.1, 0.25)$, $s_1, s_2 \in (0, 2)$, $c \in (3, 5)$, $t_1 = t_2 \in (6, 8)$, $r \in (10, 14)$, $\tau \in (1, 2)$, and $\theta_1 = \theta_2 \in (0.1, 0.3)$. In these problems, the average increase in a retailer's expected profit with optimal sharing over no sharing is 3.3%. Although inventory sharing does not always decrease the total orders and safety stocks, it does on average by 1.27% and 5.3%, respectively.

3.2. Manufacturers' Benefits from Transshipments

Intuitively, manufacturers would like retailers to transship as it would increase sales to consumers. However, extensive transshipments may decrease manufacturer sales to retailers. So both total manufacturer sales, which determines the short-term manufacturer profit (Dong and Rudi 2004), and total retailer sales (Anupindi and Bassok 1999), are important, especially if unsold products are returned to the manufacturer or cleared with manufacturer rebates. We combine these two measures to define the total expected profit of the manufacturer. When "sales" is used without a qualifier, it refers to retailer sales in the remainder.

The expected total sales is denoted by $E[TS]$ and is related to the expected total lost sales $E[TL]$ via $E[TS] + E[TL] = N(p_1 + p_2)$. Notations $E[TS^e]$ and $E[TS^{NS}]$ denote the expected total sales when the optimal sharing and no sharing policies are used, respectively. The percent increase in expected total sales from inventory sharing is $\Delta E[TS] = (E[TS^e]/E[TS^{NS}] - 1) \cdot 100$. The expected profit of the manufacturer is $\Pi = (S_1 + S_2)(c - c') - (S_1 + S_2 - E[TS])s$, where $S_1 + S_2$ is the total manufacturer sales, c' is the per unit production cost of the manufacturer, and the salvage price s is the manufacturer's buyback price. The increase in the expected profit of the manufacturer with optimal transshipments compared with no sharing is calculated as $\Delta \Pi = (\Pi^e/\Pi^{NS} - 1) \cdot 100$. The expected total lost sales $E[TL^e]$, the improvement $\Delta E[TS]$ in the expected total sales, and the improvement $\Delta \Pi$ in the expected profit of the manufacturer are reported in Table 2 for $c' = 1$.

From $\Delta E[TS] > 0$ throughout Table 2, a manufacturer enjoys increased expected retailer sales from transshipments. On the contrary, the expected profit of the manufacturer is not necessarily higher under

Table 2 Benefit of the Retailers' Optimal Transshipment Policy for the Manufacturer

	$E[TL^e]$	$\Delta E[TS]$	$\Delta \Pi$
P0	0.689	2.92	1.33
	Increasing p_1		
P1	0.622	3.02	1.36
P2	0.771	1.30	-1.41
P3	0.514	2.22	1.04
	Increasing s_1		
P4	0.690	2.92	0.64
P5	0.447	2.75	1.95
P6	0.279	2.72	2.72
	Increasing c		
P7	0.081	1.55	1.55
P8	1.491	2.97	0.92
P9	3.475	7.64	7.26
	Increasing r		
P10	0.985	6.13	5.82
P11	0.664	3.07	1.40
P12	0.458	1.20	-2.01
	Increasing τ		
P13	0.690	2.92	1.33
P14	0.690	2.92	1.33
P15	0.437	4.42	4.74
	Increasing θ_1		
P16	0.680	3.53	1.61
P17	0.697	2.62	1.20
P18	0.720	2.02	0.93
	Increasing t_1		
P19	0.785	2.35	1.07
P20	0.735	2.65	1.21
P21	0.672	3.02	1.38
P22	0.425	4.49	4.77

optimal transshipments; see $\Delta\Pi$ for P2 and P12. This profit is bound to be higher when the inventory anomaly occurs, in which case both manufacturer and retailer sales increase. $\Delta E[TS]$ and $\Delta\Pi$ are higher when sale price r and overflow probability θ_1 are lower and when purchase cost c , transshipment price t_1 , transportation cost τ , and salvage price s_1 are higher.

Instead of offering costly incentives to the requested retailer (Zhao et al. 2005) to induce more inventory sharing, a manufacturer, in view of Table 1 and Theorem 3, can encourage retailers to set transshipment prices as high as possible. As transshipment price increases, more requests are accepted and sales increase. Besides, the requesting retailer, who wants to avoid stockouts, may buy more inventory from the manufacturer. This is why the manufacturer should prefer high transshipment prices rather than incentives. Shao et al. (2011) reach a similar conclusion in a single-period setting.

The effect of the overflow probability θ_i on sales is emphasized by Anupindi and Bassok (1999). They conclude that the expected total sales in a no sharing system is higher than sales in a complete sharing system for values of θ_i greater than a threshold level. From our numerical studies, this conclusion does not extend to the comparison of optimal sharing with no sharing. In particular, $\Delta E[TS]$ is always nonnegative in Table 2.

The 3,000 instances, introduced above for quantifying retailers' benefit, are now reconsidered. On average, the optimal sharing policy increases sales by 2.14%, which corresponds to a 49.53% decrease in total lost sales. Total sales decreased in only 8 instances out of 3,000. On the other hand, manufacturer sales decreased under the optimal sharing system in almost one third of the instances. Because sales are more

important in the long run, a manufacturer engaged in a long-term relationship with retailers benefits more from transshipments.

4. Multiretailer System

In a system with $M (>2)$ retailers, a transshipment policy based on holdback levels that are a function of only time and the inventory level at the requested retailer is no longer optimal, which is illustrated by an example in the e-companion appendix. A policy that is based on inventory levels at all retailers is hard to compute and implement. So we address the multiretailer problem with a heuristic. Huang and Sošić (2010) note the difficulty of analyzing transshipments among many retailers and introduce several heuristics.

A heuristic needs to make two important decisions. First, the requesting retailer must decide which retailer to request a transshipment from. A requested retailer with more inventory and less expected demand is more likely to accept a request. So in our multiretailer heuristic, the requesting retailer requests from the retailer whose index j maximizes x_j/p_j over $1 \leq j \leq M$. The second decision is the acceptance or rejection of a request. Our heuristic is based on the pairwise-optimal holdback levels for two retailers from §2. For requested retailer j and requesting retailer i , the holdback level is denoted by $\tilde{x}_n^{j,i}$. In the heuristic, requested retailer j accepts the request of retailer i if $x_j > \tilde{x}_n^{j,i}$. When requested retailer j rejects the request, the customer of requesting retailer i overflows only once with probability θ_{ik} to retailer k . This and the two decisions discussed above specify our multiretailer heuristic detailed in Table 3. For brevity, let c , s , x , and S be cost, salvage value, inventory level, and order quantity vectors, respectively. Let e_i be the i th unit vector and τ_{ji} be the transportation cost from retailer j to retailer i . Let $\mathbb{1}_{\mathcal{S}} = 1$ if statement \mathcal{S}

Table 3 Multiretailer Heuristic Pseudocode

Initialize: Set $\rho_0^i(x) = s \cdot x$. Compute pairwise-holdback levels \tilde{x}_n^{ij} and no-sharing order quantities S .
 $\mathcal{M} = \{1, \dots, M\}$ and $\mathcal{M}_{-i} = \{1, \dots, M\} \setminus \{i\}$. /* ρ_n^i is profit under heuristic; analogous to π_n^i */

Iterate:

For $n = 1, \dots, N$, For $m = 1, \dots, M$, For $x_m = 0, \dots, S_m$,

For $i = 1, \dots, M$, /* Retailer i is visited by a customer and then W_i^l is profit of retailer l */

If $x_i \geq 1$, $W_i^l := r_i + \rho_{n-1}^l(x - e_i)$ and $W_i^l := \rho_{n-1}^l(x - e_i)$, $l \in \mathcal{M}_{-i}$;

else /* Retailer i requests from retailer j */

$j := \arg \max_{j \in \mathcal{M}} \{x_j/p_j\}$.

If $x_j \geq 1$ and $x_j > \tilde{x}_n^{j,i}$, $W_i^l := r_i - \tau_{ji} - t_j + \rho_{n-1}^l(x - e_j)$ and $W_i^l := \mathbb{1}_{l=j} t_j + \rho_{n-1}^l(x - e_j)$, $l \in \mathcal{M}_{-i}$

else /* Retailer j rejects request of i whose customer overflows to k */

$W_i^l := \sum_{k=1, k \neq i}^M \theta_{ik} \{\mathbb{1}_{x_k \geq 1} (\mathbb{1}_{l=k} r_k + \rho_{n-1}^l(x - e_k)) + \mathbb{1}_{x_k=0} \rho_{n-1}^l(x)\} + (1 - \sum_{k=1, k \neq i}^M \theta_{ik}) \rho_{n-1}^l(x)$, $l \in \mathcal{M}$.

If $x_j = 0$, $W_i^l := \rho_{n-1}^l(x)$, $l \in \mathcal{M}$. /* No inventory is left in system */

EndFor i .

$\rho_n^i(x) := (1 - \sum_{l=1}^M p_l) \rho_{n-1}^i(x) + \sum_{l=1}^M p_l W_l^i$, $l \in \mathcal{M}$.

EndFor x_m . EndFor m . EndFor n .

Output: $J_n^H(S) = \rho_n^H(S) - c_i S_i$.

is correct; otherwise, it is zero. Let “ \cdot ” denote scalar multiplication of two vectors.

To assess the performance of the heuristic, the sum of retailer profits $\sum_{m=i}^M J_i^H(S)$ is compared with the optimal profit of the centralized system $J(S)$, which can be computed directly without characterizing the optimal transshipment policy from $J(S) = \pi_N(S) - c \cdot S$, where $\pi_0(x) = s \cdot x$ and

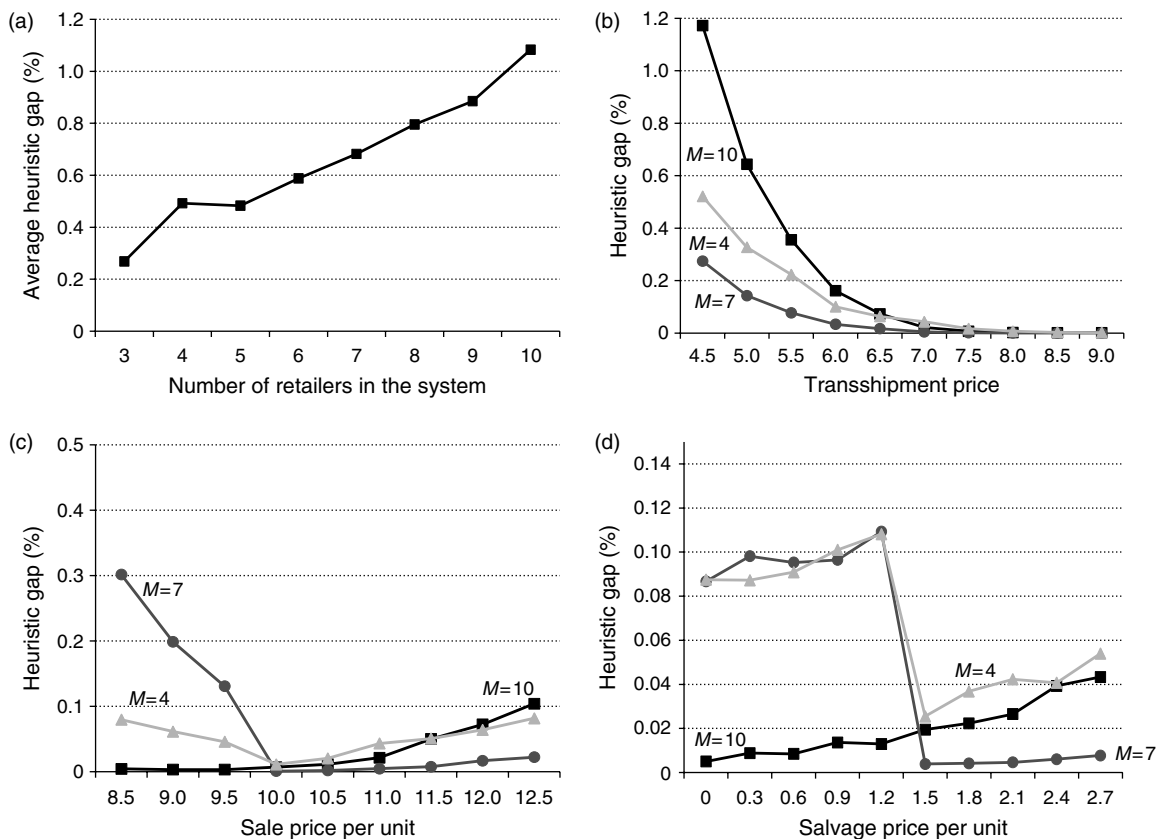
$$\begin{aligned} \pi_n(x) &= \left(1 - \sum_{i=1}^M p_i\right) \pi_{n-1}(x) \\ &\quad + \sum_{i=1}^M p_i \{ \mathbb{1}_{x_i \geq 1} (r_i + \pi_{n-1}(x - e_i)) \\ &\quad \quad + \mathbb{1}_{x_i = 0} \max\{TP_i, OP_i\} \}, \\ TP_i &= r_i + \max_{1 \leq j \leq M, x_j \geq 1} \{ \pi_{n-1}(x - e_j) - \tau_{ji} \}, \\ OP_i &= \sum_{k=1, k \neq i}^M \theta_{ik} [\mathbb{1}_{x_k \geq 1} (r_k + \pi_{n-1}(x - e_k)) + \mathbb{1}_{x_k = 0} \pi_{n-1}(x)] \\ &\quad + \left(1 - \sum_{k=1, k \neq i}^M \theta_{ik}\right) \pi_{n-1}(x), \end{aligned}$$

where TP_i is the profit with a transshipment from retailer j , and OP_i is the profit with a customer either overflowing to retailer k or out of the system.

Both profits are for a customer arrival to stocked-out retailer i . Because inventory and transshipment decisions of the decentralized system are feasible in the centralized system, $J(S)$ is an upper bound for the total profits of a decentralized system under any policy. In particular, $\sum_{i=1}^M J_i^H(S) \leq J(S)$.

The heuristic gap $(1 - \sum_{i=1}^M J_i^H(S)/J(S)) \cdot 100$ is computed with $N = 50$ and $M \in \{3, \dots, 10\}$. For every M , 50 instances are generated by setting $c_i = \hat{c}$, $t_i = \hat{t}$, $r_i = \hat{r}$, $\tau_{ij} = \hat{\tau}$, and $\theta_{ij} = \hat{\theta}$ for $1 \leq i, j \leq M$, where \hat{c} , \hat{t} , \hat{r} , $\hat{\tau}$, and $\hat{\theta}$ are sampled along with p_i and s_i from a uniform distribution over the following ranges: $p_i \in (0, 1/M)$, $s_i \in (0, 2)$, $\hat{c} \in (3, 5)$, $\hat{t} \in (6, 8)$, $\hat{r} \in (10, 14)$, $\hat{\tau} \in (1, 2)$, and $\hat{\theta} \in (0, 1/(M-1))$. In each instance, retailers can have different demand probabilities and different salvage values while each of the other parameters is the same across retailers. With p_i uniformly distributed over $(0, 1/M)$, total expected system demand per period is $1/2$. Because the total demand does not change with M , the heuristic gaps can be compared across different values of M . To focus on transshipment decisions, optimal orders under no sharing are used in the heuristic and centralized solutions. Note that orders differ slightly from no sharing to optimal sharing; see the ΔS column in Table 1. The average heuristic gap over 50 instances is illustrated in Figure 2(a) for $M \in \{3, \dots, 10\}$. Because the average is

Figure 2 (a) Average Heuristic Gap vs. M ; (b) Gap vs. t_i for $r_i = 11$ and $s_i = 2$; (c) Gap vs. r_i for $t_i = 7$ and $s_i = 2$; (d) Gap vs. s_i for $r_i = 11$ and $t_i = 7$



less than 1% in all but the $M = 10$ retailer case, our multiretailer heuristic appears to perform well.

Another issue to investigate is how the heuristic gap changes with monetary parameters such as market price, transshipment price, and salvage value. The only other monetary parameter is purchase cost, which can always be set equal to one by scaling monetary units. In Figures 2(b), 2(c), and 2(d), M varies over $\{4, 7, 10\}$ and the identical retailers have $c_i = 5$, $p_i = 0.7/M$, $\tau_{ij} = 1$, and $\theta_{ij} = 0.63/(M - 1)$ for $i, j \in \{1, \dots, M\}$. In Figures 2(b), 2(c), and 2(d), one of r , t , and s varies while the other two parameters are fixed as in the figure caption.

Figure 2(b) shows that the heuristic gap reduces with transshipment prices, which also reduce holdback levels by Theorem 3. Thus, retailers guard less of their inventory and share more with others as transshipment prices increase. This cooperative tendency brings the heuristic solution closer to the centralized solution. Increasing r or s has two effects, i.e., higher ordering quantities and higher holdback levels. The first effect increases the cooperative tendency, while the second decreases it. The direction of the combined effect is not clear while its magnitude is small from Figures 2(c) and 2(d), where the y -axes have ranges of $[0, 0.4]$ and $[0, 0.12]$. Hence, the market price or the salvage value has little effect on the heuristic gap.

5. Concluding Remarks

Many independent retailers do not want to commit to extreme transshipment policies such as complete sharing or no sharing and prefer flexibility in deciding whether to accept a transshipment request. We provide this flexibility by delegating the acceptance decision to the requested retailer who bases that decision on the current time and inventory levels. A realistic model is formulated in both cost/revenue structure and sequence of events. For example, retailers decide on orders before the sales season. In-season transshipment requests happen when demands occur at a stocked-out retailer. The optimal transshipment policy has built-in flexibility that allows the retailer to reject a request one day, but accept another request a few days later. Finally, our optimal transshipment policy for two-retailer systems is used to develop an effective heuristic for many-retailer systems.

An important ingredient of our transshipment model is the customer overflow probability, which is higher for retailers that are geographically close to each other, and explains competition between close retailers. Some manufacturers have retailers with large customer overflow probabilities. Their sales increase, marginally, when retailers shift to complete inventory sharing from no sharing. On the other hand, many manufacturers whose retailers have

a small customer overflow probability expect an increase in total sales with complete inventory sharing. Yet they find it difficult to convince independent retailers to completely share their inventory. These manufacturers can suggest our optimal sharing policy to their retailers who should be more sympathetic to optimal sharing because of its flexibility, which is absent both in complete and no sharing.

Implementing a model in practice depends on the ease of computations and data availability in addition to sharpness of the managerial insights. Our model requires the computation of optimal holdback levels, which depend only on retailers' costs and demand parameters, and can be easily computed using spreadsheets. Once the holdback levels and, accordingly, profits for each pair of order quantities are computed, the equilibria for the ordering game can be found. If an inventory manager does not believe or understand the rationality assumptions of game theory, he or she can still implement the optimal holdback levels with any order quantities, as these levels do not depend on the order quantities or demands that have been realized. The robustness of the transshipment policy against order quantities further facilitates implementation in practice when retailers do not receive their exact orders because of capacity/yield problems or inventory loss/shrinkage.

Electronic Companion

An electronic companion to this paper is available as part of the online version that can be found at <http://msom.journal.informs.org/>.

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