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Naveen Jindal School of Management

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Strategies*

UTD AUTHOR(S): Nanda Kumar and Ram C. Rao

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# Consumer Stockpiling and Competitive Promotional Strategies

Manish Gangwar

Indian School of Business, Hyderabad, India 500032, manish\_gangwar@isb.edu

Nanda Kumar, Ram C. Rao

Naveen Jindal School of Management, University of Texas at Dallas, Richardson, Texas 75080  
(nkumar@utdallas.edu, rrao@utdallas.edu)

An examination of brand prices in several categories reveals that the distribution of prices is multimodal, with firms offering shallow and deep discounts. Another interesting feature of these distributions is that they may have holes in the interior of the support. These pricing distributions do not occur in extant theoretical models of price promotions. We develop a dynamic model of competition in which some price-sensitive consumers stockpile during periods of deep discounts. A game-theoretic analysis of our model generates a multimodal pricing distribution with a hole in the interior of the support. Consumer stockpiling in our model also gives rise to negative serial correlation in prices. This is consistent with our empirical observation of the pricing distribution of several brands across multiple categories in the IRI marketing data set.

We generate several interesting insights into firms' optimal promotional strategies and their interplay with the clientele mix, market structure, and other market factors. We find that, in equilibrium, stockpiling by price-sensitive consumers neither harms nor benefits firms when they adopt equilibrium strategies. Interestingly, when price-sensitive consumers stockpile, even increased consumption as a result of stockpiling does not lead to higher profits for firms.

*Key words:* promotional strategies; consumer stockpiling; endogenous stockpiling threshold; interior modes; multiple modes; hole in distribution; negative serial correlation

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## 1. Introduction

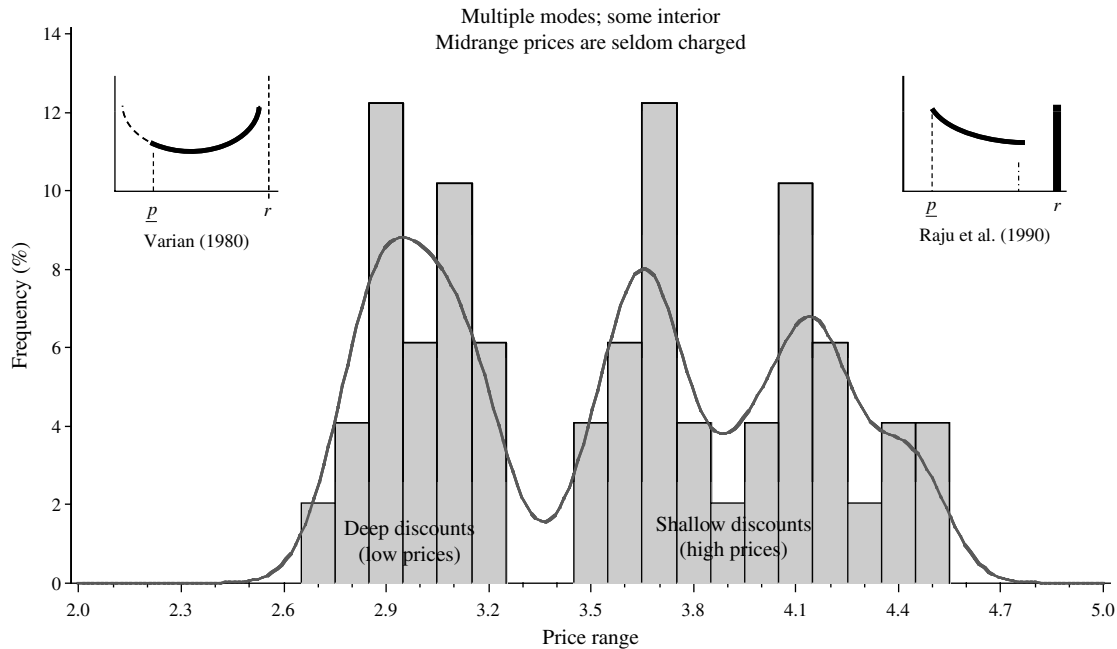
There is a rich literature in both economics and marketing that views price promotions as a mixed strategy equilibrium (Shilony 1977, Varian 1980, Narasimhan 1988, Raju et al. 1990, Rao 1991). The scant empirical work that examines whether price promotions are consistent with the mixed strategy interpretation has largely focused on testing whether price distributions of competing products exhibit independence (Rao et al. 1995). Although Rao et al. find that prices across brands are independent, they note that in many cases distributions have more than two modes. Our examination of the distribution of prices of multiple stock-keeping units (SKUs) across several categories in the IRI marketing data set (Bronnenberg et al. 2008) is consistent with the authors' finding that the distribution of prices is often multimodal and that prices frequently exhibit negative serial correlation.

The salient features of a typical distribution of prices of a brand (a single stock-keeping unit) in a frequently purchased category in a store are shown in the Figure 1. The  $x$  axis in Figure 1 represents

the range of prices, and the  $y$  axis represents the frequency (in percentage) at which the various prices are charged. First, note that the pricing distribution is multimodal. Second, often the modes are in the interior of the support. Finally, there is a range of prices in the interior of the support that are not charged very often.<sup>1</sup> The extant theory on price dispersion predicts a U-shaped distribution that is bimodal with the modes at the ends rather than in the interior of the support (as depicted in the top left and right corners of Figure 1). One of the goals of this study is to offer an explanation for promotional policies that is consistent with the multimodal distribution patterns and the negative serial correlation in prices documented in Tables A.3 and A.4 in the appendix.

<sup>1</sup> Although this type of pricing distribution is pervasive, we are unaware of any published work that documents this phenomenon. Please refer to Table A.3 in the appendix for a summary of pricing distributions of several brands across several categories that exhibit multiple modes. The left-hand-side satellite picture in Figure 1 is the equilibrium pricing density function from Varian (1980) depicting at most two modes. The right-hand-side satellite picture is the same from Raju et al. (1990), depicting the presence of a hole close to the reservation price.

Figure 1 Price Distribution in a Frequently Purchased Category



Notes. The distribution of prices is for a single SKU (Pepsi 24-pack, 8 oz cans/bottles) between July 2001 and June 2002. The data are from a single large chain store in the Midwest. A time-series plot of the same SKU reveals a possible serial correlation in prices with no obvious trend or structural break. See Figure A.1 in the appendix for a time-series plot.

Let us begin by examining the motivation and the consequence of offering price promotions. All else being equal, offering a price promotion lowers the margin. Consequently, in the absence of cost decreases and/or other externalities, firms can benefit only if demand expands in promotional periods. Heerde et al. (2003) and Steenburgh (2007) show that primary demand effects account for about two-thirds of the sales increase. It is therefore helpful to examine the sources of primary demand expansion. Primary demand expansion may occur if consumers increase their rate of consumption in promotional periods. However, in many categories such as toilet tissue, coffee, etc., the consumption rate is known to be relatively constant, and thus demand expansion in promotional periods is most likely a result of intertemporal shifts in consumers' purchases. Indeed, this is consistent with the postpromotional dip in sales that is documented in numerous empirical studies.

We consider a symmetric oligopoly in which brands compete to serve the needs of consumers who differ in two dimensions: (a) price sensitivity and (b) propensity to stockpile. All consumers have the same reservation price and consume at a rate of one unit each period. A fraction of the market is price insensitive and purchases a unit from one of the brands at random as long as the price is not greater than the reservation price. The remaining consumers are price sensitive and make purchases based solely on

prices; they buy from the lowest-priced firm. Consumers also differ in their ability to stockpile. Some consumers do not stockpile because of high storage costs, space constraints, or other nonstrategic reasons. The remaining price-sensitive consumers with low holding costs are willing to stockpile and purchase one additional unit for the future whenever prices are low enough. Firms set prices to maximize discounted profits over an infinite horizon. We characterize the Markov perfect equilibrium of this game. The mixed-strategy equilibrium is state contingent; a firm's pricing strategies in any period depend not only on its pricing decisions but also on those of its rivals' past decisions. More specifically, the pricing strategy in any given period depends critically on whether any brand has offered a deep enough promotion in the previous period to trigger stockpiling.

A novel feature of our model is that the price threshold below which stockpiling consumers engage in forward buying is determined endogenously. This is important because the promotional policies (depth and frequency of discounts) differ substantially across product categories. For instance, consider two categories, one in which a discount of 50% off the regular price is common and another in which such discounts are seldom offered. Consumers may not stockpile in the first category when they see a 30% discount, but they may be willing to stockpile in the second category for the same discount. This implies that the

firms' pricing policies will have a bearing on the consumers' stockpiling threshold, which in turn affects the quantity purchased. It is clear that firms' optimal pricing policies should take into account the demand effects, which depend on the stockpiling threshold. Therefore, in characterizing the equilibrium pricing policy, the stockpiling threshold must be endogenously determined. Said differently, the firms must recognize that any change in market characteristics will affect their pricing policy not only directly but also indirectly through the effect on the stockpiling threshold.

### 1.1. Literature Review

Our work builds on two streams of literature: (a) the rich and established literature on price promotions/dispersion and (b) the recent but growing literature that analyzes consumers' stockpiling behavior. Much of the extant literature on price promotion/dispersion (Shilony 1977, Varian 1980, Narasimhan 1988, Raju et al. 1990, Rao 1991) proposes a static model to explain price dispersion, both contemporaneous and intertemporal. Specifically, these studies show that there is no equilibrium in pure strategies. Given that stockpiling is intrinsically a dynamic problem, we extend this literature by examining the pricing decisions of firms in an oligopoly where they maximize profits over an infinite horizon. The demand dynamics in our model are also distinct from the extant literature in this area. The aggregate demand in these models is assumed to be constant, so the short-term demand expansion resulting from promotions always comes at the expense of competing firms. However, recent empirical work by Heerde et al. (2003) and Steenburgh (2007) demonstrates that the aggregate demand expansion in promotional periods accounts for about two-thirds of the sales increase, which is consistent with the long-established postpromotion dip in sales. In our model we allow some consumers to be forward looking and purchase for future consumption when they find the prices in the current period to be sufficiently attractive. In this way, the aggregate demand in every period is not the same.

The second stream of research that our paper builds on examines the impact of stockpiling on firms' pricing decision. Before discussing the findings from this literature, we highlight certain institutional characteristics that we feel are important to incorporate in a model that examines consumers' stockpiling behavior. First, the problem is inherently dynamic, as stockpiling by consumers in a given period will have an effect on their demand in the following period. This should be internalized by the firms in formulating their pricing strategies. Second, consumers seldom know future prices. They may have some expectations

of prices in future periods, but they face uncertainty about the exact price that will be offered. Finally, given the dynamic nature of the decision problem, (rational forward-looking) consumers will stockpile only if the consumer surplus from stockpiling in a given period exceeds that from not stockpiling or delaying purchase to the next period. This comparison of utility from stockpiling with the expected utility from not stockpiling yields a price threshold. If the current period price is lower than this threshold, consumers will prefer to stockpile. If, however, the current period price is above this threshold, there is no incentive to stockpile. These characteristics of the decision environment call for a model that (a) is dynamic, (b) yields equilibrium prices that are not deterministic so that consumers do not know the future prices with certainty, and (c) explicitly incorporates the endogenous relationship between current and future period prices and its effect on triggering stockpiling behavior, thereby endogenously determining the stockpiling threshold.

Salop and Stiglitz (1982) consider a market in which consumers may stockpile and show that imperfect price information leads to an equilibrium in mixed strategies. Bell et al. (2002) consider a setting in which consumers may stockpile for future consumption but allow for flexible consumption so that consumers who stockpile may reenter the market with some probability. Both studies characterize the equilibrium mixing distribution that has two price points in its support. Anton and Das Varma (2005) and Guo and Villas-Boas (2007) also consider settings in which consumers stockpile and characterize the pricing equilibrium in pure strategies. The paper that comes closest to our work is Hong et al. (2002), who consider an oligopoly setting in which firms compete for captives and shoppers. All shoppers in their model engage in stockpiling when prices fall below a certain threshold, which is exogenously determined. However, none of these theories can jointly explain pricing patterns that exhibit multiple modes, interior holes, and negative serial correlation.

Our model takes into account (a) consumers' optimal response to promotions, (b) firms' optimal response to consumer stockpiling, and (c) competitive intensity in a single framework, which allows us to capture both the direct and indirect effects of market characteristics on equilibrium pricing strategies. By doing so, our model not only generates predictions that are consistent with the pricing patterns observed in the frequently purchased categories but also delineates the conditions when stockpiling may (not) occur in equilibrium. The model also generates interesting insights on the optimal promotional strategies of firms and their interplay with the clientele mix, market structure, and other market factors.

The remainder of the paper is structured as follows. We present the model in §2. We begin with a description of the model primitives and develop some essential results in this section. We present our main results in §3, where we highlight how the equilibrium outcomes vary with the clientele mix and the market structure. In §4 we document some empirical regularities exhibited by the distribution of prices that are consistent with those documented in earlier work, and we demonstrate that our model predictions are consistent with the empirical observations. In §5 we extend our base model to allow for increase in consumption after stockpiling and examine how it affects firms' profits. We conclude with a discussion in §6.

## 2. Model

We consider a market in which  $n$  symmetric firms, selling similar products, compete to maximize profits over an infinite horizon. All consumers have a reservation price of  $r$  for a unit of the product and consume one unit in each period but differ in their price sensitivity and holding cost.<sup>2</sup> A fraction  $\nu$  of the consumers are price insensitive and purchase one unit each period from one of the  $n$  firms at random as long as the price is not greater than their reservation price  $r$ . The remaining fraction of consumers of size  $(1 - \nu)$  are price sensitive and purchase the lowest-priced product. Consumers are also heterogeneous in their ability to stockpile. A fraction  $\lambda$  of the consumers have a relatively low holding cost, here assumed to be zero; the rest have very high holding cost such that stockpiling is prohibitively expensive.

Because price-insensitive consumers do not make purchase decisions based on prices, deep discounts do not affect their purchase decision. Price-sensitive consumers with a low holding cost of size  $(1 - \nu)\lambda$  stockpile at deep discounts. We can think of these consumers as exhibiting *forward-looking* or *strategic* behavior.<sup>3</sup> This is consistent with the empirical findings of Dillon and Gupta (1996), who find that the majority of the increase in category sales during promotional periods comes from price-sensitive consumers.

To keep the state space manageable, we assume that when forward-looking consumers stockpile, they purchase only one additional unit.<sup>4</sup> Although there

are an infinite number of periods, there are only two possible states,  $s = \{0, 1\}$ , in our model. The state  $s = 0$  corresponds to the case in which none of the  $n$  firms offered a low enough price in the previous period to trigger stockpiling. Consequently, forward-looking consumers are left with no inventory in the current period. The state  $s = 1$  corresponds to the case when at least one of the  $n$  firms offered a deep enough discount to trigger stockpiling in the previous period, such that strategic consumers purchased an additional unit for future consumption, leaving them with an inventory of one in the current period. The example in Figure 2 helps to clarify the state space and demand dynamics.

Consider the market at time  $t$  and suppose forward-looking consumers have no inventory so that  $s = 0$ . If none of the  $n$  firms offers a price low enough to trigger stockpiling (high price branch in the figure), consumers purchase one unit for consumption in period  $t$  and end up with no inventory ( $s = 0$ ) in  $t + 1$ . In period  $t + 1$ , they are faced with the same decision problem. Now suppose at least one of the  $n$  firms offers a deep enough discount in  $t + 1$  so that these consumers stockpile. In our model, when consumers stockpile in  $s = 0$ , they purchase two units: one for current consumption and one for future consumption. Now consider the case when these consumers have an inventory of one unit at time  $t$  (state  $s = 1$ ). If no firm offers a deep enough discount, they use the product in their inventory and do not purchase. On the other hand, if one of the firms offers a deep enough discount, consumers stockpile, but given that they have an inventory of one unit for current consumption, they purchase one unit for future consumption. Thus, the key difference in  $s = 0$  and  $s = 1$  is that at high prices, strategic consumers purchase one unit or none, respectively; at deep discounts, they purchase two units or one, respectively.

A relevant question to ask is whether the discount required to trigger stockpiling (i.e., the low price in Figure 2) depends on the state of consumers' inventory. We formally analyze the consumers' decision of whether to stockpile at any given price in each state and show (in Lemma 1) that the stockpiling threshold in our model does not depend on the state. Let  $U_s$  denote the expected continuation utilities of strategic consumers net of price at the time of purchase in state  $s$ ,  $s = \{0, 1\}$ , and let  $\delta_c$  denote the consumers' discount factor.

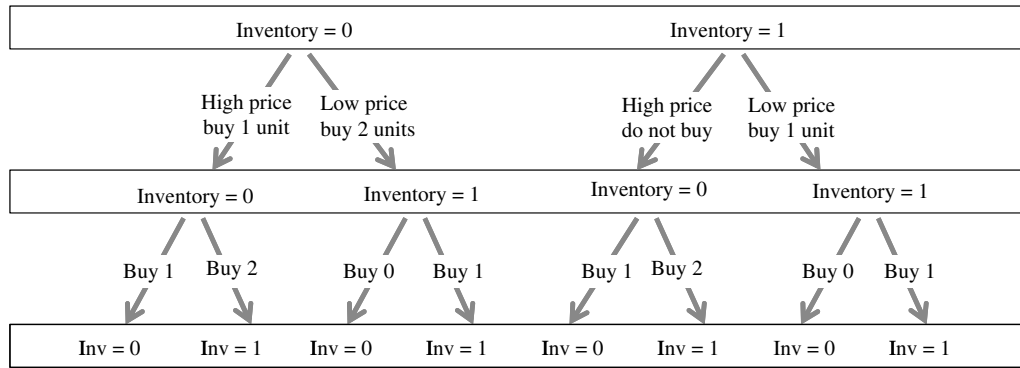
**LEMMA 1.** *There exists a single price threshold,  $t$ , regardless of strategic consumers' state of inventory, such that it is optimal for strategic consumers to stockpile for the next period whenever the minimum price in the market is less than or equal to the threshold:  $p_s^{\min} \leq t = \delta_c r - \delta_c(U_0 - U_1)$ .*

<sup>2</sup> In our model, without loss of generality, all prices are normalized with respect to marginal cost,  $c$ , and reservation price,  $r'$ . The normalized price is  $p = r \cdot (p' - c)/(r' - c)$ , where  $r'$ ,  $p'$ , and  $c$  are the actual reservation price, the actual price, and marginal cost, respectively;  $r$  is any arbitrary constant. This normalization maps the  $[r', c]$  space to  $[r, 0]$  space.

<sup>3</sup> We use the terms "forward-looking," "strategic," and "stockpiling" interchangeably throughout to refer to this segment of consumers.

<sup>4</sup> Niraj et al. (2008) find that 95% of quantity purchases are concentrated in one or two units.

Figure 2 Forward-Looking Consumers' Behavior



PROOF. See the appendix.

The strategic consumer's decision problem is whether to buy an additional unit for future consumption at current minimum price and move to (or remain in) state  $s = 1$  in the next period or to buy for future consumption and move to (or remain in) state  $s = 0$  in the next period. It is helpful to rewrite the above condition as  $\delta_c r - p_s^{\min} \geq \delta_c (U_0 - U_1)$ .<sup>5</sup> The left-hand side of this inequality,  $\delta_c r - p_s^{\min}$ , denotes the benefit (i.e., discounted consumer surplus) from buying for future at the current minimum price; the right-hand side denotes the cost of stockpiling, moving from state  $s = 0$  to state  $s = 1$ . Note that to compute the stockpiling threshold we need to know the continuation utilities  $U_0$  and  $U_1$ , which in turn depend on the firms' pricing strategies. Consequently, we now analyze the firms' decision problem.

Given that firms are symmetric, let  $V_s$  denote the expected continuation payoff to firms in state  $s$ ,  $s = \{0, 1\}$ ; let firms' discount factor be  $\delta_F$ .

## 2.1. Demand Characterization

After establishing the unique stockpiling threshold for both states in Lemma 1, we now compute the demand facing firms in each state for a given stockpiling threshold,  $t$ . Let  $D_s(p_i, p_{-i})$  denote the demand facing firm  $i$  in state  $s$ , given the vector of prices  $p_{-i}$  charged by all the other firms.<sup>6</sup> Because the behavior of strategic consumers in any given state depends on how the price compares to the stockpiling threshold  $t$ ,

we characterize the demand for each state conditional on these events as follows:

$$D_0(p_i, p_{-i} | p_i > t) = \begin{cases} \frac{\nu}{n} + 1 - \nu & \text{if } p_i < \min(p_{-i}), \\ \frac{\nu}{n} & \text{if } p_i > \min(p_{-i}). \end{cases} \quad (1)$$

If the focal firm charges a price greater than  $t$  in state  $s = 0$ , strategic consumers will purchase a maximum of one unit. If the focal firm's price is the lowest, then it sells to a fraction  $\nu/n$  of the price-insensitive consumers and to all  $(1 - \nu)$  price-sensitive consumers. If it is not the lowest-priced firm, it only sells to price-insensitive customers. Next, we specify the demand in  $s = 0$  when the focal firm charges a price below the stockpiling threshold:

$$D_0(p_i, p_{-i} | p_i \leq t) = \begin{cases} \frac{\nu}{n} + (1 - \nu)(1 + \lambda) & \text{if } p_i < \min(p_{-i}), \\ \frac{\nu}{n} & \text{if } p_i > \min(p_{-i}). \end{cases} \quad (2)$$

If the focal firm has the lowest price, then the forward-looking consumers of size  $(1 - \nu)\lambda$  stockpile and purchase two units, whereas the remaining price-sensitive consumers of size  $(1 - \nu)(1 - \lambda)$  purchase one unit resulting in a demand of  $2(1 - \nu)\lambda + (1 - \nu)(1 - \lambda) = (1 - \nu)(1 + \lambda)$  units. In addition, the firm sells to a fraction  $\nu/n$  of the price-insensitive consumers. If it is not the lowest-priced firm, then it sells only to the price-insensitive consumers. The demand in  $s = 1$  is computed in a similar fashion:

$$D_1(p_i, p_{-i} | p_i > t) = \begin{cases} \frac{\nu}{n} + (1 - \nu)(1 - \lambda) & \text{if } p_i < \min(p_{-i}), \\ \frac{\nu}{n} & \text{if } p_i > \min(p_{-i}); \end{cases} \quad (3)$$

<sup>5</sup>  $U_0$  and  $U_1$  are explicitly characterized in terms of firms' pricing strategies in (11) and (12). The formal condition that the equilibrium stockpiling threshold must satisfy is defined in (13). Since  $U_s$  represents the continuation utilities at the time of purchase,  $U_0$  is always greater than  $U_1$  because  $U_0$  accounts for the possibility of buying and consuming an extra unit.

<sup>6</sup> Notation is summarized in Table A.1 in the appendix.

$$D_1(p_i, p_{-i} | p_i \leq t) = \begin{cases} \frac{\nu}{n} + (1 - \nu) & \text{if } p_i < \min(p_{-i}), \\ \frac{\nu}{n} & \text{if } p_i > \min(p_{-i}). \end{cases} \quad (4)$$

Given the discontinuities in demand in (1) to (4) as in Varian (1980) and Narasimhan (1988), there is no equilibrium in pure strategies. Consequently, we proceed with the characterization of the symmetric Markov perfect equilibrium of this game.<sup>7</sup>

## 2.2. Equilibrium Demand Analysis

Let  $F_s(p)$  and  $[l_s, r]$  denote the equilibrium mixing distribution and corresponding support of any firm in state  $s$ , respectively. Given (1) and (2), the equilibrium mixing distributions in  $s = 0$  must satisfy the following conditions:

$$\begin{aligned} p \left[ \frac{\nu}{n} + (1 - \nu)(1 - F_0(p))^{(n-1)} \right] + \delta_F [V_0(1 - F_0(t))^{(n-1)} \\ + V_1(1 - (1 - F_0(t))^{(n-1)})] &= V_0 \quad \forall p \in (t, r], \quad (5) \\ p \left[ \frac{\nu}{n} + (1 - \nu)(1 + \lambda)(1 - F_0(p))^{(n-1)} \right] + \delta_F V_1 &= V_0 \\ \forall p \in [l_0, t]. \quad (6) \end{aligned}$$

Equations (5) and (6) require that any price charged in the support must yield the same expected profit in equilibrium. Specifically, in  $s = 0$  the firms' expected profit for any price in the support of  $F_0(p)$  is  $V_0$ . Equations (5) and (6) also recognize the difference in behavior of strategic consumers when firms charge prices higher or lower than the stockpiling threshold  $t$ . Following (1), the first term in the left-hand side of (5) represents the expected profit from charging a price greater than  $t$ . The firm is guaranteed price-insensitive consumers of size  $\nu/n$ , but price-sensitive consumers of size  $(1 - \nu)$  purchase its product only if it happens to be the lowest-priced firm, which happens with probability  $(1 - F_0(p))^{(n-1)}$ . The second term represents the continuation payoffs. Given that the focal firm is charging a price higher than  $t$ , the state in the next period will depend on rival firms' pricing strategies in the current period. If none of the rival firms charges a price lower than  $t$ —an event that occurs with probability  $(1 - F_0(t))^{(n-1)}$ —strategic consumers do not have the opportunity to stockpile and transition to  $s = 0$ , where the firms' continuation payoff is  $V_0$ . With complementary probability,  $1 - (1 - F_0(t))^{(n-1)}$ , at least one of the firms charges a price lower than  $t$  allowing consumers to stockpile

and transition to  $s = 1$  with firms' continuation payoff of  $V_1$ . The second term in (5) thus represents the firm's expected continuation payoff given the probabilities of transitioning to  $s = 0$  or 1.

The left-hand side of (6) defines the expected profit of firms in  $s = 0$  from charging any price less than or equal to  $t$  in the support. The first term follows from (2), and the second term represents the continuation payoffs. In contrast to (5), given that the focal firm charges a price lower than  $t$ , transition to  $s = 1$  is guaranteed and is independent of rival firms' pricing decisions. Equations (7) and (8) similarly require that the expected payoffs from charging any price in the support in  $s = 1$  are  $V_1$  and follow from (3) and (4), respectively:

$$\begin{aligned} p \left[ \frac{\nu}{n} + (1 - \nu)(1 - \lambda)(1 - F_1(p))^{(n-1)} \right] \\ + \delta_F [V_0(1 - F_1(t))^{(n-1)} + V_1(1 - (1 - F_1(t))^{(n-1)})] &= V_1 \\ \forall p \in (t, r], \quad (7) \\ p \left[ \frac{\nu}{n} + (1 - \nu)(1 - F_1(p))^{(n-1)} \right] + \delta_F V_1 &= V_1 \\ \forall p \in [l_1, t]. \quad (8) \end{aligned}$$

**PROPOSITION 1.** *Firms' continuation profits are independent of state and the proportion of stockpiling price-sensitive consumers:  $V_0 = V_1 = (r\nu)/(n(1 - \delta_F))$ .*

**PROOF.** Substitute  $p = r$  in (5) and (7) to get

$$\begin{aligned} \frac{\nu \cdot r}{n} + \delta_F [V_0(1 - F_0(t))^{(n-1)} + V_1(1 - (1 - F_0(t))^{(n-1)})] &= V_0, \\ \frac{\nu \cdot r}{n} + \delta_F [V_0(1 - F_1(t))^{(n-1)} + V_1(1 - (1 - F_1(t))^{(n-1)})] &= V_1. \end{aligned}$$

We solve the above equations simultaneously to obtain  $V_0$  and  $V_1$ . Q.E.D.

We find that equilibrium profits are not only independent of state but also independent of the proportion of stockpiling consumers,  $\lambda$ . It is noteworthy that the equilibrium profits (given the equilibrium mixing distributions characterized here) are identical to the case in which price-sensitive consumers do not stockpile. In other words, if we set  $\lambda = 0$ , then our model is identical to the dynamic version of Varian (1980) where the equilibrium profits are identical to that stated in Proposition 1. What this means is that if price-sensitive consumers stockpile and firms adopt equilibrium strategies, then firms can obtain the same profits as they would if there was no stockpiling.

Before characterizing the equilibrium mixing distribution, we provide the rationale for the existence of dominated prices (a hole) in the interior of the support. If the stockpiling threshold is in the interior of the support, then as noted earlier, firms will face different demand at prices above and below the stockpiling threshold  $t$  in each state. This implies that firms'

<sup>7</sup> Symmetric equilibrium is also the unique subgame perfect equilibrium among all possible symmetric and asymmetric equilibria (Baye et al. 1992).



mixing distribution will be different for prices above and below the stockpiling threshold  $t$ . To see this, substitute  $V = V_0 = V_1$  from Proposition 1 in (5) and (6) to obtain

$$\begin{aligned} p \left[ \frac{\nu}{n} + (1-\nu)(1-F_0(p))^{(n-1)} \right] + \delta_F V &= V \\ \forall p \in (t, r], & \\ p \left[ \frac{\nu}{n} + (1-\nu)(1+\lambda)(1-F_0(p))^{(n-1)} \right] + \delta_F V &= V \\ \forall p \in [l_0, t]. & \end{aligned} \quad (9)$$

We denote the cumulative distribution function above and below  $t$  by superscript  $a$  and  $b$ , respectively, in each state  $s = \{0, 1\}$ :

$$\begin{aligned} F_s^a(p) \quad \forall p \in (t, r], \\ F_s^b(p) \quad \forall p \in [l_s, t]. \end{aligned}$$

To verify the existence of dominated prices, notice that the cumulative distribution function should be nondecreasing, which implies that at the stockpiling threshold,  $t$ ,  $F_s^b(t) \leq F_s^a(t)$ , must hold. However, by using the equiprofit condition in (9), it is easy to see that  $F_s^a(t) < F_s^b(t)$ ,  $\forall \lambda > 0$ , which violates the nondecreasing property of the cumulative distribution function. This contradiction can only be resolved if prices in the neighborhood of  $t$  are not in the support of the equilibrium mixing distribution. We establish this formally in Lemma 2.

**LEMMA 2.** *There is a hole in the firms' mixing distribution whenever the stockpiling threshold is in the interior of the support of the mixing distribution in any state. In state  $s = 0$ , the hole extends from stockpiling threshold  $t$  to  $h_0 = t(r+r\lambda)/(r+t\lambda)$ , and in state  $s = 1$ , the hole extends from stockpiling threshold  $t$  to  $h_1 = t \cdot r/(r-\lambda(r-t))$ .*

**PROOF.** See the appendix.

To understand the intuition behind this result, note that prices at or below  $t$  result in a discontinuous increase in demand in both states.<sup>8</sup> Consider the case when a firm charges  $t$  while all other firms charge a price strictly greater than  $t$ . In equilibrium, the firm's expected profit from charging  $t$  and  $t + \varepsilon$  must be equal. Given the discontinuous increase in demand as a result of consumer stockpiling at prices at or below  $t$ , the price  $t + \varepsilon$  is strictly dominated. Figure 3 illustrates the cumulative distribution function of prices in any state  $s = \{0, 1\}$  when the stockpiling threshold  $t$  is in the interior of the support.

<sup>8</sup>Note that as  $\lambda \rightarrow 0$ ,  $h_0 \rightarrow t$ , and  $h_1 \rightarrow t$ . In other words, in the absence of stockpiling, price-sensitive consumers, the hole disappears.

The cumulative distribution of prices in any state  $s$  has two intervals of prices,  $[l_s, t]$  and  $[h_s, r]$ , which firms charge with positive probability. The former interval denotes *deep discounts* that trigger stockpiling behavior. Prices in the interval  $[h_s, r]$  represent *shallow discounts* that firms offer to compete for price-sensitive consumers without triggering any stockpiling. The third interval,  $(t, h_s)$ , is the flat region of the cumulative distribution function (CDF) (or the hole) with zero mass and represents the interval of prices that firms will not charge in equilibrium. The mixing distribution takes this form when the stockpiling threshold  $t$  is in the interior of the support.

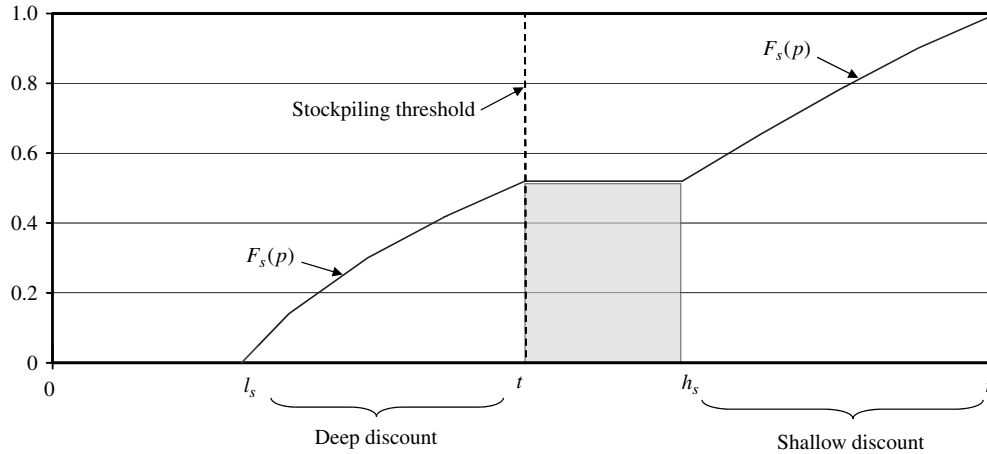
We present the mixing distribution  $F_s(p|t)$  for a given stockpiling threshold  $t$  in each state in (10) when  $t$  is in the interior of the support (details can be found in the appendix):

$$\begin{aligned} F_0(p|t) = & \\ \begin{cases} 1 & \forall p \geq r, \\ 1 - \left[ \frac{\nu(r-p)}{np(1-\nu)} \right]^{\frac{1}{n-1}} & \forall p \in \left[ \frac{rt(1+\lambda)}{r+t\lambda}, r \right), \\ 1 - \left[ \frac{\nu(r-t)}{nt(1-\nu)(1+\lambda)} \right]^{\frac{1}{n-1}} & \forall p \in \left[ t, \frac{rt(1+\lambda)}{r+t\lambda} \right), \\ 1 - \left[ \frac{\nu(r-p)}{np(1-\nu)(1+\lambda)} \right]^{\frac{1}{n-1}} & \forall p \in \left[ \frac{r\nu}{\nu+n(\lambda+1)(1-\nu)}, t \right), \\ 0 & \forall p < \frac{r\nu}{\nu+n(\lambda+1)(1-\nu)}; \end{cases} \\ F_1(p|t) = & \end{aligned} \quad (10)$$

$$\begin{aligned} \begin{cases} 1 & \forall p \geq r, \\ 1 - \left[ \frac{\nu(r-p)}{np(1-\nu)(1-\lambda)} \right]^{\frac{1}{n-1}} & \forall p \in \left[ \frac{rt}{r(1-\lambda)+t\lambda}, r \right), \\ 1 - \left[ \frac{\nu(r-t)}{nt(1-\nu)} \right]^{\frac{1}{n-1}} & \forall p \in \left[ t, \frac{rt}{r(1-\lambda)+t\lambda} \right), \\ 1 - \left[ \frac{\nu(r-p)}{np(1-\nu)} \right]^{\frac{1}{n-1}} & \forall p \in \left[ \frac{r\nu}{\nu+n(1-\nu)}, t \right), \\ 0 & \forall p < \frac{r\nu}{\nu+n(1-\nu)}. \end{cases} \end{aligned}$$

The interested reader can easily verify from (10) that as  $\nu \rightarrow 1$ , the lower bound in both states goes to reservation price,  $l_s \rightarrow r$ , and the equilibrium mixing distribution degenerates to a single price  $r$ . In this case firms do not offer promotions and always charge the reservation price,  $r$ , because there are no price-sensitive consumers to compete for in the market. On the other hand, as  $n \rightarrow \infty$  or  $\nu \rightarrow 0$ ,  $F_0(p|t) \rightarrow 1$  and  $F_1(p|t) \rightarrow 1$  for all positive prices  $p > 0$ . In other words, when all consumers are price sensitive, the pricing distribution degenerates to marginal cost,

Figure 3 Cumulative Distribution Function in State  $s = \{0, 1\}$  with Stockpiling



which is normalized to zero in our model. This is the well-known Bertrand result where firms earn zero profits in equilibrium.<sup>9</sup>

Given (10), we can now analyze the strategic consumers' decision problem and compute the continuation utilities that will allow us to endogenously determine the stockpiling threshold.

### 2.3. Endogenous Stockpiling Threshold

Recall from Lemma 1 that the stockpiling threshold  $t = \delta_c r + \delta_c U_1 - \delta_c U_0$  is a function of the strategic consumers' expected continuation utilities in both states 0 and 1. Specifically, the expected continuation utility of the strategic consumer at the time of purchase in state  $s = 0$  is

$$\begin{aligned}
 U_0 = & \underbrace{(1 - F_0(t))^n}_{\text{Probability that no firm charges a price below the stockpiling threshold}} \underbrace{(r - E_{t < p_0^{\min}}[p_0^{\min}])}_{\text{Utility from buying one unit at expected minimum price, when all firms charge } p > t} + \underbrace{\delta_c U_0}_{\text{Discounted continuation utility in state } s=0} \\
 & + \underbrace{(1 - (1 - F_0(t))^n)}_{\text{Probability that at least one firm charges a price below the stockpiling threshold}} \underbrace{((1 + \delta_c)r - 2E_{p_0^{\min} \leq t}[p_0^{\min}])}_{\text{Utility from buying two units, at expected minimum price, when at least one firm charges } p < t} + \underbrace{\delta_c U_1}_{\text{Discounted continuation utility in state } s=1}
 \end{aligned} \quad (11)$$

and that in state  $s = 1$  is

$$\begin{aligned}
 U_1 = & (1 - F_1(t))^n (\delta_c U_0) + (1 - (1 - F_1(t))^n) \\
 & \cdot (\delta_c r - E_{p_1^{\min} \leq t}[p_1^{\min}]) + \delta_c U_1.
 \end{aligned} \quad (12)$$

Here, expected minimum prices in state  $s = \{0, 1\}$  are

$$\begin{aligned}
 E_{p_s^{\min} \leq t}[p_s^{\min}] &= \int_{l_s}^t \frac{npf_s(p)(1 - F_s(p))^{n-1} dp}{(1 - (1 - F_s(t))^n)} \quad \text{and} \\
 E_{t < p_s^{\min}}[p_s^{\min}] &= \int_{h_s(t)}^r \frac{npf_s(p)(1 - F_s(p))^{n-1} dp}{(1 - F_s(t))^n}.
 \end{aligned}$$

<sup>9</sup> We thank the associate editor for encouraging us to generate the known results by varying  $\nu$  to bolster the face validity of our findings.

The expected continuation utility of strategic consumers in each state depends on whether they have had the opportunity to stockpile.<sup>10</sup> The first term on the right-hand side of (11) and (12) represents the expected utility if all firms charge a price above the stockpiling threshold. In (11), when the lowest price is greater than  $t$ , strategic consumers purchase one unit, pay the expected minimum price, and transition to  $s = 0$  in the next period with continuation utility of  $\delta_c U_0$ . In (12), when the lowest price is greater than  $t$ , strategic consumers do not purchase because they already have inventory for current consumption and thus transition to  $s = 0$  in the next period with continuation utility of  $\delta_c U_0$ . The second term on the right-hand side of (11) and (12) represents the expected utility if at least one firm charges a price less than the stockpiling threshold. In  $s = 0$ , strategic consumers purchase two units, pay the expected minimum price if it is less than  $t$ , and transition to  $s = 1$  in the next period with a continuation utility of  $\delta_c U_1$ . In  $s = 1$ , consumers purchase one unit, as they have inventory for current consumption, and transition to  $s = 1$  in the next period with continuation utility of  $\delta_c U_1$ . Using (11) and (12) in conjunction with Lemma 1, we state the equilibrium condition for the stockpiling threshold,  $t^*$ , in terms of firms' mixing distribution characterized in (10). The stockpiling threshold is the solution to the following equation:

$$\begin{aligned}
 t^* = & [\delta_c ((1 - F_0(t))^n E_{t < p_0^{\min}}[p_0^{\min}] \\
 & + 2(1 - (1 - F_0(t))^n) \cdot E_{p_0^{\min} \leq t}[p_0^{\min}] \\
 & - (1 - (1 - F_1(t))^n) E_{p_1^{\min} \leq t}[p_1^{\min}]) \\
 & \cdot [1 + \delta_c ((1 - F_1(t))^n - (1 - F_0(t))^n)]^{-1}.
 \end{aligned} \quad (13)$$

<sup>10</sup> Effects of exogenous market parameters on continuation utilities are reported in Table A.2 in the appendix.

This completes the characterization of the equilibrium distributions.

Endogenizing the stockpiling threshold by explicitly accounting for strategic behavior on the part of consumers presents nontrivial methodological challenges. It is reasonable therefore to ask what the value is of engaging in this exercise. Are the predictions from our model any different from a model that does not endogenize the threshold? We have shown that in a market where consumers are willing to stockpile, in equilibrium, the firm would offer shallow and deep discounts. As a practical matter, a manager may be interested in the frequency and depth of discounts and how these *metrics* vary with market parameters. If we let  $M$  denote a metric of interest and ask how  $M$  is affected by a market parameter  $\theta$ , then to obtain the net effect, we need to examine<sup>11</sup>

$$\frac{dM}{d\theta} = \frac{\partial M}{\partial \theta} + \frac{\partial M}{\partial t} \frac{\partial t}{\partial \theta}.$$

The first term on the right-hand side is the *direct* effect of the market parameter on the metric of interest and the second term represents the *indirect* effect of the market parameter on  $M$  through its effect on the stockpiling threshold. If the stockpiling threshold is not endogenous, then the indirect effect  $(\partial M/\partial t)(\partial t/\partial \theta) = 0$ . Consequently, depending on the sign of  $(\partial M/\partial t)(\partial t/\partial \theta)$ , we may over- or underestimate the effect of the market parameter on the metric of interest. Given that the equilibrium is fully characterized, we now present the main results.

### 3. Results

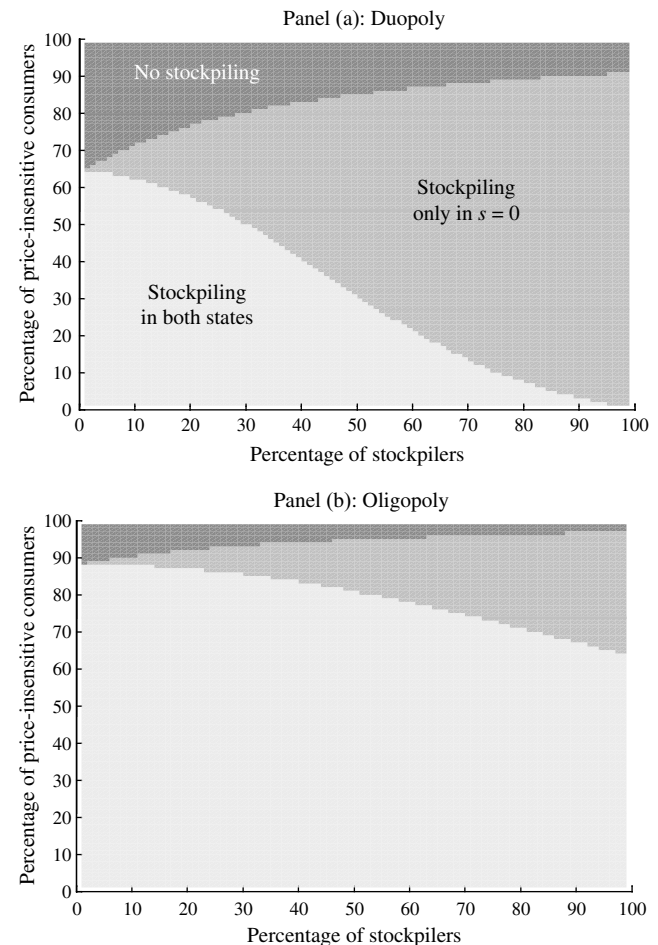
In (10) we characterize the mixing distributions in each state conditional on a given stockpiling threshold,  $t$ . The stockpiling threshold in equilibrium is endogenously determined and is the solution to (13), which is a *transcendental equation*. All the parameters of interest in our model, with the exception of the number of firms,  $n$ , are in a compact set. Specifically,  $\nu$ ,  $\lambda$ , and  $\delta$  all lie in the  $[0, 1]$  interval. As noted earlier, the reservation price  $r$  can be normalized to any arbitrary constant without loss of generality (refer to footnote 3). By choosing different values of the market parameters over the compact set, we can characterize the equilibrium strategies under all market conditions. We begin by highlighting the role of the clientele mix and market structure on the equilibrium outcomes.

Note from (10) that the lower bound in state  $s = 0$  is lower than that in state  $s = 1$ ,  $l_0 < l_1$ . Depending on market conditions, it is possible that the endogenous equilibrium threshold,  $t^*$ :  $t^* < l_0 < l_1$ , is below

the lower bound in  $s = 0$ , in which case there will be no stockpiling in either state in equilibrium. It is also possible that  $t^*$ :  $l_0 < t^* < l_1$  such that it is below the lower bound of the support in  $s = 1$  but greater than the lower bound in  $s = 0$ . In this case, stockpiling can occur only when consumers do not have an inventory. Figure 4 highlights market conditions under which stockpiling may (not) result in equilibrium.

In Figure 4, the  $y$  axis represents the proportion of price-insensitive consumers,  $\nu$ , and the  $x$  axis represents the size of the segment willing to stockpile,  $\lambda$ . Panel (a) represents a duopoly market, whereas panel (b) represents an oligopoly with five firms,  $n = 5$ . The effect of market structure on the equilibrium outcome is best understood by comparing the two panels in Figure 4. As expected, the parameter space over which stockpiling occurs is larger when competition is more intense in panel (b) vis-à-vis panel (a). In both cases, consistent with intuition, we find that when the size of the price-insensitive consumer segment  $\nu$  is large, stockpiling does not occur in equilibrium. In contrast, when  $\nu$  is not too large, then stockpiling is more likely to result in equilibrium.

Figure 4 Equilibrium Outcomes



Note. Discount factors are fixed at  $\delta_c = \delta_f = 0.8$ .

<sup>11</sup> The metric of interest may include frequency and average depth of deep or shallow discounts, the average price in each state, etc.

We find it interesting that when  $\nu$  is not too large as the proportion of stockpiling consumers  $\lambda$  increases, stockpiling is more likely to occur only when consumers have no inventory,  $s = 0$  (see panel (a) in Figure 4). The rationale for this is that when the proportion of consumers willing to stockpile is large, firms compete more intensely in periods when they do not have inventory. In state  $s = 0$  the expected benefit resulting from increased demand offsets the cost of acquiring sales at deep discounts. In the subsequent period after stockpiling, when the clientele mix is skewed toward more price-insensitive consumers, firms harvest by charging higher prices. Indeed, for values of  $\nu$  exceeding a certain threshold, it is suboptimal for firms to charge prices less than  $t^*$  in  $s = 1$ ; the cost of subsidizing price-insensitive and price-sensitive consumers who purchase only for current consumption dominates the expected benefit of the demand expansion that may result from strategic consumers who purchase for future consumption. Thus, when  $\nu$  is not too large but  $\lambda$  is large, firms compete intensely to sell to strategic consumers when they do not have inventory and harvest in the subsequent period.<sup>12</sup>

### 3.1. Equilibrium Pricing Strategy

So far we have delineated the effect of the clientele mix and market structure on the equilibrium outcome. We now demonstrate how the conditional and unconditional expected prices, the depth and frequency of discounts, and the equilibrium stockpiling threshold vary with respect to market parameters. In particular, we will highlight the effect of consumers' discount factor,  $\delta_c$ , and the proportion of price-sensitive strategic consumers,  $\lambda$ , on the equilibrium promotional strategies.<sup>13</sup> For practitioners, this offers some guidance on how the depth and frequency of shallow and deep promotions should be adjusted to different market conditions. In the left panel in Figure 5,  $E_1$ ,  $E_0$ ,  $E_1^{\min}$ , and  $E_0^{\min}$  ( $E$  and  $E_{-min}$ ) denote the conditional (unconditional) expected prices and the expected minimum prices. In the right panel,  $\Pr(s = 0)$  denotes the unconditional probability that strategic consumers have low inventory;  $t$  denotes the stockpiling threshold;  $F_0(t)$  and  $F_1(t)$  denote probabilities of firms charging prices below the stockpiling threshold in state  $s = 0$  and  $s = 1$ , respectively; and  $F(t)$  denotes the unconditional probability of firms charging prices below the stockpiling threshold.<sup>14</sup>

We begin by highlighting the effect of consumers' discount factor on the firms' equilibrium strategies.

As one might expect, stockpiling does not occur in equilibrium when consumers' discount factor is low. As consumers' discount factor increases, they first only stockpile in the low inventory state,  $s = 0$ , and when the discount factor is large enough, stockpiling occurs with positive probability in both states.

Not surprisingly, the stockpiling threshold  $t$  increases with consumers' discount factor. What is surprising is that in equilibrium with stockpiling, the unconditional expected ( $E$ ) and expected minimum prices ( $E^{\min}$ ) are increasing even though the expected prices in each state are decreasing in the consumers' discount factor (see the left panel in Figure 5). The rationale behind this finding is that the stockpiling threshold and the probability of firms charging prices below the stockpiling threshold in each state,  $F_0(t)$  and  $F_1(t)$ , are all increasing in consumers' discount factor (see the right panel in Figure 5). This means that when consumers' discount factor is high, firms are more likely to offer deep discounts, and the probability that consumers will end up in the high inventory state,  $s = 1$ , is greater relative to that when discount factor is low. Consequently, although the expected prices in each state ( $E_0$  and  $E_1$ ) are decreasing, the unconditional expected price ( $E$ ) will increase, because the expected price in  $s = 1$  is higher than that in  $s = 0$  and consumers are more often in state  $s = 1$  than in  $s = 0$  as  $\delta_c$  increases. We highlight that none of these findings could have been obtained without endogenizing the stockpiling threshold.<sup>15</sup>

We now turn to the effect of  $\lambda$ , the proportion of stockpiling price-sensitive consumers, on the firms' equilibrium strategies. One might expect that as the proportion of stockpiling consumers increases, market firms would compete more intensely for their business resulting in lower expected prices, ( $E$ ). Interestingly counter to this intuition, firms' expected prices increase with the proportion of stockpiling consumers. This occurs because in the state with low inventory,  $s = 0$ , as  $\lambda$  increases, the potential demand from stockpiling increases and firms compete more intensely for this demand; consequently, expected prices ( $E_0$ ) and ( $E_0^{\min}$ ) decrease in  $\lambda$  (see the left panel in Figure 6). Furthermore, if consumers' stockpile in  $s = 0$ , they transition to  $s = 1$ , where the expected minimum prices are higher ( $E_1^{\min} > E_0^{\min}$  and  $E_1 > E_0$ ) and increasing in  $\lambda$ . In markets with higher  $\lambda$ , the firm's ability to charge higher prices in periods after stockpiling increases significantly and more than offsets the decrease in price resulting from intense price

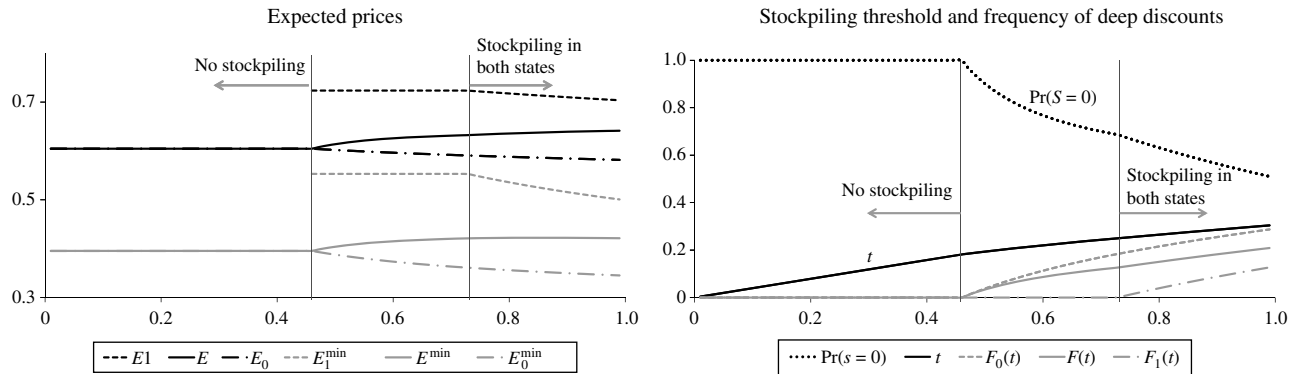
<sup>12</sup> We thank an anonymous reviewer for encouraging us to highlight this interesting result.

<sup>13</sup> Firms' discount factor does not affect equilibrium outcomes except the value function  $V_s$ .

<sup>14</sup> Refer to the state transition probability section in the appendix for details on how unconditional probabilities are obtained.

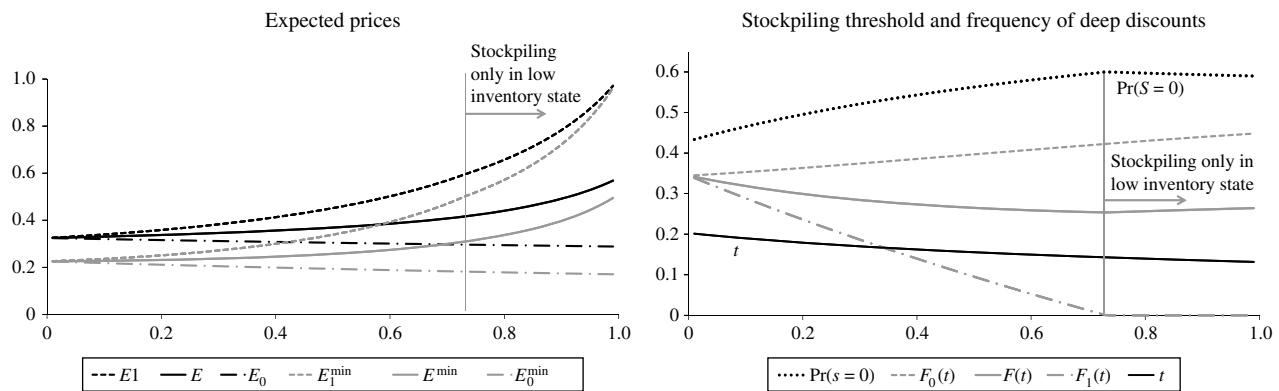
<sup>15</sup> Note that if the stockpiling threshold were exogenous, the unconditional expected prices would be obtained by pricing distributions characterized in Equation (10), which are independent of the consumers' discount factor.

**Figure 5** Equilibrium Outcomes with Respect to Consumer Discount Factor  $\delta_c$



*Notes.* This is a representative picture and is drawn based on exogenous parameter values of  $r = 1$ ,  $n = 3$ ,  $\lambda = 0.5$ ,  $\nu = 0.5$ , and  $\delta_r = 0.9$ . The  $x$  axis represents the consumer discount factor  $\delta_c$ , and the  $y$  axis represents the relevant metric of interest.

**Figure 6** Equilibrium Outcomes with Respect to the Proportion of Stockpiling Consumers  $\lambda$



*Notes.* This is a representative picture and is drawn based on exogenous parameter values of  $r = 1$ ,  $n = 2$ ,  $\nu = 0.25$ ,  $\delta_c = 0.9$ , and  $\delta_r = 0.9$ . The  $x$  axis represents the proportion of stockpiling consumers  $\lambda$ , and the  $y$  axis represents the relevant metric of interest.

competition in low inventory states. Hence the unconditional expected prices ( $E$ ) are higher in markets with a larger proportion of stockpiling consumers.

We also note that as the proportion of strategic price-sensitive consumers  $\lambda$  increases, the stockpiling threshold decreases. The intuition for this is that stockpiling consumers can remain in the state of low prices,  $s = 0$ , if they do not stockpile. Said differently, consumers must be offered a sufficiently good deal to exit this state and enter the high inventory state where the expected minimum prices ( $E_1^{\min}$ ) are significantly higher. As  $\lambda$  increases, the difference in  $E_1^{\min}$  and  $E_0^{\min}$  increases, and consequently, the stockpiling threshold decreases.

The above discussion highlights the sensitivity of the equilibrium outcome and the optimal promotional strategies to various market conditions and can serve as a guide to practitioners. In the next section, we highlight pricing patterns predicted by the model and demonstrate that they are consistent with commonly observed pricing patterns.

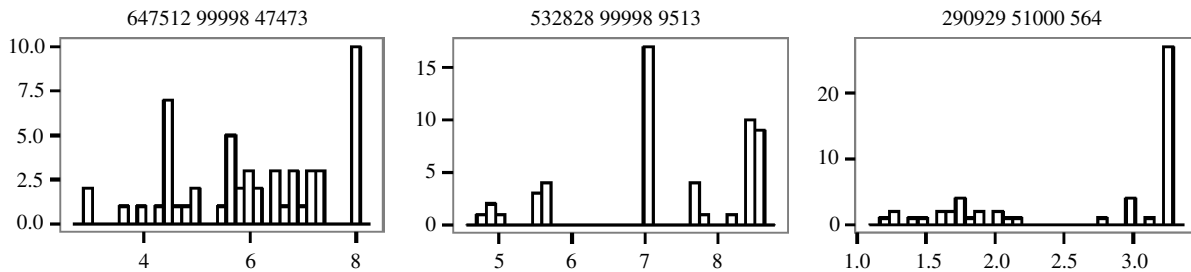
## 4. Empirical Regularities and Model Predictions

Recall from Figure 1 that the pricing distribution is often multimodal with modes and a hole in the interior of the support. With the exception of Rao et al. (1995), we are unaware of any other paper that documents the prevalence of multiple modes. Hence, in §4.1, we first document our empirical findings as they pertain to the prevalence of multiple modes in pricing distributions using data on multiple brands across several product categories. We also demonstrate that our model predictions are consistent with these commonly observed patterns. Another empirical regularity is that the serial correlation in prices is often negative. In §4.2 we empirically demonstrate that the serial correlation in prices is often negative and also show that our model predictions are consistent with this empirical regularity.

### 4.1. Multiple Modes

Our empirical examination of the IRI marketing data set is consistent with the finding in Rao et al. (1995)

**Figure 7** Histograms of Weekly Prices for Three Distinct SKUs from the IRI Marketing Data Set



that the distribution of prices is multimodal. A detailed description of the data is available in Bronnenberg et al. (2008). The evidence presented here is from the 2006 in the data set. We plotted the histogram of weekly prices for several SKUs across all categories available in the data set to understand the shape of pricing distributions. We found that several SKUs have small price variation and, consequently, have a single mode. Given our goal to understand the pricing patterns that occur as a result of promotions, we focus on SKUs that have sufficient price variability in the weekly prices. Given this objective, we choose the top 15 SKUs with the most price variation from each of the 28 categories (totaling 420 SKUs). For the purpose of illustration, in Figure 7 we provide the histograms from a SKU from three different categories: blades, coffee, and soup, from left to right. The  $x$  axis represents the price, and the  $y$  axis represents the frequency (in percentage).

To determine the number of modes in the pricing distribution, we count only distinct prices that are charged with more than 5% frequency. So in Figure 7, for example, the picture in the middle has only two modes: one around price 9 and another one around 7. A summary of the number of SKUs out of the top 15 with one, two, or more than two modes in their pricing distribution in each category can be found in Table A.3 in the appendix. Across all categories, 8% of the SKUs have a single mode, 38% have two modes, and 54% have more than two modes. Furthermore, of the SKUs with only two modes, 71% have one internal mode. If we include SKUs with more than two modes, then across all categories, 81% have an internal mode.

In conclusion, the finding that the pricing distribution is multimodal and that there are interior modes is a prevalent phenomenon.

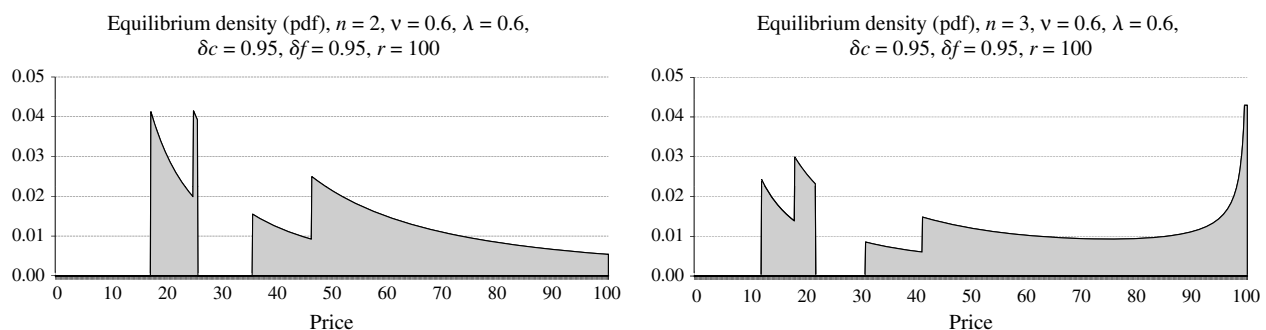
Given the prevalence of the pricing distributions that are multimodal with interior modes, we now demonstrate that the equilibrium pricing distributions implied by our theory can explain these patterns. In our model, the unconditional distribution, which is the convex combination of state-dependent distribution, will yield internal modes in the equilibrium pricing distribution because the lower bounds of the pricing distributions and the upper bound of the hole are different across two states. For illustration, in Figure 8 we plot the unconditional equilibrium density of prices for the following set of parameter values:  $r = 100$ ,  $n = 2$  and 3,  $\nu = 0.6$ ,  $\lambda = 0.6$ , and  $\delta_c = 0.95$ .

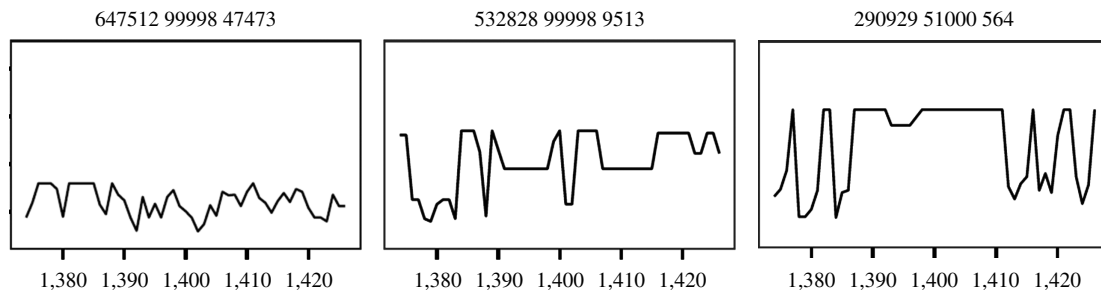
It is worth noting that the equilibrium mixing density implied by our theory (see Figure 8) is consistent with the multimodal pricing patterns with interior modes often observed in practice. For a wide range of parameters, the equilibrium density predicted by our model is multimodal with interior modes.

#### 4.2. Negative Serial Correlation in Prices

Although the negative serial correlation in prices has been documented by earlier empirical studies (Blattberg and Neslin 1990, Pesendorfer 2002), in this section we provide additional evidence of negative serial correlation using the IRI marketing data set. First, we examine the prevalence of negative serial correlation in prices by plotting the weekly prices of all 420 SKUs over the 52-week period. Visual inspection of these time series shows that there are no structural breaks or trends in the pricing patterns.

**Figure 8** Predicted Equilibrium Pricing Density



**Figure 9** Weekly Time-Series Plot of Three Distinct SKUs from the IRI Marketing Data Set

Furthermore, the data suggest the possible presence of negative serial correlation. As an example, we present the time series of SKUs from the same three categories—blades, coffee, and soup—from left to right in Figure 9. The  $x$  axis represents the week number, and the  $y$  axis represents the price.

Recall that in our model consumers enter the market in every period. As a result, the interpurchase time assumed in our model is a single period. To formally test for negative serial correlation in prices, we use the purchase cycle data in the IRI data set as a proxy for a unit of time in our model. We estimate the random coefficient regression model at the SKU level for each category:  $p_t = \alpha + \rho p_{t-l} + \varepsilon_t$ , where  $t$  is week index and  $l$  is the purchase cycle (lag) in weeks for that category. The parameter  $\rho$  is the serial correlation in prices.

In Table A.4 in the appendix, we report the mean serial correlation and its variance across the SKUs in each category. Although in several categories the mean estimates are positive, several SKUs in the category still exhibit negative serial correlation. For instance, in the peanut butter category, the average serial correlation across SKUs is 0.07, and yet 8 of 15 SKUs in the category have a negative estimate for the serial correlation parameter. On the other hand, in the soup category, the mean serial correlation is 0.27 and the variance across SKUs is 0.03, which is rather small relative to the mean. All 15 SKUs in this category exhibit positive serial correlation. An examination of the time-series plot of SKUs in this category reveals that the frequency of promotions is quite low and prices are relatively constant (please refer to the third plot in Figure 9 as an example), which explains the positive serial correlation.

We should also note that the mean estimate of the serial correlation is negative in many of the 28 product categories. For example, in the laundry detergent category, the average serial correlation is  $-0.17$ ; in the frozen dinner category, it is  $-0.19$ . Likewise, 13 of 15 SKUs in the former category and 12 of 15 SKUs in the latter category exhibit negative serial correlation. If we look at the individual estimates of all 420 SKUs across all categories, approximately half of the SKUs exhibit negative serial correlation (see Table A.4 in the appendix).

It is worth noting that visual inspection of the histograms of prices and the time-series plots across SKUs (see Figures 7 and 9) reveal that the SKUs with multiple modes tend to exhibit negative serial correlation in prices, whereas SKUs that do not have multiple modes tend to exhibit positive serial correlation. We now provide the intuition for why our model predicts the negative serial correlation in prices.

In the presence of stockpiling, the state-independent pricing distribution implied by our model generates a negative serial correlation in prices. Note from (10) that  $F_1(p)$  first-order stochastically dominates  $F_0(p)$  for all  $\lambda > 0$  in any equilibrium with stockpiling, which implies that the expected price in  $s = 1$  is greater than the expected price in  $s = 0$ . The first-order stochastic dominance also implies that the probability that a firm will charge prices below any price, including the stockpiling threshold, in state  $s = 1$  is lower than that in state  $s = 0$ ,  $F_1(t) < F_0(t) \forall \lambda > 0$ . This means that when a firm is in state  $s = 0$ , it offers deep discounts more often (relative to when it is in state  $s = 1$ ) and is therefore more likely to transition to state  $s = 1$  where expected prices are higher. Conversely, when a firm is in state  $s = 1$ , it offers shallow discounts more often (relative to when it is in state  $s = 0$ ) and is therefore more likely to transition to state  $s = 0$ , where expected prices are lower. Thus, the equilibrium promotional strategy in our model generates negative serial correlation in prices.

To the best of our knowledge, our model is the first to offer an explanation for both the presence of multiple modes in pricing distribution and the negative serial correlation of prices. In the next section, we extend our base model to incorporate the possibility of increased consumption when consumers have excess inventory at home and show that qualitative findings of the base model do not change.

## 5. Extension: Stockpiling with Flexible Consumption

In the base model, we assume that the consumption rate is constant so that strategic consumers carry inventory forward in the event that they stockpile. In this extension we relax this assumption to allow for

flexible consumption. We assume that when strategic consumers have inventory, they may consume the additional unit with probability  $\theta \in (0, 1)$ . Note that  $\theta = 1$  corresponds to our base model with no flexible consumption while  $\theta = 0$  corresponds to the case in which strategic consumers exhaust all inventory and are perpetually in a state of zero inventory at the time of purchase irrespective of whether they bought an additional unit in the previous period. The demand dynamics in the flexible consumption case is described in Figure 10. The stockpiling threshold in this case is computed in a manner similar to the base case. It is determined by comparing the strategic consumers' continuation utility from buying an extra unit for the future but consuming it in the same period with probability  $\theta$  versus that from not stockpiling.

The solution procedure is very similar to our base model. Additional details can be found in the appendix. In Proposition 2, we present the main finding from incorporating the possibility of increased consumption due to stockpiling.

**PROPOSITION 2.** *Firms' profits are independent of the probability of increased consumption as a result of stockpiling by strategic price-sensitive consumers:*

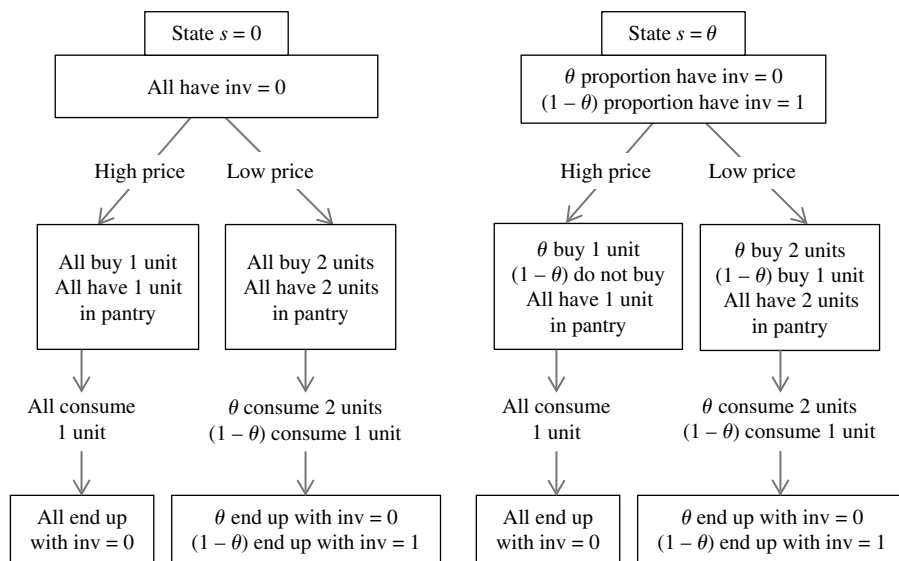
$$V_0 = V_\theta = \frac{r\nu}{n(1 - \delta_F)}$$

**PROOF.** See the appendix.

We find it interesting that the firms' equilibrium profits do not depend on the probability of increased consumption as a result of stockpiling,  $\theta$ , by the strategic price-sensitive consumers. In fact, the expected profits with flexible consumption are identical to that obtained in our base model. This is surprising because firms' profits are unaffected despite increased consumption by consumers. To see the intuition behind this result, consider the case when  $\theta = 1$  so that all consumers who stockpile consume both units in the current period. In this case consumers are perpetually in state  $s = 0$ . As in the base model, competition for price-sensitive consumers will result in firms' mixing over a range of prices with the reservation price  $r$  in the support of the mixing distribution. The equiprofit condition requires that the expected profit from charging  $r$  be the same as from charging any other price in the support of the mixing distribution. The expected profit from charging  $r$  is independent of  $\theta$ , which yields the above result. It is easy to extend this argument to the case when  $\theta \in (0, 1)$ . Intuitively, all potential profits from price-sensitive consumers are competed away in equilibrium; hence increased consumption as a result of stockpiling by strategic consumers has no impact on firms' equilibrium profits.

Other findings from this setup are qualitatively similar to the base model. Recall that in our base model with no flexible consumption, firms compete relatively more intensely in the state in which strategic

**Figure 10** State Space and Demand Dynamics with Flexible Consumption



*Notes.* If strategic consumers do not have any inventory, they purchase at least one unit for current consumption and may stockpile if prices are lower than the stockpiling threshold. If they do not stockpile, they transition to a state with no inventory in the next period. If they do stockpile, then a fraction  $\theta$  consume both units and end up with no inventory in the next period, whereas the remaining  $(1 - \theta)$  consume only one unit and end up with an inventory of one unit. In this case, the market has a fraction  $\theta$  with no inventory and a fraction  $(1 - \theta)$  with an inventory of one unit. In the event that strategic consumers do have inventory, they do not purchase for current consumption if prices are high and transition to a state with no inventory in the next period. If firms offer a deep discount, consumers stockpile and purchase one unit. Again, a fraction  $\theta$  consume both units and end up with no inventory in the next period, whereas the remaining  $(1 - \theta)$  consume just one unit and end up with an inventory of one.



consumers do not have inventory and harvest in the subsequent period. With flexible consumption, the force to harvest weakens as some of the strategic consumers reenter the market and purchase a unit even when firms offer a shallow discount and purchase two units when firms offer deep discounts. This is to be expected, as  $s = \theta$  in this extension can be viewed as a convex combination of  $s = 0$  and  $s = 1$  in the base model, where firms compete intensely in  $s = 0$  and harvest in  $s = 1$ .

## 6. Concluding Remarks

In this paper, we developed a model that incorporates the dynamic effects of price promotions. Specifically, we allow for the category demand to expand in promotional periods by allowing for intertemporal shifts. We do this by assuming that in the promotional periods when the firms offer deep discounts (below the stockpiling threshold), some consumers purchase not only for current consumption but also stockpile for future consumption. An intertemporal shift in demand induces state dependence such that firms' pricing strategies in any given period depend not only on their strategy in the previous period but also on that of their competitors in the previous period. The demand dynamics lead to several interesting findings. First, and most important, the equilibrium pricing distributions implied by our theory are consistent with the pricing distributions observed in practice. We find that in the equilibrium mixing distribution, firms offer shallow discounts or deep discounts. The depth and frequency of these discounts depends on whether a deep discount was offered by any firm in the previous period (i.e., it depends on the state). Second, endogenizing the stockpiling threshold allows us to highlight market conditions in which stockpiling may or may not occur in equilibrium. Under the market conditions in which stockpiling can occur, in equilibrium, the pricing distribution is multimodal with interior modes. When market conditions are such that the equilibrium pricing strategies do not permit stockpiling, the equilibrium pricing distribution is at most bimodal, with modes at the end of the support, which is consistent with the predictions from extant theory (Varian 1980, Narasimhan 1988).

Our analysis also demonstrates the importance of explicitly incorporating strategic behavior on the part of the consumer to endogenously determine the stockpiling threshold. Strategic price-sensitive consumers purchase an extra unit when deep discounts are offered. If firms offer shallow discounts in the subsequent period, they stay out of the market and consume from their inventory. This induces an interesting cycle in which firms compete intensely by

offering deep discounts in one period and harvest in the following period by offering shallow discounts. The dependence on past pricing decisions is particularly interesting, even if only one firm offers a deep discount in the previous period, it is in the best interest of all the firms to charge higher average prices in the following period as the clientele mix is now skewed more toward price-insensitive consumers. This is because a deep discount (below the stockpiling threshold), even if it is offered by only one firm, serves to clear the demand from strategic price-sensitive consumers that stockpile. The benefit of offering a deep discount in the following period to induce them to stockpile (i.e., purchase only for future consumption) is dominated by the cost of subsidizing price-insensitive consumers. Another benefit (to firms) from offering a deep discount is that by selling in advance to all price-sensitive strategic consumers, competition for price-sensitive consumers is mitigated in later periods. Finally, we highlight the interplay of market characteristics with promotional strategies as it relates to the frequency and depth of shallow versus deep discounts, which in turn offers many interesting insights for managers.

As a caveat, note that price-insensitive consumers in our model are indistinguishable from consumers who are loyal or variety seeking but price insensitive. In this light, our model can be viewed as one that examines a market where only some switchers stockpile. Interested readers may refer to Gangwar et al. (2012), who investigate what happens when loyal consumers engage in stockpiling.

The key findings from incorporating flexible consumption are qualitatively similar. Recall that in our base model with no flexible consumption, firms compete relatively more intensely in the state in which strategic consumers do not have inventory and harvest in the subsequent period. With flexible consumption, the force to harvest gets weaker as some of the strategic consumers reenter the market and purchase a unit even when firms offer a shallow discount, and they purchase two units when firms offer deep discounts. However, extra consumption as a result of stockpiling by strategic price-sensitive consumers does not generate additional revenue for the firm.

## Acknowledgments

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Appendix

Table A.1 Notation

Variable	Description
$s$	Subscript for state = 0 or 1
Exogenous	
$n$	Number of firms
$r$	Reservation price
$c$	Marginal cost
$\nu$	Fraction of price-insensitive consumers
$\lambda$	Fraction of strategic consumers
$\theta$	Probability of consuming additional unit
$\delta_c$	Consumer discount factor
$\delta_F$	Firms' discount factor
Endogenous	
$t$	Stockpiling threshold
$h_s$	Upper bound of the hole in state $s$
$l_s$	Lower bound of price in state $s$
$F_s$	CDF in state $s$ for prices
$U_s$	Consumer's continuation utility in state $s$
$V_s$	Firms' continuation payoff in state $s$

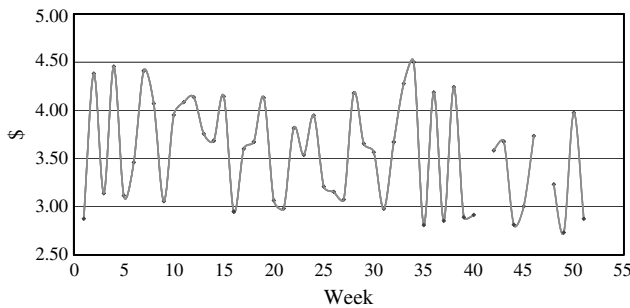
PROOF OF LEMMA 1. We begin by examining the decision problem of forward-looking consumers in each state. In  $s = 0$ , these consumers have no inventory for current-period consumption. Consequently, they have one of two options: (1) purchase only one unit for current-period consumption and end up in  $s = 0$  (with no inventory) next period or (2) stockpile—purchase two units, consume one unit, and end up in  $s = 1$  (with an inventory of one unit). Given the lowest price,  $p_0^{\min}$ , in the current period, strategic consumers will engage in stockpiling only if the following inequality holds:

$$(1 + \delta_c)r - 2p_0^{\min} + \delta_c U_1 \geq r - p_0^{\min} + \delta_c U_0.$$

The left-hand side (LHS) of this inequality represents the utility from stockpiling and transitioning to  $s = 1$ , in which case the consumers' expected continuation utility is  $U_1$ . The right-hand side (RHS) of the inequality represents the utility from not stockpiling and transitioning to  $s = 0$ , in which case the expected continuation utility is  $U_0$ . Stockpiling is the preferred option if  $p_0^{\min} \leq \delta_c r + \delta_c U_1 - \delta_c U_0$ .

In  $s = 1$ , strategic consumers have an inventory of one unit. Given that they have inventory for current period consumption, they have two options: (1) purchase nothing, consume the unit in inventory, and end up in  $s = 0$  (with no inventory); or (2) stockpile—purchase one unit and end up in  $s = 1$  (with an inventory of one unit) next

Figure A.1 Weekly Time-Series Plot of Prices



period. Once again given the lowest price,  $p_1^{\min}$ , in the current period, strategic consumers will engage in stockpiling iff  $\delta_c r - p_1^{\min} + \delta_c U_1 \geq \delta_c U_0$  or  $p_1^{\min} \leq \delta_c r + \delta_c U_1 - \delta_c U_0$ .

We define the *stockpiling threshold*  $t$  as the price where this inequality binds or  $t = \delta_c r + \delta_c U_1 - \delta_c U_0$ . Note that  $t$  and the expected continuation utilities  $U_0$  and  $U_1$  are all endogenously determined and are the same in both states. Q.E.D.

Equilibrium Mixing Distributions

We solve for symmetric equilibrium, which is also unique subgame perfect (Baye et al. 1992). We use the same logic as Narasimhan (1988) to rule out mass points in the interior of the support. Intuitively, suppose there was a mass point in the interior of the support. Then all firms will charge that price with positive probability in equilibrium. In that case one firm can profitably deviate by lowering its price, which contradicts the equilibrium. Refer to Narasimhan (1988) for a detailed proof.

Recall in (1)–(4) that a firm's demand depends not only on the state but also on whether a firm charges a price above or below the stockpiling threshold. When she sees a price equal to the threshold, she stockpiles; hence substitute  $p = t$  in (6) and (8) and use Proposition 1 to evaluate the probability of a firm offering a price less than or equal to the stockpiling threshold through following equations:

$$t \left[ \frac{\nu}{n} + (1 - \nu)(1 + \lambda)(1 - F_0(t))^{n-1} \right] = \frac{r\nu}{n(1 - \delta_F)},$$

$$t \left[ \frac{\nu}{n} + (1 - \nu)(1 - F_1(t))^{n-1} \right] = \frac{r\nu}{n(1 - \delta_F)}.$$

Further algebraic manipulation of the above equations yields the firms' probability of offering a price below the stockpiling threshold in each state as follows:

$$F_0(t) = 1 - \left[ \frac{\nu(r - t)}{nt(1 - \nu)(1 + \lambda)} \right]^{1/(n-1)},$$

$$F_1(t) = 1 - \left[ \frac{\nu(r - t)}{nt(1 - \nu)} \right]^{1/(n-1)}.$$

To solve for the cumulative distribution functions above and below the stockpiling threshold in each state, we substitute values of  $V_s$  from Proposition 1 and  $F_s(t)$  from above equalities in Equations (5)–(8). We get the following CDF:

$$F_0^a(p) = 1 - \left[ \frac{\nu(r - p)}{np(1 - \nu)} \right]^{1/(n-1)} \quad \forall p \in (t, r],$$

$$F_0^b(p) = 1 - \left[ \frac{\nu(r - p)}{np(1 - \nu)(1 + \lambda)} \right]^{1/(n-1)} \quad \forall p \in [l_0, t],$$

$$F_1^a(p) = 1 - \left[ \frac{\nu(r - p)}{np(1 - \nu)(1 - \lambda)} \right]^{1/(n-1)} \quad \forall p \in (t, r],$$

$$F_1^b(p) = 1 - \left[ \frac{\nu(r - p)}{np(1 - \nu)} \right]^{1/(n-1)} \quad \forall p \in [l_1, t],$$

where superscript  $a$  ( $b$ ) denotes the price range above (below) the stockpiling threshold.

Given the functional form of CDFs in (14) and (15), we evaluate the lower bounds of support in each state. We

know that the CDFs at the lower bounds in each state must evaluate to zero. Hence,

$$F_0^b(l_0) = 1 - \left[ \frac{\nu(r - l_0)}{nl_0(1 - \nu)(1 + \lambda)} \right]^{1/(n-1)} = 0, \quad \text{and}$$

$$F_1^b(l_1) = 1 - \left[ \frac{\nu(r - l_1)}{nl_1(1 - \nu)} \right]^{1/(n-1)} = 0.$$

This leads to following lower bounds in each state:

$$l_0 = \frac{r\nu}{\nu + n(1 + \lambda)(1 - \nu)}, \quad l_1 = \frac{r\nu}{\nu + n(1 - \nu)}.$$

**PROOF OF LEMMA 2.** Using the equiprofit condition in (9), it is easy to see that  $F_s^a(t) < F_s^b(t) \forall \lambda > 0$ , which violates the nondecreasing property of the cumulative distribution function. This contradiction can only be resolved if prices in the neighborhood of  $t$  are not in the support of the equilibrium mixing distribution. Suppose in an equilibrium the hole starts at some lower bound  $k$ —that is, below the stockpiling threshold  $t$ . Knowing that the continuation payoffs are same for all prices below  $t$ , we focus on only current-period payoffs. In such an equilibrium, consider the focal firm’s deviation of shifting a little mass from below the hole to inside the hole. This deviation will result in higher profits for the focal firm because the firm can charge a higher price without affecting the probability of attracting price-sensitive consumers and the stockpiling decision of strategic consumers. This profitable deviation violates the equilibrium condition. Hence, the hole must extend above the stockpiling threshold  $t$ . Equating the value of CDF above the stockpiling threshold at the upper bound of the hole,  $F_s^a(h_s)$ , to the values of CDF below the stockpiling threshold at  $t$ ,  $F_s^b(t)$ , respectively, in each state  $s = \{0, 1\}$  provides the following equalities:

$$F_0^a(h_0) = F_0^b(t) \Leftrightarrow$$

$$1 - \left[ \frac{\nu(r - h_0)}{nh_0(1 - \nu)} \right]^{1/(n-1)} = 1 - \left[ \frac{\nu(r - t)}{nt(1 - \nu)(1 + \lambda)} \right]^{1/(n-1)},$$

$$F_1^a(h_1) = F_1^b(t) \Leftrightarrow$$

$$1 - \left[ \frac{\nu(r - h_1)}{nh_1(1 - \nu)(1 - \lambda)} \right]^{1/(n-1)} = 1 - \left[ \frac{\nu(r - t)}{nt(1 - \nu)} \right]^{1/(n-1)},$$

from which we solve for the upper bounds of holes in each state:

$$h_0 = \frac{rt(1 + \lambda)}{r + t\lambda}, \quad h_1 = \frac{rt}{r + t\lambda - r\lambda}. \quad \text{Q.E.D.}$$

By appropriately defining the support of the equilibrium mixing distribution above and below the stockpiling threshold,  $F_s^a(p | t)$  and  $F_s^b(p | t)$  in (14) and (15) by substituting the lower bound of support,  $l_s$ , and the upper bound of hole,  $h_s$ , in each state  $s = \{0, 1\}$ , we fully characterize the equilibrium mixing distribution in (10) when  $t$  is in the interior of the support.

### State Transition Probabilities

We need to know the state transition probabilities to compute the unconditional CDFs and probability density functions (pdfs) under different market conditions. Recall that although the stockpiling threshold is same across both states, the mixing probabilities are different. However, the state in any given period depends only on whether the lowest price in the previous period was above or below the stockpiling threshold. Therefore, if the lowest price in the previous period was below the stockpiling threshold,  $p_s^{\min} \leq t$ , the current period state will be  $s = 1$ . Formally stated,  $\Pr(s = 1) = \Pr(p_0^{\min} \leq t)\Pr(s = 0) + \Pr(p_1^{\min} \leq t)\Pr(s = 1)$ .

Substitute the appropriate state dependent conditional probabilities for minimum prices,  $\Pr(p_0^{\min} \leq t) = 1 - (1 - F_0(t))^n$  and  $\Pr(p_1^{\min} \leq t) = 1 - (1 - F_1(t))^n$  in the foregoing equation to solve for unconditional state probability:

$$\Pr(s = 0) = \frac{(1 - F_1(t))^n}{1 - (1 - F_0(t))^n + (1 - F_1(t))^n} = 1 - \Pr(s = 1).$$

### Consumer Utilities

**Table A.2** First Derivatives of Discounted Utilities with Respect to Market Parameters

$d(\text{row})/d(\text{column})$	Consumer discount factor ( $\delta_c$ )	No. of firms ( $n$ )	Proportion of stockpiling consumers ( $\lambda$ )	Proportion of price-insensitive consumers ( $\nu$ )
Strategic consumer’s expected utility ( $U$ )	+	+	+	–
Strategic consumer’s expected utility in state 0 ( $U_0$ )	+	+	+	–
Difference of strategic consumer’s expected utility in states 0 and 1, ( $U_0 - U_1$ )	+	+	+	–
Strategic consumer’s expected utility in state 1 ( $U_1$ )	+	+	+	–

### Flexible Consumption: Unique Stockpiling Threshold

Similar to our base model, we redefine two states as state  $s = 0$  and state  $s = \theta$ . However, unlike the base case, in this case states do not directly correspond to strategic consumers’ inventory position. In state  $s = \theta$ ,  $\theta$  proportion of strategic consumers have no inventory (with continuation utility of  $U_{\theta 0}$ ) and  $(1 - \theta)$  fraction have inventory of one unit (with continuation utility of  $U_{\theta 1}$ ).

In state  $s = 0$ , when the strategic consumer has no inventory, he will buy two units and stockpile only if the following inequality holds:

$$\theta[2r - 2p_0^{\min} + \delta_c U_{\theta 0}] + (1 - \theta)[(1 + \delta_c)r - 2p_0^{\min} + \delta_c U_{\theta 1}] \geq r - p_0^{\min} + \delta_c U_0.$$

The LHS and RHS represent the utility, at the time of purchase, from stockpiling and not stockpiling, respectively. The first term on the LHS represents the utility from buying and consuming two units, which happens with the probability  $\theta$ ; the second term on the LHS is the utility from

buying two units but consuming only one, which happens with the complementary probability  $(1 - \theta)$ . The RHS is the utility from buying only one unit (not stockpiling) and staying in state  $s = 0$ . The strategic consumer in state  $s = 0$  will stockpile only if the minimum price in the market,  $p_0^{\min}$ , is below  $t$ :

$$p_0^{\min} \leq t = (\theta + (1 - \theta)\delta_c)r + \delta_c(\theta U_{\theta 0} + (1 - \theta)U_{\theta 1} - U_0). \quad (16)$$

Analogous comparison of the utilities for the strategic consumer with zero inventory in state  $s = \theta$  provides exactly the same condition for the stockpiling decision as mentioned above. On the other hand, utility comparison for consumers with inventory of 1 in state  $s = \theta$  yields the following inequality:  $\theta(r - p_0^{\min} + \delta_c U_{\theta 0}) + (1 - \theta) \cdot (\delta_c r - p_0^{\min} + \delta_c U_{\theta 1}) > \delta_c U_0$ . Notice that the RHS of the above inequality does not include the utility from the current-period consumption, which is already accounted for in the last period's purchase decision. However, the strategic consumer's stockpiling threshold is the same as (16):  $t = (\theta + (1 - \theta)\delta_c)r + \delta_c(\theta U_{\theta 0} + (1 - \theta)U_{\theta 1} - U_0)$ .

PROOF OF PROPOSITION 2. Given the demand dynamics described in Figure 10, we can write the equiprofit conditions in both states  $s = \{0, \theta\}$ . In particular, all prices in the support of the equilibrium mixing distribution must provide the same continuation profit. In state  $s = 0$  this leads to

$$p \left[ \frac{\nu}{n} + (1 - \nu)(1 - F_0(p))^{n-1} \right] + \delta_F [V_0(1 - F_0(t))^{n-1} + V_\theta(1 - (1 - F_0(t))^{n-1})] = V_0 \quad \forall p \in (t, r],$$

$$p \left[ \frac{\nu}{n} + (1 - \nu)(1 + \lambda)(1 - F_0(p))^{n-1} \right] + \delta_F V_\theta = V_0 \quad \forall p \in [l_0, t].$$

In state  $s = \theta$  the equiprofit conditions are

$$p \left[ \frac{\nu}{n} + (1 - \nu)(1 - \lambda + \lambda\theta)(1 - F_\theta(p))^{n-1} \right] + \delta_F [V_0(1 - F_\theta(t))^{n-1} + V_\theta(1 - (1 - F_\theta(t))^{n-1})] = V_\theta \quad \forall p \in (t, r],$$

$$p \left[ \frac{\nu}{n} + (1 - \nu)(1 + \lambda\theta)(1 - F_\theta(p))^{n-1} \right] + \delta_F V_\theta = V_\theta \quad \forall p \in [l_\theta, t].$$

Notice that these equiprofit conditions are similar to (5)–(8) outlined in the main paper. Similar to Proposition 1, simultaneously solving the above equiprofit conditions at the reservation price  $p = r$  in both states  $s = \{0, \theta\}$  leads directly to following equality:  $V_0 = V_\theta = (r\nu)/(n(1 - \delta_F))$ . Q.E.D.

The solution procedure to identify the equilibrium pricing distribution in the flexible consumption case is similar to the base case. For any given stockpiling threshold, the

equilibrium distributions for the flexible consumption case in both states are

$$F_0(p|t) = \begin{cases} 1 & \forall p \geq r, \\ 1 - \left[ \frac{\nu(r-p)}{np(1-\nu)} \right]^{\frac{1}{n-1}} & \forall p \in \left[ \frac{rt(1+\lambda)}{r+t\lambda}, r \right), \\ 1 - \left[ \frac{\nu(r-t)}{nt(1-\nu)(1+\lambda)} \right]^{\frac{1}{n-1}} & \forall p \in \left[ t, \frac{rt(1+\lambda)}{r+t\lambda} \right), \\ 1 - \left[ \frac{\nu(r-p)}{np(1-\nu)(1+\lambda)} \right]^{\frac{1}{n-1}} & \forall p \in \left[ \frac{r\nu}{\nu+n(\lambda+1)(1-\nu)}, t \right), \\ 0 & \forall p < \frac{r\nu}{\nu+n(\lambda+1)(1-\nu)}; \end{cases}$$

$$F_1(p|t) = \begin{cases} 1 & \forall p \geq r, \\ 1 - \left[ \frac{\nu(r-p)}{np(1-\nu)(1-\lambda)} \right]^{\frac{1}{n-1}} & \forall p \in \left[ \frac{rt}{r(1-\lambda)+t\lambda}, r \right), \\ 1 - \left[ \frac{\nu(r-t)}{nt(1-\nu)} \right]^{\frac{1}{n-1}} & \forall p \in \left[ t, \frac{rt}{r(1-\lambda)+t\lambda} \right), \\ 1 - \left[ \frac{\nu(r-p)}{np(1-\nu)} \right]^{\frac{1}{n-1}} & \forall p \in \left[ \frac{r\nu}{\nu+n(1-\nu)}, t \right), \\ 0 & \forall p < \frac{r\nu}{\nu+n(1-\nu)}. \end{cases}$$

Table A.3 Number of Modes

Category	No. of modes above 5%		
	1	2	2+
Beer	1	3	11
Blades	0	8	7
Coffee	0	10	5
Cold cereal	1	4	10
Deodorant	2	0	13
Diapers	4	4	7
Facial tissue	0	5	10
Frozen dinners/entrees	0	3	12
Frozen pizza	0	2	13
Household cleaner	1	9	5
Laundry detergent	1	8	6
Margarine/spreads/butter	3	5	7
Mayonnaise	3	2	10
Milk	2	4	9
Mustard ketchup	1	7	7
Paper towels	2	3	10
Peanut butter	0	10	5
Photography supplies	1	5	3
Razors	0	2	1
Salty snacks	2	6	7
Shampoo	0	6	9
Soup	6	8	1
Spaghetti/Italian sauce	0	2	13
Sugar substitutes	2	8	5
Toilet tissue	1	8	6
Toothbrush	0	6	9
Toothpaste	1	9	5
Yogurt	0	5	10
Total percentages (%)	8	38	54

**Table A.4** Negative Serial Correlation

Category	Purchase cycle	Lag in weeks	Average serial correlation	Variance of serial correlation	No. of SKUs (out of 15) with negative serial correlation
Beer	67	10	0.10	0.04	6
Blades	106	15	-0.07	0.04	8
Coffee	65	9	-0.02	0.01	10
Cold cereal	48	7	-0.08	0.02	11
Deodorant	94	13	0.05	0.05	6
Diapers	55	8	0.15	0.02	3
Facial tissue	70	10	-0.06	0.04	9
Frozen dinners/entrees	51	7	-0.19	0.06	12
Frozen pizza	64	9	0.05	0.04	6
Household cleaner	82	12	0.17	0.05	4
Laundry detergent	80	11	-0.17	0.04	13
Margarine/spreads/butter	65	9	0.17	0.04	3
Mayonnaise	95	14	-0.11	0.03	12
Milk	29	4	0.19	0.03	3
Mustard ketchup	91	13	-0.08	0.03	11
Paper towels	78	11	-0.03	0.02	9
Peanut butter	82	12	0.07	0.04	8
Photography supplies	104	15	0.16	0.02	2
Razors	87	12	0.05	0.09	2
Salty snacks	41	6	-0.06	0.02	9
Shampoo	87	12	0.07	0.03	6
Soup	45	6	0.27	0.03	0
Spaghetti/Italian sauce	72	10	0.11	0.12	6
Sugar substitutes	82	12	0.10	0.03	6
Toilet tissue	67	10	0.05	0.03	6
Toothbrush	87	12	0.04	0.02	6
Toothpaste	89	13	0.00	0.03	10
Yogurt	50	7	0.07	0.07	9

Given the equilibrium pricing distributions, we can compute strategic consumers' net discounted utilities in each state. The following equations are analogous to (11) and (12) of the base case:

$$U_{\theta_0} = (1 - F_{\theta}(t))^n (r - E_{t < p_{\theta}^{\min}}[p_{\theta}^{\min}] + \delta_c U_0) \\ + (1 - (1 - F_{\theta}(t))^n) [(1 + \theta + (1 - \theta)\delta_c)r \\ - 2E_{p_{\theta}^{\min} \leq t}[p_{\theta}^{\min}] + \theta\delta_c U_{\theta_0} + (1 - \theta)\delta_c U_{\theta_1}],$$

$$U_{\theta_1} = (1 - F_{\theta}(t))^n \delta_c U_0 + (1 - (1 - F_{\theta}(t))^n) \\ \cdot [(\theta + (1 - \theta)\delta_c)r - E_{p_{\theta}^{\min} \leq t}[p_{\theta}^{\min}] + \theta\delta_c U_{\theta_0} \\ + (1 - \theta)\delta_c U_{\theta_1}],$$

$$U_0 = (1 - F_0(t))^n (r - E_{t < p_0^{\min}}[p_0^{\min}] + \delta_c U_0) \\ + (1 - (1 - F_0(t))^n) [(1 + \theta + (1 - \theta)\delta_c)r \\ - 2E_{p_0^{\min} \leq t}[p_0^{\min}] + \theta\delta_c U_{\theta_0} + (1 - \theta)\delta_c U_{\theta_1}].$$

We obtain  $U_{\theta_0}$ ,  $U_{\theta_1}$ , and  $U_0$  as a function of the stockpiling threshold and firms pricing strategy by simultaneously solving the foregoing equations. The endogenous stockpiling threshold  $t^*$  in the flexible consumption case is the solution of the following equality obtained by substituting  $U_{\theta_0}$ ,  $U_{\theta_1}$ , and  $U_0$  in (16):

$$t^* = (\theta + (1 - \theta)\delta_c)r + \delta_c(\theta U_{\theta_0}(t^*) \\ + (1 - \theta)U_{\theta_1}(t^*) - U_0(t^*)).$$

### Empirical Regularities in IRI Marketing Data Set

The number in each row in Table A.3 represents the number of SKUs in the category with one, two, and more than two modes in the pricing distribution of 15 SKUs with most price variation in each of the 28 categories in the IRI marketing 2006 data set.

To test for the negative serial correlation in prices, we use the purchase cycle data in the IRI data set as a proxy for a unit of time in our model. We estimate the random coefficient regression model at the SKU level for each category:  $p_t = \alpha + \rho p_{t-l} + \varepsilon$ , where  $t$  is a week index and  $l$  is the purchase cycle (lag) in weeks for that category. In Table A.4 we report the mean serial correlation and its variance (across the SKUs) in each category. The last column in Table A.4 denotes the number of SKUs out of 15 in each category in which the serial correlation estimates are negative.

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