Supplemental Material for "Tunable Spin-Orbit Coupling via Strong Driving in Ultracold Atom Systems"

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NUMERICAL MODEL OF RAMAN DRIVEN BEC

To simulate the dynamics of the BEC under modulated Raman coupling, we numerically solve the time-dependent Gross-Pitaevskii equation (TDGPE)

$$i\hbar\frac{\partial\Psi}{\partial t} = \left[H(t) + V(\mathbf{r}) + H_I\right]\Psi, \tag{S1}$$

where $\Psi = (\Psi_\downarrow, \Psi_\uparrow)^T$ is a two-component wave function. We numerically simulated 3D BECs with $N = 10^5$ atoms in a harmonic confining potential

$$V(\mathbf{r}) = \frac{m}{2}(\omega_x^2 x^2 + \omega_y^2 y^2 + \omega_z^2 z^2), \tag{S2}$$

and with atomic density-density interactions described by

$$H_I = \mathrm{diag}\left(g_{\uparrow\downarrow}|\Psi_\uparrow|^2 + g_{\downarrow\downarrow}|\Psi_\downarrow|^2, g_{\uparrow\uparrow}|\Psi_\uparrow|^2 + g_{\uparrow\downarrow}|\Psi_\downarrow|^2\right). \tag{S3}$$

The interaction constants $g_{\downarrow\downarrow} = g_{\uparrow\downarrow} = 4\pi\hbar^2 N(c_0 + c_2)/m$ and $g_{\uparrow\uparrow} = 4\pi\hbar^2 N c_0/m$ are derived from ^{87}Rb's s-wave scattering lengths $c_0 = 100.86a_\mathrm{B}$, $c_2 = -0.46a_\mathrm{B}$ (a_B is the Bohr radius).

Initial state preparation for time evolution of GPE. We obtained the $t = 0$ initial state (before modulation) using imaginary time-evolution (ITE) with either the BEC initially polarized as $|\downarrow\rangle$ at $q_x = -1$ ($\hbar\delta_0 = -0.5E_L$), or as a balanced spin superposition at $q_x = \pm 1$ ($\hbar\delta_0 = 0$). For the latter case, we stop ITE until the BEC radius does not change significantly while the spins are still balanced. We then explicitly time-evolved with the TDGPE [1] including the full time-dependent Raman coupling $\Omega(t) = \Omega_0 + \Omega\cos(\omega t)$, where $\hbar\Omega_0 = 0.3E_L$ is the small constant offset and Ω is slowly ramped on as in the experimental procedure.

Spin "reflection" at $q_x = 0$. Our simulations show that the spin "reflection" is present when the Raman coupling offset $\hbar\Omega_0$ and detuning $\hbar\delta_0$ are small but non-zero. In the the single spin simulation, we included an additional small Raman coupling offset $\hbar\Omega_0 \sim 0.3E_L$ necessary to induce the "reflection" (otherwise the two bands are decoupled and switched in the fast modulation) and used a small detuning $\hbar\delta_0 = -0.5E_L$; furthermore, using $\hbar\delta_0 = -0.4E_L$ and $-0.2E_L$, the quasimomentum also displayed a reflection at $q_x = 0$. However, for $\delta_0 = 0$ the "reflection" depended on the actual value of the Raman coupling and driving frequency.

INSTABILITY OF THE MODULATION DYNAMICS

In experiment we observe that the spin-orbit coupled BECs exhibit heating under the fast modulation of Raman coupling (Fig. 2). This behavior is captured by a Floquet description

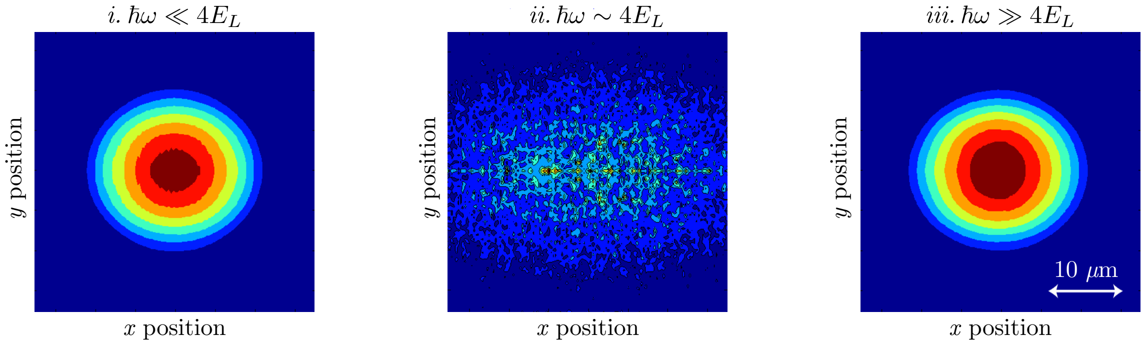

FIG. S1. Spatial density distributions $|\Psi_{\downarrow}|^2(x, y)$ at fixed coupling Ω_R for the modulation frequencies $\omega/2\pi = 1$ kHz, 7.36 kHz and 20 kHz.

of the system; for simplicity, we focus on a minimal three-state system spanned by the basis

$$\{|\downarrow, n = 0\rangle, |\uparrow, n = +1\rangle, |\uparrow, n = -1\rangle\},$$

where $|\sigma, n\rangle$ denotes the state with spin σ and Floquet side band index n. In such basis the Floquet Hamiltonian is

$$H = \begin{pmatrix} (k+1)^2 & \Omega/4 & \Omega/4 \\ \Omega/4 & (k-1)^2 + \hbar\omega & 0 \\ \Omega/4 & 0 & (k-1)^2 - \hbar\omega \end{pmatrix}. \tag{S4}$$

Figure 4c shows representative Floquet band structures of the periodic Raman driven system for low driving frequency $\hbar\omega \ll 4.0E_L$ (left), $\hbar\omega \sim 4E_L$ (middle), $\hbar\omega \gg 4E_L$ (right). The corresponding space density distributions $|\Psi_{\downarrow}|^2(x, y)$ of the BEC for the different modulation frequencies $\omega/2\pi = 1$ kHz, 7.36 kHz, 20 kHz at fixed coupling Ω_R are shown in Fig. S1.

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